AUXETIC METAMATERIALS

A Brief study into auxetics

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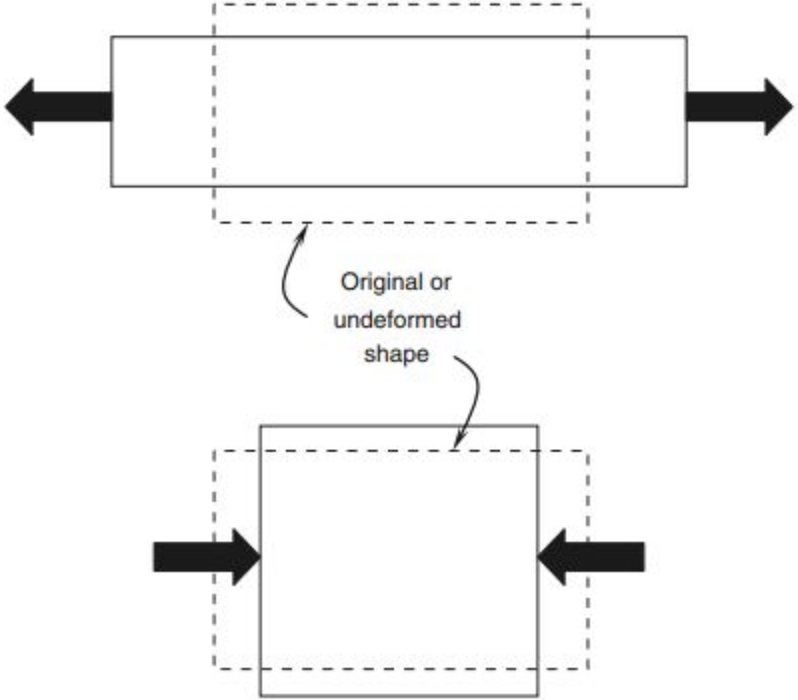
Mechanical Metamaterials

- Conventional materials have mechanical, thermal, and optical properties that are determined by their molecular or atomic composition
- Metamaterials are purposely designed to produce characteristics that a conventional material can't exhibit
- Negative Poisson's Ratio (NPR) materials, also known as auxetic materials, have attracted attention due to their unique behavior

Poisson’s Ratio

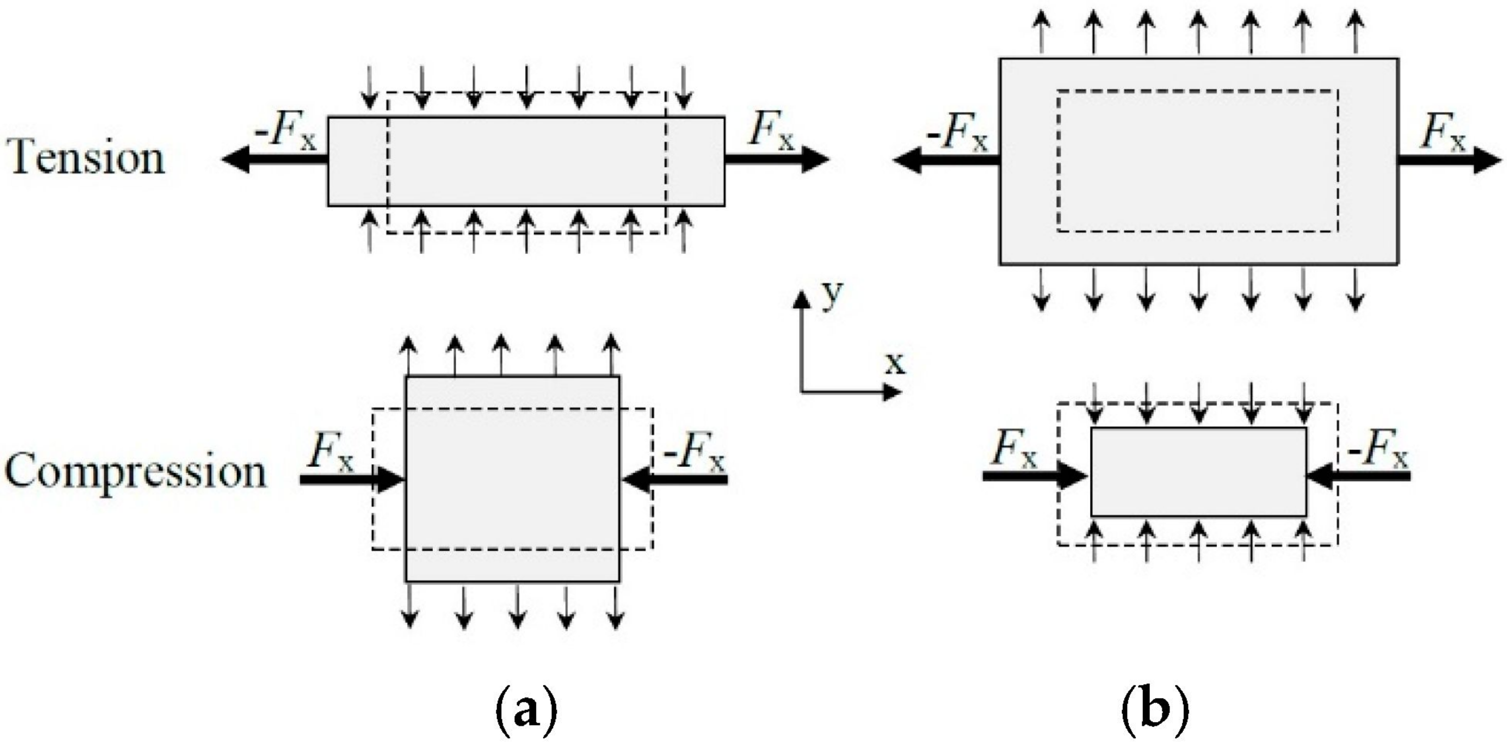
- Poisson's ratio is a measure of the Poisson effect, the phenomenon in which a material tends to expand in directions perpendicular to the direction of compression
- Mathematically ,it is the ratio of the transverse strain to the axial strain

$$\nu = -\frac{\epsilon_{trans}}{\epsilon_{axial}}$$


Original or
undeformed
shape

What are auxetics?

- Auxetic materials/structures are materials that exhibit an unexpected behaviour when they are subjected to mechanical stresses and strains
- When they're stretched in the longitudinal direction, they become thicker in one or several of the perpendicular width-wise directions
- The same logic applies in the opposite way
- This behaviour relates to having an negative Poisson's Ratio
- Due to this property the materials have many benefits which includes being ultra-light, higher stiffness, better strength-to-weight ratio, and a very good energy absorption capacity from loads, especially the types of loads that makes the material to vibrate
- Therefore, Auxetics are useful in applications such as body armor,packaging material, knee and elbow pads, shock absorbing material, and sponge mops



(a) Normal material.
(b) Auxetic material

Auxetics

- Auxetic materials have a negative poisson’s ratio because of its microstructure
- One of the earliest and most common structure is the re-entrant honeycomb structure
- We have used this structure to derive the poisson’s ratio of auxetic materials in both - 2D and 3D cases

Common structures for Auxetics

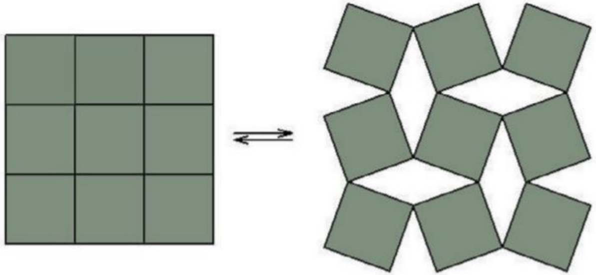
The re-entrant structure is the most commonly used structure for design and manufacturing purposes.Here the specific property is achieved by unfolding the re-entrant units i.e they are achieved by depending on the deformation of diagonals and the rotation around alternate hinge points. Here the re-entrant honeycomb/hexagonal structure is the most widely used.

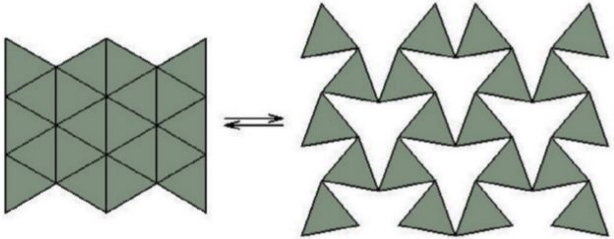
Other models include the :

Rotating Model which is mainly based on zeolites and silicates. Here when stretched in a certain direction, angles between the edges of adjacent or triangles alter and units rotate expanding in both directions with the increasing angles

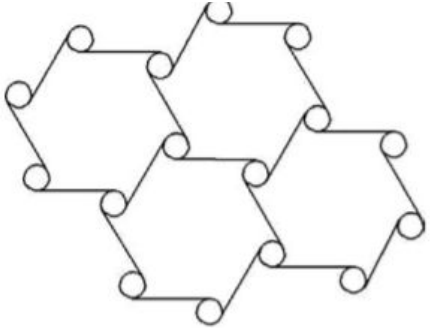
Nodule and Fibril model, here the transition of nodes connected by fibrils under loading gives rise to its auxetic properties.

Chiral Model, here the wrapping and unwrapping of the ligaments around its nodes gives rise to its Auxetic properties.





