

2023

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer from **all** the Parts as directed.*

Part – A

1. Choose correct answer in each of the following : $1 \times 5 = 5$

   (a) The moment of inertia of a uniform rod of mass M and length L about a perpendicular bisector is :

   (i) $\frac{ML^2}{12}$ (ii) $\frac{M^2L}{12}$

   (iii) $\frac{ML^2}{6}$ (iv) $\frac{M^2L}{6}$

(b) The perimeter of the curve $r = a(\cos\theta)$ is :

(i) πa (ii) $2\pi a$

(iii) $3\pi a$ (iv) $4\pi a$

(c) Stream cipher is a kind of encryption that converts plain text by taking following number of bytes of the plain text at a time :

(i) 32 bytes (ii) 16 bytes

(iii) 8 bytes (iv) 1 byte

(d) The general solution of $y = px + p^2$; $p = \frac{dy}{dx}$ is :

(i) $y = x^2 + c$ (ii) $y = x^2 + x$

(iii) $y = c^2 + cx$ (iv) None of these

(e) Which of the following is a linear function ?

(i) $f : R^2 \to R$ defined as $f(x, y) = 3x + 5y$

(ii) $f : R^2 \to R$ defined as $f(x, y) = 3x + 5y + 2$

(iii) $f : R^2 \to R$ defined as $f(x, y) = 3x + 2$

(iv) None of these

2. Fill in the blanks : $1 \times 5 = 5$

(a) The value of $\int \cos^5 dx$ is _____.

(b) AES is an example of a _____ cipher.

(c) _____ of two convex sets is a convex set.

(d) The solution of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$ is _____.

(e) The curve $y^2=x(x-1)^2$ is symmetrical about _____ axis.

Part – B

Answer any **four** questions of the following : $5\times4=20$

3. Integrate $\int\frac{dx}{(2+x)\sqrt{(1+x)}}$.

4. Find the reduction formula for $\int\sin^n x\,dx$.

5. Solve : $y\,dx-x\,dy=xy\,dx$.

6. Solve : $\frac{d^2y}{dx^2}+9y=x^2$.

7. Explain private key encryption algorithm DES.

8. Prove that the intersection of two convex sets is also a convex set.

Part – C

Answer any **four** questions of the following :

$10 \times 4 = 40$

9. Find the perimeter of the cardioid $r = a(1 + \cos\theta)$.

10. Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

11. Solve : $y = 2px + y^2p^3, p = \frac{dy}{dx}$.

12. Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin(2x)$.

13. Explain private key encryption algorithm AES.

14. Solve the following LPP using simplex method :

Max $Z = 4x + 10y$

Subject to the constraints

$2x + y \leq 50$

$2x + 5y \leq 100$

$2x + 3y \leq 90$

$x, y \geq 0$