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PRAGMA

NUMERICAL ANALYSIS OF FLUID FLOW IN A PIPE WITH ANNULAR FINS USING FINITE VOLUME METHOD

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ABSTRACT

In this project, a numerical solver is developed to solve the convection-diffusion equation for fluid flow through a pipe. The pipe in consideration has multiple annular fins attached along its length. The governing equations are solved in the axisymmetric coordinate system. The discretisation is performed using the upward differencing scheme (UDS), and the system of equations is solved using the line-by-line tri-diagonal matrix algorithm (TDMA). The solver was validated by comparing the temperature profile of the first fin along its length with analytical solutions. The fluid's mean temperature along the pipe's length was also compared with analytical solutions for validation. The parameters of the fin, viz. fin length, thickness and spacing, are varied, and their effects on the fin's effectiveness are analysed. It is observed that the fin effectiveness increases as each of these parameters is increased.

NOMENCLATURE

ϕ An arbitrary scalar variable.
ρ_f Density of the fluid.
ρ_w Density of the pipe wall.
$\vec{u}$ Velocity vector.
Γ_f Diffusion coefficient of the fluid.
Γ_w Diffusion coefficient of the pipe wall.
$\vec{A}_f$ Face area vector.
R_1 Inner radius of the pipe.
R_2 Outer radius of the pipe.
R_f Outer radius of the annular fin.
L_f Fin length.
t_f Fin thickness.
s_f Fin spacing.
$c_{p,f}$ Specific heat capacity of the fluid.
$c_{p,w}$ Specific heat capacity of the pipe wall.
T_{in} Inlet temperature.
u_{mean} Mean velocity of the fluid flowing in the pipe.
$_f$ Fin effectiveness.

INTRODUCTION

Fourier's law of thermal conduction states that the rate of heat conducted through a medium is directly proportional to the negative gradient of temperature and the area perpendicular to the direction of heat flow. Heat exchangers are one broad set of devices wherein the heat transfer rate needs to be maximised. Most heat exchangers consist of 2 working fluids separated by a wall made of highly conducting material. In most real-life scenarios, the temperatures of these fluids are fixed. Hence, fins are attached to the wall to increase the wall's effective area, thereby increasing the heat transfer rate. There are numerous ways of attaching fins to a heat exchanger. The fin arrangement needs to be optimised to get the highest returns. Numerical solvers play their part in this regard. Different geometric parameters and material properties can be changed, and their effects can be easily studied using these solvers.

Similar Studies

Esmail et al. [1] presented a paper on annular fins with different profiles. They reported the deviation between the fin efficiency calculated based on constant and variable heat transfer coefficients with different radius ratios and dimensionless parameters. Mi Sandar and Ulrich [2] conducted a 3-D numerical study wherein the effects of different arrangements of bundles of annular-finned tubes were studied. Kundu and Das [3] developed a model to carry out the performance analysis for annular fins with different rectangular and trapezoidal cross-sections. Pipes with annular fins are commonly used in latent heat thermal energy storage (LHTES) systems. Phase change materials (PCMs) absorb or release large amounts of latent heat, making them perfect for energy storage. Lacroix [4] used TDMA to perform numerical simulations on a shell and tube storage unit. He concluded that annular fins were the most effective for moderate mass flow rates and small inlet temperatures.

Present study

This project analyses the heat transfer occurring when a hot fluid flows through a pipe fitted with annular fins. The fluid is assumed to be incompressible and newtonian, and the flow is laminar. Further, it is assumed that the entry length of the flow in the pipe is minimal compared to the length of the whole pipe. For such a case, an analytical solution exists for the flow velocity in the pipe. Hence for this analysis, the velocity field is assumed to be known apriori. The convection-diffusion equation has to be solved to find out the temperatures in the system.

GOVERNING EQUATION

The governing equation used is the steady-state convection-diffusion equation with no source terms. The equation for a scalar variable ϕ is given by

$$\vec{\nabla} \cdot (\rho \vec{u} \phi) = \vec{\nabla} \cdot (\Gamma \vec{\nabla} \phi) \tag{1}$$

where ρ is the density of the medium, $\vec{u}$ is a given flow field and Γ is diffusion coefficient of the medium.