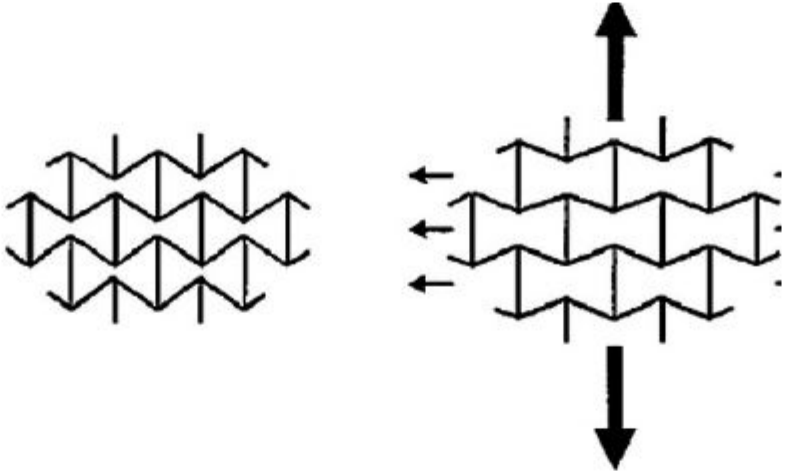
Rotating model 2D



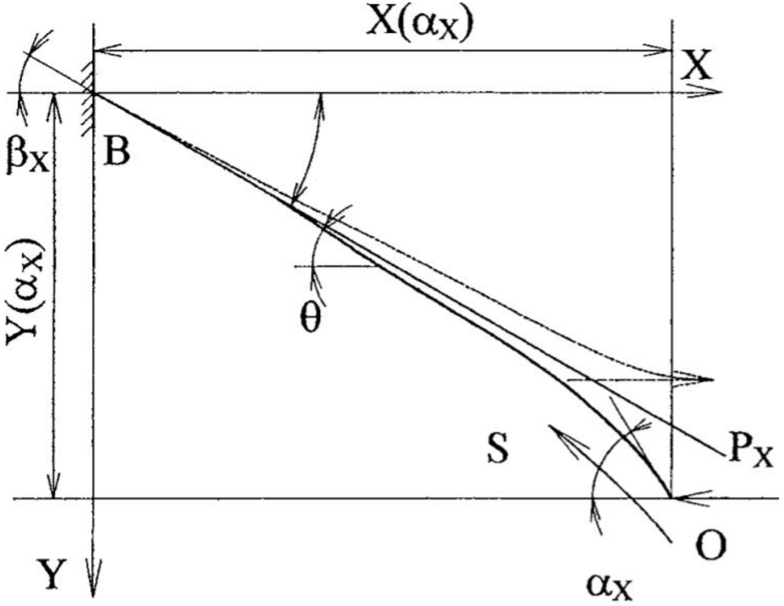
Three-bar circular-node chiral structure

2D unit cell

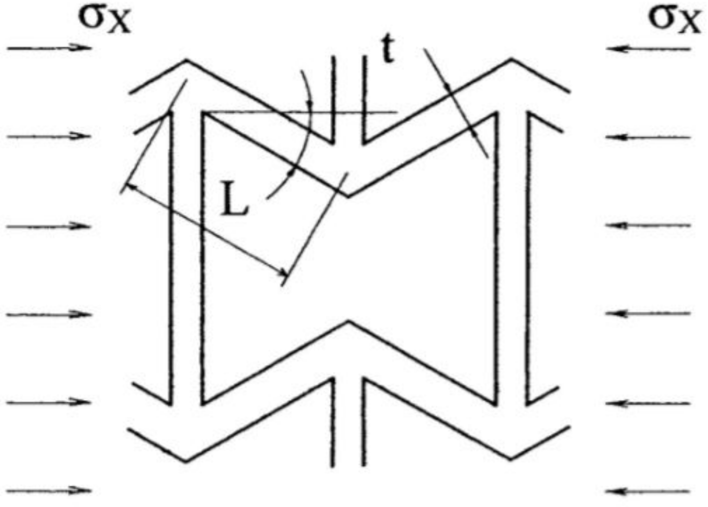
- As said previously we are choosing a re-entrant honeycomb structure for our Auxetic material
- Here the simple change in geometry from a regular honeycomb to a re-entrant honeycomb gives enhances the materials properties including increased resistance to indentation and increased energy absorption under compression, etc.



2D Unit Cell

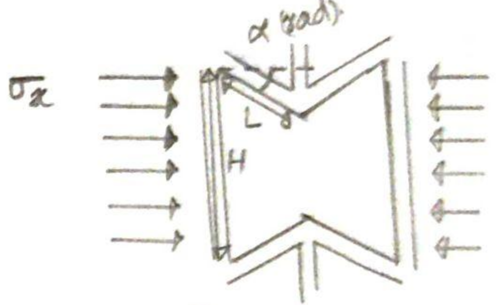


For loading in X direction



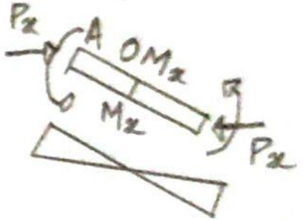
2D Poisson's Ratio derivation :

taking a Re-entrant honeycomb that is uniaxially loaded in the x-direction.



From the external equilibrium equations,

- $b \rightarrow$ breadth of cell member
- $H \rightarrow$ vertical length of cell member
- $L \rightarrow$ length of Inclined cell member.
- $\alpha \rightarrow$ angle b/w undeformed cell member and x-Axis.



P_x : Force applied on Inclined member.

$$P_x = \sigma_x \times b \times (H - L\sin\alpha)$$

$$M_x = P_x \times L\sin\alpha/2$$

As we can see there is zero bending moment at the midpoint, thus one half of the Inclined member is similar to a Cantilever beam loaded at the free end by P_x.

taking $E \rightarrow$ Young's modulus of Material. $I = bt^3/12$

the differential equation for the deflection curve is.

$$EI \times \frac{d\theta}{ds} = - P_x \times y$$

Boundary conditions:

$(EI) \times d\theta/ds \,|_{s=L/2} = M_x \Rightarrow \theta = \beta_x$

$\theta \rightarrow$ Bending Angle at any point along the deformed member.

$d\theta/ds \,|_{s=0} = 0 \Rightarrow \theta = \alpha_x$

further differentiate this equation, to get

$$(EI) \times d^2\theta/ds^2 = - P_x \times \frac{dy}{ds}$$

$$\Rightarrow (EI) \times d^2\theta/ds^2 = - P_x \times \sin\theta \Rightarrow (EI) \times \frac{d^2\theta}{ds^2} \times \frac{d\theta}{ds} \times ds = - P_x \sin\theta \, d\theta$$

Integrating both sides $\Rightarrow$ $EI \times 1/2 \times \left[d\theta/ds\right]^2 = P_x\left[\cos\theta - \cos\alpha_x\right]$

take $k_x = \sqrt{P_x/EI}$

$$\Rightarrow \frac{d\theta}{ds} = -\sqrt{\frac{P_x}{(EI) \times 2 \times (\cos\theta - \cos\alpha_x)}}$$

taking variable separable method and Integrating,

$\cos\theta = 1 - 2\sin^2\theta/2$

$$L/2 = \int_{B_x}^{\alpha_x} d\theta / k_x\sqrt{2(\cos\theta - \cos\alpha_x)}$$

$$\Rightarrow L = \int_{B_x}^{\alpha_x} \frac{d\theta}{k_x\sqrt{\sin^2\alpha_x/2 - \sin^2\theta/2}}$$

Now, taking $A_x = \sin \alpha_x/2$ and as $\beta_x < \theta < \alpha_x \Rightarrow \delta_x < \phi < \pi/2$

$\sin \theta/2 = A_x \sin \phi$

So, we have

$$L = \frac{2}{k_x} \int_{\delta_x}^{\pi/2} \frac{d\phi}{\sqrt{1 - A_x^2 \sin^2\phi}} = 2/k_x \, F(\alpha_x, \beta_x)$$

$$d\theta = \frac{2 A_x \cos\phi \, d\phi}{\sqrt{1 - A_x^2 \sin^2\phi}}$$

↳ elliptic Integral whose values depends on α_x and β_x

Now from our initial statement,

$$P_x = (EI)\, k_x^2 = \frac{(EI)}{L^2} \times 4 F^2(\alpha_x, \beta_x)$$

As we saw initially,

$$dL_{xx} = ds(\cos\theta)$$

$$\Rightarrow L_{xx}(\theta) = \frac{1}{2k_x}\int_{\beta_x}^{\theta} \frac{\cos\theta \, d\theta}{\sqrt{\sin^2(\alpha_x/2) - \sin^2\theta/2}}$$

$\sin\varepsilon_x = \dfrac{\sin\theta/2}{\sin\alpha_x/2}$

$\cos\theta = 1 - 2\sin^2\theta/2$

and converting equation in terms of ϕ $= \dfrac{1}{k_x}\displaystyle\int_{\delta_x}^{\varepsilon_x} \frac{1 - 2A_x^2 \sin^2\phi \, d\phi}{\sqrt{1 - A_x^2\sin^2\phi}}$

$$= {}^1/_{k_x}\left[\underbrace{2\int_{\delta_x}^{\varepsilon_x}\sqrt{1 - A_x^2\sin^2\phi}\, d\phi}_{\text{2nd kind Elliptic Integral}} - \underbrace{\int_{\delta_x}^{\varepsilon_x} {}^1/\sqrt{1 - A_x^2\sin^2\phi}\, d\phi}_{\text{1st kind Elliptic Integrals}}\right]$$

