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MATHEMATICAL PHYSICS

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CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: VECTOR ALGEBRA & VECTOR CALCULUS

CSIR PREVIOUS YEAR QUESTIONS

1. A vector perpendicular to any vector that lies on the plane defined by $x+y+z=5$, is

[CSIR June - 2012]

(a) $\vec{i}+\vec{j}$ (b) $\vec{j}+\vec{k}$ (c) $\vec{i}+\vec{j}+\vec{k}$ (d) $2\vec{i}+3\vec{j}+5\vec{k}$

2. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is

[CSIR Dec - 2012]

(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

3. Let $\vec{r}$ be the position vector of any point in three dimensional space and $r=|\vec{r}|$. Then

[CSIR Dec - 2014]

(a) $\vec{\nabla}.\vec{r}=0$ and $\vec{\nabla}\times\vec{r}=\frac{\vec{r}}{r}$ (b) $\vec{\nabla}.\vec{r}=0$ and $\vec{\nabla}^2\vec{r}=0$

(c) $\vec{\nabla}.\vec{r}=3$ and $\vec{\nabla}^2\vec{r}=\frac{\vec{r}}{r^2}$ (d) $\vec{\nabla}.\vec{r}=3$ and $\vec{\nabla}\times\vec{r}=0$

ANSWER KEY

1. (c) 2. (a) 3. (d)

GATE PREVIOUS YEAR QUESTIONS

1. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n}dS$ is: **[GATE 2001]**

   (a) V (b) 2V (c) 0 (d) 3V

2. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals **[GATE 2001]**

   (a) 1 (b) 3 (c) 0 (d) -3

3. A vector $\vec{A} = (5x+2y)\hat{i} + (3y-z)\hat{j} + (2x-az)\hat{k}$ is solenoidal if the constant a has a value: **[GATE 2002]**

   (a) 4 (b) –4 (c) 8 (d) –8

4. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b (a≠b) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors? **[GATE 2002]**

   (a) $-b\hat{i}+a\hat{j}$ (b) $-a\hat{i}+b\hat{j}$ (c) $-a\hat{i}-b\hat{j}$ (d) $-b\hat{i}-a\hat{j}$

5. The unit vector normal to the surface $3x^2+4y=z$ at the ponit (1, 1, 7) is: **[GATE 2002]**

   (a) $\left(-6\hat{i}+4\hat{j}+\hat{k}\right)/\sqrt{53}$ (b) $\left(4\hat{i}+6\hat{j}-\hat{k}\right)/\sqrt{53}$

   (c) $\left(6\hat{i}+4\hat{j}-\hat{k}\right)/\sqrt{53}$ (d) $\left(4\hat{i}+6\hat{j}+\hat{k}\right)/\sqrt{53}$

6. The curl of the vector $\vec{A} = z\,\hat{i} + x\,\hat{j} + y\,\hat{k}$ is given by: **[GATE 2003]**

   (a) $\hat{i}+\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}+\hat{k}$ (c) $\hat{i}+\hat{j}-\hat{k}$ (d) $-\hat{i}-\hat{j}-\hat{k}$

7. For the function $\phi = x^2y + xy$ the value of $\left|\vec{\nabla}\phi\right|$ at $x=y=1$ is **[GATE 2004]**

   (a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

8. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)$ is **[GATE 2005]**

   (a) 0 (b) $\hat{i}$ (b) $2\hat{j}$ (d) $3\hat{k}$

9. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}, u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly dependent pair is: **[GATE 2005]**

   (a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

10. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2, 0) is: **[GATE 2005]**

   (a) $\dfrac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

11. If $\vec{r} = x\hat{i} + y\hat{j}$, then: [GATE 2007]

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}|\vec{r}| = \vec{r}$
(b) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \vec{r}$
(c) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \frac{\vec{r}}{r}$
(d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

12. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement: [GATE 2009]

(a) Circulation of $\vec{E}$ is zero.
(b) $\vec{E}$ can always be expressed as the gradient of a scalar field.
(c) The potential difference between any two arbitary points in the region R is zero.
(d) The work done in a closed path lying entirely in R is zero.

13. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that [GATE 2011]

(a) $\vec{\nabla} \times \vec{F} = 0$
(b) $\vec{\nabla}.\vec{F} = 0$
(c) $\vec{\nabla} V = 0$
(d) $\nabla^2 V = 0$

14. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point P(1, 1, 1) is: [GATE 2011]

(a) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$
(b) $\frac{2\hat{i} + \hat{j} - \hat{k}}{\sqrt{6}}$
(c) $\frac{\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}$
(d) $\frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

15. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$ [GATE 2012]

(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent
(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent
(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal
(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

16. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is [GATE 2014]

(a) $\frac{\hat{x} + \hat{y} - \hat{z}}{\sqrt{3}}$
(b) $\frac{\hat{x} - \hat{y} - \hat{z}}{\sqrt{3}}$
(c) $\frac{\hat{x} - \hat{y} + \hat{z}}{\sqrt{3}}$
(d) $\frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}}$

17. The direction of $\vec{\nabla} f$ for a scalar field $f(x, y, z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point $P(1, 1, 2)$ is

(a) $\frac{(-\hat{j} - 2\hat{k})}{\sqrt{5}}$
(b) $\frac{(-\hat{j} + 2\hat{k})}{\sqrt{5}}$
(c) $\frac{(\hat{j} - 2\hat{k})}{\sqrt{5}}$
(d) $\frac{(\hat{j} + 2\hat{k})}{\sqrt{5}}$ [GATE 2016]

ANSWER KEY

1. (d)	2. (c)	3. (c)	4. (a)	5. (c)	6. (a)	7. (d)
8. (a)	9. (d)	10. (d)	11. (c)	12. (c)	13. (a)	14. (d)
15. (a)	16. (d)	17. (b)				

MISCELLANEOUS QUESTIONS

1. A particle is moved quasi-statically from the point $(-3,0)$ to $(3,0)$, along a path $y = x^2 - 9$ in an external force field given by $\vec{F} = yi + 3yj$. Give that all physical quantities are in SI units, the magnitude of the work done on the particle is given by
(a) 36 J (b) 18 J (c) 9 J (d) 0

2. The value of t for which three vevtors
$[(1-t),0,0],[1,(1-t),0]$ & $[1,1,(1-t)]$ are linearly dependent is
(a) 1 (b) 0 (c) 2 (d) –1

3. The value of $\oint \vec{A} \cdot \vec{d\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is:
(a) 0 (b) 2LA (c) 4LA (d) L^2A

4. The necessary and sufficient condition that $\oint_c \vec{A}.d\vec{r} = 0$, for any closed curve C is
(a) $\vec{\nabla}.\vec{A} = 0$ (b) $\vec{\nabla} \times \vec{A} = 0$ (c) $\vec{\nabla}.\vec{\nabla} \times \vec{A} = 0$ (d) $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = 0$

5. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:
(a) Arbitrary number (b) 1 (c) 2 (d) 3

6. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to
(a) Curl of a vector function (b) Gradient of a scalar function
(c) Gradient of a vector function (d) Zero

7. Given the vector $\vec{A}(y,-x,0)$, the line integral $\oint_c \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with its centre at the origin. (correct to the first decimal place) is
(a) 172.8 (b) 157.1 (c) –146.3 (d) 62.8

8. The value of the integral $I = \int_s \vec{r}.d\vec{s}$ where S is the surface enclosing the volume V is
(a) 3 (b) V (c) 3V (d) 0

ANSWER KEY

1. (a) 2. (a) 3. (a) 4. (b) 5. (d) 6. (b) 7. (b)
8. (c)

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MATHEMATICAL PHYSICS

SECTION: MATRIX ALGEBRA

CSIR PREVIOUS YEAR QUESTIONS

1. The matrices

$$A=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}, B=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, C=\begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}$$

satisfy the commutation relation **[CSIR June - 2014]**

(a) $[A,B]=B+C,[B,C]=0,[C,A]=B+C$

(b) $[A,B]=C,[B,C]=A,[C,A]=B$

(c) $[A,B]=B,[B,C]=0,[C,A]=A$

(d) $[A,B]=C,[B,C]=0,[C,A]=B$

2. The matrix $M=\begin{pmatrix}1&3&2\\3&-1&0\\0&0&1\end{pmatrix}$ satisfies the equation **[CSIR Dec-2016]**

(a) $M^3-M^2-10M+12I=0$

(b) $M^3+M^2-12M+10I=0$

(c) $M^3-M^2-10M+10I=0$

(d) $M^3+M^2-10M+10I=0$

ANSWER KEY

1. (d) **2.(c)**

GATE PREVIOUS YEAR QUESTIONS

1. For any operator A, $i(A^{+} - A)$ is: **[GATE 2001]**
(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

2. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$? **[GATE 2002]**

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

3. A 3×3 matrix has eigenvalues $0, 2+i$ and $2-i$. Which one of the following statements is correct? **[GATE 2003]**
(a) The matrix is Hermitian (b) The matrix is unitary
(c) The inverse of the matrix exists (d) The determinant of the matrix is zero

4. A real traceless 4×4 unitary matrix has two eigen values -1 and 1. The other eigenvalues are: **[GATE 2004]**
(a) zero and +2 (b) -1 and +1 (c) zero and +1 (d) +1 and +1

5. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are: **[GATE 2004]**
(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) -1 and +1

Common data for Q. 6 and Q. 7:

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5. **[GATE 2006]**

6. The other two eigen values are:
(a) 0 and 0 (b) 1 and 1 (c) 1 and -1 (d) -1 and -1

7. The normalized eigen vector corresponding to the eigen value 5 is:

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

8. The eigenvalues of a matrix are i, $-2i$ and $3i$. The matrix is: **[GATE 2007]**
(a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian

9. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are: **[GATE 2007]**

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

10. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is

[GATE 2008]

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

11. The eigenvalues of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ are [GATE 2008]

(a) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 45^0$

(b) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 30^0$

(c) ± 1 since, the matrix is unitary

(d) $\frac{1}{2}(1 \pm i)$ when $\theta = 30^0$

12. The eigenvalues of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are [GATE 2009]

(a) real and distinct

(b) complex and distinct

(c) complex and coinciding

(d) real and coinciding

13. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are [GATE 2010]

(a) 5, 2, -2 (b) -5, -1, 1 (c) 5, 1, -1 (d) -5, 1, 1

14. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are [GATE 2012]

(a) 0, 1, 1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

15. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is [GATE 2014]

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

ANSWER KEY

1. (a)	2. (c)	3. (d)	4. (b)	5. (c)	6. (c)	7. (d)
8. (d)	9. (a)	10. (d)	11. (b)	12. (b)	13. (c)	14. (b)
15. (d)						

JEST PREVIOUS YEAR QUESTIONS

1. For an $N \times N$ matrix consisting of all ones, **[JEST 2012]**

(a) all eigenvalues = 1
(b) all eigenvalues = 0
(c) the eigenvalues are 1, 2, ..., N
(d) one eigenvalue = N, the others = 0

ANSWER KEY

1. (d)

MISCELLANEOUS QUESTIONS

1. The eigenvalues of the matrix $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$ are :

(a) 1 and –1 (b) 16 and 16 (c) 16 and –16 (d) 1 and 256

2. Given the three matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } [\sigma_i, \sigma_j] \equiv \sigma_i\sigma_j - \sigma_j\sigma_i,$$

then $[\sigma_1,[\sigma_2,\sigma_3]]+[\sigma_2,[\sigma_3,\sigma_1]]+[\sigma_3,[\sigma_1,\sigma_2]]$ is

(a) $\sigma_1^2+\sigma_2^2+\sigma_3^2$ (b) $\sigma_1+\sigma_2+\sigma_3$ (c) 0 (d) Identity

3. Which one of the following is not hermitian?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$

4. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is:

(a) orthogonal (b) hermitian (c) symmetric (d) anti-symmetric

5. A given $(n \times n)$ nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore,

(a) Exactly k eigenvalues of A must be zero. (b) Exactly $(n-k)$ eigenvalue of A must be zero.

(c) Every eigenvalue of A is zero. (d) A can have $(n-i)$ non-zero eigenvalues.

6. If $L(x)$ is a linear differential operator and $y_1(x)$, $y_2(x)$ are two arbitrary functions. Then

(a) $L(x)(y_1+y_2) = Ly_1 + Ly_2$ (b) $L(x)(y_1+y_2) = y_1Ly_2 + y_2Ly_1$

(c) $L(x)(y_1y_2) = Ly_1 + Ly_2$ (d) $L(x)(y_1-y_2) = Ly_1 + Ly_2$

7. For which of the following values of a, the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?

(a) –4 (b) 4 (c) 0 (d) 1

8. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (c) $\begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

9. The number of independent real parameters of a most general hermitian matrix of order 4 is:

(a) 4 (b) 8 (c) 16 (d) 32

10. The eigen values of the matrix, $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are

(a) Purely imaginary (b) Complex (c) Real (d) Zero

11. The trace and the determinant of a 2×2 matrix are denoted by T and D. The eigenvalues of the matrix are given by

(a) $\frac{T \pm \sqrt{T^2 - 4D}}{2}$ (b) $\pm\frac{\sqrt{T^2 - 4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm\frac{T}{2}$

ANSWER KEY

1. (c)	2. (c)	3. (d)	4. (a)	5. (c)	6. (a)	7. (b)
8. (b)	9. (a)	10. (c)	11. (a)			

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MATHEMATICAL PHYSICS

SECTION: COMPLEX ANALYSIS

CSIR PREVIOUS YEAR QUESTIONS

1. The first few terms in the Taylor series expansion of the function f(x) = sinx around $x=\frac{\pi}{4}$ are [CSIR Dec 2011]

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.........\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.........\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.........\right]$

(d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+.......\right]$

2. Let $u(x,y)=x+\frac{1}{2}\left(x^2-y^2\right)$ be the real part of an analytic function $f(z)$ of the complex variable $z=x+iy$. the imaginary part of $f(z)$ is [CSIR June 2012]

(a) $y+xy$ (b) xy (c) y (d) y^2-x^2

3. The value of the integral $\oint_C \frac{z^3}{z^2-5z+6}dz$, where C is closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is [CSIR Dec 2012]

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

4. Which of the following functions cannot be the real part of a complex analytic function of $z=x+iy$? [CSIR Dec 2013]

(a) x^2y (b) x^2-y^2 (c) x^3-3xy^2 (d) $3x^2y-y-y^3$

5. The principal value of the integral $\int_{-\infty}^{\infty}\frac{\sin(2x)}{x^3}dx$ is [CSIR Dec 2014]

(a) -2π (b) $-\pi$ (c) π (d) 2π

6. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$ is **[CSIR June 2015]**

(a) $\pi/\sqrt{2}$ (b) $\pi/2$ (c) $\sqrt{2}\pi$ (d) 2π

ANSWER KEY

1. (b) **2. (a)** **3. (a)** **4. (a)** **5. (a)** **6.(a)**

GATE PREVIOUS YEAR QUESTIONS

1. The value of the integral $\int_C z^{10} dz$, where C is the unit circle with the origin as the centre is: [GATE 2001]

(a) 0 (b) $z^{11}/11$ (c) $2\pi i z^{11}/11$ (d) 1/11

2. The value of the residue of $\frac{\sin z}{z^6}$ is [GATE 2001]

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

3. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true? [GATE 2002]

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

4. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is: [GATE 2003]

(a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

5. The inverse of the complex number $\frac{3+4i}{3-4i}$ is: [GATE 2004]

(a) $\frac{7}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{24}{25}$

6. The value of $\int_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is: [GATE 2004]

(a) π for $a = 2$ (b) zero for $a = \frac{1}{2}$ (c) 4π for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$

7. The value of the integral $\int_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z| = 4$, is: [GATE 2005]

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

8. All solutions of the equation $e^z = -3$ are [GATE 2005]

(a) $in\pi \ln 3, n = \pm1, \pm2$............ (b) $\ln 3 + i(2n+1)\pi, n = 0, \pm1, \pm2$............

(c) $\ln 3 + i2n\pi, n = 0, \pm1, \pm2$............ (d) $i3n\pi, n = \pm1, \pm2$............

