

2024

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from all the Groups as directed.

Group – A**(Objective Type Questions)**

1. Choose the correct answer of the following :

1×5 = 5

(a) The equation of the curve which passes through the point $(a, 0)$ and whose slope at any point $P(x, y)$ is x/y , is :

- (i) $x^2 - y^2 = a^2$ ~~(ii) $x^2 + y^2 = a^2$~~
(iii) $x^2 \cdot y^2 = a^2$ (iv) $x^2 / y^2 = a^2$

(b) A curve passes through the origin and at any point (x, y) on it, the product of its slope and the ordinate is a constant (i. e. $y \, dy/dx = c$). The equation of the curve is :

- (i) Circle

~~(ii) Parabola~~

~~(iii) Hyperbola~~

(iv) Ellipse

(c) The solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right) \text{ is :}$$

(i) $x = c \sin\left(\frac{y}{x}\right)$ ~~(ii) $y = c \sin\left(\frac{y}{x}\right)$~~

~~(iii) $x = c \tan\left(\frac{y}{x}\right)$~~ (iv) $y = c \tan\left(\frac{y}{x}\right)$

(d) The union of two convex sets :

~~(i) Is always convex~~

~~(ii) May not be convex~~

(iii) Can not be determined

(iv) None of these

(e) The value of $\Gamma(n)$ is equal to :

~~(i) $(n-1)!$~~

(ii) $(n+1)!$

(iii) $n!$

(iv) None of these

2. Fill in the blanks in each of the following : $1 \times 5 = 5$

(a) The solution of the differential equation $(1+y^2)dx + (1+x^2)dy = 0$ is _____

(b) The curve for which the normal at every point passes through a fixed point is a _____

(c) The solution of the differential equation

$$x \frac{dy}{dx} + \frac{y^2}{x} = y \text{ is } \underline{\hspace{2cm}}.$$

(d) The perimeter of the cardioid $r = a(1 + \cos\theta)$ is _____.

(e) The moment of inertia of a solid sphere about a diameter is _____.

Group - B

(Short-answer Type Questions)

Answer any four questions of the following :

$$5 \times 4 = 20$$

3. Integrate $\int \frac{1}{x^2 + a^2} dx$.

4. Find the perimeter of the curve $r = a \sin\theta$.

5. Solve $ydx - xdy = xydx$.

6. Solve $y = 2px + p^2$.

7. Differentiate between stream ciphers and block ciphers.

8. Solve graphically the LPP,

$$\text{Max } Z = 3x + 5y$$

$$\text{Subject to } x + 2y \leq 20,$$

$$x + y \leq 15$$

$$y \leq 8, x, y \geq 0$$

Group – C

(Long-answer Type Questions)

Answer any four questions of the following :

$$10 \times 4 = 40$$

9. Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about $\theta = 0$.

10. Solve (i) $y = 2px + y^2 p^3$ (ii) $\frac{dy}{dx} = \cos(x + y)$.

11. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$.

12. Find the orthogonal trajectory of the family of curve $x^{2/3} + y^{2/3} = a^{2/3}$.

13. Explain the public key encryption algorithm RSA with example.

14. Solve the following using simplex method:

$$\text{Max } Z = 7x + 5y$$

Subject to the constraints

$$x + 2y \leq 6$$

$$4x + 3y \leq 15$$

$$x, y \geq 0.$$