So, we have

$$L_{xx}(\theta) = {}^1/_{k_x}\left[2E(\theta, \beta_x) - F(\theta, \beta_x)\right]$$

as we know,

$$\varepsilon_{xx} = \frac{L_{xx}(\alpha_x) - L/_2\cos\alpha}{L/_2\cos\alpha}$$

Similarly for L_{yx}

$$dL_{yx} = ds\,(\sin\theta)$$

$$\sin\theta = 2\sin\theta/2\,\cos\theta/2 \qquad = \frac{1}{2k_x}\int_{\beta_x}^{\theta}\frac{\sin\theta\, d\theta}{\sqrt{\sin^2\alpha_x/2 - \sin^2\theta_x/2}}$$

$$= \frac{2A_x}{k_x}\int_{\delta_x}^{\varepsilon_x}\sin\phi\, d\phi = \frac{2A_x}{k_x}\left[\cos\delta_x - \cos\varepsilon_x\right]$$

and as we know, $\varepsilon_{yx} = \dfrac{H/2 - L_{yx}(\alpha_x) - [H/2 - L/2\sin\alpha]}{H/2 - L/2\sin\alpha}$

Now taking θ as α_x,

$$\theta = \alpha_x \Rightarrow \mathcal{E}_x = \pi/2$$

So, we get

$$\mathcal{E}_{xx} = \frac{\left[2E(\alpha_x, B_x) - F(\alpha_x, B_x)\right] 1/k_x - L/2 \cos\alpha}{L/2 \cos\alpha}$$

and,

$$\mathcal{E}_{yx} = \frac{L\sin\alpha - \frac{4A_x(\cos\delta_x)}{k_x}}{H - L\sin\alpha}$$

So, the poisson's ratio can be calculated as,

$$\nu_x = -\varepsilon_{yx}/\varepsilon_{xx} = \left(\frac{L\sin\alpha - 4A_x\cos\delta_x/k_x}{H - L\sin\alpha}\right) \Bigg/ \frac{\frac{2E(\alpha_x, \beta_x) - F(\alpha_x, \beta_x)}{k_x} - 4_2\cos\alpha}{4_2\cos\alpha}$$

This formula is for large deviations

Now, for very small deviations : $\alpha_x \to \beta_x$

hence for small deviations, the formula simplifies to,

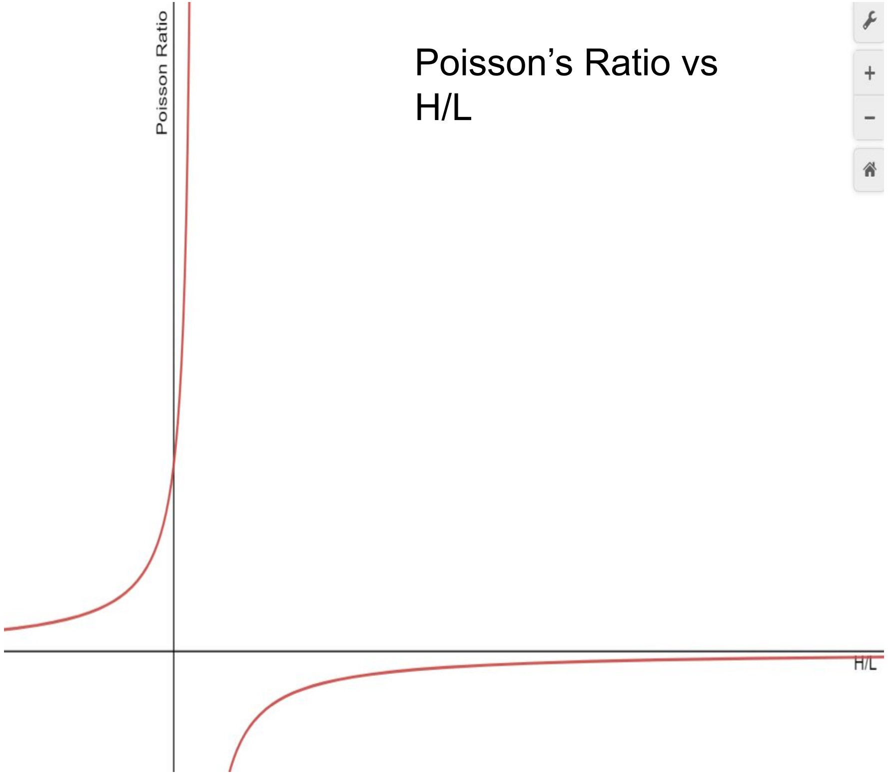
$$\nu_x = \frac{-L\cos^2\alpha}{(H - L\sin\alpha)\sin\alpha}$$

and by symmetry, we can write,

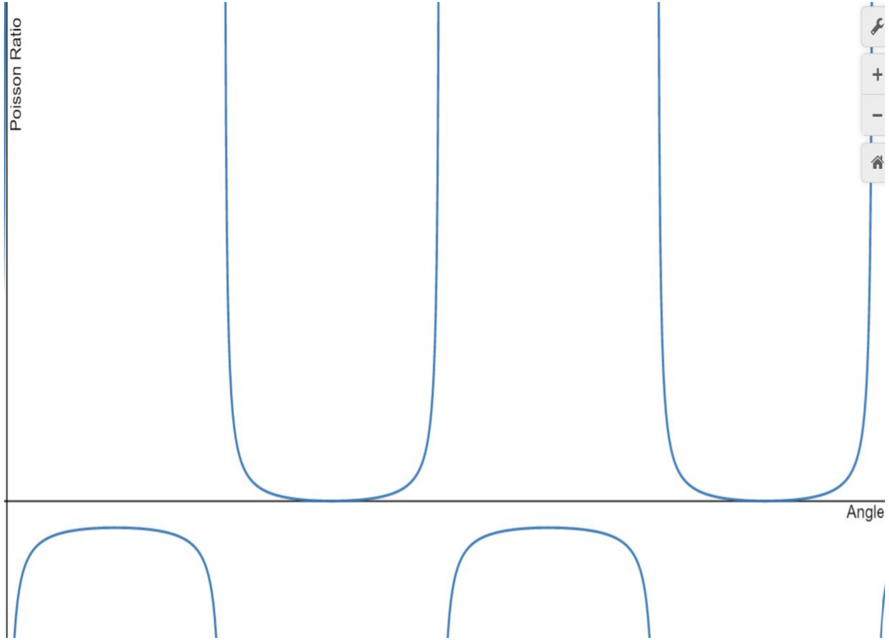
$$\nu_y = -\frac{(H - L\sin\alpha) \times \sin\alpha}{L\cos^2\alpha}$$

Graphs :

For Loading in X - direction



Poisson’s Ratio vs Angle subtended by Inclined member with the X axis



3D unit cell

- One of the advantages of this structure is that it can be readily extended into 3D.
- Figure a illustrates the 3D re-entrant unit cell, and Fig b shows a 3D lattice block array based on the unit cell.

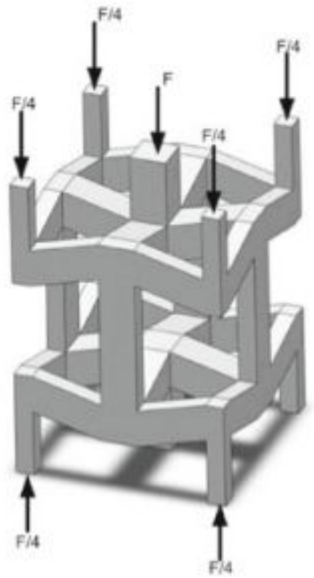


Fig a: Unit cell of 3D re-entrant lattice

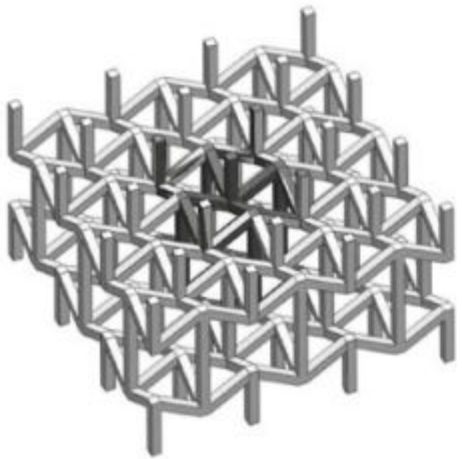
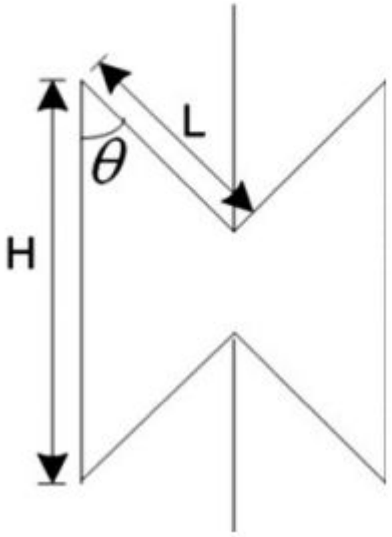


Fig b: 3D re-entrant lattice

3D unit cell

- For this study, there are four primary design parameters for the re-entrant lattice structure: length of the vertical struts (H), the length of the re-entrant struts (L), the re-entrant angle (θ), and the square strut cross section thickness (t).