NUMERICAL METHOD

Discretizing the Governing Equation

The finite volume method (FVM) is applied to discretise the governing equation [5]. Integrating the equation over a control volume yields

$$\int_{cv} \vec{\nabla} \cdot (\rho \vec{u} \phi) dv = \int_{cv} \vec{\nabla} \cdot (\Gamma \vec{\nabla} \phi) dv \tag{2}$$

Using the gauss-divergence theorem, the integrals are simplified into

$$\int_{cs} (\rho \vec{u} \phi) \cdot d\vec{A} = \int_{cs} (\Gamma \vec{\nabla} \phi) \cdot d\vec{A} \tag{3}$$

Considering a rectangular cell to be the control volume, the above equation can be approximated to

$$\sum_f ((\rho \vec{u} \phi)_f - (\Gamma \vec{\nabla} \phi)_f) \cdot d\vec{A}_f = 0 \tag{4}$$

Upward Differencing Scheme (UDS). Since the values of the variables are stored in the cell centroids, ϕ at the face should be represented in terms of ϕ at the cell centroids. In the upwind differencing scheme, ϕ_f is assumed to be equal to $\phi_{centroid}$ of the adjacent cell upstream to the face. Hence

$$\phi_e = \phi_P \qquad \phi_w = \phi_W$$
$$\phi_n = \phi_P \qquad \phi_s = \phi_S$$

Equation 1 can be rearranged to form the general discretized equation.

$$a_P \phi_P = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + b \tag{5}$$

where

$$\begin{aligned} a_E &= D_e + max(-F_e, 0) \\ a_W &= D_w + max(F_w, 0) \\ a_N &= D_n + max(-F_n, 0) \\ a_S &= D_s + max(F_s, 0) \end{aligned}$$

and

$$\begin{aligned} F_e &= \rho_e u_e \Delta y & De &= \tfrac{\Gamma_e \Delta y}{\Delta x} \\ F_w &= \rho_w u_w \Delta x & Dw &= \tfrac{\Gamma_w \Delta x}{\Delta y} \\ F_n &= \rho_n u_n \Delta y & Dn &= \tfrac{\Gamma_n \Delta y}{\Delta x} \\ F_s &= \rho_s u_s \Delta x & Ds &= \tfrac{\Gamma_s \Delta x}{\Delta y} \end{aligned}$$

Geometry and Mesh Generation

The pipe in consideration has an inner radius ($R_1 = 5cm$), an outer radius ($R_2 = 6cm$) and annular fins with an outer radius ($R_f = 56cm$). A $1m$ long pipe was considered. The equations were discretized over a structured mesh in the axisymmetric coordinate system. For this problem, $\Delta x = 0.002$ and $\Delta r = 0.002$, resulting in a mesh of size 280×500.

Solution Method

Tri-Diagonal Matrix Algorithm The TDMA algorithm exploits the fact that in a 1-D grid, the value of ϕ_{i-1} is only a function of ϕ_i. The function is of the form

$$\phi_{i-1} = P_{i-1}\phi_i + Q_{i-1} \tag{6}$$

However, the general form of a discretized equation is

$$a_i T_i = b_i T_{i+1} + c_i T_{i-1} + d_i \tag{7}$$

A recurrence relation is obtained from equations 6 and 7.

$$P_i = \frac{b_i}{a_i - c_i P_{i-1}} \qquad Q_i = \frac{d_i + c_i Q_{i-1}}{a_i - c_i P_{i-1}} \tag{8}$$

Using equation 8, forward substitution is performed to get the values of P_i and Q_i. Using these values and equation 6, backward substitution is performed to obtain the ϕ values

Line-by-Line TDMA The problem discussed in this project is solved using line-by-line TDMA. Consider one line from the grid at a time, and assume ϕ is known for every other cell (either from an initial guess or the previous iteration). Solve for ϕ in that line and then move on to the next line. The traverse direction is the direction along the line in which TDMA is performed, and the sweep direction is the direction orthogonal to the traverse direction.

