ME6230
Mechanics of Human Movement

Assignment 2

Nilesh Balu
me20b121@smail.iitm.ac.in
Instructor: **Dr. Sujatha Srinivasan**

Indian Institute of Technology, Madras

Sunday 9th February, 2025

Contents

List of Figures

Abstract

In this assignment, an academic research paper, viz. **Equivalent system based inverse dynamics analysis of transfemoral prosthetic legs: Validation and application** [1] *is studied and summarised. Further, five questions are framed, the answers to which would bring out the main ideas behind the methods discussed in the paper*

1 Introduction

Inverse dynamics analysis is a method by which one estimates the joint forces and the moments about the joints based on three inputs - the kinematic data, the external force on the terminal link and the body segment parameters (BSP). A critical application of inverse dynamics analysis is in analysing the performance of prostheses. The effectiveness of a prosthesis can be determined by analysing the kinetics at the knee and the hip joints. For instance, a good passive transfemoral prosthesis would have the user apply the minimal moment needed at the hip to control the prosthesis.

2 Proposed Method

With the onset of polycentric prostheses, performing inverse dynamics analysis becomes complex. In inverse dynamics analysis, each link would have three unknowns - 2 components of joint forces and the joint moment (for one of the joints). In the case of polycentric knees, there are more joints than links. As a result, the system of equations obtained is indeterminate. The equivalent system (ES) approach simplifies this analysis. In the ES method, the knee kinetics is computed about the instantaneous centre of the ABL and PBL links. This instantaneous centre is called the functional knee centre (FKC).

2.1 Background

Consider a case shown in figure 2.1, where a load P1 is acting on Link 1 at the joint joining Link 1 and L. Similarly, there is a load P2 at the joint joining Link 2 and L. We can replace these forces with an equivalent force and moment acting at their instantaneous centre. These equivalent forces are not the same as the original forces. The relationship between the equivalent and actual forces is given by force transforms (FT) and moment transforms (MT), and these transforms depend on the intermediate link.

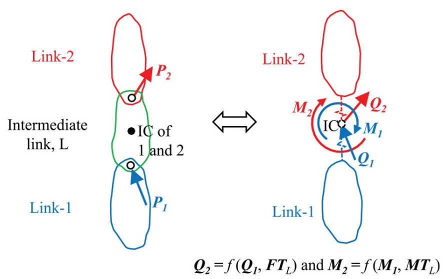

Figure 2.1: Equivalent forces and moments

2.2 ES analysis

Instead of considering the actual polycentric knee for our analysis, we will consider two equivalent links - link T and link S (refer figure 2.2 for the free body diagrams). The forces and moments at the polycentric knee joints can be substituted by equivalent forces and moments on links T and S, given by equations (1) to (4).