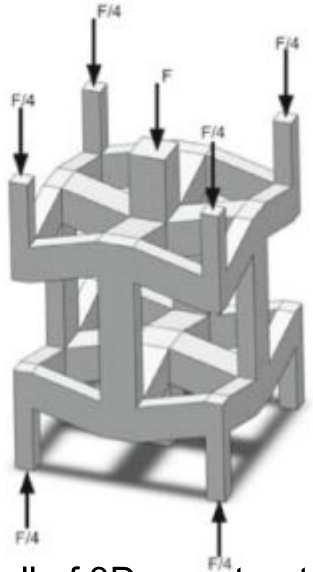


Defining the design parameters for unit cell

Derivation 3D

Case 1: Compression in z direction

- Consider a remote compressive stress applied in the z direction of an infinite 3D lattice structure



Unit cell of 3D re-entrant lattice

σ = compressive stress

→ Net force on the upper (or lower) face of the unit cell $= \sigma \times A$

$$= \sigma \times (2 . L . \sin\theta)^2$$

Derivation 3D

- If F is the force in one of the vertical struts, then

$$2F = \sigma (2L \sin\theta)^2$$

$$\boxed{F = 2\sigma L^2 \sin^2\theta} \quad \text{—} (1)$$

Derivation 3D

- The deformation of the simplified structure is the linear superposition of three components:
  - Compressive deformation of the vertical struts
  - Bending-induced deflection from the re-entrant strut
  - Shear-induced deflection from the re-entrant strut

$$\Rightarrow \Delta z_1 = \frac{2\,\sigma H L^2 \sin^2\theta}{E t^2} \quad \text{—— (2)}$$

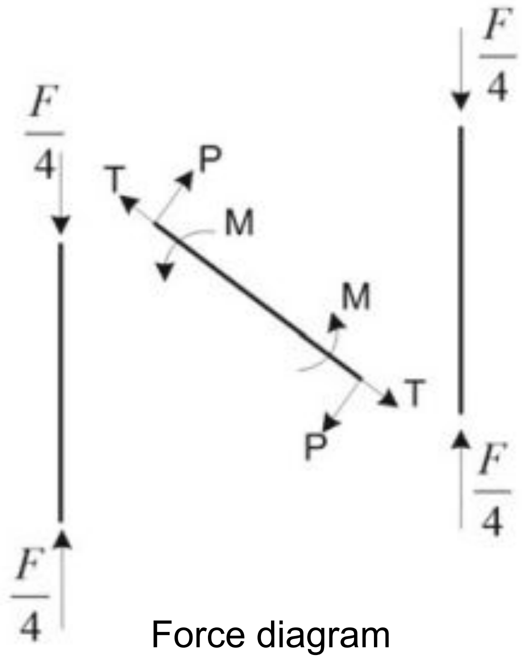
$$\left(\because \Delta l = \frac{FL}{AE}\right)$$

Force diagram:

$$P = \frac{F}{4} \sin\theta \quad \text{——} \quad (3)$$

$$T = \frac{F}{4} \cos\theta \quad \text{——} \quad (4)$$

$$M = \frac{PL}{2} = \frac{FL}{8} \sin\theta \quad \text{——} \quad (5)$$



Force diagram

→ From Timoshenko beam theory, the angle of deflection of the re-entrant strut can be expressed as

$$\theta = \frac{dw}{dx} + \gamma \qquad \text{--- (6)}$$

where $w \rightarrow$ deflection of the strut

$\gamma \rightarrow$ shear strain

$$\frac{dw}{dx} = \frac{ML}{6EI} \qquad \text{--- (7)}$$

$$\gamma = \frac{P}{kGA} \qquad \text{--- (8)}$$

where k is the geometrical factor.

→ for a rectangular cross section, $k = \frac{5}{6}$

$$\therefore \theta_1 = \theta_2 = \frac{ML}{6EI} + \frac{6P}{5GA} \quad \text{— (9)}$$

– Since the joint is assumed to be rigid, the reentrant angle remains almost unchanged following deformation (i.e. $\theta_1 = \theta_2$ = very small)

$$\Rightarrow \Delta x = L\sin(\theta+\theta_1) - L\sin\theta$$
$$= L\sin\theta_1 \cos\theta \simeq \boxed{L_1\theta_1\cos\theta} \quad \text{— (10)}$$

$$\Delta z_2 = L\cos\theta - L\cos(\theta+\theta_1)$$
$$= L\sin\theta_1 \sin\theta \simeq \boxed{L\theta_1 \sin\theta} \quad \text{— (11)}$$

★ Poisson's Ratio:

Due to symmetry of structure,

$$\mu_{zy} = \mu_{zx} = \frac{-\varepsilon_x}{\varepsilon_z}$$

$$= -\frac{\left(\dfrac{\Delta x}{2L\sin\theta}\right)}{\left(\dfrac{\Delta z_1 + 2\Delta z_2}{2(H - L\cos\theta)}\right)}$$

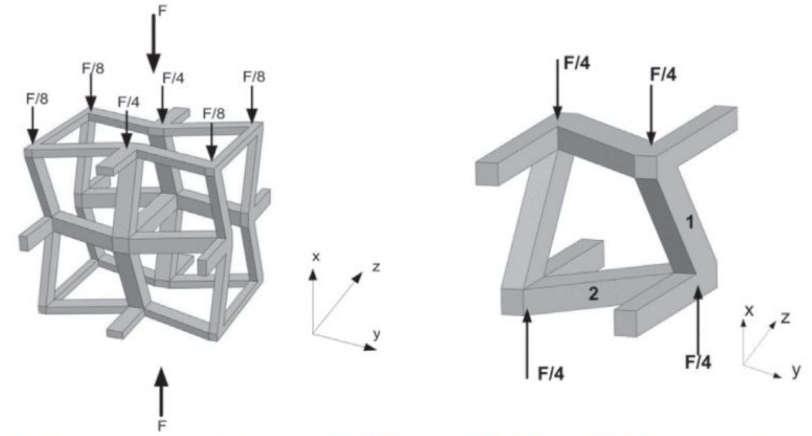
→ substituting the values from the previous equations, we get

$$\mu_{zy} = \mu_{zx} = \frac{-\left(\dfrac{L^2}{4Et^4} + \dfrac{3}{10Gt^2}\right)\cos\theta(\alpha - \cos\theta)}{\dfrac{\alpha}{Et^2} + \left(\dfrac{L^2}{4Et^4} + \dfrac{3}{10Gt^2}\right)\sin^2\theta} \qquad (12)$$

$$\left(\text{where } \alpha = \frac{H}{L}\right)$$

Case 2: Compression in y direction

→ Due to symmetry, the unit cell can be further simplified as shown in the figure.



(a) A max-symmetry unit cell (b) Simplified structure

★ <u>Compression in y-direction:</u>

→ The compressive force applied on each reentrant strut is $F/4$

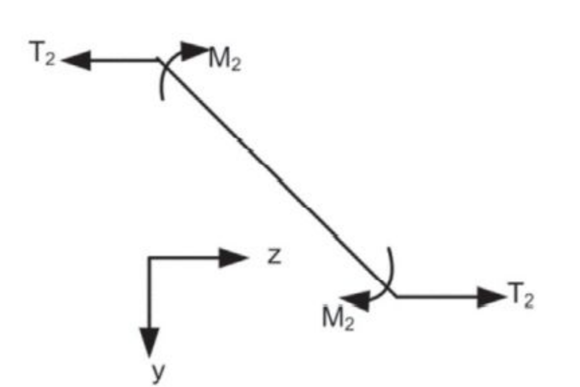
$$\therefore \quad \frac{F}{4} = \sigma\left[(H - L\cos\theta)(L\sin\theta)\right]$$

$$F = 4\sigma(H - L\cos\theta)(L\sin\theta) \quad \text{—— (13)}$$

→ P_1 and P_2 (shear forces of each re-entrant strut) can be determined from force equilibrium as

$$P_1 = \frac{F}{4} \cos\theta - T_1 \sin\theta \quad \text{—— (14)}$$

$$P_2 = T_2 \sin\theta \quad \text{—— (15)}$$



Force balance in re-entrant strut

- The deflections in both kinds of struts are identical as no global torsional effects must appear on the simplified structure

$$\Rightarrow M_1 = M_2$$

$$\Rightarrow P_1 = P_2 \Rightarrow \frac{F}{4}\cos\theta - T_1 \sin\theta = T_2 \sin\theta \quad \text{--- (16)}$$

from force equilibrium of vertical struts,

$$T_1 = T_2$$

$$\Rightarrow T_1 = T_2 = \frac{F}{8}\cot\theta \quad \text{--- (17)}$$

$$\Rightarrow P_1 = P_2 = \frac{F}{8}\cos\theta \quad \text{--- (18)}$$

★ Poisson's Ratio:

$$\mu_{yz} = -\frac{\varepsilon_z}{\varepsilon_y} = \frac{-2\Delta z\,(L\sin\theta)}{2\Delta y\,(H - L\cos\theta)} = -\frac{\Delta z \sin\theta}{\Delta y\,(\alpha - \cos\theta)}$$

$$= -\frac{L\theta_1 \sin^2\theta}{L\theta_1 \cos\theta\,(\alpha - \cos\theta)}$$

$$\boxed{\mu_{yz} = \frac{-\sin^2\theta}{\cos\theta\,(\alpha - \cos\theta)}} \quad \text{——} (19)$$

where $\alpha = \dfrac{H}{L}$

* Effective Modulus:

$$E_z = \frac{\sigma}{\varepsilon_z} = \frac{\sigma}{\frac{\Delta z}{H - L\cos\theta}} = \frac{\sigma(H - L\cos\theta)}{(\Delta z_1 + 2\Delta z_2)}$$

substituting values, we get

$$\boxed{E_z = \frac{(\alpha - \cos\theta)}{\frac{2\alpha L^2 \sin^2\theta}{Et^2} + \left(\frac{L^4}{2Et^4} + \frac{3L^2}{5Gt^2}\right)\sin^4\theta}} \quad \text{—(20)}$$

similarly,

$$E_y = E_x = \frac{\sigma}{\varepsilon_y} = \frac{\sigma}{2\Delta y}(L\sin\theta)$$

substituting values

$$\Rightarrow \boxed{E_y = \frac{1}{L^2\cos^2\theta(\alpha-\cos\theta)}\frac{1}{\left(\frac{L^2}{Et^4}+\frac{6}{Gt^2}\right)}} \quad \text{---} (21)$$