VALIDATION

Temperature profile of the fin

Consider a $1m$ long pipe with an inner radius of $5cm$, an outer radius of $6cm$, fin length of $50cm$, fin thickness of $2cm$ and fin spacing of $4cm$. The pipe flow was solved under the following conditions:

$\vec{u} = 2u_{mean}(1 - \frac{r^2}{R_1^2})\hat{i}$, where $u_{mean} = 5m/s$
$T_{in} = 500°C$, $\rho_f = 1000kg/m^3$, $\rho_w = 8993kg/m^3$,
$\Gamma_f = 1kg/m.s$ and $\Gamma_w = 40kg/m.s$
$c_{p,f} = 50$, $c_{p,w} = 0.1$
The boundary condition on the pipe and fin outer walls is the Robin boundary condition, where there is convection from the surface to the surrounding air at $T_\infty = 25°C$, with a heat transfer rate of $h = 37W/m^2K$.

Acosta-Iborra and Campo, in their paper [6], arrived at an approximate analytical solution for the temperature of the midline of the fin along the length of the fin. The normalised temperature profile ($\theta = \frac{T - T_\infty}{T_{base} - T_\infty}$) obtained from the solver was plotted

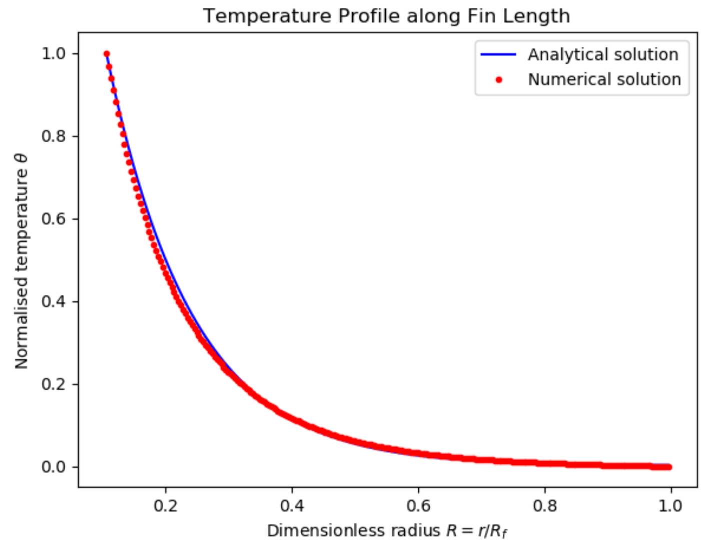

FIGURE 1. VALIDATING DIFFUSION IN FINS.

as a function of the fin length for the first fin. The values obtained from the solver are very close ($RMSE = 1.346 \times 10^{-2}$) to the analytical approximation of the true solution, as seen in Fig. 1.

Mean temperature along flow direction

A pipe flow problem was solved under the following conditions:

$\vec{u} = 2u_{mean}(1 - \frac{r^2}{R_1^2})\hat{i}$, where $u_{mean} = 5m/s$
$T_{in} = 500°C$, $\rho_f = 1000kg/m^3$, $\rho_w = 8993kg/m^3$,
$\Gamma_f = 1kg/m.s$ and $\Gamma_w = 40kg/m.s$
$c_{p,f} = 50$, $c_{p,w} = 0.1$
The boundary condition on the pipe and fin outer walls is the Neumann boundary condition, where constant heat flux is taken out from the pipe. $q'' = 100W/m^2$

The mean temperature was plotted as a function of distance along the length of the pipe. The result was compared with the analytical solution in Incropera's book [7]. The results match almost perfectly, with an $RMSE = 1.59 \times 10^{-4}$. In this way, the solver has been completely validated.

RESULTS AND DISCUSSION

The geometric parameters of the fin, viz. fin length, fin thickness and fin spacing, were varied, and the effects were studied. The amount of heat exchange, the mean temperatures along the centerline and the heatmaps are analysed for each case.

