PROJECT

SEE609: Mathematical and Computational Tools for Engineering

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Discretization of equation:

$$\int_S \boldsymbol{\rho\phi v \cdot n\, dS} = \int_S \boldsymbol{\Gamma\nabla\phi \cdot n\, dS} + \int_\Omega \boldsymbol{q\phi\, d\Omega}$$

u = x, v = -y

UDS APPROCH:

For interior node

$$\int_S \rho\phi v \cdot n\, dS = \int_{Se} \rho\phi exi \cdot i\, dS + \int_{Sw} \rho\phi wxi \cdot -i\, dS + \int_{Sn} \rho\phi nyj \cdot j\, dS + \int_{Ss} \rho\phi syj \cdot -jdS$$

$$\int_S \rho\phi v \cdot n\, dS = \rho x(\phi_P - \phi_W)\Delta y + \rho y(\phi_P - \phi_N)\Delta x + \rho\Delta x\Delta y(\phi_P - \phi_N)/2$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}}\right) + \Gamma\Delta x\left(\frac{d\phi}{dy}|_{\{n\}} - \frac{d\phi}{dy}|_{\{s\}}\right)$$

$$\frac{d\phi}{dx}|_{\{e\}} = \frac{(\phi_e - \phi_w)}{\Delta x} \quad , \frac{d\phi}{dx}|_{\{w\}} = \frac{(\phi_p - \phi_w)}{\Delta x} \quad , \frac{d\phi}{dx}|_{\{s\}} = \frac{(\phi_s - \phi_p)}{\Delta y} \quad , \frac{d\phi}{dx}|_{\{n\}} = \frac{(\phi_p - \phi_n)}{\Delta y}$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_E + \phi_W - 2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)$$

$$\int_\Omega q\phi\, d\Omega = \frac{(2x * \Delta x + \Delta x^2)(2y * \Delta y + \Delta y^2)}{4}$$

Equation:

- $\rho x(\phi_P - \phi_W)\,\Delta y + \rho y(\phi_P - \phi_N)\Delta x + \rho\Delta x\Delta y(\phi_P - \phi_N)/2 - \Gamma\Delta y\left(\frac{\phi_E + \phi_W - 2\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

- $\boldsymbol{\left(-\rho\Delta yx - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W + \left(\rho x\Delta y + \rho\Delta xy + \frac{2\Gamma\Delta y}{\Delta x} + \frac{2\Gamma\Delta x}{\Delta y}\right)\phi_P + \left(-\rho y\Delta x - \frac{\Gamma\Delta x}{\Delta y} - \rho\Delta x\Delta y\right)\phi_N + \left(-\frac{\Gamma\Delta x}{\Delta y}\right)\phi_S + \left(-\frac{\Gamma}{\Delta x}\Delta y\right)\phi_E = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}}$

APPLYING BOUNDARY CONDITION AT DIFFERENT FACES

FOR NORTH:

$\phi_n = 0$

$$\int_S \rho\phi v \cdot n\, dS = [\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_0) - \rho\Delta y\phi_N]\Delta x$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_W + \phi_E - 2\phi_P}{\Delta x}\right)\Delta y + \Gamma\left(\frac{\phi_S + 2\phi_0 - 3\phi_P}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_0) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(\frac{\phi_W+\phi_E-2\phi_P}{\Delta x}\right)\Delta y - \Gamma\left(\frac{\phi_S+2\phi_0-3\phi_P}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\boldsymbol{\left(-\frac{\Gamma}{\Delta x} - \rho x\right)\Delta y\phi_W + \left[\left(\rho x + \rho\Delta x + \frac{2\Gamma}{\Delta x}\right)\Delta y + \left(\rho y + \frac{3\Gamma}{\Delta y}\right)\Delta x\right]\phi_P + \left(-\frac{\Gamma}{\Delta x}\right)\Delta y\phi_E + \left(-\frac{\Gamma}{\Delta y}\right)\Delta x\phi_S = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4} + \left[(\rho y + \rho\Delta y)\Delta x + \frac{2\Gamma}{\Delta y}\Delta x\right]\phi_0}$