★ Cross-checking with 2-D case:

→ for compression in z-direction,

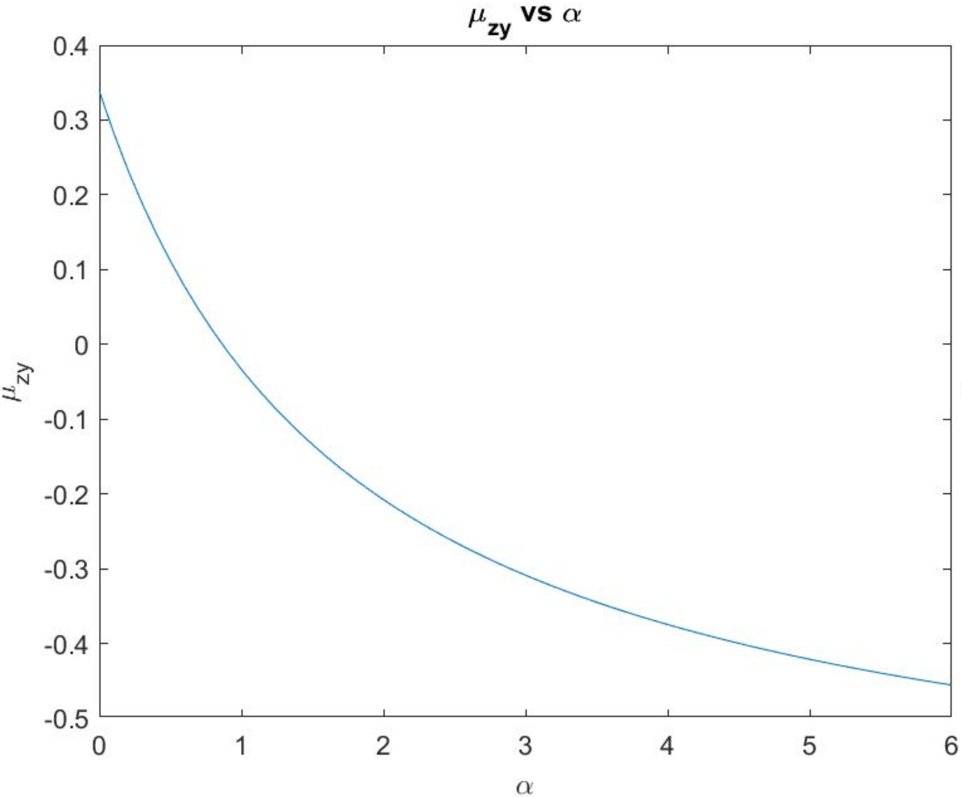
$$\mu_{zy} = \frac{\left(\frac{L^2}{4E} + \frac{3t^2}{10G}\right)\cos\theta \cdot (\alpha - \cos\theta)}{\frac{\alpha t^2}{E} + \left(\frac{L^2}{4E} + \frac{3t^2}{10G}\right)\sin^2\theta}$$

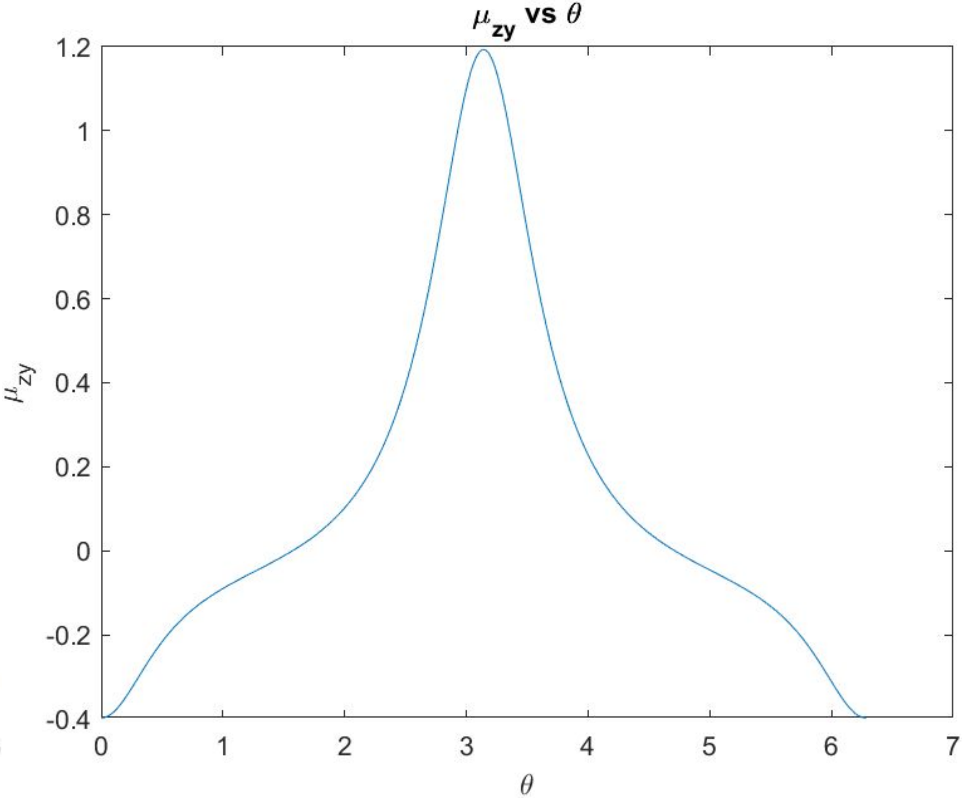
$2D \Rightarrow t \to 0.$

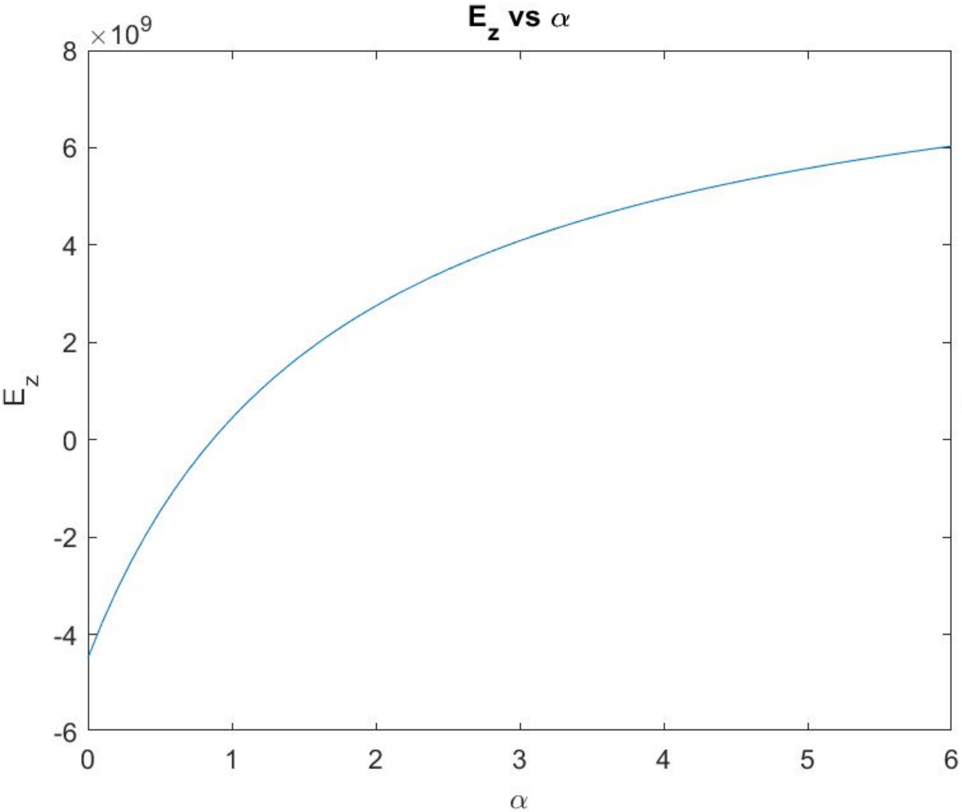
$$\therefore \boxed{\mu_{zy} = \frac{\cos\theta\,(\alpha - \cos\theta)}{\sin^2\theta}}$$

- It matches!
- The shear terms have become nullified
- This also tells us the euler-bernoulli beam theory is a good enough approximation for 2D lattice