The parameters remaining constant in this analysis are as follows:

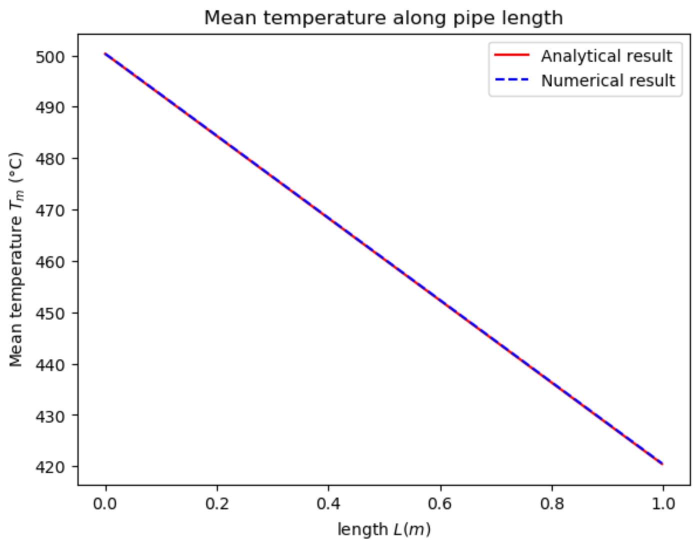

FIGURE 2. VALIDATING FLUID TEMPERATURE.

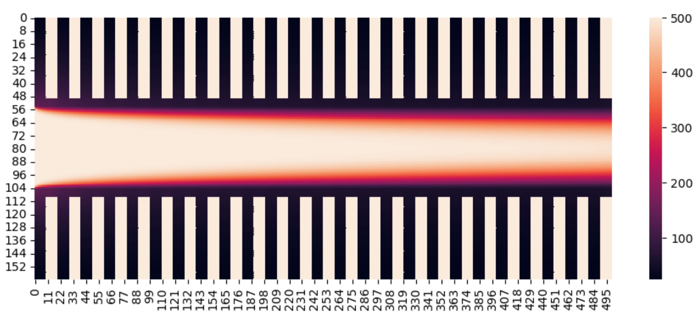

FIGURE 3. HEAT MAP FOR A FINNED PIPE WITH $L_f = 10cm$.

$\vec{u} = 2u_{mean}(1 - \frac{r^2}{R_1^2})\hat{i}$, where $u_{mean} = 5m/s$
$T_{in} = 500°C$, $\rho_f = 1000kg/m^3$, $\rho_w = 8993kg/m^3$,
$\Gamma_f = 1kg/m.s$ and $\Gamma_w = 40kg/m.s$
$c_{p,f} = 50$, $c_{p,w} = 0.1$
The boundary condition on the pipe and fin outer walls is the Robin boundary condition, where there is convection from the surface to the surrounding air at $T_\infty = 25°C$, with a heat transfer rate of $h = 37W/m^2K$.
The pipe has an inner radius of $5cm$, an outer radius of $6cm$, and is $1m$ long

Fin length (L_f)

The analysis is done for a pipe with no fins and pipes with $L_f = 2, 10, 20$ and $50cm$. The other geometric parameters of the fins are $s_f = 2cm$ and $t_f = 2cm$.

The heat map of temperature distribution in the pipe obtained from the numerical solver is shown in Fig3.

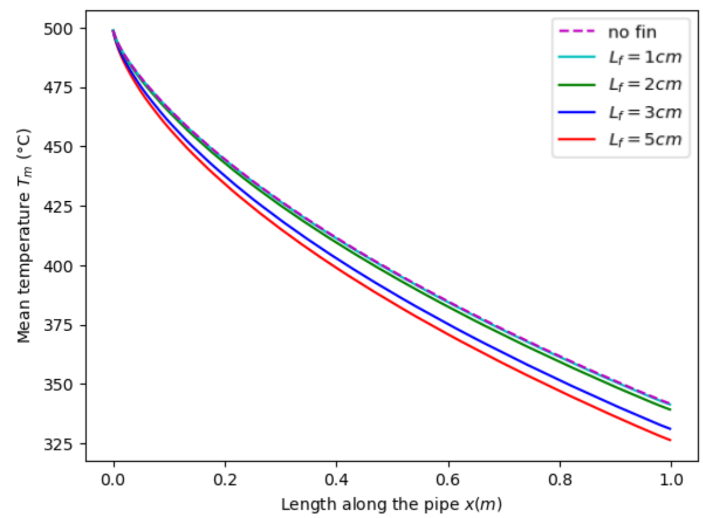

FIGURE 4. MEAN TEMPERATURE FOR DIFFERENT FIN LENGTHS.