9. The contour integral $\oint \frac{dz}{z^2+a^2}$ is to be evaluated on a circle of radius 2a centered at the origin. It will have contributions only from the points. **[GATE 2007]**

(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) ia and $-ia$

(c) $ia, -ia, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

10. The value of $\int_{-i}^{i} \pi(z+1)\,dz$ is **[GATE 2008]**

(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) $(-1+2i)\pi$

11. The value of the integral $\int_C \frac{e^z}{z^2-3z+2}dz$, where the contour C is the circle $|z|=\frac{3}{2}$ is **[GATE 2009]**

(a) $2\pi ie$ (b) πie (c) $-2\pi ie$ (d) $-\pi ie$

12. The value of the integral $\oint_C \frac{e^z \sin z}{z^2}dz$, where the contour C is the unit circle: $|z-2|=1$, is **[GATE 2010]**

(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

13. The value of the integral $\oint_C e^{1/z}dz$, using the contour C of circle with unit radius $|z|=1$, is **[GATE 2012]**

(a) 0 (b) $1-2\pi i$ (c) $1+2\pi i$ (d) $2\pi i$

14. For the function $f(z)=\frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z=1$ ....

(Your answer should be an integer) **[GATE 2013]**

15. The value of the integral $\oint_C \frac{z^2}{e^z+1}dz$, where C is the circle $|z|=4$, is **[GATE 2014]**

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

16. Which of the following is an analytic function of z everywhere in the complex plane? **[GATE 2016]**

(a) z^2 (b) $(z^*)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

ANSWER KEY

1. (a)	**2. (b)**	**3. (b)**	**4. (a)**	**5. (d)**	**6. (b)**	**7. (c)**
8. (b)	**9. (b)**	**10. (b)**	**11. (c)**	**12. (a)**	**13. (d)**	**14. (3)**
15. (c)	**16. (a)**					

TIFR PREVIOUS YEAR QUESTIONS

1. If $z = x+iy$ then the function $f(x,y) = (1+x+y)(1+x-y)+a(x^2-y^2)-1+2iy(1-x-ax)$ where a is a real parameter, is analytic in the complex z plane if a is equal to **[TIFR 2013]**

   (a) -1 (b) $+1$ (c) 0 (d) i

2. The integral $\int_0^{\infty} \frac{dx}{4+x^4}$ evalautes to **[TIFR 2014]**

   (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

3. The integral $\int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta + a^2}$ where $0 < a < 1$, evaluates to **[TIFR 2015]**

   (a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

4. The value of the integral $\oint_C \frac{\text{Sin } z}{z^6} dz$, where C is the circle with centre $z = 0$ and radius 1 unit **[TIFR 2016]**

   (a) $i\pi$ (b) $\frac{i\pi}{120}$ (c) $\frac{i\pi}{60}$ (d) $-\frac{i\pi}{6}$

5. The value of the integral $\int_0^{\infty} \frac{dx}{x^4+4}$, is **[TIFR 2017]**

   (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

6. The value of the integral $\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\cos x}{x^2+a^2}$ is **[TIFR 2018]**

   (a) $1/2a$ (b) $1/2\pi a$ (c) $\pi a \exp(-a)$ (d) $\exp(-a)/a$

ANSWER KEY

1. (a) 2. (d) 3. (a) 4. (c) 5. (d) 6. (d)

JEST PREVIOUS YEAR QUESTIONS

1. The value of integral $I = \oint_c \frac{\sin z}{2z-\pi} dz$

with c a is circle $|z| = 2$, is **[JEST 2014]**

(a) 0 (b) $2\pi i$ (c) πi (d) $-\pi i$

2. The value of limit $\lim_{z\to i} \frac{z^{10}+1}{z^6+1}$ is equal to **[JEST 2014]**

(a) 1 (b) 0 (c) –10/3 (d) 5/3

3. Given an analytic function $f(x,y) = \phi(x,y) + i\,\psi(x,y)$ where $\phi(x,y) = x^2 + 4x - y^2 + 2y$. If C is a constant, then which of the following relations is true? **[JEST 2015]**

(a) $\psi(x,y) = x^2 y + 4y + C$ (b) $\psi(x,y) = 2xy - 2x + C$

(c) $\psi(x,y) = 2xy + 4y - 2x + C$ (d) $\psi(x,y) = x^2 y - 2x + C$

ANSWER KEY

1. (c) **2. (d)** **3. (c)**

MISCELLANEOUS QUESTIONS

1. The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is
(a) πi (b) $2\pi i$ (c) 0 (d) 2π

2. The complex function $f(z)=e^{\frac{1}{(z-1)^2}}$ has singular point at $z=1$ which is:
(a) a pole of order 1
(b) a pole of order 2
(c) an isolated essential singularity
(d) a non-isolated essential singularity

3. The function tan $(1/z)$ does not have a pole at $z=$
(a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$

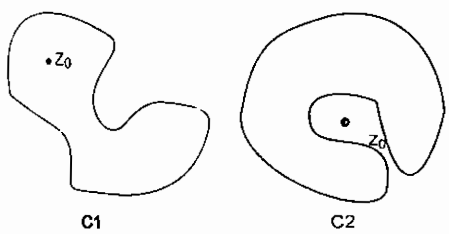
4. The real part of a complex function $f(z)$, analytic in a region is given by $u(x,y)=x^2-y^2$. If the function vainshes at $z=0$, the imaginary part of the function is
(a) $2xy$ (b) $-2xy$ (c) x^2+y^2 (d) y^2-x^2

5. The residue of the function, $f(z)=\frac{z}{(2z+1)(5-z)}$ at $z=-1/2$ is:
(a) 1/11 (b) $-1/11$ (c) 1/22 (d) $-1/22$

6. Two functions of a complex variable $z=x+iy$ are given as (i) $|z|$ and (ii) z^2. which of the following statements is correct?
(a) Both (i) and (ii) are analytic
(b) (i) is analytic and (ii) is nonanalytic
(c) Both are nonanalytic
(d) (i) is nonanalytic and (ii) is analytic

7. A function $f(Z)$ of a complex variable Z is given as $f(Z)=Z\,exp\left(\frac{1}{Z}\right)$. At $Z=0$.
(a) It has no singularity
(b) It has a simple pole and residue is 1
(c) It has an essential sigularity and residue is 1/2
(d) It has a pole of order 2 and residue is 1/6

8. A function $f(Z)$ of a complex variable Z is analytic everywhere in the complex plane except at $Z=Z_0$. Two contours $C1$ and $C2$ around Z_0 are shown in the figure.



The values of the integrals $I_1=\oint_{C1}\frac{f(Z)dZ}{Z-Z_0}$, $I_2=\oint_{C2}\frac{f(Z)dZ}{Z-Z_0}$ are
(a) $I_1=0, I_2=2\pi i f(Z_0)$
(b) $I_1=2\pi i f(Z_0), I_2=0$
(c) $I_1=\pi i f(Z_0), I_2=\pi i f(Z_0)$
(d) $I_1=2\pi i f(Z_0), I_2=2\pi i f(Z_0)$

9. The value of the integral $\oint\limits_C \frac{z^3}{(z-1)^2}dz$, where C is the circle $|z| = 4$ is

(a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero

10. The leading term in Laurent expansion of $f(z) = \frac{1}{z(1-z)^2}$ around $z = 0$ is

(a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1

11. The value of $\oint\limits_c \frac{e^z(z+3)}{z-2}dz$ where c is a unit circle with centre at origin is:

(a) $5e^2$ (b) $\frac{5}{2\pi i}e^2$ (c) 0 (d) $10\pi i e^2$

12. If $f(z) = \frac{Z^2 + a^2 - 2aZ}{(Z-a)^3(Z-b)}$, then $Z = a$ is:

(a) Pole of order 1 (b) Pole of order 2 (c) Pole of order 3 (d) An essential singularity.

13. Which one of the following functions is not analytic?

(a) $f(z) = |z|$ (b) $f(z) = z^2$ (c) $f(z) = \cos z$ (d) $f(z) = \sin z$

14. The value of the integral $\oint\limits_c \frac{\cos \pi z}{z(z+1)}$ where C is the circle $|z-1| = 3$ is:

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

ANSWER KEY

1. (b)	2. (c)	3. (a)	4. (a)	5. (d)	6. (d)	7. (c)
8. (b)	9. (a)	10. (c)	11. (c)	12. (a)	13. (a)	14. (d)

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MATHEMATICAL PHYSICS

SECTION: DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS

CSIR PREVIOUS YEAR QUESTIONS

1. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tend to **[CSIR June -2013]**

(a) 1 (b) 2 (c) 1/2 (d) ∞

2. The function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$ satisfies the differential equation **[CSIR Dec -2014]**

(a) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$

(b) $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$

(c) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$

(d) $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

3. Consider the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$. If x = 0 at t = 0 and x = 1 at t = 1, the value of x at t =2 , will be **[CSIR June -2015]**

(a) $e^2 + 1$ (b) $e^2 + e$ (c) $e + 2$ (d) $2e$

4. The function $y(x)$ satisfies the differential equation $x\frac{dy}{dx} + 2y = \frac{\cos \pi x}{x}$. If $y(1) = 1$, the value of $y(2)$ is

(a) π (b) 1 (c) 1/2 (d) 1/4 **[CSIR June -2017]**

ANSWER KEY

1. (a) 2. (c) 3. (b) 4. (d)

GATE PREVIOUS YEAR QUESTIONS

1. Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x = 2$ is **[GATE 2016]**

(a) e^{-2} (b) $2e^{-2}$ (c) e^{2} (d) $2e^{2}$

2. The points, where the series solution of the Legendre differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0$ will diverge, are located at: **[GATE 2007]**

(a) 0 and 1 (b) 0 and -1 (c) -1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

3. Solution of the differential equation $x\frac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$ at $x = 1$, is **[GATE 2007]**

(a) $y = 5x^4 - 4$ (b) $y = \frac{x^4}{5} + \frac{4x}{5}$ (c) $y = \frac{4x^4}{5} + \frac{1}{5x}$ (d) $y = \frac{x^4}{5} + \frac{4}{5x}$

Statement for Linked Answer Q. 4 and Q. 5:

For the differential equaiton $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ **[GATE 2005]**

4. One of the solutions is:

(a) e^x (b) $ln\, x$ (c) e^{-x^2} (d) e^{x^2}

5. The second linearly independent solution is:

(a) e^{-x} (b) xe^x (c) x^2e^x. (d) x^2e^{-x}

6. Consider the differential equation $d^2x/dt^2 + 2\, dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by: **[GATE 2003]**

(a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$

ANSWER KEY

1.(d) **2. (c)** **3. (d)** **4. (a)** **5. (b)** **6. (b)**

TIFR PREVIOUS YEAR QUESTIONS

1. The differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ has the complete solution, in terms of arbitrary constants A and B as **[TIFR 2013]**

(a) $A\exp x + Bx\exp x$

(b) $A\exp x + B\exp(-x)$

(c) $A\exp x + Bx\exp(-x)$

(d) $x\{A\exp x + B\exp(-x)\}$

ANSWER KEY

1. (a)

JEST PREVIOUS YEAR QUESTIONS

1. What are the solutions to $f''(x)-2f'(x)+f(x)=0$? **[JEST 2014]**

(a) c_1e^x/x (b) c_1x+c_2/x (c) $c_1xe^x+c_2$ (d) $c_1e^x+c_2xe^x$

ANSWER KEY

1. (d)

MISCELLANEOUS QUESTIONS

1. The solution of the differential equation

$$\frac{dy}{dx}+\frac{2}{3}\frac{y}{x}=0$$

with the condition y=2 when x=3 is given by

(a) $x^2y^3=72$ (b) $x^2y^3=108$ (c) $x^3y^2=108$ (d) $x^3y^2=72$

2. If $f(x)=x^4-2=\sum_{n=0}^{\infty}a_nP_n(x)$, then a_3 is

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 0

3. The value of $\frac{J_{\frac{1}{2}}(x)}{J_{-\frac{1}{2}}(x)}$ is:

(a) 0 (b) *tan x* (c) *cot x* (d) *tan h x*.

4. Legendre's differential equation:

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n\left(n+1\right)y=0$$

has an ordinary point at $x=$

(a) -1 (b) 0 (c) 1 (d) ∞

5. For the differential equation: $z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+y=0$

(a) $z=\infty$ is an ordinary point
(b) $z=0$ is a regular singular and $z=\infty$ is an irregular singular point
(c) $z=0$ and $z=\infty$ both are regular singular points
(d) $z=0$ is an irregular singular point

6. The Legendre polynomial $P_n(x)$. where $-1\le x\le +1$

(a) Is singular at $x=\pm 1$ (b) Satisfies $\int_{-1}^{1}dxP_n^2(x)=1$

(c) Satisfies $\int_{-1}^{1}dxP_n(x)=0$ for $n\ge 1$ (d) Is always an even function of x.

7. In order to obtain the solution of the initial value problem of the equation of motion, $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+16x=0$.

How many initial conditions are required?

(a) 0 (b) 1 (c) 2 (c) 3

8. The general solution of the linear differential equation, $\frac{d^4u}{dx^4}=0$ is, u(x) equal to

(a) 0 (b) c_3t+c_4 (c) $c_2t^2+c_3t+c_4$ (d) $c_1t^3+c_2t^2+c_3t+c_4$

The regular singular points of the associated Legendre differential equation in the finite domain are

(a) $1, -1$ (b) 0 (c) ∞ (d) 2

. The Wronskian of the linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}+4y=0$ is:

(a) Constant (b) 0 (c) ∞ (d) Undetermined

. Given the Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)\frac{3x^2-1}{2}$, then the polynomial $(3x^2+x-1)$ can be expressed as

(a) $P_2(x)-P_1(x)$ (b) $2P_2(x)+P_1(x)$

(c) $P_2(x)+P_1(x)$ (d) $2P_2(x)+P_1(x)+P_0(x)$

2. The solution of the differential equation $x\frac{dy}{dx}+y=1$, given the condition $y=0$ at $x=1$ is

(a) $y=(1-x^2)\ell n\, x$ (b) $y=\frac{x^2-1}{x}e^{-x}$ (c) $y=\frac{x-1}{x}$ (d) $y=\sinh(x)$

ANSWER KEY

1. (a)	**2. (d)**	**3. (b)**	**4. (b)**	**5. (a)**	**6. (c)**	**7. (c)**
8. (d)	**9. (a)**	**10. (a)**	**11. (b)**	**12. (c)**		

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MATHEMATICAL PHYSICS

SECTION: FOURIER SERIES, FOURIER TRANSFORM, LAPLACE TRANSFORM

CSIR PREVIOUS YEAR QUESTIONS

1. Fourier Transform of the derivative of the dirac δ -function, namely $\delta'(x)$ is proportional to **[CSIR Dec 2013]**

(a) 0 (b) 1 (c) $\sin k$ (d) ik

2. The Fourier transform of $f(x)$ is $\tilde{f}(k)=\int_{-\infty}^{\infty} dx\, e^{ikx} f(x)$. If $f(x)=\alpha\,\delta(x)+\beta\,\delta'(x)+\gamma\,\delta''(x)$, where $\delta(x)$ is the Dirac Delta function (and prime denotes derivative), what is $\tilde{f}(k)$? **[CSIR Dec 2015]**

(a) $\alpha+i\beta k+i\gamma k^2$ (b) $\alpha+\beta k-\gamma k^2$ (c) $\alpha-i\beta k-\gamma k^2$ (d) $i\alpha+\beta k-i\gamma k^2$

3. The inverse Laplace tranform of $\frac{1}{s^2(s+1)}$ is **[CSIR June 2013]**

(a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2+1-e^{-t}$ (c) $t-1+e^{-t}$ (d) $\frac{1}{2}t^2\left(1-e^{-t}\right)$

4. The Laplace transform of $6t^3+3\sin 4t$ is **[CSIR June 2015]**

(a) $\frac{36}{s^4}+\frac{12}{s^2+16}$ (b) $\frac{36}{s^4}+\frac{12}{s^2-16}$ (c) $\frac{18}{s^4}+\frac{12}{s^2-16}$ (d) $\frac{36}{s^3}+\frac{12}{s^2+16}$

5. The Laplace transform of $f(t)=\begin{cases}\frac{t}{T}, & 0<t<T\\ 1 & t>T\end{cases}$, is **[CSIR Dec. 2016]**

(a) $-\frac{(1-e^{-sT})}{s^2T}$ (b) $\frac{(1-e^{-sT})}{s^2T}$ (c) $\frac{(1+e^{-sT})}{s^2T}$ (d) $\frac{(1-e^{sT})}{s^2T}$

ANSWER KEY

1. (d) 2. (c) 3. (c) 4. (a) 5. (b)

GATE PREVIOUS YEAR QUESTIONS

9.