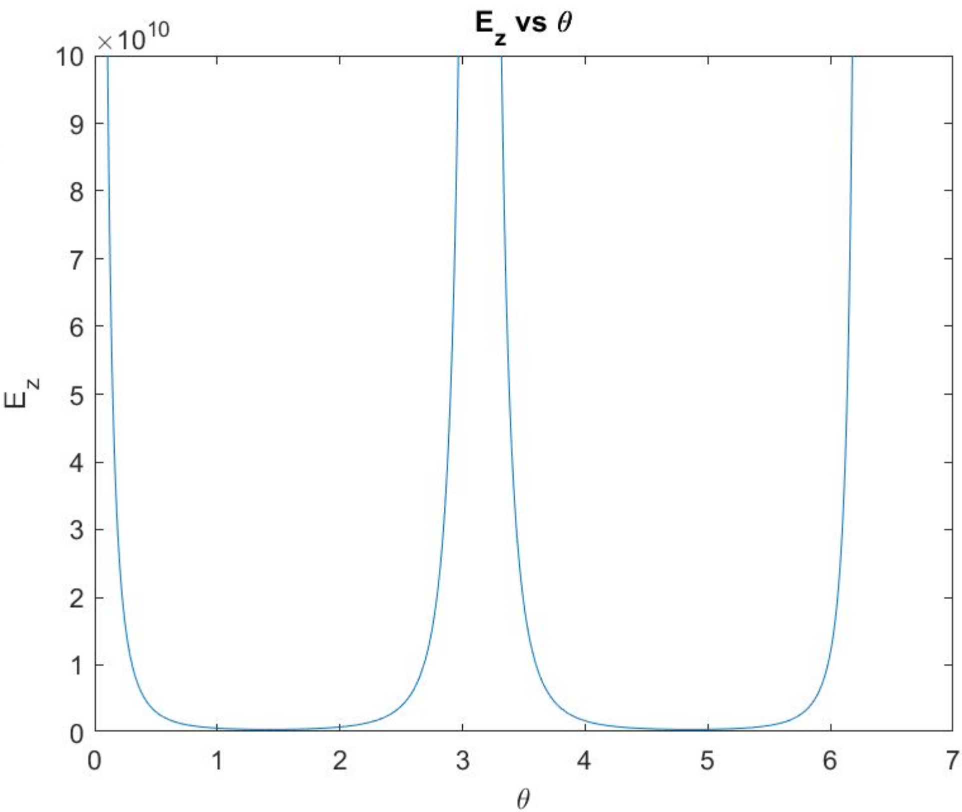
Plots: (compression in Z direction)





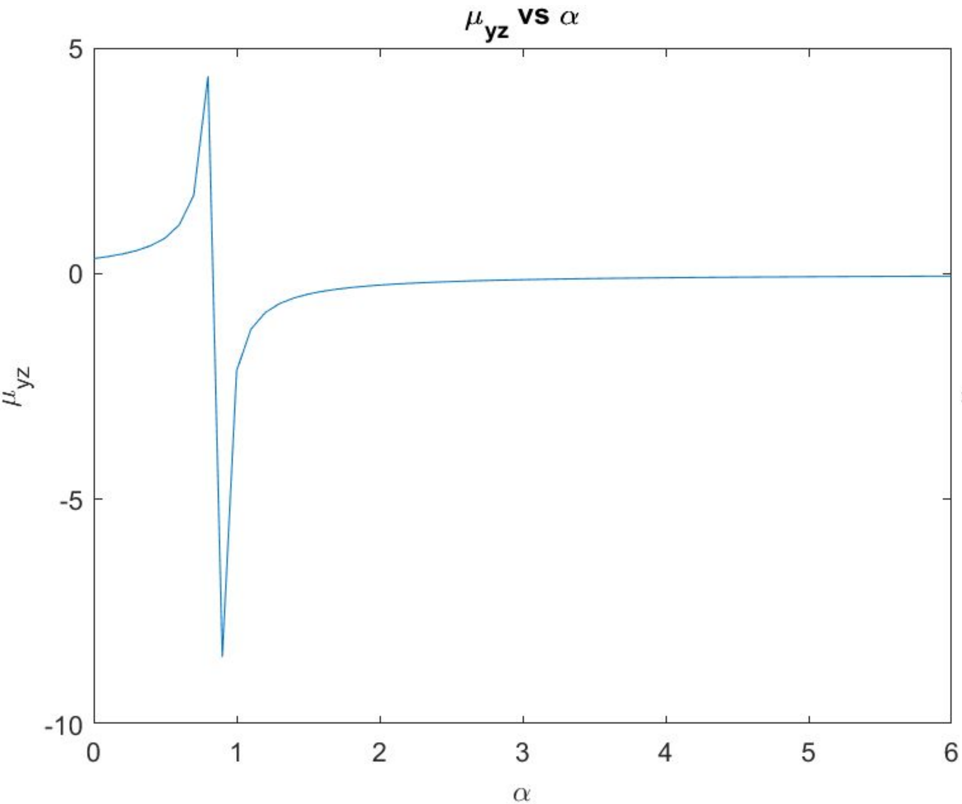


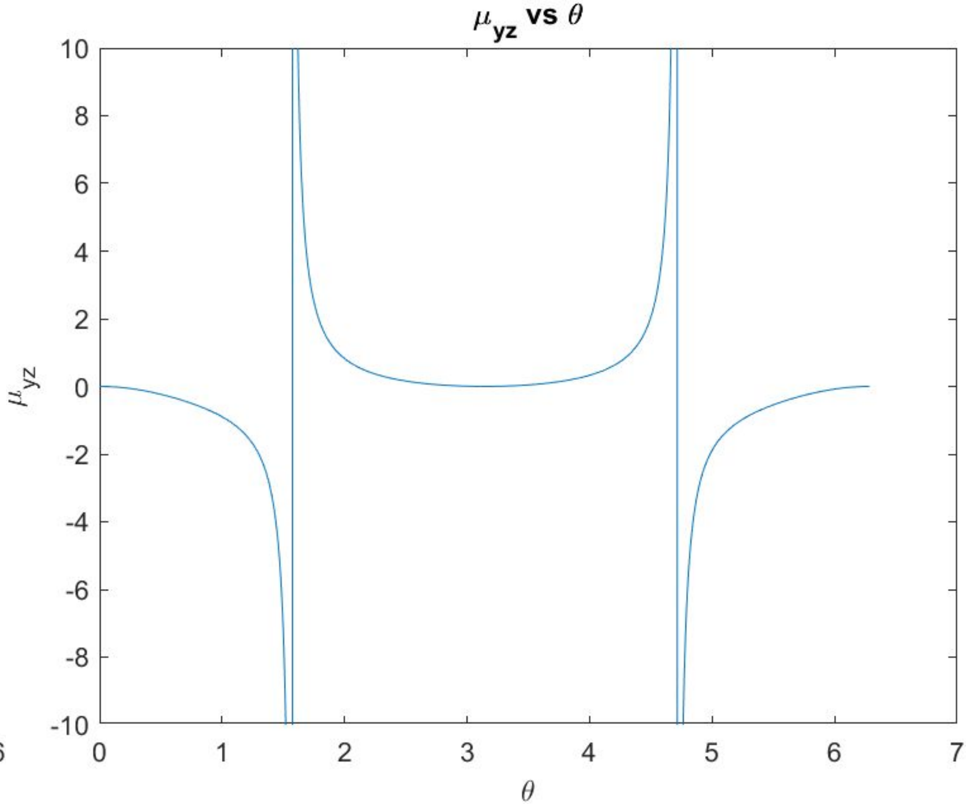
E_z vs α
$\times 10^9$
8
6
4
2
0
-2
-4
-6
0
1
2
3
4
5
6
E_z
α