FOR WEST:

$\phi_E = \phi_P, \frac{d\phi}{dx}|_{\{e\}} = 0$

$$\int_S \rho\phi v \cdot n\, dS = [\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_W - \phi_P}{\Delta x}\right)\Delta y + \Gamma\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(\frac{\phi_W-\phi_P}{\Delta x}\right)\Delta y - \Gamma\left(\frac{\phi_S+\phi_N-2\phi_P}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(-\frac{\Gamma}{\Delta x} - \rho x\right)\Delta y\phi_W + \left[\left(\rho x + \rho\Delta x + \frac{\Gamma}{\Delta x}\right)\Delta y + \left(\rho y + \frac{2\Gamma}{\Delta y}\right)\Delta x\right]\phi_P + \left(-\frac{\Gamma}{\Delta x}\right)\Delta y\phi_E + \left(-\rho y - \rho\Delta y - \frac{\Gamma}{\Delta y}\right)\Delta x\phi_N + \left(-\frac{\Gamma}{\Delta y}\right)\Delta x\phi_S = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}}$

FOR EAST:

$\phi_E = 1 - y$

$$\int_S \rho\phi v \cdot n\, dS = [\rho x(\phi_P - 1 + y) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_E - 3\phi_P + 2 - 2y}{\Delta x}\right)\Delta y + \Gamma\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - 1 + y) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(\frac{\phi_E - 3\phi_P + 2 - 2y}{\Delta x}\right)\Delta y + \Gamma\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$
- $\left(-\frac{\boldsymbol{\Gamma}}{\Delta x} - \boldsymbol{\rho x}\right)\Delta \boldsymbol{y}\boldsymbol{\phi_W} + \left[\left(\boldsymbol{\rho x} + \boldsymbol{\rho}\Delta \boldsymbol{x} + \frac{3\boldsymbol{\Gamma}}{\Delta x}\right)\Delta\boldsymbol{y} + \left(\boldsymbol{\rho y} + \frac{2\boldsymbol{\Gamma}}{\Delta y}\right)\Delta\boldsymbol{x}\right]\boldsymbol{\phi_P} + \left(-\frac{\boldsymbol{\Gamma}}{\Delta x}\right)\Delta\boldsymbol{y}\boldsymbol{\phi_E} + \left(-\boldsymbol{\rho y} - \boldsymbol{\rho}\Delta\boldsymbol{y} - \frac{\boldsymbol{\Gamma}}{\Delta y}\right)\Delta\boldsymbol{x}\boldsymbol{\phi_N} + \left(-\frac{\boldsymbol{\Gamma}}{\Delta y}\right)\Delta\boldsymbol{x}\boldsymbol{\phi_S} = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

FOR SOUTH

<u>If x= 0 to 0.2 and 0.8 to 1</u>

$\frac{d\boldsymbol{\phi}}{d\boldsymbol{x}}|_{\{s\}} = \mathbf{0}\,,\ \boldsymbol{\phi_s} = \boldsymbol{\phi_P}$

$$\int_S \boldsymbol{\rho\phi v} \cdot \boldsymbol{n\, dS} = [\boldsymbol{\rho x}(\boldsymbol{\phi_P} - \boldsymbol{\phi_W}) + \boldsymbol{\rho}\Delta\boldsymbol{x}\boldsymbol{\phi_P}]\Delta\boldsymbol{y} + [\boldsymbol{\rho y}(\boldsymbol{\phi_P} - \boldsymbol{\phi_N}) - \boldsymbol{\rho}\Delta\boldsymbol{y}\boldsymbol{\phi_N}]\Delta\boldsymbol{x}$$

$$\int_S \boldsymbol{\Gamma\nabla\phi} \cdot \boldsymbol{n\, dS} = \boldsymbol{\Gamma}\left(\frac{\boldsymbol{\phi_W} + \boldsymbol{\phi_E} - 2\boldsymbol{\phi_P}}{\Delta\boldsymbol{x}}\right)\Delta\boldsymbol{y} + \boldsymbol{\Gamma}\left(\frac{\boldsymbol{\phi_N} - \boldsymbol{\phi_P}}{\Delta\boldsymbol{y}}\right)\Delta\boldsymbol{x}$$