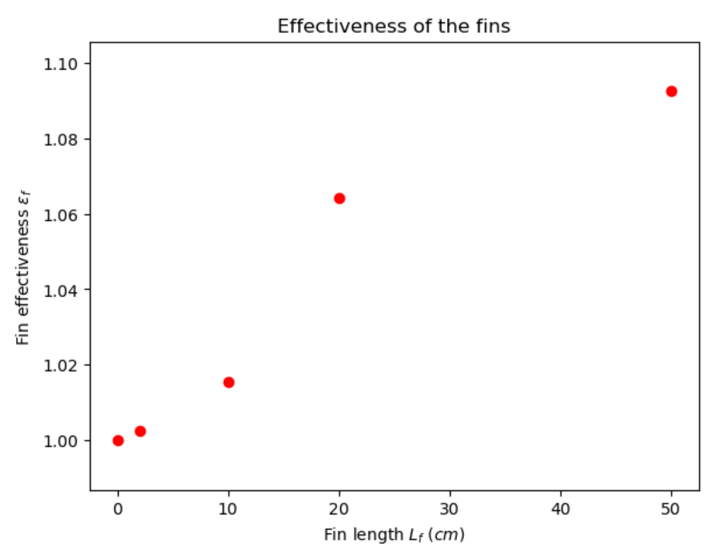

FIGURE 5. FIN EFFECTIVENESS VS FIN LENGTH.

The mean temperature and the fin effectiveness are plotted in Fig.4 and Fig.5. It is clear that there is an improvement in the amount of heat transfer with the addition of fins. As the length of the fin increases, the heat transferred increases and saturates to a constant value.

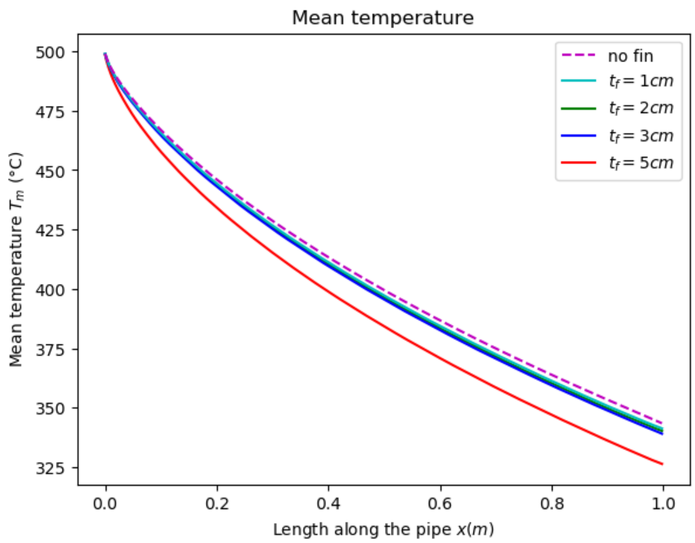

FIGURE 6. MEAN TEMPERATURE FOR DIFFERENT FIN THICKNESS.

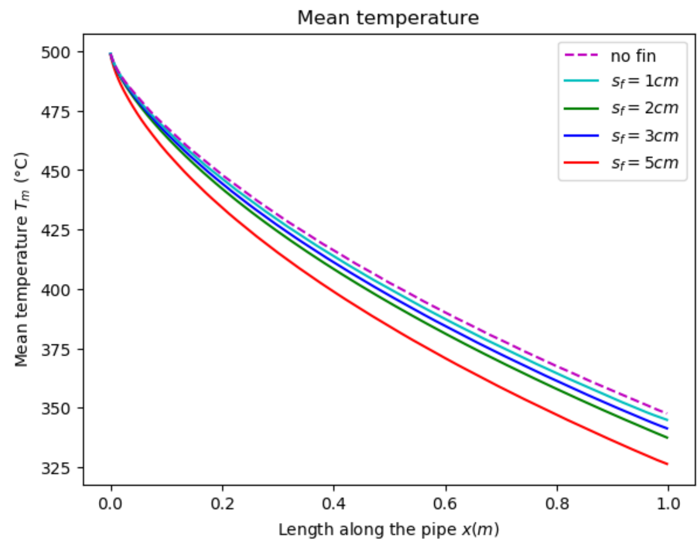

FIGURE 8. MEAN TEMPERATURE FOR DIFFERENT FIN SPACINGS.

