



**2021**

*Time : 3 hours*

*Full Marks : 70*

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

*Answer from **all** the Parts as directed.*

**Part – A**

1. Choose the correct answer in each of the following :

$$1 \times 5 = 5$$

(a)  $\int \frac{x dx}{x^4 - x^2 - 2}$  is equal to :

(i)  $\frac{1}{6} \log k \frac{x^2 - 1}{x^2 + 1}$



(ii)  $\frac{1}{4} \log k \frac{x^2 - 2}{x^2 + 2}$

(iii)  $\frac{1}{2} \log k \frac{x^2 - 4}{x^2 + 4}$

• (iv) None of these

(b) Area of loop of the curve  $y^2 = x(x - 1)^2$  is :

(i)  $\frac{2}{3}$

(ii)  $\frac{4}{11}$

✓ (iii)  $\frac{8}{15}$

(iv) None of these

(c) The differential equation  $(x^2 - 4xy - y^2)dx + (y^2 - 4xy - x^2)dy = 0$  is of :

(i) First order and first degree

(ii) First order and second degree

(iii) Second order and first degree

(iv) None of these

(d) Solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = x^3 \text{ is :}$$

☒ (i)  $yx = \frac{x^4}{4} + c$

(ii)  $yx = \frac{x^5}{5} + c$

(iii)  $yx = \frac{x^6}{6} + c$

☒ (iv) None of these

(e) Which of the following is a mode of operation for Block Ciphers in cryptography ?

☒ (i) Electronic Code Book (ECB)

☒ (ii) Cipher Block Chaining (CBC)

(iii) Counter Mode

(iv) All of these

2. Fill in the blanks :

1×5 = 5

(a) Data Encryption Standard (DES) is a

\_\_\_\_\_.



(b) An equation of the form  $\frac{dy}{dx} + py = Q$ , where

P and Q are functions of x or constants is known as \_\_\_\_\_.

(c) The limit of

$$n \left[ \frac{1}{n^2} + \frac{1}{n^2 + 1^2} + \frac{1}{n^2 + 2^2} + \dots + \frac{1}{n^2 + (n-1)^2} \right] \text{ is}$$

\_\_\_\_\_.

(d) The area bounded by the curve  $r = f(\theta)$  and the radii vectors  $\theta = \alpha$  and  $\theta = \beta$  is given by the definite integral \_\_\_\_\_.

(e) If the convex set of feasible solutions of  $Ax=b, x \geq 0$  is a convex polyhedron, then at least one of the extreme points gives \_\_\_\_\_.

### Part - B

Answer any **four** questions of the following :

$$5 \times 4 = 20$$

3. (a) Examine whether the set

$$S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 9\} \text{ is a convex set.}$$

(b) Solve the following linear programming problem by graphical method :

$$\text{Max } Z = x_1 + 3x_2$$

Subject to the constraints

$$3x_1 + 6x_2 \leq 8$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

✓ 4. Evaluate  $\int_0^{\pi/2} \cos^n x \cos nx \, dx$ , where  $n$  is a positive integer.

✓ 5. Solve :  $x \frac{dy}{dx} + y = y^2 \log x$ .

✓ 6. Explain RC<sup>4</sup> Encryption Algorithm.

7. How can you combine hash function with public key encryption for message authentication ?

✓ 8. Find the perimeter of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ .



### Part – C

Answer any **four** questions of the following :

$$10 \times 4 = 40$$

9. Solve the following linear programming problem by simplex method :

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

$$\text{Subject to } 2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

10. Find the volume and surface area of the solid generated by revolving the cycloid  $x = a(\theta + \sin\theta)$ ,  $y = a(1 + \cos\theta)$  about its base.

11. What is RSA encryption and how does it work ?  
Is RSA encryption secure ?

12. Find the moment of inertia of a thin hollow spherical shell of radius  $a$  and mass  $M$  about a diameter.

13. (a) Find the orthogonal trajectories of  
 $r = a(1 - \cos\theta)$ , where  $a$  is the parameter.

(b) Solve :  $(x - a)p^2 + (x - y)p - y = 0$

14. Solve :

(a)  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$

(b)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x + xe^{2x} - 2\sin x$