1. $f(x)$ symmetric periodic function of x i.e $f(x)=f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form **[GATE 2013]**

(a) $f(x)=\sum_{n=1}^{\infty}(a_n\cos nkx+b_n\sin nkx)$ (b) $f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nkx)$

(c) $f(x)=\sum_{n=1}^{\infty}(b_n\sin nkx)$ (d) $f(x)=a_0+\sum_{n=1}^{\infty}(b_n\sin nkx)$

2. If the fourier transform $F[\delta(x-a)]=\exp(-i2\pi\nu a)$, then $F^{-1}(\cos 2\pi\nu a)$ will correspond to **[GATE 2008]**

(a) $\delta(x-a)-\delta(x+a)$ (b) a constant

(c) $\frac{1}{2}[\delta(x-a)+i\delta(x+a)]$ (d) $\frac{1}{2}[\delta(x-a)+\delta(x+a)]$

3. The k^{th} Fourier component of $f(x)=\delta(x)$ is **[GATE 2006]**

(a) 1 (b) zero (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

4. The Fourier transform of the function $f(x)$ is $F(k)=\int e^{ikx}f(x)\,dx$. The Fourier transform of $df(x)/dx$ is: **[GATE 2003]**

(a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$

5. Fourier transform of which of the following functions does not exist? **[GATE 2002]**

(a) $e^{-|x|}$ (b) xe^{-x^2} (c) e^{x^2} (d) e^{-x^2}

6. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is not correct? **[GATE 2013]**

(a) $F(t)=\delta(t), f(s)=1$ (singularity at 0) (b) $F(t)=1, f(s)=\frac{1}{s}(s>0)$

(c) $F(t)=\sin kt, f(s)=\frac{s}{s^2+k^2}(s>0)$ (d) $F(t)=te^{kt}, f(s)=\frac{1}{(s-k)^2}(s>k, s>0)$

7. If $f(x)=\begin{cases}0 & for\ x<3\\ x-3 & for\ x\geq 3\end{cases}$ then, the laplace transform of $f(x)$ is **[GATE 2010]**

(a) $s^{-2}e^{3s}$ (b) $s^{2}e^{-3s}$ (c) s^{-2} (d) $s^{-2}e^{-3s}$

8. If $\bar{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constnat, is **[GATE 2005]**

(a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$

9. The Laplace transform of $f(t) = \sin \pi t$ is $F(s) = \frac{\pi}{\left(s^2+\pi^2\right)}$, $s > 0$. Therefore, the Laplace transform of $t \sin \pi t$ is: **[GATE 2004]**

(a) $\frac{\pi}{s^2\left(s^2+\pi^2\right)}$ (b) $\frac{2\pi}{s^2\left(s^2+\pi^2\right)^2}$ (c) $\frac{2\pi s}{\left(s^2+\pi^2\right)^2}$ (d) $\frac{2\pi}{\left(s^2+\pi^2\right)^2}$

ANSWER KEY

1. (b) 2. (d) 3. (c) 4. (c) 5. (c) 6. (c) 7. (d)
8. (b) 9. (c)

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MATHEMATICAL PHYSICS

SECTION: PROBABILITY

CSIR PREVIOUS YEAR QUESTIONS

1. An unbiased dice is thrown three times successively. The probability that the number of dots on the uppermost surface add up to 16 is **[CSIR Dec 2011]**

   (a) 1/16 (b) 1/36 (c) 1/108 (d) 1/216

2. A ball is picked at random from one of the two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white? **[CSIR June 2012]**

   (a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

3. In a series of five cricket matches, one of the captains calls 'heads' every time when the toss is taken. The probability that he will win 3 times and lose 2 times is **[CSIR Dec 2012]**

   (a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

4. A random variable n obeys Poisson statistics. The probability of finding $n = 0$ is 10^{-6}. The expectation value of n is nearest to **[CSIR June 2017]**

   (a) 14 (b) 10^6 (c) e (d) 10^{-}

ANSWER KEY

1. (b) 2. (b) 3. (d) 4. (a)

TIFR PREVIOUS YEAR QUESTIONS

1. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle/\hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is **[TIFR 2014]**

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

ANSWER KEY

1. (a)

JEST PREVIOUS YEAR QUESTIONS

1. An unbiased die is cast twice. The probability that the positive difference (bigger-smaller) between the two numbers is 2 is **[JEST 2012]**

(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/3

2. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, then mean value of x is **[JEST 2013]**

(a) λ (b) 2λ (c) $\lambda/2$ (d) 0

3. If two ideal dice are rolled once, what is the probability of getting atleast one '6'? **[JEST 2015]**

(a) 11/36 (b) 1/36 (c) 10/36 (d) 5/36

4. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(x^2 + \mu x\right)/2\sigma^2\right]$, is **[JEST 2016]**

(a) 0 (b) $\mu/2$ (c) $-\mu/2$ (d) σ

ANSWER KEY

1. (b) 2. (b) 3. (a) 4. (c)

LEVEL-2

1.

2.

3.

4.

5.

6

CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: VECTOR ALGEBRA & VECTOR CALCULUS

CSIR PREVIOUS YEAR QUESTIONS

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by **[CSIR June - 2011]**

(a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ (b) $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$ (c) $\frac{(\vec{a}.\vec{b})\vec{b}}{b^2}$ (d) $\frac{(\vec{b}.\vec{a})\vec{a}}{a^2}$

2. The equation of the plane that is tangent to the surface $xyz=8$ at the point $(1,2,4)$ is **[CSIR Dec - 2011]**

(a) $x+2y+4z=12$ (b) $4x+2y+z=12$
(c) $x+4y+2z=12$ (d) $x+y+z=7$

3. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120° with respect to $\hat{i}$. The angle between the vectors $\vec{u}=a\hat{i}+b\hat{n}$ and $\vec{v}=a\hat{n}+b\hat{i}$ will be 60° if **[CSIR June - 2013]**

(a) $b=\sqrt{3}a/2$ (b) $b=2a/\sqrt{3}$ (c) $b=a/2$ (d) $b=a$

4. If $\vec{A}=yz\,\hat{i}+xz\,\hat{j}+xy\,\hat{k}$, then the integral $\oint_C \vec{A}.d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by x = 0, x = a and y = 0, y = b) is **[CSIR Dec - 2013]**

(a) $\frac{1}{2}(a^3+b^3)$ (b) $\pi(ab^2+a^2b)$ (c) $\pi(a^3+b^3)$ (d) 0

5. If $\vec{A}=yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.d\vec{l}$ is **[CSIR June - 2014]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

6. Consider the three vectors $\vec{v}_1=2\hat{i}+3\hat{k}$, $\vec{v}_2=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{v}_3=5\hat{i}+\hat{j}+\alpha\hat{k}$, where $\hat{i},\hat{j}$ and $\hat{k}$ are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of α is **[CSIR June - 2018]**

(a) 31/4 (b) 23/4 (c) 27/4 (d) 0

ANSWER KEY

1. (a) **2. (b)** **3. (c)** **4. (d)** **5. (d)** **6. (a)**

GATE PREVIOUS YEAR QUESTIONS

1. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$ **[GATE 2001]**

(a) The three vectors are orthonormal (b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space.

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

Common data for Q.2 and Q.3 **[GATE 2003]**

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$

2. The surface integral of this vector over the surface of a cube of size and centered at the origin.

(a) 0 (b) 2π (c) $2\pi\ a^3$ (d) 4π

3. Which one of the following is NOT correct?

(a) Value of the line integral of this vector around any closed curve is zero

(b) This vector can be written as the gradient of some scalar function

(c) The line integral of this vector from point P to point Q is independent of the path taken.

(d) This vector can represent the magnetic field of some current distribution

4. The two vectors $\vec{p} = \hat{i}$, $\vec{q} = (\hat{i} + \hat{j})\ /\ \sqrt{2}$ are: **[GATE 2003]**

(a) related by a rotation (b) related by a reflection through the xy-plane

(c) related by an inversion (d) not linearly independent

5. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is: **[GATE 2007]**

(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

6. Cosnider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is: **[GATE 2007]**

(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$ (b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$ (d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

7. The curl of the vector field is $\vec{F}$ is $2\hat{x}$. Identity the appropriate vector field $\vec{F}$ from the choices given below: **[GATE 2008]**

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2x\hat{y} + 5y\hat{z}$

8. The value of the contour integral $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with centre at the origin is

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r **[GATE 2009]**

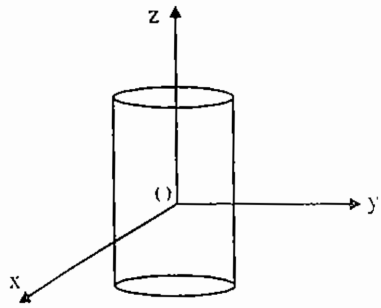
9. Consider the set of vectors in three-dimensional real vector space $\mathbf{R}^3$, $S=\{(1,1,1),(1,-1,1),(1,1,-1)\}$. Whch of the following statement is true? [GATE 2009]

(a) S is not a linearly independent set (b) S is a basis for $\mathbf{R}^3$

(c) The vectors in S are orthogonal

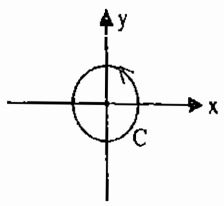
(d) An orthogonal set of vectors cannot be generated from S

10. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is: [GATE 2011]



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero

11. Given $\vec{F}=\vec{r}\times\vec{B}$, where $\vec{B}=B_0(\hat{i}+\hat{j}+k)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is, [GATE 2012]



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

12. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\vec{\nabla}\left[\vec{A}.\left(\vec{B}\times\vec{r}\right)\right]$ is [GATE 2013]

(a) $\vec{A}.\vec{B}$ (b) $\vec{A}\times\vec{B}$ (c) $\vec{r}$ (d) zero

13. Four forces are given below in Cartesian and spherical polar coordinates. [GATE 2015]

(i) $\vec{F}_1=K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2=K\left(x^3\hat{y}-y^3\hat{z}\right)$

(iii) $\vec{F}_3=K\left(x^3\hat{x}+y^3\hat{y}\right)$ (iv) $\vec{F}_4=K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

(a) (iii) and (iv) are conservative but (i) and (ii) are not

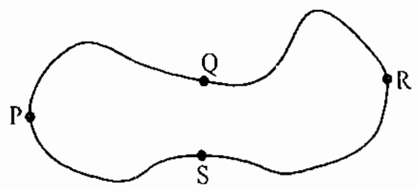
(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

14. Given that magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot d\vec{l} = \phi_1$ along PQR, the value of

$\int_P^R \vec{A}\cdot d\vec{l}$ along PSR is **[GATE 2015]**



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$ (c) $-\phi_1$ (d) ϕ_1

15. In spherical polar coordinate (r,θ,ϕ), the unit vector $\hat{\theta}$ at $\left(10,\frac{\pi}{4},\frac{\pi}{2}\right)$ is **[GATE 2018]**

(a) $\hat{k}$ (b) $\frac{1}{\sqrt{2}}\left(\hat{j}+\hat{k}\right)$ (c) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (d) $\frac{1}{\sqrt{2}}\left(\hat{j}-\hat{k}\right)$

16. Given : $\vec{V}_1 = \hat{i}-\hat{j}$ and $\vec{V}_2 = -2\hat{i}+3\hat{j}+2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1,\vec{V}_2,\vec{V}_3)$ a complete set for a three dimensional real linear vector space ? **[GATE 2018]**

(a) $\vec{V}_3 = \hat{i}+\hat{j}+4\hat{k}$ (b) $\vec{V}_3 = 2\hat{i}-\hat{j}+2\hat{k}$ (c) $\vec{V}_3 = \hat{i}+2\hat{j}+6\hat{k}$ (d) $\vec{V}_3 = 2\hat{i}+\hat{j}+4\hat{k}$

ANSWER KEY

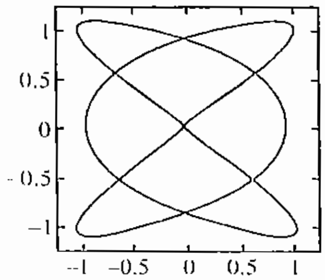
1. (b)	2. (d)	3. (d)	4. (a)	5. (a)	6. (a)	7. (b)
8. (a)	9. (b)	10. (b)	11. (c)	12. (b)	13. (d)	14. (b)
15. (d)	16. (d)					

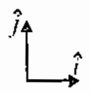
TIFR PREVIOUS YEAR QUESTIONS

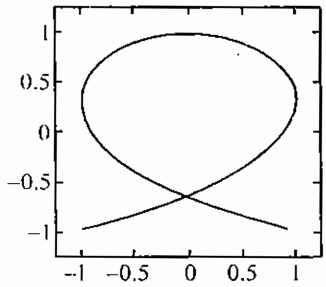
1. A two-dimensional vector $\vec{A}(t)$ is given by $\vec{A}(t) = \hat{i}\sin 2t + \hat{j}\cos 3t$

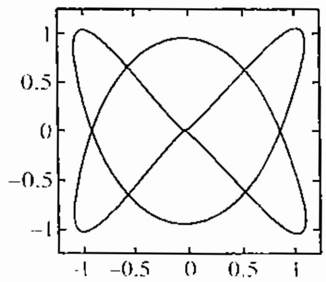
Which of the following graphs best describes the locus of the tip of the vector, as t is varied from 0 to 2π?

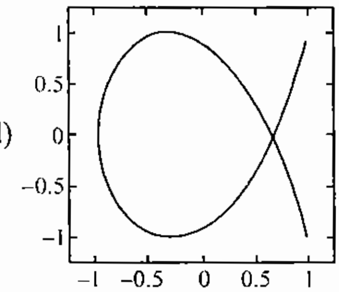
[TIFR 2013]

(a) 



(b) 

(c) 

(d) 

2. Consider the surface corresponding to the equation $4x^2 + y^2 + z = 0$. A possible unit tangent to this surface at the point (1, 2, –8) is **[TIFR 2013]**

(a) $\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$

(b) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$

(c) $\frac{4}{9}\hat{i} - \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k}$

(d) $-\frac{1}{\sqrt{5}}\hat{i} + \frac{3}{\sqrt{5}}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

3. Which of the following vectors is parallel to the surface $x^2y + 2xz = 4$ at the point (2,-2,3)?

[TIFR 2015]

(a) $-6\hat{i} - 2\hat{j} + 5\hat{k}$ (b) $6\hat{i} + 2\hat{j} + 5\hat{k}$ (c) $6\hat{i} - 2\hat{j} + 5\hat{k}$ (d) $6\hat{i} - 2\hat{j} - 5\hat{k}$

ANSWER KEY

1. (c) **2. (a)** **3. (c)**

JEST PREVIOUS YEAR QUESTIONS

The vector field $xz\hat{i} + y\hat{j}$ in cylinderical polar coordinates is **[JEST 2013]**

(a) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi\left(1-z\right)\hat{e}_\phi$

(b) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi\left(1+z\right)\hat{e}_\phi$

(c) $\rho\left(z\sin^2\phi + \cos^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi\left(1+z\right)\hat{e}_\phi$

(d) $\rho\left(z\sin^2\phi + \cos^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi\left(1-z\right)\hat{e}_\phi$

2. What is the equation of the plane which is tangent to the surface $xyz = 4$ at the point $(1, 2, 2)$?

[JEST 2017]

(a) $x+2y+4z=12$ (b) $4x+2y+z=12$ (c) $x+4y+z=0$ (d) $2x+y+z=6$

ANSWER KEY

1. (a) **2. (d)**

MISCELLANEOUS QUESTIONS

1. Given that the co-ordinates of a particle are $y(t) = A\cos(2\omega t)$ and $x(t) = \sin(\omega t)$ the trajectory of the particle is a
(a) Circle (b) Ellipse (c) Hyperbola (d) Parabola

2. A point particle is moving in the (x,y) plane on a trajectory given in polar coordinates by the equation $r\sin\left(\theta + \frac{\pi}{4}\right) = 5$. The trajectory of the particle is:
(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

3. A point particle is moving the (x, y) plane on a trajectory given in polar coordinates by the equation
$$r^2 - 2r\sin\left(\theta + \frac{\pi}{4}\right) - 3 = 0$$
The trajectory of the particle is
(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

4. The normal to the surface given by the equation $z = \cos x \cosh y$ at the point $x = \frac{\pi}{2}$ and $y = 0$ lies in
(a) (x,y) plane (b) (x, z) plane
(c) (y, z) plane (d) On the plane given by the equation $x + y + z = \frac{\pi}{2} + 1$

5. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation
$$25 + r^2\cos 2\theta = 0$$
The trajectory of the particle is a
(a) parabola (b) circle (c) ellipse (d) hyperbola

6. The value of the line integral
$$\oint \frac{xdy - ydx}{x^2 + y^2}$$
along a circle of radius 3 centered at the origin in the counter clockwise direction is given by

(a) 0 (b) $\frac{3}{2\pi}$ (c) 2π (d) 6π

7. $\nabla^2\left(\frac{1}{r}\right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

8. $\iiint \nabla^2\left(\frac{1}{r}\right)dV \quad r \neq 0$ is:

(a) 0 (b) -4π (c) 4π (d) 1

9. The value of $\oint_S \frac{\vec{r}\cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is:

(a) 0 (b) π (c) 4π (d) 8π

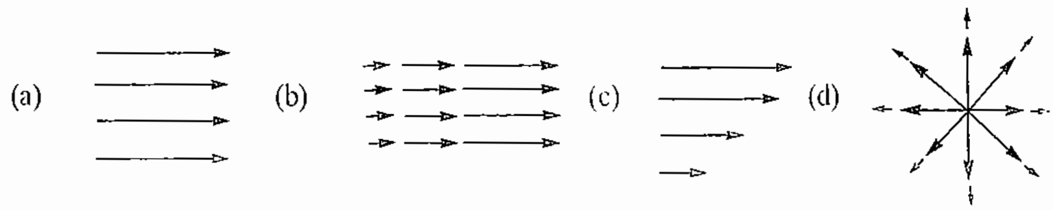
10. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?

(a) $\frac{d\vec{A}}{dt}=0$ (b) $\frac{d^2\vec{A}}{dt^2}=0$ (c) $\frac{d\vec{A}}{dt}\cdot\vec{A}=0$ (d) $\frac{d\vec{A}}{dt}\times\vec{A}=0$

11. The function $r^n\,\hat{r}$ $(r>0)$, where r refers to spherical coordinate system, is:

(a) an irrotational vector for $n=-2$
(b) a solenoidal vector for $n=-2$
(c) an irrotational vector for $n=-3$
(d) a solenoidal vector for $n=-3$

12. Identify the vector field given below which has a finite curl



13. Given any three non-zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, their triple product $\vec{A}.(\vec{B}\times\vec{C})$ vanishes if

(a) They are perpendicular to each other.
(b) Any two of them are perpendicular
(c) Any two of them are parallel.
(d) They are non-coplanar.