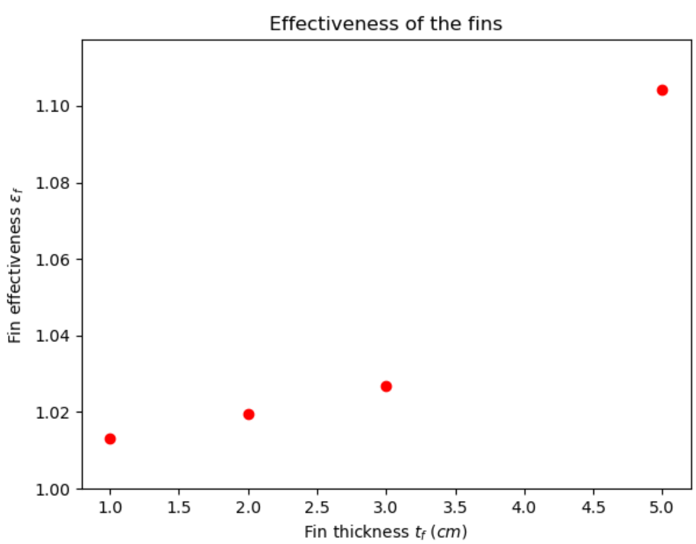

FIGURE 7. FIN EFFECTIVENESS VS FIN THICKNESS.

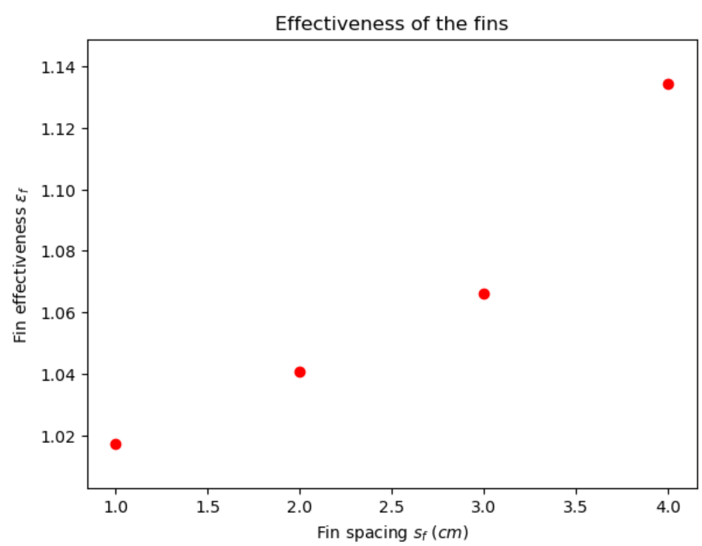

FIGURE 9. FIN EFFECTIVENESS VS FIN SPACING.

Fin thickness (t_f)

The analysis is done for fin thickness $t_f = 1, 2, 3$ and 5. The other geometric parameters of the fins are $L_f = 20cm$ and $s_f = 2cm$

The mean temperature and the fin effectiveness are plotted in Fig.6 and Fig.7. It is observed that the effectiveness increases with an increase in fin thickness. The rate of increase of effectiveness increases with an increase in fin thickness.

Fin spacing (s_p)

In this project, the fin spacing is defined as the distance between one fin's right face and the adjacent right fin's left face. The analysis is done for small fin spacings $s_f = 3, 4, 5$, and $6cm$. The other geometric parameters of the fins are $L_f = 20cm$ and $t_f = 2cm$.

The mean temperature and the fin effectiveness are plotted in Fig.8 and Fig.9. It is evident that there is an increase in the amount of heat transfer with the increase in the spacing between fins. This, however, is only valid for smaller fin spacings.

CONCLUSION

A $C++$ code is developed to solve the convection-diffusion equation using the finite volume method on a structured mesh in axisymmetric coordinates. The solver solves for the temperature of the fluid and the pipe when the fluid flows through it. The solution obtained from the solver was validated by comparing it with approximate analytical solutions. Using the solver, the effects of changing the geometric parameters of the fin are studied. The effectiveness of the fin increases when the

1. fin length increases
2. fin thickness increases, and
3. fin spacing increases (for smaller fin thicknesses).

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