- $[\boldsymbol{\rho x}(\boldsymbol{\phi_P} - \boldsymbol{\phi_W}) + \boldsymbol{\rho}\Delta\boldsymbol{x}\boldsymbol{\phi_P}]\Delta\boldsymbol{y} + [\boldsymbol{\rho y}(\boldsymbol{\phi_P} - \boldsymbol{\phi_N}) - \boldsymbol{\rho}\Delta\boldsymbol{y}\boldsymbol{\phi_N}]\Delta\boldsymbol{x} - \boldsymbol{\Gamma}\left(\frac{\boldsymbol{\phi_W} + \boldsymbol{\phi_E} - 2\boldsymbol{\phi_P}}{\Delta x}\right)\Delta\boldsymbol{y} + \boldsymbol{\Gamma}\left(\frac{\boldsymbol{\phi_N} - \boldsymbol{\phi_P}}{\Delta y}\right)\Delta\boldsymbol{x} = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$
- $\left(-\frac{\boldsymbol{\Gamma}}{\Delta x} - \boldsymbol{\rho x}\right)\Delta\boldsymbol{y}\boldsymbol{\phi_W} + \left[\left(\boldsymbol{\rho x} + \boldsymbol{\rho}\Delta\boldsymbol{x} + \frac{2\boldsymbol{\Gamma}}{\Delta x}\right)\Delta\boldsymbol{y} + \left(\boldsymbol{\rho y} + \frac{\boldsymbol{\Gamma}}{\Delta y}\right)\Delta\boldsymbol{x}\right]\boldsymbol{\phi_P} + \left(-\boldsymbol{\rho y} - \boldsymbol{\rho}\Delta\boldsymbol{y} - \frac{\boldsymbol{\Gamma}}{\Delta y}\right)\Delta\boldsymbol{x}\boldsymbol{\phi_N} + \left(-\frac{\boldsymbol{\Gamma}}{\Delta x}\right)\Delta\boldsymbol{y}\boldsymbol{\phi_E} = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

<u>For x= 0.2 to 0.8</u>

$\phi_S = \phi_P + a * \Delta y / \Gamma$

$$\int_S \rho\phi v \cdot n\, dS = [\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_W + \phi_E - 2\phi_P}{\Delta x}\right)\Delta y + \Gamma\left(\frac{a}{\Gamma} - \frac{(\phi_P - \phi_N)}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(\frac{\phi_W+\phi_E-2\phi_P}{\Delta x}\right)\Delta y - \Gamma\left(\frac{a}{\Gamma} - \frac{(\phi_P-\phi_N)}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(-\frac{\Gamma}{\Delta x} - \rho x\right)\Delta y\phi_W + \left[\left(\rho x + \rho\Delta x + \frac{2\Gamma}{\Delta x}\right)\Delta y + \left(\rho y + \frac{\Gamma}{\Delta y}\right)\Delta x\right]\phi_P + \left(-\rho y - \rho\Delta y - \frac{\Gamma}{\Delta y}\right)\Delta x\phi_N + \left(-\frac{\Gamma}{\Delta x}\right)\Delta y\phi_E = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4} - a\Delta x + \frac{y*a*\Delta y*\Delta x}{2\Gamma}}$

<u>For Corner points:</u>

Top point left face:

$\phi_n = 0\,, \phi_w = 0\,,\ as\ y = 1$

X direction

$[\rho x(\phi_e - \phi_w) + \rho\Delta x\phi_e]\Delta y$ = $[\rho x(\phi_e) + \rho\Delta x\phi_P]\Delta y$