14. When the fluid is incompressible, the equation of continuity can be reduced to $\frac{\partial\rho}{\partial t}=0=\rho(\vec{\nabla}.\vec{v})$. Since the density ρ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as

(a) $\vec{\nabla}.\vec{\nabla}\phi=0$ (b) $\vec{\nabla}\times\vec{\nabla}\phi=0$ (c) $\vec{\nabla}\phi=0$ (d) $\vec{v}=\text{constant}$

ANSWER KEY

1. (d)	2. (b)	3. (c)	4. (b)	5. (d)	6. (c)	7. (c)
8. (a)	9. (c)	10. (c)	11. (b)	12. (c)	13. (c)	14. (b)

CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: MATRIX ALGEBRA

CSIR PREVIOUS YEAR QUESTIONS

Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ **[CSIR June - 2011]**

1. The eigenvalues of M are

(a) 0,1,2 (b) 0,0,3 (c) 1,1,1 (d) -1,1,3

2. The exponential of M simplifies (I is the 3 x 3 identity matrix)

(a) $e^M = I + \left(\frac{e^3 - 1}{3}\right)M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e-1)M$

3. A 3 x 3 matrix M has $Tr[M] = 6, Tr[M^2] = 26, Tr[M^3] = 90$. Which of the following can be possible set of eigenvalues of M? **[CSIR Dec - 2011]**

(a) {1,1,4} (b) {-1,0,7} (c) {-1,3,4} (d) {2,2,2}

4. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ **[CSIR June - 2012]**

(a) (1,4,9) (b) (0,7,7) (c) (0,1,13) (d) (0,0,14)

5. The eiegenvalues of the anti-symmetric matrix $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2, n_3 are components of an unit vector, are **[CSIR June - 2012]**

(a) 0, -i, i (b) 0, 1, -1 (c) 0, 1+i, -1-i (d) 0, 0, 0

6. A 2 x 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = I$ **[CSIR Dec - 2012]**

(a) 20 (b) 30 (c) 60 (d) 120

7. Given a 2 x 2 unitary matrix satisfying $U^\dagger U = UU^\dagger = I$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix V $\left(V^\dagger V = VV^\dagger = I\right)$ with det $V = 1$ from it by **[CSIR Dec - 2012]**

(a) Multiplying U by $e^{-i\varphi/2}$ (b) Multiplying a single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column by $e^{-i\varphi/2}$ (d) Multiplying U by $e^{-i\varphi}$.

8. Consider a $n \times n$ $(n > 1)$ matrix A, in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$) The matrix A **[CSIR Dec - 2013]**

(a) has one degenerate eigenvalue with degeneracy $(n-1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalues with degeneracy n

(d) does not have any degenerate eigenvalues

9. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are **[CSIR June - 2014]**

(a) -5,-2,7 (b) -7,0,7 (c) -4i,2i,2i (d) 2,3,6

10. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

[CSIR Dec-2014]

(a) $b = 0$ and $a = 0$ (b) $b = a$ and $b = -2a$

(c) $b = 2a$ and $b = -a$ (d) $b = a/2$ and $b = -a/2$

11. Which of the following cannot be eigen values of a real 3 × 3 matrix **[CSIR June-2017]**

(a) $2i, 0, -2i$ (b) 1, 1, 1 (c) $e^{i\theta}, e^{-i\theta}, 1$ (d) $i, 1, 0$

12. Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and **[CSIR June-2017]**

$$x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times \left[x\sigma_x + y\sigma_y + z\sigma_z\right] \exp\left(-\frac{i\theta\sigma_z}{2}\right)$$

Then the coordinates are related as follows

(a) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ (b) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

13.

14

15

(c) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ (d) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

13. Consider the matrix equation **[CSIR Dec. 2017]**

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution, and the corresponding normalised solution (upto a sign) is

(a) $b = 2c$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$ (b) $c = 2b$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, 1, -2)$

(c) $c = b + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(2, -1, -1)$ (d) $b = c + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$

14. Let A be a non-singular 3×3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $(A^T)^{-1}$. Which of the following is true? **[CSIR Dec. 2017]**

(a) $\vec{u} \cdot \vec{a} = 0, \quad \vec{u} \cdot \vec{b} = 0, \quad \vec{u} \cdot \vec{c} = 1$ (b) $\vec{u} \cdot \vec{a} = 0, \quad \vec{u} \cdot \vec{b} = 1, \quad \vec{u} \cdot \vec{c} = 0$

(c) $\vec{u} \cdot \vec{a} = 1, \quad \vec{u} \cdot \vec{b} = 0, \quad \vec{u} \cdot \vec{c} = 0$ (d) $\vec{u} \cdot \vec{a} = 0, \quad \vec{u} \cdot \vec{b} = 0, \quad \vec{u} \cdot \vec{c} = 0$

15. Which of the following statements is true for a 3×3 real orthogonal matrix with determinant $+1$?

(a) the modulus of each of its eigenvalues need not be 1, but their product must be 1. **[CSIR June 2018]**

(b) at least one of its eigenvalues is $+1$.

(c) all of its eigenvalues must be real.

(d) none of its eigenvalues need be real.

ANSWER KEY

1. (b)	**2. (a)**	**3. (c)**	**4. (d)**	**5. (a)**	**6. (c)**	**7. (a)**
8. (a)	**9. (b)**	**10. (b)**	**11. (d)**	**12. (b)**	**13. (d)**	**14. (c)**
15. (b)						

GATE PREVIOUS YEAR QUESTIONS

1. If two matrices A and B can be diagonalized simultaneously, which of the following is true? **[GATE 2002]**

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

2. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are: **[GATE 2005]**

(a) $-2, -1, 1, 2$ (b) $-1, 1, 0, 2$ (c) $1, 0, 2, 3$ (d) $-1, 1, 0, 3$

3. The determinant of a 3×3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvlaue is: **[GATE 2005]**

(a) 4 (b) 6 (c) 8 (d) 9

4. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\underset{\sim}{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is: **[GATE 2006]**

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

5. For arbitary matrices E, F, G and H, if $EF - FE=0$, then Trace $(EFGH)$ is equal to **[GATE 2008]**

(a) Trace $(HGFE)$ (b) Trace (E).Trace (F).Trace (G).Trace (H)

(c) Trace $(GFEH)$ (d) Trace $(EGHF)$

6. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is *NOT TRUE*? **[GATE 2011]**

(a) Det A = Det B (b) Trace of A = Trace of B

(c) A and B have the same eigenvectors (d) A and B have the same eigenvalues.

7. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is: **[GATE 2011]**

(a) 18 (b) 12 (c) 9 (d) 6

8. The degenerate eigenvalues of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ is : .... **[GATE 2013]**

(your answer should be an integer)

9. The eigenvalues of a Hermitian matrix are all **[GATE 2018]**

(a) real (b) imaginary (c) of modulus one (d) real and positive

ANSWER KEY

1. (c) **2. (a)** **3. (b)** **4. (a)** **5. (a)** **6. (c)** **7. (d)**

8. (5) **9. (a)**

TIFR PREVIOUS YEAR QUESTIONS

1. The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ **[TIFR 2010]**

can be relateed by a similarity transformation to the matrix.

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -i \\ 0 & -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

2. Consider the matrix, $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$

A 3-dimensional basis formed by eigenvectors of M is **[TIFR 2011]**

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

3. The trace of the real 4×4 matrix $U = \exp(A)$, where $A = \begin{pmatrix} 0 & 0 & 0 & \pi/4 \\ 0 & 0 & -\pi/4 & 0 \\ 0 & \pi/4 & 0 & 0 \\ -\pi/4 & 0 & 0 & 0 \end{pmatrix}$

is equal to **[TIFR 2011]**

(a) $2\sqrt{2}$ (b) $\pi/4$ (c) $\exp(i\varphi)$ for $\varphi = 0, \pi$ (d) zero

4. Two different 2×2 matrices A and B are found to have the same eigenvalues. It is then correct to state that $A = SBS^{-1}$ where S can be a **[TIFR 2012]**

(a) traceless 2×2 matrix (b) Hermitian 2×2 matrix

(c) unitary 2×2 matrix (d) arbitrary 2×2 matrix

5. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that **[TIFR 2014]**

(a) $\{M, N\} = 0$ (b) $[M, N] = 0$ (c) $M^{\dagger} = N$ (d) $M^{\dagger} = N^{-1}$

6. If the eigenvalues of a symmetric 3×3 matrix A are 0, 1, 3 and the corresponding eigenvectors can be written as **[TIFR 2016]**

$$\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}1\\-2\\1\end{bmatrix}$$

respectively, then the matrix A^4 is

(a) $\begin{bmatrix}41 & -81 & 40\\-81 & 0 & -81\\40 & -81 & 41\end{bmatrix}$ (b) $\begin{bmatrix}-82 & -81 & 79\\-81 & 81 & -81\\79 & -81 & 83\end{bmatrix}$ (c) $\begin{bmatrix}14 & -27 & 13\\-27 & 54 & -27\\13 & -27 & 14\end{bmatrix}$ (d) $\begin{bmatrix}14 & -13 & 27\\-13 & 54 & -13\\27 & -13 & 14\end{bmatrix}$

7. Denote the commutator of two matrices A and B by $[A, B] = AB - BA$ and the anti-commutator by $\{A, B\} = AB + BA$. If $\{A, B\} = 0$, we can write $[A, BC] =$ **[TIFR 2017]**

(a) $-B[A,C]$ (b) $B\{A,C\}$ (c) $-B\{A,C\}$ (d) $[A,C]B$

8. The matrix $\begin{pmatrix}100\sqrt{2} & x & 0\\-x & 0 & -x\\0 & x & 100\sqrt{2}\end{pmatrix}$, where $x > 0$, is known to have two equal eigenvalues. Find the value of x. **[TIFR 2017]**

9. A unitary matrix U is expanded in terms of a Hermitian matrix H, such that $U = e^{i\pi H/2}$. If we know that

$$H = \begin{pmatrix}\frac{1}{2} & 0 & \frac{\sqrt{3}}{2}\\0 & 1 & 0\\\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2}\end{pmatrix}$$

then U must be **[TIFR 2017]**

(a) $\begin{pmatrix}i & \frac{1}{2} & \frac{\sqrt{3}}{2}\\\frac{1}{2} & i & \frac{1}{2}\\\frac{\sqrt{3}}{2} & \frac{1}{2} & i\end{pmatrix}$ (b) $\begin{pmatrix}\frac{i}{2} & 0 & \frac{i\sqrt{3}}{2}\\0 & i & 0\\\frac{i\sqrt{3}}{2} & 0 & -\frac{i}{2}\end{pmatrix}$ (c) $\begin{pmatrix}1 & 0 & \sqrt{3}\\0 & 2 & 0\\\sqrt{3} & 0 & -1\end{pmatrix}$ (d) $\begin{pmatrix}2i & 1 & \frac{\sqrt{3}}{2}\\1 & 2i & 0\\\frac{\sqrt{3}}{2} & 0 & 2i\end{pmatrix}$

10. If a 2×2 matrix $\mathbb{M}$ is given by **[TIFR 2018**

$\mathbb{M} = \begin{pmatrix}1 & \frac{(1-i)}{\sqrt{2}}\\\frac{(1+i)}{\sqrt{2}} & 0\end{pmatrix}$, then $\det\exp\mathbb{M} =$

(a) e (b) e^2 (c) $2i\sin\sqrt{2}$ (d) $\exp\left(-2\sqrt{2}\right)$

ANSWER KEY

1. (d)	2. (b)	3. (a)	4. (c)	5. (a)	6. (c)	7. (c)
8. (50)	9. (b)	10. (a)				

JEST PREVIOUS YEAR QUESTIONS

1. The coordinate transformation

$$x' = 0.8x + 0.6y, \qquad y' = 0.6x - 0.8y$$

represents **[JEST 2013]**

(a) a translation (b) a proper rotation (c) a reflection (d) none of the above.

2. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represents $\cos(\pi M/6)$? **[JEST 2016]**

(a) $\frac{1}{2}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3. Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. Similarly transformation of M to Λ can be performed by

[JEST 2017]

(a) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$ (c) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$ (d) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$

4. If $\rho = \left[I + \frac{1}{\sqrt{3}}\left(\sigma_x + \sigma_y + \sigma_z\right)\right]\Big/2$, where σ's are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is **[JEST 2017]**

(a) 2^{2017} (b) 2^{-2017} (c) 1 (d) 1/2

5. Two of the eigenvalues of the matrix **[JEST 2018]**

$$A = \begin{pmatrix} a & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are 1 and -1. What is the third eigenvalue?

(a) 2 (b) 5 (c) -2 (d) -5

ANSWER KEY

1. (c) **2. (b)** **3. (a)** **4. (c)** **5. (b)**

MISCELLANEOUS QUESTIONS

1. Consider the matrix, $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

the eigenvalues of A are, $\left(i = \sqrt{-1}\right)$

(a) $\{1,-i,-1,i\}$ (b) $\{1,-1,0,0\}$ (c) $\{1,1,1,1\}$ (d) $\{1,1,i,-i\}$

2. Let P be a $n \times n$ diagonalizable matrix which satisfies the equations

$$P^2 = P, Tr(P) = n-1$$

Det (P) is :

(a) n (b) 0 (c) 1 (d) $n-1$

3. Let M be a 3×3 Hermitian matrix which satisfies the matrix equation

$$M^2 - 5M + 6I = 0$$

Where I refers to the identity matrix. Which of the following are possible eigenvalues of M

(a) $\{1, 2, 3\}$ (b) $\{2, 2, 3\}$ (c) $\{2, 3, 5\}$ (d) 5, 5, 6$\}$

4. Given the three matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Which of the following statements is true for all positive integers n and $i = 1, 2, 3$?

(a) $\sigma_i^n = I$ (b) $\sigma_i^n = \sigma_i$ (c) $\sigma_i^{2n} = I$ (d) $\sigma_i^{2n} = \sigma_i$

5. The trace of a 2×2 matrix is 1 and its determinate is 1. Which of the following has to be true ?

(a) One of the eigenvalues is 0 (b) One of the eigenvalue is 1

(c) Both of the eigenvalues are 1 (d) Neither of the eigenvalues is 1

6. Let M by a 3×3 Hermitain matrix which satisfies the matrix equation

$$M^2 - 7M + 12I = 0$$

where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10 ?

(a) 27 (b) 36 (c) 48 (d) 64

7. Consider a 3×3 matrix of the form:

$$\begin{pmatrix} a_1^2 & a_1a_2 & a_1a_3 \\ a_1a_2 & a_2^2 & a_2a_3 \\ a_1a_3 & a_2a_3 & a_3^2 \end{pmatrix}$$

The number of zero eigenvalues for this matrix is

(a) 0 (b) 1 (c) 2 (d) 3

8. If A is an antisymmetric matrix and A^T be the transpose of this matrix, then which one of the following relations does not hold good?