$$\mathbf{F}_{eq} = -\mathbf{F}_{K1} - \mathbf{F}_{K2} \tag{1}$$

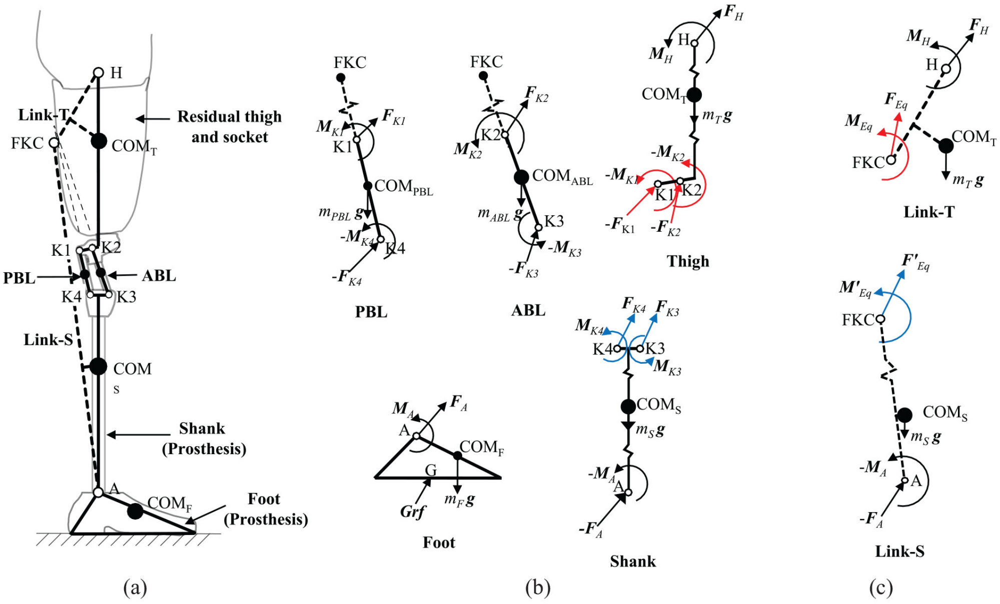

Figure 2.2: Free body diagrms

$$\mathbf{M}_{eq} = (\mathbf{L}_{K1-FKC} \times -\mathbf{F}_{K1}) + (\mathbf{L}_{K2-FKC} \times -\mathbf{F}_{K2}) - \mathbf{M}_{K1} - \mathbf{M}_{K2} \tag{2}$$

$$\mathbf{F}'_{eq} = \mathbf{F}_{K3} + \mathbf{F}_{K4} \tag{3}$$

$$\mathbf{M}'_{eq} = (\mathbf{L}_{K3-FKC} \times \mathbf{F}_{K3}) + (\mathbf{L}_{K4-FKC} \times \mathbf{F}_{K4}) + \mathbf{M}_{K3} + \mathbf{M}_{K4} \tag{4}$$

Now, the normal inverse dynamics analysis is carried of on the equivalent leg. This equivalent leg consists of the foot, link T and link S.

Foot:

$$\mathbf{F}_A + \mathbf{F}_G + m_F\mathbf{g} = m_F\mathbf{a}_{COM_F} \tag{5}$$

$$\mathbf{L}_{A-COM_F} \times \mathbf{F}_A + \mathbf{L}_{G-COM_F} \times \mathbf{F}_G + \mathbf{M}_A = I_{COM_F}\boldsymbol{\alpha}_F \tag{6}$$

Link T:

$$-\mathbf{F}_A + \mathbf{F}'_{eq} + m_S\mathbf{g} = m_S\mathbf{a}_{COM_S} \tag{7}$$

$$(\mathbf{L}_{A-COM_S} \times -\mathbf{F}_A) + (\mathbf{L}_{FKC-COM_S} \times \mathbf{F}'_{eq}) - \mathbf{M}_A + \mathbf{M}'_{eq} = I_{COM_S}\boldsymbol{\alpha}_S \tag{8}$$

Link S:

$$\mathbf{F}_H + \mathbf{F}_{eq} + m_T\mathbf{g} = m_T\mathbf{a}_{COM_T} \tag{9}$$

$$(\mathbf{L}_{H-COM_T} \times F_H) + (\mathbf{L}_{FKC-COM_T} \times \mathbf{F}_{eq}) + \mathbf{M}_H + \mathbf{M}_{eq} = I_{COM_T}\boldsymbol{\alpha}_T \tag{10}$$

It is important to note that the relation $\mathbf{F}_{eq} = -\mathbf{F}'_{eq}$ does not hold. The equivalent forces are not action-reaction pairs. The relation between $\mathbf{F}_{eq}$ and $\mathbf{F}'_{eq}$ is given by force transforms and moment transforms (FTs and MTs respectively). To compute this, analysis is done on the polycentric knee using kinematic data of the ABL and PBL. This relation can be written in the form

$$\mathbf{F}_{eq} = -\mathbf{F}'_{eq} - (\mathbf{FT}_{ABL} + \mathbf{FT}_{PBL}) \tag{11}$$

where

$$\mathbf{FT}_{ABL} = m_{ABL}(\mathbf{a}_{COM_{ABL}} - \mathbf{g}) \tag{12}$$
$$\mathbf{FT}_{PBL} = m_{PBL}(\mathbf{a}_{COM_{PBL}} - \mathbf{g}) \tag{13}$$

We find the relationship between the moments by performing a similar analysis on the ABL and PBL. The final result is as follows:

$$\mathbf{M}_{eq} = -\mathbf{M}'_{eq} - (\mathbf{MT}_{ABL} + \mathbf{T}_{PBL}) \tag{14}$$

where

$$\mathbf{MT}_{ABL} = \mathbf{L}_{COM_{ABL}-FKC} \times m_{ABL}(\mathbf{a}_{FKC_{ABL}} - \mathbf{g}) + I_{FKC_{ABL}}\boldsymbol{\alpha}_{ABL} \tag{15}$$
$$\mathbf{MT}_{PBL} = \mathbf{L}_{COM_{PBL}-FKC} \times m_{PBL}(\mathbf{a}_{FKC_{PBL}} - \mathbf{g}) + I_{FKC_{PBL}}\boldsymbol{\alpha}_{PBL} \tag{16}$$

3 Validation

Validating any newly proposed method using analytical or trusted numerical methods is mandatory. In this paper, synthetic motion data was generated using ADAMS software. The input motion at the hip was taken from existing literature [2]. The BSPs were estimated either from the model or from experimental data. The kinetics computed using ES analysis were compared with that from ADAMS, taking root mean square error (RMSE) as the measure of closeness.