Y direction

$[\rho y(\phi_P)]\Delta x$

$$\frac{d\phi}{dx}|_{\{e\}} = \frac{(\phi_E - \phi_P)}{\Delta x} = \,, \frac{d\phi}{dx}|_{\{w\}} = \frac{2(\phi_p)}{\Delta x}$$

$$\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}} = \left(\frac{\phi_E - 3\phi_P}{\Delta x}\right)$$

$$\frac{d\phi}{dx}|_{\{s\}} = \frac{(\phi_S - \phi_P)}{\Delta y} = \,, \frac{d\phi}{dx}|_{\{n\}} = \frac{2(\phi_P)}{\Delta y}$$

$$, \frac{d\phi}{dy}|_{\{s\}} - \frac{d\phi}{dy}|_{\{n\}} = \frac{(\phi_S - 3\phi_P)}{\Delta y}$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_E - 3\phi_P}{\Delta x}\right)\Delta y + \Gamma\left(\frac{(\phi_S - 3\phi_P)}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_e) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P)]\Delta x - \Gamma\left(\frac{\phi_E-3\phi_P}{\Delta x}\right)\Delta y - \Gamma\left(\frac{(\phi_S- 3\phi_P)}{\Delta y}\right)\Delta x =$

  $\frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\left(-\frac{\boldsymbol{\Gamma}}{\Delta \boldsymbol{x}}\Delta \boldsymbol{y}\right)\boldsymbol{\phi_E} + \left[\left(\boldsymbol{\rho x} + \boldsymbol{\rho \Delta x} + \frac{\mathbf{3}\boldsymbol{\Gamma}}{\Delta \boldsymbol{x}}\right)\Delta \boldsymbol{y} + \left(\boldsymbol{\rho y} + \frac{\mathbf{3}\boldsymbol{\Gamma}}{\Delta \boldsymbol{y}}\right)\Delta \boldsymbol{x}\right]\boldsymbol{\phi_P} + \left(\frac{\boldsymbol{\Gamma}}{\Delta \boldsymbol{y}}\right)\Delta \boldsymbol{x}\boldsymbol{\phi_S} = \frac{(\mathbf{2}\boldsymbol{x}*\Delta \boldsymbol{x}+\Delta \boldsymbol{x}^{\mathbf{2}})(\mathbf{2}\boldsymbol{y}*\Delta \boldsymbol{y}+\Delta \boldsymbol{y}^{\mathbf{2}})}{\mathbf{4}}$

Top point right face:

$\phi_n = 0 \, , \phi_E = \phi_P$

For x direction

$[\rho x(\phi_e - \phi_w) + \rho\Delta x\phi_e]\Delta y$ = $[\rho x(\phi_P - \phi_w) + \rho\Delta x\phi_P]\Delta y$

Y direction

$[\rho y(\phi_P\ )]\Delta x$

$$\frac{d\phi}{dx}|_{\{e\}} = 0 = \ , \frac{d\phi}{dx}|_{\{w\}} = \frac{(\phi_p - \phi_w)}{\Delta x}$$

$$\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}} = \left(\frac{\phi_P - \phi_W}{\Delta x}\right)$$

$$\frac{d\phi}{dx}|_{\{s\}} = \frac{(\phi_S - \phi_P)}{\Delta y} = \ , \frac{d\phi}{dx}|_{\{n\}} = \frac{2(\phi_P)}{\Delta y}$$

$$, \frac{d\phi}{dy}|_{\{s\}} - \frac{d\phi}{dy}|_{\{n\}} = \frac{(\phi_S - 3\phi_P)}{\Delta y}$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\left(\frac{\phi_P - \phi_W}{\Delta x}\right)\Delta y + \Gamma\left(\frac{(\phi_S - 3\phi_P)}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - \phi_W) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P\ )]\Delta x - \Gamma\left(\frac{\phi_P - \phi_W}{\Delta x}\right)\Delta y - \Gamma\left(\frac{(\phi_S - 3\phi_P)}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\left(-\frac{\boldsymbol{\Gamma}}{\Delta \boldsymbol{x}} - \boldsymbol{\rho x}\right)\Delta \boldsymbol{y}\boldsymbol{\phi_W} + \left[\left(\boldsymbol{\rho x} + \boldsymbol{\rho \Delta x} + \frac{\mathbf{2}\boldsymbol{\Gamma}}{\Delta \boldsymbol{x}}\right)\Delta \boldsymbol{y} + \left(\boldsymbol{\rho y} + \frac{\mathbf{3}\boldsymbol{\Gamma}}{\Delta \boldsymbol{y}}\right)\Delta \boldsymbol{x}\right]\boldsymbol{\phi_P} + \left(\frac{-\boldsymbol{\Gamma}}{\Delta \boldsymbol{y}}\right)\Delta \boldsymbol{x}\boldsymbol{\phi_S} = \frac{(\mathbf{2}\boldsymbol{x}*\Delta \boldsymbol{x}+\Delta \boldsymbol{x}^{\mathbf{2}})(\mathbf{2}\boldsymbol{y}*\Delta \boldsymbol{y}+\Delta \boldsymbol{y}^{\mathbf{2}})}{\mathbf{4}}$