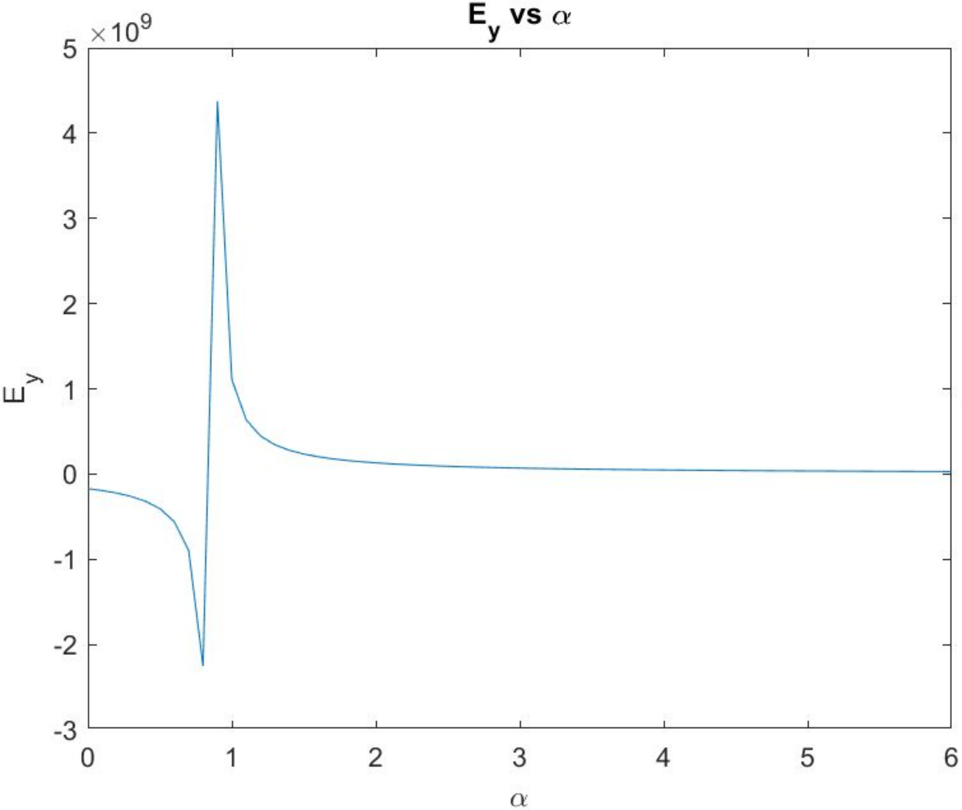
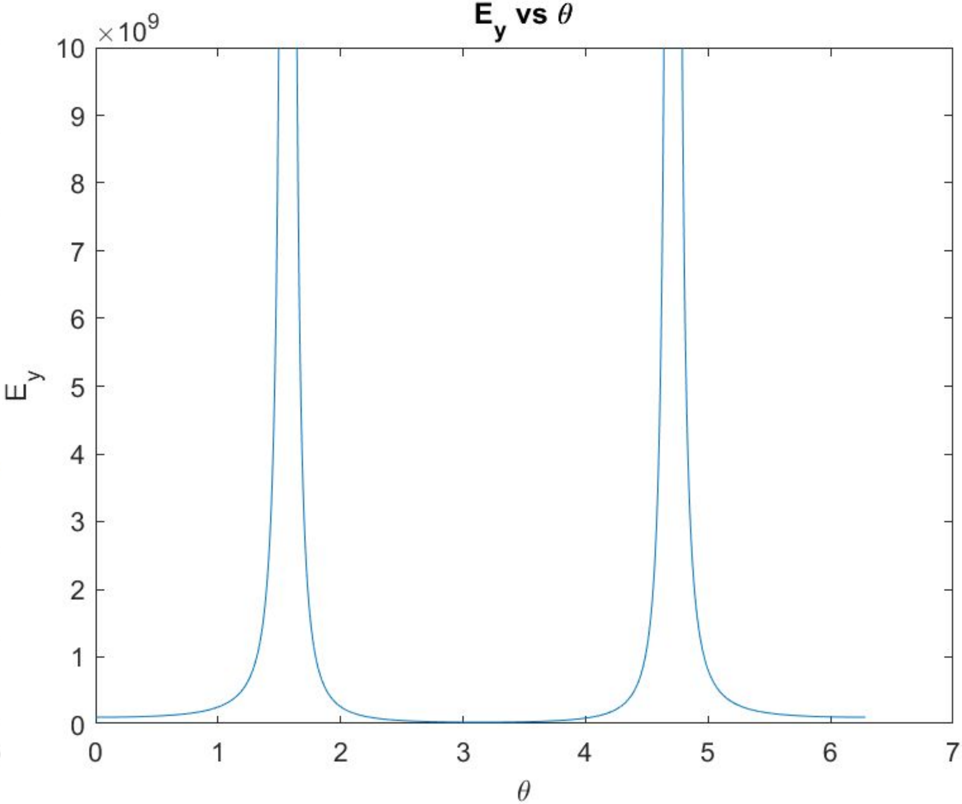


E_z vs θ
$\times 10^{10}$
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
E_z
θ

(compression in y direction)






E_y vs α
×10^9
E_y
α
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6

E_y vs θ
×10^9
E_y
θ
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7

Bending of beams

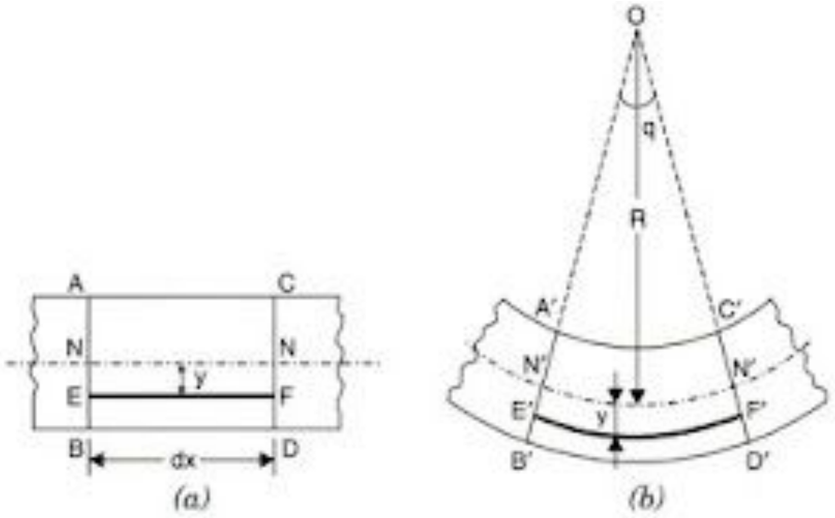
- Consider an auxetic beam that is subject to a force that causes bending moment. The stress experienced by the beam as a function of y can be calculated using the flexural formula as follows:

$$\sigma_b = \frac{My}{I}$$

Bending of beams

- This causes a curvature to be formed, having a radius of curvature:

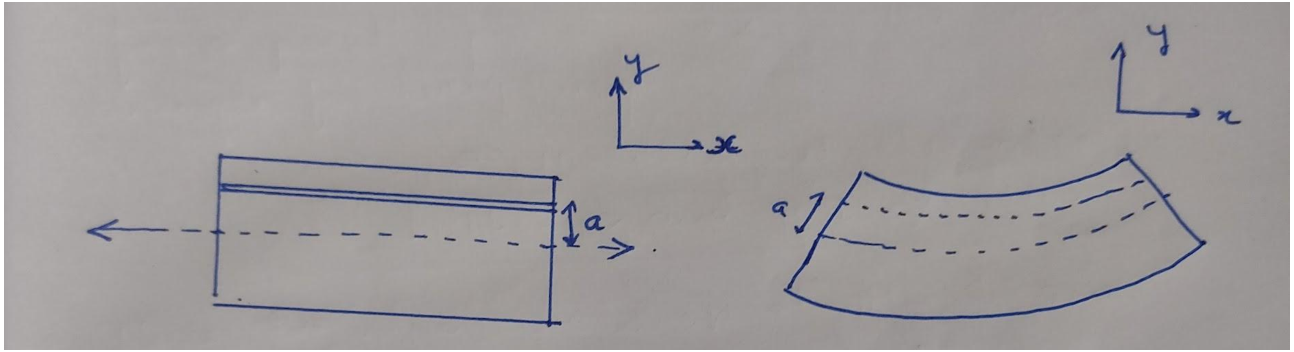
$$R = \frac{EI}{M} = \frac{Ey}{\sigma_b}$$



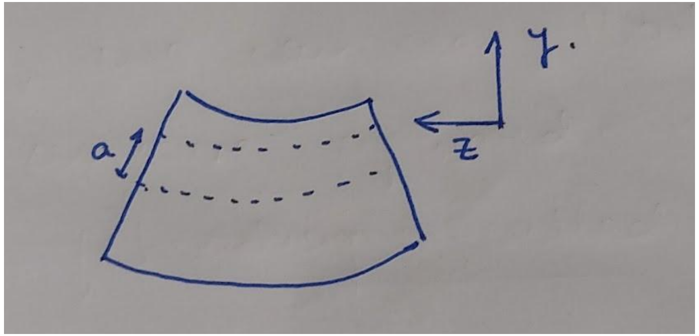
Synclastic bending

- Assume that the beam is tending to form a curvature with centre of curvature in the +ve y direction
- This will cause a compression for points lying above x-z plane ($y>0$) and elongation for points with $y<0$

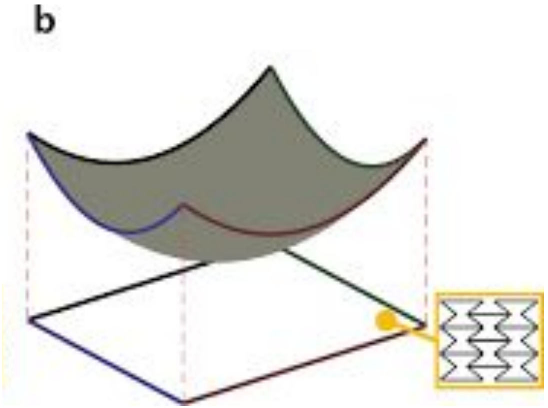
- Now, consider an element with a small height on some plane y=a (a>0)
- This element experiences a compressive bending stress given by the above mentioned flexural formula, and hence a compression
- Now consider the resultant displacement in the z direction due to the stress along x direction



- Since Poisson's Ratio is negative, strains in the z and x directions have the same sign
- The element is getting compressed in both - z and x directions
- Further, for an element with y = b (b<0), we can see that the element elongates in both - z and x directions



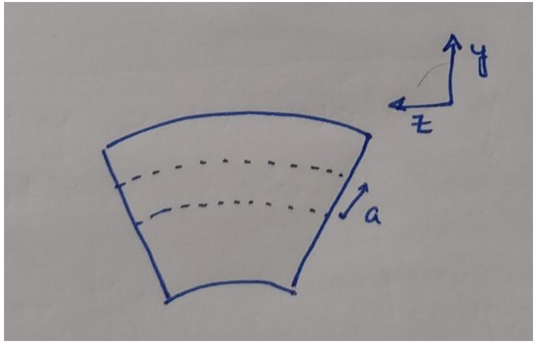
- Thus, when we look at the beam as a whole, we can see that it forms a concave upward shaped figure
- This type of curvature is called synclastic curvature.



