

Mathematics**2022**

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from **all** the Parts as directed.

Part – A

1. Choose correct answer in each of the following :

1×5 = 5

(a) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

☒ (i) πab

(ii) $\pi a^2 b$

(iii) πab^2

(iv) $\pi a^2 b^2$

(b) $\int (x+2)\sqrt{2x+1} dx$ is equal to :

(i) $\frac{1}{5}(x+3)(2x+1)^{1/2} + C$

(ii) $\frac{1}{5}(x+3)^{3/2}(2x+1) + C$

☒ (iii) $\frac{1}{5}(x+3)(2x+1)^{3/2} + C$

(iv) None of these

(c) The volume of a right circular cone of height 'h' and radius 'a' is :

(i) $\frac{1}{2}\pi a^2 h$

☒ (ii) $\frac{1}{3}\pi a^2 h$

(iii) $\frac{1}{2}\pi h^2 a$

(iv) $\frac{1}{3}\pi h^2 a$

(d) The orthogonal trajectories of the rectangular hyperbola $xy = a^2$ is :

(i) $x - y = c^2$

(ii) $x^2 - y = c^2$

(iii) $x - y^2 = c^2$

☒ (iv) $x^2 - y^2 = c^2$

(e) If A and B are two convex sets, then which of the following is also a convex set ?

☒ (i) $A \cap B$

(ii) $A \cup B$

(iii) $A - B$

(iv) All of these

2. Fill in the blanks :

$$1 \times 5 = 5$$

(a) DES is an example of a block cipher.

(b) The solution of differential equation $x \frac{dy}{dx} + \frac{y^2}{x} = y$ is $3xy^2 - 2xy^3 + 6 = c$

(c) The moment of inertia of a thin uniform circular plate about an axis through its centre perpendicular to its plate is $\frac{Mr^2}{2}$.

(d) The set of feasible solutions of a LPP is a convex set.

(e) The objective function of a LPP is necessarily a linear function.

Part - B

Answer any **four** questions of the following :

$$5 \times 4 = 20$$

✓ 3. Evaluate : $\int (2x^2 + 3) \sqrt{x+4} \, dx$

4. Obtain the reduction formula for $\int \tan^n x \, dx$ and

evaluate $\int_0^{\pi/4} \tan^5 x \, dx$.

✓ 5. Solve : $\log \left(\frac{dy}{dx} \right) = ax + by$

✓ 6. Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan \left(\frac{y}{x} \right)$

✓ 7. Write short notes on stream cipher and block cipher.

8. Solve the following LPP graphically :

$$\text{Max } Z = 5x + 3y$$

Subject to the constraints :

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x, y \geq 0$$

Part – C

Answer any four questions of the following :

$$10 \times 4 = 40$$

✓ 9. Find the perimeter of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

10. Solve : $p(p + x) = y(x + y)$; $p = \frac{dy}{dx}$

✓ 11. Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$

✓ 12. Solve : $\frac{dy}{dx} + xy = x^3$

✓ 13. Describe the RSA algorithm with an example.

14. Solve the following LPP by Simplex method :

$$\text{Max } Z = 3x + 7y + 6z$$

subject to the constraints,

$$3x + 2y + 3z \leq 8$$

$$x + y \leq 3$$

$$y + z \leq 6$$

$$x, y, z \geq 0$$