(a) $A^T = -A$ (b) $AA^T = A^TA$

(c) A^2 is an antisymmetric matrix (d) A^2 is a symmetirc matrix

9. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by λ_1, λ_2 and λ_3. Which one of th following statement is **NOT TRUE?**

(a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$
(b) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$
(c) All eigenvalues are real
(d) Product of the eigenvalues is 1

10. If a matrix A satisfies the condition $A^2 = I$ and $Tr(A) = 0$ (where I is a $n \times n$ identity matrix), then

(a) The determinant of A must be 0
(b) The determinant of A must be +1
(c) A must be odd dimensional
(d) A must be even dimensional

11. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if

(a) $a = 1, b = -1$
(b) $a = 1/\sqrt{2}, b = -1/\sqrt{2}$
(c) $a = 1/\sqrt{2}, b = -i/\sqrt{2}$
(d) $a = 1,\ b = 1$

12. If A and B are two $n \times n$ matrices, then the trace of $C = [A, B]$ is

(a) $Tr(AB)$
(b) $Tr(A)\ Tr(B)$
(c) 0
(d) $Tr(A) + Tr(B)$

13. Consider the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S, then S is

(a) diagonal
(b) symmetric
(c) orthogonal
(d) unitary

ANSWER KEY

1. (a)	2. (b)	3. (b)	4. (c)	5. (d)	6. (b)	7. (c)
8. (c)	9. (c)	10. (d)	11. (b)	12. (c)	13. (d)	

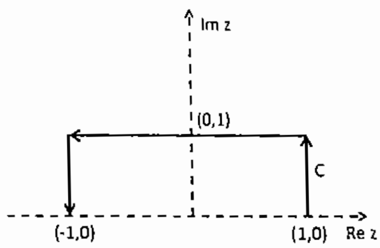
CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: COMPLEX ANALYSIS

CSIR PREVIOUS YEAR QUESTIONS

1. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: **[CSIR June 2011]**



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$ (c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

2. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z|<2$? **[CSIR June 2011]**

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

3. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$ and around $z=1$ is **[CSIR June 2012]**

(a) $\frac{1}{2}\left[1+z+z^2+....\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+.......\right]$ (b) $\frac{1}{1-z}-z-(1-z)^2+(1-z)^3+........$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+......\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+........\right]$ (d) $2(z-1)+5(z-1)^2+7(z-1)^3+......$

4. The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$ **[CSIR June 2012]**

(a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

5. The taylor series expansion of the function $\ln(\cosh x)$, where x is real, about point x = 0 starts with the following terms: **[CSIR Dec 2012]**

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+........$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+........$ (c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+........$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+........$

6. With z = x+ iy, which of the following functions f (x,y) is NOT a complex analytic function of z? **[CSIR June 2013]**

(a) $(x+iy-8)^3\left(4+x^2-y^2+2ixy\right)^7$ (b) $(x+iy)^7(1-x-iy)^3$

(c) $\left(x^2-y^2+2ixy-3\right)^5$ (d) $(1-x+iy)^4(2+x+iy)^6$

7. Given that the integral $\int_0^\infty \frac{dx}{y^2+x^2}=\frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{\left(y^2+x^2\right)^2}$ is **[CSIR Dec 2013]**

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

8. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{\sin^2 z}$ is **[CSIR June 2014]**

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

9. The Laurent series expansion of the function $f(z)=e^z+e^{1/z}$ about $z=0$ is **[CSIR Dec 2014]**

(a) $\sum_{n=-\infty}^{\infty}\frac{z^n}{n!}$ for all $|z|<\infty$ (b) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ only if $0<|x|<1$

(c) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ for all $0<|z|<\infty$ (d) $\sum_{n=-\infty}^{\infty}\frac{z^n}{n!}$, only if $|z|<1$

10. Consider the function $f(z)=\frac{1}{z}\ln(1-z)$ of a complex variable $z=re^{i\theta}(r\ge 0,-\infty<\theta<\infty)$. The singularities of $f(z)$ are as follows: **[CSIR Dec 2014]**

(a) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for $0\le\theta<2\pi$

(b) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ other than $0\le\theta<2\pi$

(c) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ

(d) branch points at $z=0$, $z=1$ and $z=\infty$

11. The function $\frac{z}{\sin \pi z^2}$ of a complex variable z has **[CSIR Dec 2015]**

(a) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ for n = 1, 2, 3 ....

(b) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ and $z=\pm i\sqrt{n}$ for n = 1, 2, 3 ....

(c) poles of order 2 at $z=\pm\sqrt{n}$ for n = 0, 1, 2, 3 ....

(d) poles of order 2 at $z=\pm n$ for n = 0, 1, 2, 3 ....

2. The radius of convergence of the Taylor series expansion of the function $\dfrac{1}{\cosh(x)}$ around x = 0, is

[CSIR June 2016]

(a) ∞ (b) π (c) $\pi/2$ (d) 1

3. The value of the contour integral

$$\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$$

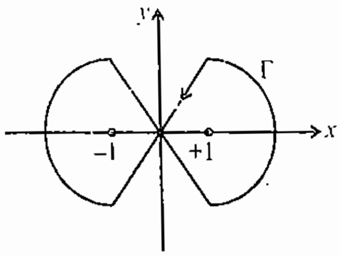
around the unit circle C traversed in the anti-clockwise direction, is **[CSIR June 2016]**

(a) 0 (b) 2 (c) $-\dfrac{8}{\sqrt{3}}$ (d) $-\tanh\left(\dfrac{1}{2}\right)$

4. Let $u(x,y)=e^{ax}\cos(by)$ be the real part of a function $f(z)=u(x,y)+iv(x,y)$ of the complex variable $z=x+iy$, where a,b are real constants and $a\neq 0$. The function $f(z)$ is complex analytic everywhere in the complex plane if and only if **[CSIR June 2017]**

(a) $b=0$ (b) $b=\pm a$ (c) $b=\pm 2\pi a$ (d) $b=a\pm 2\pi$

5. The integral $\oint_\Gamma \dfrac{z\,e^{i\pi z/2}}{z^2-1}dz$ along the closed contour Γ shown in the figure is **[CSIR June 2017]**



(a) 0 (b) 2π (c) -2π (d) $4\pi i$

6. Consider the real function $f(x)=\dfrac{1}{(x^2+4)}$. The Taylor expansion of $f(x)$ about $x=0$ converges

(a) for all values of x (b) for all values of x except $x=\pm 2$ **[CSIR Dec. 2017]**
(c) in the region $-2<x<2$ (d) for $x>2$ and $x<-2$

7. What is the value of α for which $f(x,y)=2x+3(x^2-y^2)+2i(3xy+\alpha y)$ is an analytic function of complex variable $z=x+iy$? **[CSIR June 2018]**

(a) 1 (b) 0 (c) 3 (d) 2

ANSWER KEY

1. (c)	2. (b)	3. (b)	4. (b)	5. (b)	6. (d)	7. (b)
8. (c)	9. (c)	10. (b)	11. (b)	12. (c)	13. (c)	14. (d)
15. (c)	16. (c)	17. (a)				

GATE PREVIOUS YEAR QUESTIONS

1. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is a circle defined by $|z| = 3$, is: **[GATE 2006]**

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$ (c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

2. If $I = \oint_C \ln z\, dz$, where C is the unit circle taken anticlockwise and lnz is the principal branch of the logarithmic function, which of the following is correct? **[GATE 2008]**

(a) $I = 0$ by residue theorem.
(b) I is not defined since, lnz is branch cut.
(c) $I \neq 0$
(d) $\oint_C \ln(z^2)\, dz = 2I$

3. For the complex function, $f(z) = \frac{e^{\sqrt{z}} - e^{-\sqrt{z}}}{\sin(\sqrt{z})}$, which of the following statement is correct? **[GATE 2010]**

(a) $z = 0$ is a branch point.
(b) $z = 0$ is a pole of order one
(c) $z = 0$ is a removable singularity
(d) $z = 0$ is an essential singularity

Common data for Q. 4 and Q. 5

Consider a function $f(z) = \frac{z\sin z}{(z-\pi)^2}$ of a complex variable z. **[GATE 2011]**

4. Which of the following statements is TRUE for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z = \pi$

(c) $f(z)$ has a pole of order 2 at $z = \pi$

(d) $f(z)$ has a simple pole at $z = \pi$.

5. Consider a counterclockwise circular contour $|z| = 1$ about the origin. The integral $\oint f(z)dz$ over this contour is:

(a) $-i\pi$ (b) Zero (c) $i\pi$ (d) $2i\pi$

6. Consider $w = f(z) = u(x,y) + iv(x,y)$ to be an analytic function in a domain D. Which one of the following options is NOT correct? **[GATE 2015]**

(a) $u(x,y)$ satisfies Laplace equation in D

(b) $v(x,y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

7. Consider a complex function $f(z)=\dfrac{1}{z\left(z+\dfrac{1}{2}\right)\cos(z\pi)}$. Which one of the following statements is correct?

(a) $f(z)$ has simple poles at $z=0$ and $z=-\dfrac{1}{2}$ **[GATE 2015]**

(b) $f(z)$ has a second order pole at $z=-\dfrac{1}{2}$

(c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

8. The absolute value of the integral $\int \dfrac{5z^3+3z^2}{z^2-4}\,dz$, over the circle $|z-1.5|=1$ in complex plane, is ________ (up to two decimal places). **[GATE 2018]**

ANSWER KEY

1. (a) **2. (a)** **3. (c)** **4. (d)** **5. (b)** **6. (c)** **7. (b)**

8. (81.60 to 81.80)

JEST PREVIOUS YEAR QUESTIONS

1. The value of integral $\int_0^{\infty} \frac{\ln x}{\left(x^2+1\right)^2} dx$ is **[JEST 2012]**

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

2. Compute $\lim_{z \to 0} \frac{\mathrm{Re}\left(z^2\right)+\mathrm{Im}\left(z^2\right)}{z^2}$ **[JEST 2013]**

(a) the limit does not exist (b) 1
(c) –i (d) –1

3. The value of the integral $\int_0^{\infty} \frac{\ln x}{\left(x^2+1\right)} dx$ is **[JEST 2016]**

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) π^2 (d) 0

4. Which one is the image of the complex domain $\{z \mid xy \geq 1, x+y>0\}$ under the mapping $f(z)=z^2$, if $z=x+iy$? **[JEST 2017]**

(a) $\{z \mid xy \geq 1, x+y>0\}$ (b) $\{z \mid x \geq 2, x+y>0\}$
(c) $\{z \mid y \geq 2 \;\forall\, x\}$ (d) $\{z \mid y \geq 1 \;\forall\, x\}$

5. The integral $I=\int_1^{\infty} \frac{\sqrt{x-1}}{(1+x)^2} dx$ is **[JEST 2017]**

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\sqrt{\frac{\pi}{2}}$

6. The integral, $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx$ is

(a) $\frac{\pi}{e}$ (b) πe^{-2} (c) π (d) zero **[JEST 2018]**

ANSWER KEY

1. (b)	2. (a)	3. (d)	4. (c)	5. (b)	6. (a)

MISCELLANEOUS QUESTIONS

1. Given $i=\sqrt{-1}$, then i^i is
   (a) Purely real
   (b) Purely imaginary
   (c) Of the form $x + iy$ with $x \neq 0, y \neq 0$
   (d) Not defined

2. Let z_1 and z_2 be two non-zero complex numbers. If

$$|z_1 + z_2| = |z_1| + |z_2|$$

   Which of the following is true?
   (a) $\mathrm{Re}(z_1\bar{z}_2) < 0$, and $\mathrm{Im}(z_1\bar{z}_2) = 0$
   (b) $\mathrm{Re}(z_1\bar{z}_2) > 0$, and $\mathrm{Im}(z_1\bar{z}_2) = 0$
   (c) $\mathrm{Re}(z_1\bar{z}_2) > 0$, and $\mathrm{Im}(z_1\bar{z}_2) > 0$
   (d) $\mathrm{Re}(z_1\bar{z}_2) < 0$, and $\mathrm{Im}(z_1\bar{z}_2) < 0$

3. If 'z' is a complex number, them we can express $\sin^{-1} z$ as
   (a) $-i\,\ln\left(iz \pm \sqrt{1-z^2}\right)$ (b) $-i\,\ln\left(z \pm \sqrt{z^2-1}\right)$ (c) $\ln\left(z+\sqrt{z^2+1}\right)$ (d) $\ln\left(z+\sqrt{z^2-1}\right)$

4. Two complex numbers Z_1 and Z_2 are given to satisfy $|Z_1| = |Z_2|$ necessarily follows that:
   (a) $Z_1 = Z_2$ (b) $Z_1 = e^{i\phi} Z_2$ (c) $Z_1 = Z_2$ (d) $Z_1 = Z_2^{-1}$

ANSWER KEY

1. (a) **2. (b)** **3. (a)** **4. (b)**

CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS

CSIR PREVIOUS YEAR QUESTIONS

1. Let $p_n(x)$ (where n = 0,1,2, ...) be ploynomial of order 'n' wirh real co-efficients defined in the interval $2 \le x \le 4$. If $\int_2^4 p_m(x)\, p_n(x)\, dx = \delta_{mn}$, then **[CSIR June -2011]**

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

2. The genarating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x) t^n$ for Legendre polynomials $P_n(x)$ is $F(x,t) = \left(1-2xt+t^2\right)^{-1/2}$. The value of $P_3(-1)$ is **[CSIR Dec -2011]**

(a) 5/2 (b) 3/2 (c) 1 (d) -1

3. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$. Let, $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$. If $w(0) = 1$, then $w(1)$ is given by **[CSIR Dec -2011]**

(a) 1 (b) e^2 (c) $1/e$ (d) $1/e^2$

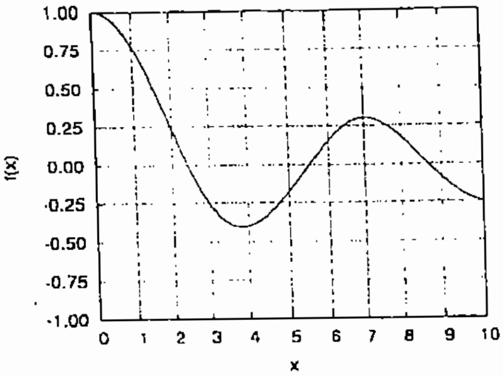
4. Let y(x) be a continuous real function in the range of 0 to 2π, satisfying the inhomogeneous differential equation: $\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$ **[CSIR June -2012]**

(a) is continuous (b) has a discontinuity of 3
(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

5. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3-4i)f = 0$ and satisfies the condition $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is **[CSIR Dec -2012]**

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi}$

6. The graph of the function f(x) shown below is best described by [CSIR Dec -2012] 11



12

(a) The bessel function $J_0(x)$ (b) $\cos x$

(c) $e^{-x}\cos x$ (d) $\frac{\cos x}{x}$

13

7. Given that $\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is [CSIR June -2013]

(a) 12 (b) 6 (c) 24 (d) -6

8. Consider the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$$ [CSIR June -2014]

with initial conditions $x(0)=0$ and $\dot{x}(0)=1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 (b) 1 (c) 2 (d) ∞

9. Given, $\sum_{n=0}^{\infty} P_n(x)t^n = (1-2xt+t^2)^{-1/2}$ for $|t|<1$, the value of $P_5(-1)$ is [CSIR June -2014]

(a) 0.26 (b) 1 (c) 0.5 (d) -1

10. The solution of the differential equation $\frac{dx}{dt}=2\sqrt{1-x^2}$, with initial condition $x=0$ at $t=0$ is 14

[CSIR Dec -2015]

(a) $x=\begin{cases}\sin 2t, & 0\le t\le \frac{\pi}{4}\\ \sinh 2t, & t\ge \frac{\pi}{4}\end{cases}$ (b) $x=\begin{cases}\sin 2t, & 0\le t< \frac{\pi}{2}\\ 1, & t\ge \frac{\pi}{2}\end{cases}$

15

(c) $x=\begin{cases}\sin 2t, & 0\le t< \frac{\pi}{4}\\ 1, & t\ge \frac{\pi}{4}\end{cases}$ (d) $x=1-\cos 2t,\quad t\ge 0$

11. The Hermite polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$$ **[CSIR Dec -2015]**

The corresponding generating function $G(t,x) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n$ satisfies the equation

(a) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$ (b) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

(c) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$ (d) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial^2 G}{\partial x \partial t} = 0$

12. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values $m = 10$ kg, $\gamma = 10$ kg/s, and $g \approx 10$ m/s^2, the distance travelled (in metres) in time t in seconds, is **[CSIR Dec -2016]**

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$ (c) $5t^2-(1-e^t)$ (d) $5t^2$

13. The generating function $G(t, x)$ for the Legendre polynomials $P_n(t)$ is **CSIR Dec -2017]**

$$G(t, x) = \frac{1}{\sqrt{1-2xt+x^2}} = \sum_{n=0}^{\infty} x^n P_n(t), \text{ for } |x| < 1.$$

If the function $f(x)$ is defined by the integral equation

$$\int_0^x f(x')\,dx' = xG(1, x),$$

it can be expressed as

(a) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m\left(\frac{1}{2}\right)$ (b) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m(1)$

(c) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(1) P_m(1)$ (d) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(0) P_m(1)$

14. Consider the following ordinary differential equation : **[CSIR June 2018]**

$$\frac{d^2x}{dt^2} + \frac{1}{x}\left(\frac{dx}{dt}\right)^2 - \frac{dx}{dt} = 0$$

with the boundary conditions $x(t=0) = 0$ and $x(t=1) = 1$. The value of $x(t)$ at $t = 2$ is

(a) $\sqrt{e-1}$ (b) $\sqrt{e^2+1}$ (c) $\sqrt{e+1}$ (d) $\sqrt{e^2-1}$

15. In the function $P_n(x)e^{-x^2}$ of a real variable x, $P_n(x)$ is a polynomial of degree n. The maximum number of extrema that this function can have is **[CSIR June 2018]**

(a) $n+2$ (b) $n-1$ (c) $n+1$ (d) n

ANSWER KEY

1. (d)	**2. (d)**	**3. (d)**	**4. (d)**	**5. (c)**	**6. (a)**	**7. (a)**
8. (b)	**9. (d)**	**10. (c)**	**11. (a)**	**12. (b)**	**13. (b)**	**14. (c)**
15. (c)						

GATE PREVIOUS YEAR QUESTIONS

1. A function $y(z)$ satisfies the ordinary differential equation $y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0$, where $m = 0, 1, 2, 3.......$

Consider the four statements P, Q, R, S as given below: **[GATE 2015]**

P: z^m and z^{-m} are linearly independent solutions for all values of m.

Q: z^m and z^{-m} are linearly independent solutions for all values of $m > 0$.

R: $\ln z$ and 1 are linearly independent solutions for $m = 0$.

S: z^m and $\ln z$ are linearly independent solutions for all values of m.

