

2022*Time : 3 hours**Full Marks : 70**Pass Marks : $31\frac{1}{2}$*

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from all the Groups as directed.

Group – A**(Objective Type Questions)**

1. Choose the correct alternative in each of the following : $1 \times 5 = 5$

(a) Which of the following is an empty set ?

- (i) $A = \{x : x \text{ is a prime number greater than } 99\}$.
- (ii) $B = \{x : x^2 = 4, x \text{ is even}\}$.
- (iii) $C = \{x : x^2 - 2 = 0 \text{ and } x \text{ is rational number}\}$.
- (iv) $D = \{x : 1 < x < 3 \text{ and } x \text{ is a natural number}\}$.

(b) If A , B and C are any three sets, then $A \times (B \cup C)$ is equal to :

- (i) $(A \times B) \cup (A \times C)$ (ii) $(A \cup B) \times (A \cup C)$
(iii) $(A \times B) \cap (A \times C)$ (iv) None of these

(c) $(A^c)^c = ?$

- (i) $U - A$ (ii) A^c
(iii) U (iv) A

(d) The inverse of $-i$ in the multiplicative group $\{1, -1, i, -i\}$ is :

- (i) 1 (ii) -1
(iii) i (iv) $-i$

(e) The number of elements in the Power set $P(S)$ of the set $S = \{1, 2, 3\}$ is :

- (i) 4 (ii) 8
(iii) 2 (iv) None of these

2. Fill in the blanks in each of the following :

$$1 \times 5 = 5$$

(a) A Square matrix M is said to be Symmetric if _____.

(b) For unique solution of $AX = B$, $|A|$ must be _____, where A is a square matrix.

(c) The highest power of 3 in $80!$ is _____.

- (d) If a and b are relatively prime integers then $\gcd(a, b) = \underline{\hspace{2cm}}$.
- (e) 4^{th} roots of unity are $\underline{\hspace{2cm}}$.

Group – B
(Short-answer Type Questions)

Answer any **four** questions of the following :

$5 \times 4 = 20$

3. Find all the partitions of $A = \{a, b, c\}$.
4. Define a Relation and an equivalence relation on any set A .
5. If A, B, C are three sets, then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
6. Show that $\text{Tr. } (A + B) = \text{Tr. } (B + A)$, where $A = [a_{ij}]_{n \times n}$ and $B = [b_{ij}]_{n \times n}$.
7. In a group G , prove that the identity element is unique.
8. Give an example of 3×3 matrices to show that matrix multiplication is not commutative.

Group – C
(Long-answer Type Questions)

Answer any **four** questions of the following :

$10 \times 4 = 40$

9. Let ' R ' be the relation on the set of real numbers given by aRb if $|a - b| \leq \frac{1}{2}$.
Prove that ' R ' is not an equivalence relation.

10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Determine : (i) $f \circ g$
(ii) $g \circ f$.

11. Find the inverse of the matrix A where

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 6 & 7 & 9 \end{bmatrix}$$

12. Expand $e^{\sin x}$ by Maclaurin's theorem as far as the term involving x^4 .

13. Prove that the set of all integers under addition is an Abelian group.

14. Test the consistency and solve the following system of simultaneous linear equation by matrix method : $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$.