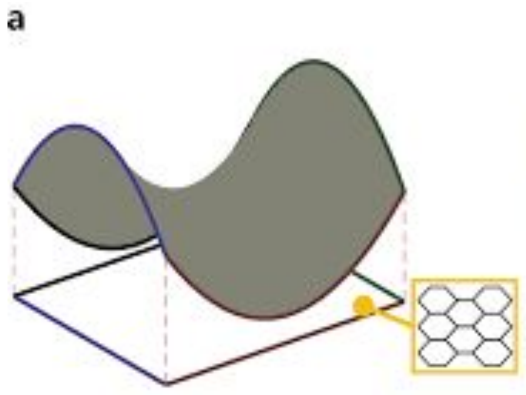
Synclastic bending

Anticlastic bending

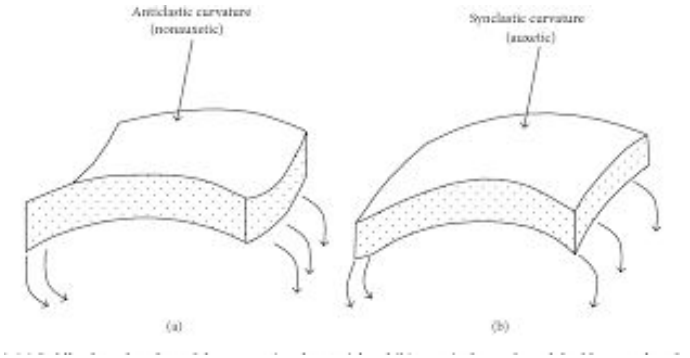
- In the case of non-auxetic beams, the poisson's ratio is positive, and hence the strains in z and x directions differ in signs
- This means:
    - Element in the top half -
        - Compressed in x direction
        - Elongated in z direction
    - Element in the bottom half -
        - Elongated in the x direction
        - Compressed in the z direction



- This leads to the formation of a saddle shaped figure
- This type of curvature is called anticlastic curvature



Anticlastic bending



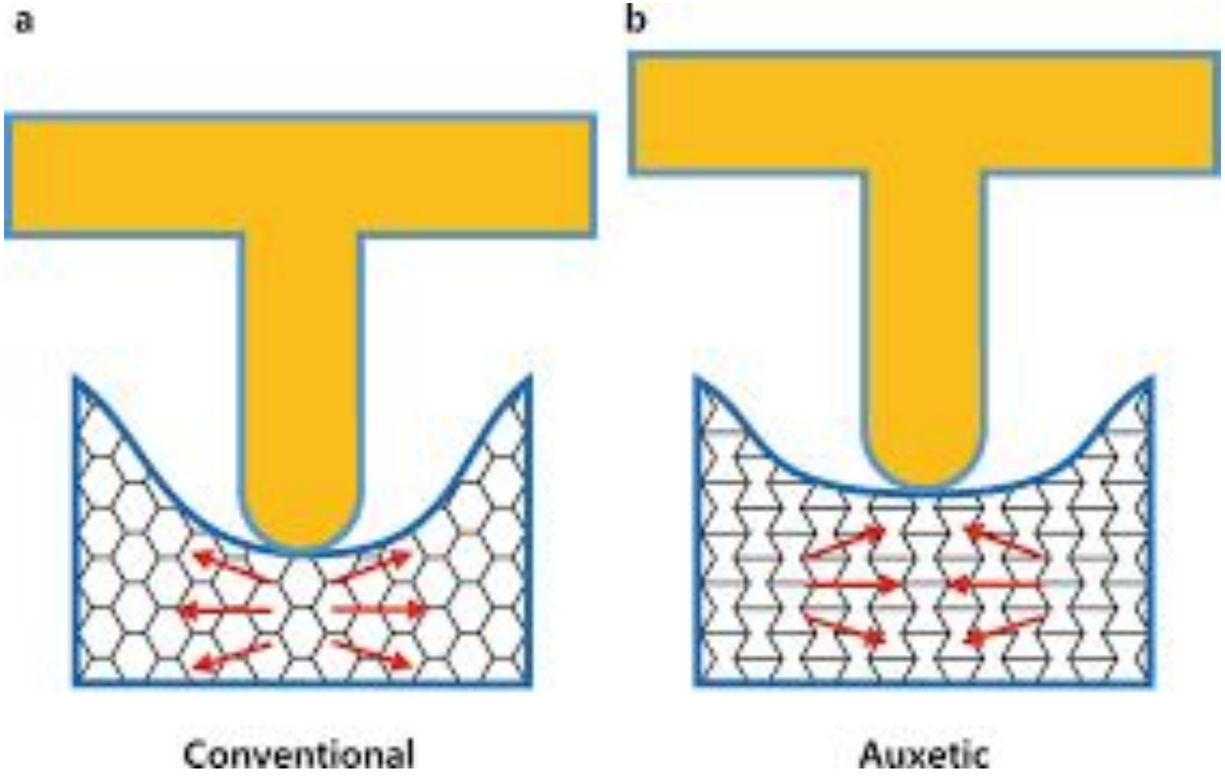
Normal material Auxetic material

- This concept can be used to define an important term, which comes into play in the manufacture of protective equipment

Indentation resistance

- When an impact is applied to non-auxetic materials in a certain region, these materials tend to spread from the region to relieve the pressure.
- Thus, this behaviour causes a decrease in density at the impact region.
- On the contrary, in the same case, isotropic auxetic materials move towards the region where the impact is applied.
- This leads to an increase in indentation resistance by increasing the relative density at the region of impact

Indentation resistance



Application of this property

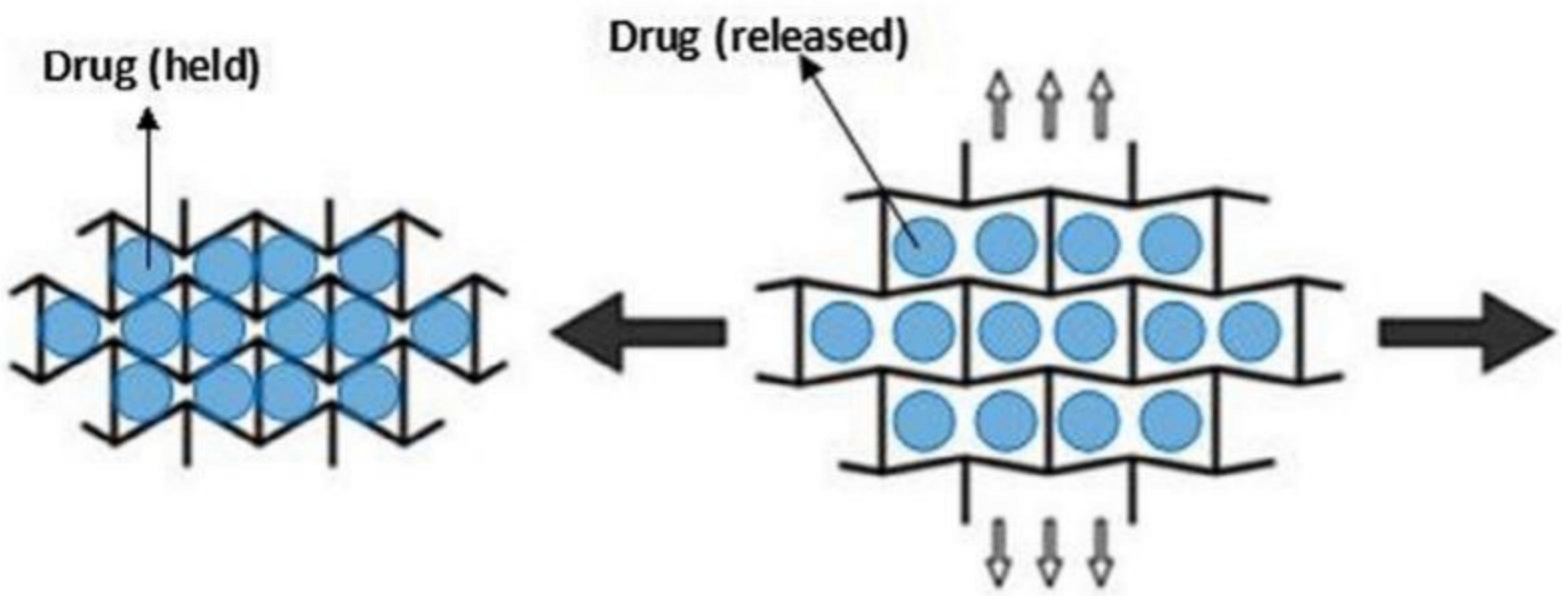
- This property is one of the reasons why auxetic materials are used as protective equipment
- Also, unlike foam (common ingredient in protective gear) which has inadequate air permeability, the Auxetics are much more comfortable
- When there is an impact on the auxetic material, it tries to enclose the impacted area, thus minimizing the damage of the impact to the body

Smart bandages :

This is a recently developed technology that uses the unusual behaviour of Auxetics to store the healing medicine as a part of the bandaid. When the auxetic material is used to bind up a swollen wound, it expands under tensile to release the medicine and after the wound has been healed and the swelling lightens, the bandage shrinks stopping the release of the medicine.

Here the property used is that once you apply stress to the material as you pull it, the material’s pores get bigger.Here with the uniform enlargement of the pores an adjustable filter can also be made, which can range from very small holes to very large holes. So now these Auxetic foams can be impregnated with the required medicine.

In addition to this, due to the synclastic nature, auxetic fabrics can turn into a circular arch under bending and fit the shape of elbows or knees properly, giving a better protection and body flexibility. This makes it easier to fit these bandaids and can sometimes be also worn by athletes as precautions.



Conclusion and further developments:

- We have conducted a brief study on auxetic materials with a re-entrant honeycomb structure, wherein we have derived the poisson’s ratio and have gone through a few properties and applications of these materials.
- Further, we can perform computational analysis on a model of an auxetic material by FEA, using softwares like matlab/ansys and compare our results with the theoretical results
- We can also perform experiments to further confirm our results.

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