The correct option for the combination of valid statements is

(a) P, R and S only (b) P and R only (c) Q and R only (d) R and S only

2. The solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is **[GATE 2014]**

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

3. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of **[GATE 2011]**

(a) Circles with different radii (b) Circles with different centres.

(c) Straight lines with different slopes (d) Straight lines with different intercepts on the y-axis

4. The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions

$y(0) = 0$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$ is **[GATE 2010]**

(a) $\frac{1}{2}\cosh(t) + t\sinh(t)$ (b) $-\sinh(t) + t\cosh(t)$

(c) $t\cosh(t)$ (d) $t\sinh(t)$

5. Given the recurrence relation for the Legendre polynomials:

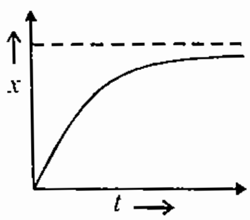
$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

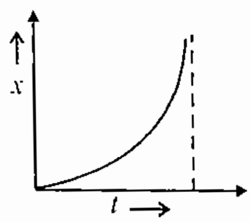
Which of the following integrals has a non-zero value? **[GATE 2010]**

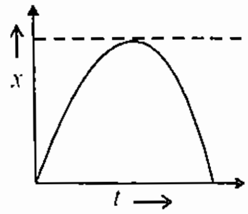
(a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\,dx$ (b) $\int_{-1}^{1} xP_n(x) P_{n+2}(x)\,dx$

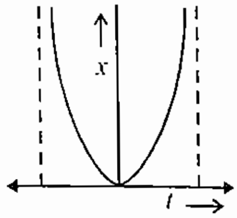
(c) $\int_{-1}^{1} x[P_n(x)]^2\,dx$ (d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\,dx$

6. Which one of the following curves gives the solution of the differential equation $k_1\frac{dx}{dt} + k_2x = k_3$, where k_1, k_2 and k_3 are positive real constants with initial conditions $x = 0$ at $t = 0$? **[GATE 2009]**

(a) 

(b) 

(c) 

(d) 

7. Consider the bessel equation: $\frac{d^2y}{dz^2}+\frac{1}{z}\frac{dy}{dz}+y=0 \ (v=0)$

Which of the following statements is correct? **[GATE 2008]**

(a) Equation has regular singular points at $z=0$ and $z=\infty$.

(b) Equation has 2 linearly independent solutions that are entire.

(c) Equation has an entire solution and a second linearly independent solution is singular at $z=0$.

(d) Limit $z\to\infty$, taken along X- axis, exists for both the linearly independent solutions.

8. The solution of the differential equation,

$$(1+x)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}-y(x)=0 \text{ is:}$$ **[GATE 2002]**

(a) Ax^2+B (b) $Ax+Be^{-x}$ (c) $Ax+Be^{x}$ (d) $Ax+Bx^2$

where A and B are constants.

9. The solution of the system of differential equations,

$$\frac{dy}{dx}=y-z \ and \ \frac{dz}{dx}=-4y+z$$

is given by (for A and B are arbitrary constants) **[GATE 2001]**

(a) $y(x)=Ae^{3x}+Be^{-x}; z(x)=-2Ae^{3x}+2Be^{-x}$ (b) $y(x)=Ae^{3x}+Be^{-x}; z(x)=2Ae^{3x}+2B^{-x}$

(c) $y(x)=Ae^{3x}+Be^{-x}; z(x)=2Ae^{3x}-2Be^{-x}$ (d) $y(x)=Ae^{3x}+Be^{-x}; z(x)=-2Ae^{3x}-2Be^{-x}$

10. Given: $\frac{d^2f(x)}{dx^2}-2\frac{df(x)}{dx}+f(x)=0$, and boundary conditions $f(0)=1$ and $f(1)=0$, the value of $f(0.5)$ is _______ (up to two decimal places). **[GATE 2018]**

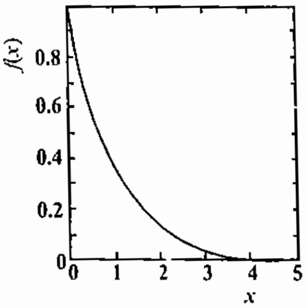
ANSWER KEY

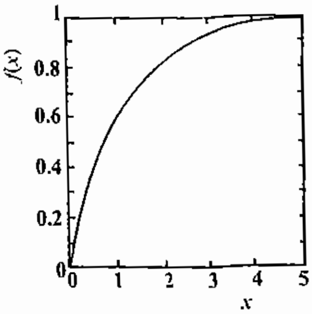
1. (c)	**2. (d)**	**3. (a)**	**4. (d)**	**5. (d)**	**6. (a)**	**7. (c, d)**
8. (b)	**9. (a)**	**10. (0.81 to 0.84)**				

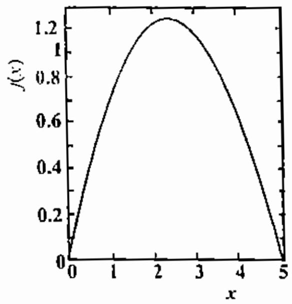
TIFR PREVIOUS YEAR QUESTIONS

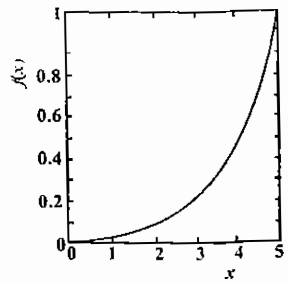
1. The solution of the integral equation $f(x)=x-\int_0^x dt\, f(t)$

has the graphical form **[TIFR 2014]**

(a)


(b)


(c)


(d)


2. Consider the differential equation $\frac{d^2y}{dx^2}=-4\left(y+\frac{dy}{dx}\right)$ with the boundary conditions that $y(x)=0$ at $x=\frac{1}{5}$.

When plotted as a function of x, for $x\geq 0$, we can say with certainity that the value of y

(a) first increases, then decreases to zero. **[TIFR 2015]**

(b) first decreases, then increases to zero.

(c) has an extremum in the range 0 < x < 1.

(d) oscillates from positive to neagtive with amplitude decreasing to zero.

3. The generating function for a set of polynomials in x is given by

$$f(x,t)=\left(1-2xt+t^2\right)^{-1}$$

The third polynomial (order x^2) in this set is **[TIFR 2015]**

(a) $2x^2+1$ (b) $4x^2+1$ (c) $2x^2-x$ (d) $4x^2-1$

4. The function $y(x)$ satisfies the differential equation $x\frac{dy}{dx}=y(\ln y-\ln x+1)$ with the initial condition $y(1)=3$. What will be the value of $y(3)$? **[TIFR 2015]**

5. Write down $x(t)$, where $x(t)$ is the solution of the following differential equation

$$\left(\frac{d}{dt}+2\right)\left(\frac{d}{dt}+1\right)x=1,$$

with the boundary conditions $\left.\frac{dx}{dt}\right|_{t=0}=0,\ \left.x(t)\right|_{t=0}=-\frac{1}{2}.$ **[TIFR 2017]**

6. If $y(x)$ satisfies the differential equation $y'' - 4y' + 4y = 0$ with boundary conditions $y(0) = 1$ and $y'(0) = 0$, then $y\left(-\frac{1}{2}\right) =$ **[TIFR 2018]**

(a) $\frac{2}{e}$ (b) $\frac{1}{2}\left(e + \frac{1}{e}\right)$ (c) $\frac{1}{e}$ (d) $-\frac{e}{2}$

ANSWER KEY

1. (b) **2. (c)** **3. (d)** **4. (81)** **5.** $\left(\exp(2t) - 2\exp(-t) + \frac{1}{2}\right)$

6. (a)

JEST PREVIOUS YEAR QUESTIONS

1. Consider the differential equation

$$\frac{dG(x)}{dx} + kG(x) = \delta(x)$$

where k is a constant. Which of the following statement is true? **[JEST 2013, 2015]**

(a) Both G(x) and G'(x) are continuous at $x = 0$

(b) G(x) is continuous at $x = 0$ but G'(x) is not.

(c) G(x) is discontinuous at $x = 0$

(d) The continuity properties of G(x) and G'(x) at $x = 0$ depend on the value of k.

2. For which of the following conditions does the integral $\int_0^1 P_m(x)P_n(x)dx$ vanish for $m \neq n$, where $P_m(x)$ and $P_n(x)$ are the Legendre polynomials of order m and n respectively? **[JEST 2018]**

(a) all $m, m \neq n$ (b) $m - n$ is an odd integer

(c) $m - n$ is a non-zero even integer (d) $n = m \pm 1$

3. If $y(x)$ satisfies **[JEST 2018]**

$$\frac{dy}{dx} = y\left[1 + (\log y)^2\right]$$

and $y(0) = 1$ for $x \geq 0$, then $y\left(\frac{\pi}{2}\right)$ is

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) infinity

ANSWER KEY

1. (c) 2. (c) 3. (d)

MISCELLANEOUS QUESTIONS

1. Given the equations $H' = -aG$ and $G' = bH, G$ can have oscillatory solutions

(a) $\forall\, a, b$
(b) for no choice of a, b
(c) If $a < 0$ and $b > 0$
(d) $a < 0$ and $b < 0$

2. Consider a forced harmonic oscillator which obeys the differential equation,

$$\frac{d^2y}{dt^2} + y = \sin t.$$

Which one of the following is the solution of the differential equation with initial condition y (0) = 0?

(a) $y(t) = 6\sin t$
(b) $y(t) = 12\sin t + \frac{t}{2}\cos t$
(c) $y(t) = 12\sin t - \frac{t}{2}\cos t$
(d) $y(t) = 6\cos t - \frac{t}{2}\cos t$

3. From the solution of the differential equation: $\frac{dy}{dx} = \frac{1}{1+x^2}$

what can you regarding the following series?

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \ldots$$

(a) The series is divergent
(b) The series is absolutely convergent
(c) The series converges to $\frac{\pi}{4}$
(d) The series converges to 0

4. Consider the system of differential equations

$$\frac{dx}{dt} = -y, \qquad \frac{dy}{dt} = x$$

Plotting the various solutions in the x, y-plane one obtains

(a) Hyperbola
(b) Parabola
(c) Circles
(d) Straight lines

5. For the Bessel equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at x = 0 is

(a) zero
(b) one
(c) two
(d) three

ANSWER KEY

1. (c) **2. (c)** **3. (c)** **4. (c)** **5. (b)**

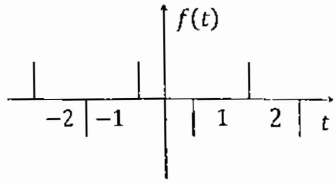
CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: FOURIER SERIES, FOURIER TRANSFORM, LAPLACE TRANSFORM

CSIR PREVIOUS YEAR QUESTIONS

. Consider the periodic function $f(t)$ with time period T as shown in the figure below:

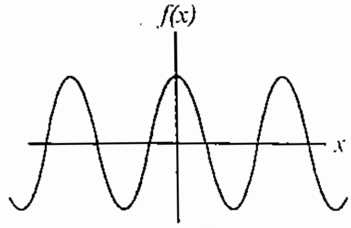


[CSIR JUNE 2015]

The spikes are located at $t = \frac{1}{2}(2n-1)$, where $n = 0, \pm 1, \pm 2....$, are Dirac delta functions of strength ± 1. The amplitudes a_n in the fourier expansion $f(t) = \sum_{n=-\infty}^{\infty} a_n e^{i2\pi nt/T}$ are given by

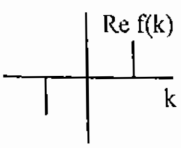
(a) $(-1)^n$ (b) $\frac{1}{n\pi}\sin\frac{n\pi}{2}$ (c) $i\sin\frac{n\pi}{2}$ (d) $n\pi$

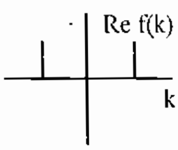
. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

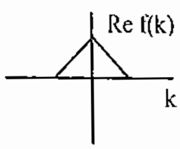


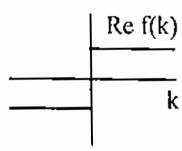
Which of the following graphs represents the real part of its Fourier transform?

[CSIR June 2014]

(a)


(b)


(c)


(d)


. A solution $y(x)$ satisfies the following differential equation:

$$\frac{d^2y}{dx^2} - \omega^2 y = -\delta(x-a)$$

where ω is positive. The Fourier transform $\tilde{f}(k) = \int_{-\infty}^{\infty} dx\, e^{ikx} f(x)$ of f and the solution of the equation are, respectively

[CSIR Dec 2015]

(a) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}\left[e^{-\omega|x-a|}+e^{\omega|x-a|}\right]$ (b) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}e^{-\omega|x-a|}$

(c) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left[e^{-i\omega|x-a|}+e^{i\omega|x-a|}\right]$ (d) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2i\omega}\left[e^{-i\omega|x-a|}-e^{i\omega|x-a|}\right]$

4. What is the Fourier Transform $\int dx\, e^{ikx} f(x)$ of

$$f(x)=\delta(x)+\sum_{n=1}^{\infty}\frac{d^n}{dx^n}\delta(x)$$

where $\delta(x)$ is the Dirac delta function? **[CSIR June 2016]**

(a) $\frac{1}{1-ik}$ (b) $\frac{1}{1+ik}$ (c) $\frac{1}{k+i}$ (d) $\frac{1}{k-i}$

5. The graph of the function $f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$ (where n = 0,1,2........). Its laplae transform $\tilde{f}(s)$ is **[CSIR Dec 2011]**

(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s\left(1+e^{-s}\right)}$ (d) $\frac{1}{s\left(1-e^{-s}\right)}$

6. The Fourier transform $\int_{-\infty}^{\infty} dx\, f(x)e^{ikx}$ of the function $f(x)=\frac{1}{x^2+2}$ is **[CSIR Dec. 2016]**

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$ (b) $\sqrt{2}\pi e^{-\sqrt{2}k}$ (c) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2}k}$ (d) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2}|k|}$

7. Consider the differential equation $\frac{dy}{dx}+ay=e^{-bt}$ with the initial condition $y(0)=0$. Then the Laplace transform $Y(s)$ of the solution $y(t)$ is **[CSIR Dec. 2017]**

(a) $\frac{1}{(s+a)(s+b)}$ (b) $\frac{1}{b(s+a)}$ (c) $\frac{1}{a(s+b)}$ (d) $\frac{e^{-a}-e^{-b}}{b-a}$

8. The fourier transform $\int_{-\infty}^{\infty} dx\, f(x)e^{ikx}$ of the function $f(x)=e^{-|x|}$ is **[CSIR June 2018]**

(a) $-\frac{2}{1+k^2}$ (b) $-\frac{1}{2(1+k^2)}$ (c) $\frac{2}{1+k^2}$ (d) $\frac{2}{(2+k^2)}$

ANSWER KEY

1. (c) 2. (b) 3. (b) 4. (a) 5. (c) 6. (d) 7. (a)
8. (c)

GATE PREVIOUS YEAR QUESTIONS

. A periodic function $f(x)=x$ for $-\pi<x<\pi$ has the Fourier series representation

$$f(x)=\sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$$ 1.

Using this, one finds the sum $\sum_{n=1}^{\infty}\frac{1}{n^2}$ to be **[GATE 2004]**

(a) $2\ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi\ln 2$

. A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi<x<\pi)$ as: 2.

$$f(x)=\begin{cases}-1 & \text{for} \quad -\pi<x<0\\ 1 & \text{for} \quad 0<x<\pi\end{cases}$$ **[GATE 2016]**

The approximate Fourier series expansion for $f(x)$ is

(a) $f(x)=\frac{4}{\pi}\left[\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\text{.........}\right]$

(b) $f(x)=\frac{4}{\pi}\left[\sin x-\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x-\text{.........}\right]$

(c) $f(x)=\frac{4}{\pi}\left[\cos x+\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x+\text{.........}\right]$

(d) $f(x)=\frac{4}{\pi}\left[\cos x-\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x-\text{.........}\right]$

The Heaviside function is defined as $H(t)=\begin{cases}+1 & for \quad t>0\\ -1 & for \quad t<0\end{cases}$

and its fourier transform is given by $-2i/\omega$. The fourier transform of $\frac{1}{2}\left[H\left(t+\frac{1}{2}\right)-H\left(t-\frac{1}{2}\right)\right]$ is **[GATE 2015]** 3.