Results: The RMSE values varied from 3×10^{-6}N to 17×10^{-6}N for hip reaction forces and 2.1×10^{-6}Nm to 2.6×10^{-6}Nm for the moment at the hip. These RMSE values are infinitesimally small! Hence, the ES approach is validated. **Note:** The joint reaction forces and moments at FKC cannot be compared with the actual force and moment at the knee. The knee kinetics at FKC is indirectly validated when the hip kinetics are compared.

4 Practical applications

The ES approach provides a way to analyse, understand and compare the performance of different prostheses. This paper, for instance, compares a single-axis knee to a polycentric knee by collecting kinematic data through user trials. Based on the hip kinetics and other quantities derived from the hip kinetics, it was concluded that the polycentric knee is more efficient than the single-axis knee. In this way, the ES approach can analyse prostheses' performance and help improve the design.

5 Questions and Answers

1. What is the significance of hip kinetics in the analysis of prostheses? Compare a polycentric knee to a single-axis knee by analysing the hip kinetics.

   Ans: Hip kinetics is significant in prosthetic gait analysis as it clearly brings out the human-mechanism interaction. It is important to avoid excessive forces at the hip joint as it can lead to joint degradation in the long term. A higher moment at the hip indicates increased muscle effort. This paper used the ES analysis to conclude that the polycentric knee reduces peak hip power and energy consumption more than the single-axis knee. This means that the four-bar knee can be helpful for prosthesis users to walk farther with reduced effort and energy expenditure.

2. Why are the two equivalent forces at the FKC, not action-reaction pairs? In other words, why are they not equal to each other in magnitude and opposite in direction?

   Ans: The two equivalent forces obtained at the FKC are not action-reaction pairs. This is because, when writing the equivalent forces and moments, it must be written to preserve the law of energy conservation. As a result, the equivalent kinetics will depend on the kinematics of the intermediate links. The equivalent forces and moments are hence related through Force transforms and Moment transforms, which are functions of the kinematics of the ABL and PBL (the intermediate links in the polycentric knee)

3. What are some applications of ES and inverse dynamics?

   Ans: Inverse dynamics analysis is used to calculate the forces and moments acting on the joints of the prosthetic leg during walking or other activities. This analysis can provide insight into the functioning of the prosthetic leg and help identify any issues or areas for improvement. The ES analysis is a technique to apply inverse dynamics analysis to an indeterminate system. Similar to how ES analysis makes it easier to analyse the functioning of prostheses, it could also be used in robotics. Inverse dynamics analysis is usually exploited in robotics to estimate the torque required to be applied by the motor. If a complex mechanism yields an indeterminate system of equations, then the ES method can be adopted.

4. What information does the force and moment obtained at the FKC give?

   Ans: The equivalent force and moment obtained at the FKC give more profound insights into the working mechanism of the prosthetic knee. For instance, the vertical knee reaction force's magnitude and line of action contribute to the rotational motion during the swing. When the COM of the shank is posterior to the FKC, a higher reaction would lead to a faster extension. When the COM is anterior to the FKC, it opposes the extension.

5. How will the analysis change when analysing a 6-bar or a more complex mechanism?

   Ans: The six-bar knee will have four intermediate links. This would imply that there are four transforms for force and moment. Hence, the relationship between the equivalent forces at the FKC will change. The remaining analysis remains the same. Thus, it can be seen that the ES method is a generic method that can be applied to multiple mechanisms with very few changes in the solution approach.

References

[1] S. Sudeesh, M. Shunmugam, and S. Sujatha, “Equivalent system based inverse dynamics analysis of transfemoral prosthetic legs: Validation and application,” *Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine*, vol. 237, no. 4, pp. 467–480, 2023. PMID: 36855780.

[2] D. Winter, *Biomechanics and Motor Control of Human Movement*. Wiley, 2009.