(a) $\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}$ (b) $\frac{\cos\frac{\omega}{2}}{\frac{\omega}{2}}$ (c) $\sin\frac{\omega}{2}$ (d) 0

The Fourier transform $F(k)$ of a function $f(x)$ is defined as: $F(k)=\int_{-\infty}^{\infty}dx\, f(x)\exp(ikx)$

Then $F(k)$ for $f(x)=exp(-x^2)$ is: [Given: $\int_{-\infty}^{\infty}\exp(-x^2)dx=\sqrt{\pi}$] **[GATE 2004]**

(a) $\pi\exp(-k)$ (b) $\sqrt{\pi}\exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi}\exp(-k^2)$

ANSWER KEY

1. (c) **2. (a)** **3. (a)** **4. (b)**

TIFR PREVIOUS YEAR QUESTIONS

1. A function $f(x)$ is defined in the range $-1 \le x \le 1$ by $f(x)=\begin{cases}1-x & for\ \ x\ge 0\\ 1+x & for\ \ x<0\end{cases}$

The first few terms in the Fourier series approximating this function are **[TIFR 2010]**

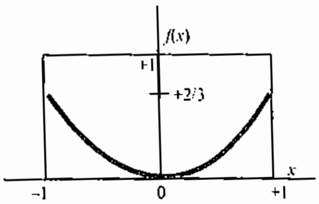
(a) $\frac{1}{2}+\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$

(b) $\frac{1}{2}+\frac{4}{\pi^2}\sin\pi x+\frac{4}{9\pi^2}\sin 3\pi x+\ldots$

(c) $\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$

(d) $\frac{1}{2}-\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x-\ldots$

2. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1\le x\le +1$, as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

$$f(x)\approx a_0+a_1\cos\frac{\pi x}{2}+a_2\sin\frac{\pi x}{2}$$ **[TIFR 2013]**

Which of the following would be the best choices for the coefficients a_0, a_1 and a_2?

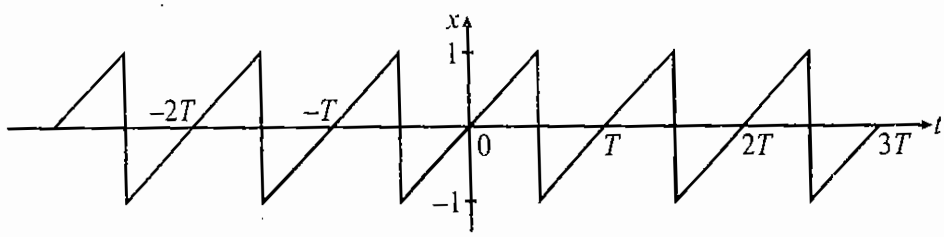
(a) $a_0=1, a_1=-\frac{1}{3}, a_2=0$

(b) $a_0=\frac{2}{3}, a_1=-\frac{2}{3}, a_2=0$

(c) $a_0=\frac{2}{3}, a_1=0, a_2=-\frac{2}{3}$

(d) $a_0=-\frac{1}{3}, a_1=0, a_2=-1$

3. Consider the waveform $x(t)$ shown in the diagram below.



The Fourier series for $x(t)$ which gives the closest approximation to this waveform is **[TIFR 2017]**

(a) $x(t)=\frac{2}{\pi}\left[\cos\frac{\pi t}{T}-\frac{1}{2}\cos\frac{4\pi t}{T}+\frac{1}{3}\cos\frac{3\pi t}{T}+\cdots\right]$

(b) $x(t)=\frac{2}{\pi}\left[-\sin\frac{\pi t}{T}+\frac{1}{2}\sin\frac{2\pi t}{T}-\frac{1}{3}\sin\frac{3\pi t}{T}+\cdots\right]$

(c) $x(t) = \frac{2}{\pi}\left[\sin\frac{\pi t}{T} - \frac{1}{2}\sin\frac{2\pi t}{T} + \frac{1}{3}\sin\frac{3\pi t}{T} + \cdots\right]$

(d) $x(t) = \frac{2}{\pi}\left[-\cos\frac{2\pi t}{T} + \frac{1}{2}\cos\frac{4\pi t}{T} - \frac{1}{3}\cos\frac{6\pi t}{T} + \cdots\right]$

4. The Fourier series which reproduces, in the interval $0 \le x < 1$, the function **[TIFR 2018]**

$$f(x) = \sum_{n=-\infty}^{+\infty} \delta(x-n)$$

where n is an integer, is

(a) $\cos\pi x + \cos 2\pi x + \cos 3\pi x + \cdots$ (to ∞)

(b) $1 + 2\cos 2\pi x + 2\cos 4\pi x + 2\cos 6\pi x + \cdots +$ (to ∞)

(c) $1 + \cos\pi x + \cos 2\pi x + \cos 3\pi x + \cdots$ (to ∞)

(d) $(\cos\pi x + \sin\pi x) + \frac{1}{2}(\cos 2\pi x + \sin 2\pi x) + \frac{1}{3}(\cos 3\pi x + \sin 3\pi x) + \cdots$ (to ∞)

ANSWER KEY

1. (a) **2. (b)** **3. (*)** **4. (b)**

JEST PREVIOUS YEAR QUESTIONS

1. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is **[JEST 2016]**

(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential

2. The Fourier transform of the function $\frac{1}{x^4+3x^2+2}$ up to a proportionality constant is **[JEST 2017]**

(a) $\sqrt{2}\exp\left(-k^2\right)-\exp\left(-2k^2\right)$ (b) $\sqrt{2}\exp\left(-|k|\right)-\exp\left(-\sqrt{2}\,|k|\right)$

(c) $\sqrt{2}\exp\left(-\sqrt{|k|}\right)-\exp\left(-\sqrt{2\,|k|}\right)$ (d) $\sqrt{2}\exp\left(-\sqrt{2}k^2\right)-\exp\left(-2k^2\right)$

3. The function $f(x)=\cosh x$ which exists in the range $-\pi \le x \le \pi$ is periodically repeated between $x=(2m-1)\pi$ and $(2m+1)\pi$, where $m=-\infty$ to $+\infty$. Using Fourier series, indicate the correct relation at $x=0$. **[JEST 2017]**

(a) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1-n^2}=\frac{1}{2}\left(\frac{\pi}{\cosh\pi}-1\right)$ (b) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1-n^2}=2\frac{\pi}{\cosh\pi}$

(c) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1+n^2}=2\frac{\pi}{\sinh\pi}$ (d) $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}=\frac{1}{2}\left(\frac{\pi}{\sinh\pi}-1\right)$

4. The Laplace transform of $\left(\sin(at)-at\cos(at)\right)/\left(2a^3\right)$ is **[JEST 2018]**

(a) $\frac{2as}{\left(s^2+a^2\right)^2}$ (b) $\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$ (c) $\frac{1}{\left(s+a\right)^2}$ (d) $\frac{1}{\left(s^2+a^2\right)^2}$

5. Consider the wavepacket defined by

$$\psi(x)=\int_{-\infty}^{\infty}dkf(k)\exp\left[i(kx)\right]$$ **[JEST 2018]**

Further, $f(k)=0$ for $|k|>k/2$ and $f(k)=a$ for $|k|\le K/2$. Then, the form of normalized $\psi(x)$ is

(a) $\frac{\sqrt{8\pi k}}{x}\sin\frac{Kx}{2}$ (b) $\sqrt{\frac{2}{\pi K}}\frac{\sin\frac{Kx}{2}}{x}$ (c) $\frac{\sqrt{8\pi K}}{x}\cos\frac{Kx}{2}$ (d) $\sqrt{\frac{2}{\pi K}}\frac{\sin\frac{Kx}{2}}{x}$

ANSWER KEY

1. (c) **2. (b)** **3. (d)** **4. (d)** **5. (b & d)**

CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: PROBABILITY

CSIR PREVIOUS YEAR QUESTIONS

1. Consider three particles A, B and C, each with attribute S that can take two values ± 1. Let $S_A = 1, S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to -S with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is **[CSIR June 2013]**

   (a) 2/3 (b) 1/3 (c) 2/9 (d) 4/9

2. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of turning up 1,2,3,4,5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even? **[CSIR Dec 2013]**

   (a) 144/441 (b) 225/441 (c) 221/441 (d) 220/441

3. A child makes a random walk on a square lattice of lattice constant 'a', taking a step in the north, east, south or west directions with probabilities 0.255, 0.255, 0.245, 0.245 respectively. After a large number of steps N, the expected position of the child w.r.t the strating point is at a distance **[CSIR Dec 2013]**

   (a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction (b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

   (c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0

4. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps? **[CSIR June 2014]**

   (a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

5. Let, $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ. Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is **[CSIR June 2014]**

   (a) 1 (b) 3/4 (c) 1/2 (d) 1/4

6. Two independent random variables m and n, which can take the integer values 0, 1, 2, ..., ∞, follow the Poisson distribution, with distinct mean values μ and ν respectively. Then **[CSIR Dec 2014]**

   (a) the probability distribution of the random variable $l = m + n$ is a binomial distribution.
   (b) the probability distribution of the random variable $r = m - n$ is also a Poisson distribution.
   (c) the variance of the random variable $l = m + n$ is equal to $\mu + \nu$.
   (d) the mean value of the random variable $r = m - n$ is equal to 0.

7. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is

[CSIR Dec 2014]

(a) $n/3$ (b) $n/8$ (c) $2n/3$ (d) 0

8. Three real variables a, b and c are each randomly chosen from a uniform probability distribution in the interval [0, 1]. The probability that $a+b>2c$ is **[CSIR June 2015]**

(a) 3/4 (b) 2/3 (c) 1/2 (d) 1/4

9. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is **[CSIR Dec 2015]**

(a) 12/27 (b) 27/64 (c) 3/8 (d) 9/16

10. Let X and Y be two independent random variables, each of which follow a normal distribution with same standard deviation σ, but with means μ and $-\mu$ respectively. Then the sum $X+Y$ follows a

[CSIR June 2016]

(a) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$

(b) normal distribution with mean 0 and standard deviation 2σ.

(c) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation 2σ.

(d) normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$.

11. A box of volume V containing N molecules of an ideal gas. is divided by wall with a hole into two conpartments. the volume of the smaller compartment $V/3$, the variance of the number of particles in it, is

[CSIR June 2016]

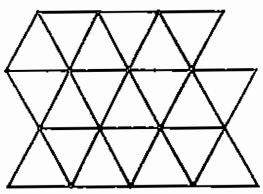
(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$ (c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$

12. Consider two radioactive atoms, each of which has a decay rate of 1 per year. The probability that at least one of them decays in the first two years is

[CSIR Dec. 2016]

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $1-e^{-4}$ (d) $(1-e^{-2})^2$

13. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps? **[CSIR Dec. 2016]**

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$ (c) $\frac{1}{18}$ (d) $\frac{1}{12}$

14. The random variable $x(-\infty < x < \infty)$ is distributed according to the normal distribution $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$.

The probability density of the random variable $y = x^2$ is **[CSIR June 2017]**

(a) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(b) $\frac{1}{2\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(c) $\frac{1}{\sqrt{2\sigma^2}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(d) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{\sigma^2}}, 0 \le y < \infty$

ANSWER KEY

1. (d)	2. (b)	3. (a)	4. (a)	5. (d)	6. (c)	7. (a)
8. (c)	9. (d)	10. (d)	11. (b)	12. (c)	13. (c)	14. (a)

TIFR PREVIOUS YEAR QUESTIONS

1. A 100 page book is known to have 200 printing errors distributed randomly through the pages. The probability that one of the pages will be found to be completely free of errors is closest to

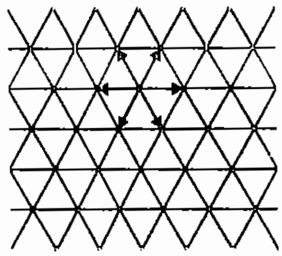
[TIFR 2011]

(a) 67% (b) 50% (c) 25% (d) 13%

2. A random number generator outputs +1 or -1 with equal probability every time it is run. After it is run 6 times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other. **[TIFR 2015]**

(a) 1/2 (b) 5/6 (c) 5/16 (d) 15/32

3. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right. **[TIFR 2016]**



The probability that the particle is back at its starting point after 3 moves is

(a) 5/18 (b) 1/6 (c) 1/18 (d) 1/36

ANSWER KEY

1. (d) **2. (c)** **3. (c)**

JEST PREVIOUS YEAR QUESTIONS

1. A box contains 100 coins out of which 99 fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one? **[JEST 2013]**

(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01

2. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to **[JEST 2013]**

(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

3. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x-axis. The probability that they meet after n steps is **[JEST 2013]**

(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

4. You receive on average 5 emails per day during a 365-days year. The number of days on average on which you do not receive any emails in that year are **[JEST 2016]**

(a) More than 5 (b) More than 2 (c) 1 (d) None of the above

5. Suppose that we toss two fair coins hundred times each. The probability that the same number of *heads* occur for both coins at the end of the experiment is **[JEST 2017]**

(a) $\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}$ (b) $2\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$

(c) $\frac{1}{2}\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$ (d) $\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$

ANSWER KEY

1. (c) 2. (d) 3. (a) 4. (b) 5. (d)

CSIR-UGC-NET/JRF Dec. 2018

MATHEMATICAL PHYSICS

SECTION: MISCELLANEOUS TOPICS

CSIR PREVIOUS YEAR QUESTIONS

1. Which of the following matrices is an element of the group $SU(2)$? **[CSIR June 2011]**

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$ (c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

2. The character table of C_{3v}, the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1\mathcal{C}_1$	1	1	b
$3\mathcal{C}_2$	1	a	c
$2\mathcal{C}_3$	1	1	d

In the above $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ denotes the three classes of C_{3v}, containing 1, 3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v}.

(A) The entries a, b, c and d in this table are, respectively

(a) 2, 1, −1, 0 (b) −1, 2, 0, −1 (c) −1, 1, 0, −1 (d) −1, 1, 1, −1

(B) The reducible representation Γ of C_{3v} with character $\chi = (4,0,1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as **[CSIR June 2011]**

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

3. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ? **[CSIR June 2012]**

(a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

4. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to **[CSIR June 2012]**

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

5. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than: (take $180°/\pi \simeq 57.29°$). **[CSIR June 2013]**

(a) 1.28° (b) 1.81° (c) 3.28° (d) 4.01°

6. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2}u(x,t)-\frac{\partial^2}{\partial x^2}u(x,t)=0$$

satisfying the boundary conditions $u(0, t) = 0 = u(L, t)$ and initial conditions $u(x, 0) = \sin(\pi x/L)$ and $\frac{\partial}{\partial t}u(x,t)|_{t=0}=\sin(2\pi x/L)$ is **[CSIR June 2013]**

(a) $\sin(\pi x/L)\cos(\pi t/L)+\frac{L}{2\pi}\sin(2\pi x/L)\cos(2\pi t/L)$

(b) $2\sin(\pi x/L)\cos(\pi t/L)-\sin(\pi x/L)\cos(2\pi t/L)$

(c) $\sin(\pi x/L)\cos(2\pi t/L)+\frac{L}{\pi}\sin(2\pi x/L)\sin(\pi t/L)$

(d) $\sin(\pi x/L)\cos(\pi t/L)+\frac{L}{2\pi}\sin(2\pi x/L)\sin(2\pi t/L)$

7. The expression **[CSIR Dec. 2013]**

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$

(b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$

(d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

8. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij}=A_iB_j$ **[CSIR Dec. 2013]**

(a) reduces to a direct sum of three 3-dimensional representations

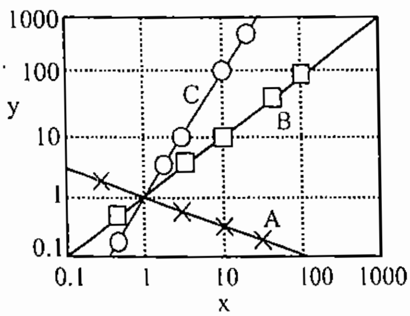
(b) is an irreducible 9-dimensional representation

(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations

(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

9. Three sets of data A, B and C from an experiment, represented by $\times$, $\square$ and $\bigcirc$, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.

[CSIR Dec. 2013]



The functional dependence $y(x)$ for the sets A, B and C are, respectively

(a) $\sqrt{x}, x$ and x^2 (b) $-\frac{x}{2}, x$ and $2x$ (c) $\frac{1}{x^2}, x$ and x^2 (d) $\frac{1}{\sqrt{x}}, x$ and x^2

10. The following data is obtained in an expriment that measures the viscosity η as a function of molecular weight M for a set of polymers. **[CSIR June 2014]**

$M\,(Da)$	$\eta\,(kPa-s)$
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is

(a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^2$ (d) $\eta \sim M^3$

11. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is **[CSIR June 2014]**

(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

12. The function $\Phi(x,y,z,t) = \cos(z - vt) + \mathrm{Re}(\sin(x + iy))$ satisfies the equation **[CSIR June 2014]**

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

13. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication? **[CSIR Dec. 2014]**

(a) $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}$, where $\alpha\beta \neq 1$

(c) $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real (d) $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

14. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is **[CSIR June 2015]**

(a) –3 (b) –2 (c) 1 (d) 4

15. The rank-2 tensor $x_i x_j$, where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set) containing **[CSIR June 2015]**

(a) 4 and 2 elements (b) 5 and 1 elements
(c) 3, 2 and 1 elements (d) 4, 1 and 1 elements

16. Consider the differential equation $\frac{dy}{dx} = x^2 - y$ with the initial condition $y = 2$ at $x = 0$. Let $y_{(1)}$ and $y_{(1/2)}$ be the solutions at $x = 1$ obtained using Euler's forward algorithm with step size 1 and 1/2 respectively. The value of $(y_{(1)} - y_{(1/2)})/y_{(1/2)}$ is **[CSIR June 2015]**

(a) $-\frac{1}{2}$ (b) –1 (c) $\frac{1}{2}$ (d) 1

17. Let $f(x,t)$ be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at $t = 0$, $f(x,0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x,0) = 0$ for all x, then $f(x,t)$ for all future times $t > 0$ is described by **[CSIR June 2015]**

(a) $e^{-(x^2 - v^2t^2)}$ (b) $e^{-(x-vt)^2}$

(c) $\frac{1}{4}e^{-(x-vt)^2} + \frac{3}{4}e^{-(x+vt)^2}$ (d) $\frac{1}{2}\left[e^{-(x-vt)^2} + e^{-(x+vt)^2}\right]$

18. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by **[CSIR Dec. 2015]**

(1) $\sqrt{\frac{hc}{E}}$ (2) $\frac{hc}{E^{3/2}}$ (3) $\left(\frac{hc}{E}\right)^2$ (4) $\frac{hc}{E}$

19. If $y = \frac{1}{\tan h(x)}$, then x is **[CSIR Dec. 2015]**

(a) $\ln\left(\frac{y+1}{y-1}\right)$ (b) $\ln\left(\frac{y-1}{y+1}\right)$ (c) $\ln\sqrt{\frac{y-1}{y+1}}$ (d) $\ln\sqrt{\frac{y+1}{y-1}}$

20. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h = 2$, is

[CSIR Dec. 2015]

(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

21. The Gauss hypergeometric function $F(a,b,c;z)$, defined by the Taylor series expansion around $z = 0$ as

$$F(a,b,c;z) = \sum_{n=0}^{\infty} \frac{a(a+1)\ldots(a+n-1)\,b(b+1)\ldots(b+n-1)}{c(c+1)\ldots(c+n-1)\,n!} z^n$$

satisfies the equation relation **[CSIR June 2016]**

(a) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a-1,b-1,c-1;z)$

(b) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a+1,b+1,c+1;z)$

(c) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a-1,b-1,c-1;z)$

(d) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a+1,b+1,c+1;z)$

22. Using dimensional analysis, Planck defined a characteristic temperature T_p from powers of the gravitional constant G, Planck's constant h, Boltzmann constant k_B and the speed of light c in vacuum. The expression for T_p is proportional to **[CSIR June 2016]**

(a) $\sqrt{\frac{hc^5}{k_B^2 G}}$ (b) $\sqrt{\frac{hc^3}{k_B^2 G}}$ (c) $\sqrt{\frac{G}{hc^4 k_B^2}}$ (d) $\sqrt{\frac{hk_B^2}{Gc^3}}$

23. The integral equation **[CSIR June 2016]**

$$\phi(x,t)=\lambda\int dx'dt' \qquad \int\frac{d\omega\, dk}{(2\pi)^2}\frac{e^{-ik(x-x')+i\omega(t-t')}}{\omega^2-k^2-m^2+i\varepsilon}\phi^3(x',t')$$

is equivalent to the differential equation

(a) $\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-m^2+i\varepsilon\right)\phi(x,t)=-\frac{1}{6}\lambda\phi^3(x,t)$

(b) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=\lambda\phi^2(x,t)$

(c) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=-3\lambda\phi^2(x,t)$

(d) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=-\lambda\phi^3(x,t)$

24. A part of the group multiplication table for a six element group $G=\{e,a,b,c,d,f\}$ is shown below. (In the following e is the identity element of G). **[CSIR June 2016]**

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d		
b	b	e	x	f	y	z
c	c					
d	d					
f	f					

The entries x, y and z should be

(a) $x=a,\ y=d$ and $z=c$
(b) $x=c,\ y=a$ and $z=d$
(c) $x=c,\ y=d$ and $z=a$
(d) $x=a,\ y=c$ and $z=d$

25. In finding the roots of the polynomial $f(x)=3x^3-4x-5$ using the iterative Newton-Raphson method, the initial guess is taken to be $x=2$. In the next iteration its value is nearest to **[CSIR June 2016]**

(a) 1.671 (b) 1.656 (c) 1.559 (d) 1.551

26. A stable asymptotic solution of the equation $x_{n+1}=1+\frac{3}{1+x_n}$ is $x=2$. If we take $x_n=2+\varepsilon_n$ and $x_{n+1}=2+\varepsilon_{n+1}$, where ε_n and ε_{n+1} are both small, the ratio $\varepsilon_{n+1}/\varepsilon_n$ is approximately

[CSIR Dec. 2016]

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

27. The 2×2 identity matrix I and the Pauli matrices $\sigma^x,\sigma^y,\sigma^z$ do not form a group under matrix multiplication. The minimum number of 2×2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is **[CSIR Dec. 2016]**

(a) 20 (b) 8 (c) 12 (d) 16

Given the values $\sin 45° = 0.7071$, $\sin 50° = 0.7660$, $\sin 55° = 0.8192$ and $\sin 60° = 0.8660$, the approximate value of $\sin 52°$, computed by Newton's forward difference method, is **[CSIR Dec. 2016]**

(a) 0.804 (b) 0.776 (c) 0.788 (d) 0.798

Let $f(x,t)$ be a solution of the heat equation $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at $t = 0$ is $f(x,0) = e^{-x^2}$ for $-\infty < x < \infty$. Then for all $t > 0$, $f(x,t)$ is given by **[CSIR Dec. 2016]**

[Useful integral: $\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$].

(a) $\frac{1}{\sqrt{1+Dt}}e^{-\frac{x^2}{1+Dt}}$ (b) $\frac{1}{\sqrt{1+2Dt}}e^{-\frac{x^2}{1+2Dt}}$ (c) $\frac{1}{\sqrt{1+4Dt}}e^{-\frac{x^2}{1+4Dt}}$ (d) $e^{-\frac{x^2}{1+Dt}}$

The Green's function satisfying $\frac{d^2}{dx^2}g(x,x_0) = \delta(x-x_0)$, with the boundary conditions $g(-L,x_0) = 0 = g(L,x_0)$, is **[CSIR June 2017]**

(a) $\begin{cases} \frac{1}{2L}(x_0-L)(x+L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0+L)(x-L), & x_0 \le x \le L \end{cases}$

(b) $\begin{cases} \frac{1}{2L}(x_0+L)(x+L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0-L)(x-L), & x_0 \le x \le L \end{cases}$

(c) $\begin{cases} \frac{1}{2L}(L-x_0)(x+L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0+L)(L-x), & x_0 \le x \le L \end{cases}$

(d) $\frac{1}{2L}(x-L)(x+L), \quad -L \le x \le L$

The interval [0, 1] is divided into $2n$ parts of equal length to calculate the integral $\int_0^1 e^{i2\pi x}dx$ using Simpson's $\frac{1}{3}$-rule. What is the minimum value of n for the result to be exact? **[CSIR June 2017]**

(a) ∞ (b) 2 (c) 3 (d) 4

Which of the following sets of 3×3 matrices (in which a and b are real numbers) form a group under matrix multiplication? **[CSIR June 2017]**

(a) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ b & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$ (b) $\left\{\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(c) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$ (d) $\left\{\begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

33. Consider an element $U(\varphi)$ of the group SU(2), where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as $U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(\varphi)$. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be **[CSIR Dec. 2017]**

(a) positive definite (b) real symmetric (c) hermitian (d) anti-hermitian

34. The differential equation $\dfrac{dy(x)}{dx} = \alpha x^2$, with the initial condition $y(0) = 0$, is solved using Euler's method. If $y_E(x)$ is the exact solution and $y_N(x)$ the numerical solution obtained using n steps of equal length, then the relative error $\left|\dfrac{(y_N(x) - y_E(x))}{y_E(x)}\right|$ is proportional to **[CSIR Dec. 2017]**

(a) $\dfrac{1}{n^2}$ (b) $\dfrac{1}{n^3}$ (c) $\dfrac{1}{n^4}$ (d) $\dfrac{1}{n}$

35. The interval [0, 1] is divided into n parts of equal length to calculate the integral $\int_0^1 e^{i2\pi x}dx$ using the trapezoidal rule. The minimum value of n for which the result is exact, is **[CSIR Dec. 2017]**

(a) 2 (b) 3 (c) 4 (d) ∞

36. The value of the integral $\int_{-\pi/2}^{+\pi/2} dx \int_{-1}^{+1} dy\, \delta(\sin 2x)\, \delta(x - y)$ is **[CSIR June 2018]**

(a) 0 (b) 1/2 (c) $1/\sqrt{2}$ (d) 1

37. The fractional error in estimating the integral $\int_0^1 x\,dx$ using Simpson's $\frac{1}{3}$-rule, using a step size 0.1, is nearest to

(a) 10^{-4} (b) 0 (c) 10^{-2} (d) 3×10^{-4} **[CSIR June 2018]**

38. The Green's function $G(x, x')$ for the equation $\dfrac{d^2y(x)}{dx^2} + y(x) = f(x)$, with the boundary values $y(0) = y\left(\dfrac{\pi}{2}\right) = 0$, is **[CSIR June 2018]**

(a) $G(x,x') = \begin{cases} x\left(x' - \dfrac{\pi}{2}\right), & 0 < x < x' < \dfrac{\pi}{2} \\ \left(x - \dfrac{\pi}{2}\right), & 0 < x' < x < \dfrac{\pi}{2} \end{cases}$ (b) $G(x,x') = \begin{cases} -\cos x' \sin x, & 0 < x < x' < \dfrac{\pi}{2} \\ -\sin x' \cos x, & 0 < x' < x < \dfrac{\pi}{2} \end{cases}$

(c) $G(x,x') = \begin{cases} \cos x' \sin x, & 0 < x < x' < \dfrac{\pi}{2} \\ \sin x' \cos x, & 0 < x' < x < \dfrac{\pi}{2} \end{cases}$ (d) $G(x,x') = \begin{cases} x\left(\dfrac{\pi}{2} - x'\right), & 0 < x < x' < \dfrac{\pi}{2} \\ x'\left(\dfrac{\pi}{2} - x\right), & 0 < x' < x < \dfrac{\pi}{2} \end{cases}$

ANSWER KEY

1. (b)	2. (c)	3. (c)	4. (a)	5. (b)	6. (d)	7. (b)
8. (c)	9. (d)	10. (d)	11. (c)	12. (a)	13. (d)	14. (a)
15. (b)	16. (b)	17. (d)	18. (c)	19. (d)	20. (a)	21. (d)
22. (a)	23. (d)	24. (d)	25. (b)	26. (c)	27. (d)	28. (c)
29. (c)	30. (a)	31. (b)	32. (c)	33. (d)	34. (d)	35. (a)
36. (b)	37. (b)	38. (b)				

GATE PREVIOUS YEAR QUESTIONS

1. The scale factors corresponding to the covariant metric tensor g_{ij} in spherical polar coordinates are

(a) $1, r^2, r^2 \sin^2\theta$ (b) $1, r^2, \sin^2\theta$ (c) 1, 1, 1 (d) $1, r, r\sin\theta$ **[GATE 2018]**

ANSWER KEY

1. (d)

TIFR PREVIOUS YEAR QUESTIONS

1. The infinite series

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\ldots$$

where $-1<x<+1$, can be summed to the value **[TIFR 2011]**

(a) tan hx (b) $\ln\left(1-\frac{4}{\pi}\tan^{-1}x\right)$ (c) $\frac{1}{2}\ln\left[\frac{(1+x)}{(1-x)}\right]$ (d) $\frac{1}{2}\ln\left[\frac{(1-x)}{(1+x)}\right]$

2. The function $f(x)$ represents the nearest integer less than x, e.g. $f(3.14)=3$. **[TIFR 2012]**

The derivative of this function (for arbitrary x) will be given in terms of the integers n as $f'(x)=$

(a) 0 (b) $\sum_n \delta(x-n)$ (c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

3. Consider the integral $\int\limits_{-p^2}^{+p^2}\frac{dx}{\sqrt{x^2-p^2}}$ **[TIFR 2012]**

where p is a constant. This integral has a real, nonsingular value if

(a) $p<-1$ (b) $p>1$ (c) $p=1$ (d) $p\to 0$

(e) $p\to\infty$

4. The value of the integral $\int\limits_0^\infty dx\, x^9\exp(-x^2)$ is **[TIFR 2013]**

(a) 20160 (b) 12 (c) 18 (d) 24

5. The integral $\int\limits_{-\infty}^{\infty} dx\,\delta(x^2-\pi^2)\cos x$, evaluates to **[TIFR 2013]**

(a) -1 (b) 0 (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

6. In spherical polar coordinates $\vec{r}=(r,\theta,\varphi)$ the delta function $\delta(\vec{r}_1-\vec{r}_2)$ can be written as **[TIFR 2014]**

(a) $\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$ (b) $\frac{1}{r_1^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$

(c) $\frac{1}{|\vec{r}_1-\vec{r}_2|^2}\delta(r_1-r_2)\delta(\cos\theta_1-\cos\theta_2)\delta(\varphi_1-\varphi_2)$ (d) $\frac{1}{r_1^2\cos\theta_1}\delta(r_1-r_2)\delta(\theta_1-\theta_2)\delta(\varphi_1-\varphi_2)$

7. The integral $\int\limits_0^\infty\frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right)-\exp\left(-\frac{x}{\sqrt{2}}\right)\right]$ evaluates to **[TIFR 2016]**

(a) zero (b) 2.03×10^{-2} (c) 2.03×10^{-1} (d) 2.03

8. Given that infinite series $y(x) = 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{(n+1)(n+2)}{2}x^n + \cdots$, find the value of $y(x)$ for $x = 6/7$. **[TIFR 2016]**

9. Evaluate the expression $n!\int_0^A dx_{n-1} \int_0^{x_{n-1}} dx_{n-2} \int_0^{x_{n-2}} dx_{n-3} \ldots \int_0^{x_3} dx_2 \int_0^{x_2} dx_1 \int_0^{x_1} dx_0$. **[TIFR 2017]**

10. Consider the two equations **[TIFR 2018]**

$\frac{x^2}{3} + \frac{y^2}{2} = 1$ and $x^3 - y = 1$

How many simultaneous real solutions does this pair of equations have ?

11. Evaluate the integral $\int_{-\infty}^{\infty} dx \exp(-x^2) \cos(\sqrt{2}x)$. **[TIFR 2018]**

ANSWER KEY

1. (c)	2. (b)	3. (d)	4. (b)	5. (d)	6. (b)	7. (c)
8. (343)	9. (A^n)	10. (2)	11. (1.074)			

JEST PREVIOUS YEAR QUESTIONS

1. If $[x]$ denotes the greatest integer not exceeding x, then $\int_0^{\infty}[x]e^{-x}dx$ **[JEST 2012]**

(a) $\frac{1}{e-1}$ (b) 1 (c) $\frac{e-1}{e}$ (d) $\frac{e}{e^2-1}$

2. As $x \to 1$, the infinite series $x-\frac{1}{3}x^3+\frac{1}{5}x^4-\frac{1}{7}x^7+.....$ **[JEST 2012]**

(a) diverges (b) converges to unity (c) converges to $\frac{\pi}{4}$ (d) none of the above.

3. What is the value of the following series? **[JEST 2012]**

$$\left(1+\frac{1}{2!}+\frac{1}{4!}+.....\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+.....\right)^2$$

(a) 0 (b) e (c) e^2 (d) 1

4. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$ for a well behaved function $f(x)$. If x has the dimension of momentum then **[JEST 2014]**

(a) $\delta(x)$ has the dimension of momentum (b) $\delta(x)$ has the dimension of (mometum)^2

(c) $\delta(x)$ is dimensionless (d) $\delta(x)$ has the dimension of (momentum)$^{-1}$.

5. The sum $\sum_{m=1}^{99}\frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to **[JEST 2015]**

(a) 9 (b) $\sqrt{99}-1$ (c) $\frac{1}{\left(\sqrt{99}-1\right)}$ (d) 11

6. The sum of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is **[JEST 2016]**

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

7. Given the condition $\nabla^2\Phi = 0$, the solution of the equation $\nabla^2\Psi = k\vec{\nabla}\Phi\cdot\vec{\nabla}\Phi$ is given by: **[JEST 2016]**

(a) $\Psi = k\Phi^2/2$ (b) $\Psi = k\Phi^2$ (c) $\Psi = k\Phi\,\ell n\,\Phi$ (d) $\Psi = k\Phi\,\ell n\,\Phi/2$

8. $\int_{-\infty}^{+\infty}(x^2+1)\,\delta(x^2-3x+2)\,dx = ?$ **[JEST 2017]**

(a) 1 (b) 2 (c) 5 (d) 7

9. $\pi \int_{-\infty}^{\infty} \exp(-|x|)\delta(\sin(\pi x))dx$, where $\delta(.....)$ is Dirac delta distribution, is **[JEST 2018]**

(a) 1 (b) $\frac{e+1}{e-1}$ (c) $\frac{e-1}{e+1}$ (d) $\frac{e}{e+1}$

10. If an abelian group is constructed with two distinct elements a and b such that $a^2 = b^2 = I$, where I is the group identity. What is the order of the smallest abelian group containing a, b and I? **[JEST 2018]**

ANSWER KEY

1. (a)	**2. (c)**	**3. (d)**	**4. (d)**	**5. (a)**	**6. (d)**	**7. (a)**
8. (d)	**9. (b)**	**10. (4)**				