Bottom point left face:

$\phi_w = 1 - y \, , \phi_P = \phi_S$

X direction

$[\rho x(\phi_P - \phi_w) + \rho\Delta x\phi_P]\Delta y$ = $[\rho x(\phi_P - 1 + y) + \rho\Delta x\phi_P]\Delta y$

Y direction

$[\rho y(\phi_P - \phi_N\ ) - \rho\Delta y\phi_N]\Delta x$

$$\frac{d\phi}{dx}|_{\{e\}} = \frac{(\phi_e - \phi_w)}{\frac{\Delta x}{2}} = \ , \frac{d\phi}{dx}|_{\{w\}} = \frac{(\phi_p - \phi_w)}{\frac{\Delta x}{2}} = \frac{(\phi_p - 1 + y)}{\frac{\Delta x}{2}}$$

$$\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}} = \left(\frac{\phi_E + 2 - 2y - 3\phi_P}{\Delta x}\right)$$

$$, \frac{d\phi}{dy}|_{\{s\}} - \quad \frac{d\phi}{dy}|_{\{n\}} = \frac{-(\phi_P - \phi_N)}{\Delta y}$$

$$\int_S \Gamma \nabla\phi \cdot n \, dS \; = \; \Gamma\left(\frac{\phi_E + 2 - 2y - 3\phi_P}{\Delta x}\right)\Delta y \; + \; \Gamma\left(\frac{-(\phi_P - \phi_N)}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - 1 + y) + \rho\Delta x\phi_P]\Delta y + [\rho y(\phi_P - \phi_N) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(\frac{\phi_E+2-2y-3\phi_P}{\Delta x}\right)\Delta y - \Gamma\left(\frac{-(\phi_P - \phi_N)}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\boldsymbol{\left(-\frac{\Gamma}{\Delta x}\Delta y\right)\phi_E + \left[\left(\rho x + \rho\Delta x + \frac{3\Gamma}{\Delta x}\right)\Delta y + \left(\rho y + \frac{2\Gamma}{\Delta y}\right)\Delta x\right]\phi_P + \left(-\rho y - \rho\Delta y - \frac{\Gamma}{\Delta y}\right)\Delta x\phi_N = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4} + (1-y)\rho x\Delta y + \frac{\Gamma}{\Delta x}(2-2y)\Delta y}$

Bottom point Right face:

$\phi_S = \phi_P, \phi_E = \phi_P$

For x direction

$[\rho x(\phi_e - \phi_w) + \rho\Delta x\phi_e]\Delta y$ = $[\rho x(\phi_P - \phi_w) + \rho\Delta x\phi_P]\Delta y$

Y direction

$[\rho x(\phi_P - \phi_w) - \rho\Delta y\phi_N]\Delta x$

$$\frac{d\phi}{dx}|_{\{e\}} = 0 = \; , \frac{d\phi}{dx}|_{\{w\}} = \frac{(\phi_p - \phi_w)}{\Delta x}$$

$$\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}} = -\left(\frac{(\phi_p - \phi_w)}{\Delta x}\right)$$

$$, \frac{d\phi}{dy}|_{\{s\}} - \quad \frac{d\phi}{dy}|_{\{n\}} = \frac{-(\phi_P - \phi_N)}{\Delta y}$$

$$\int_S \Gamma \nabla\phi \cdot n \, dS \; = \; \Gamma\left(-\frac{(\phi_p - \phi_w)}{\Delta x}\right)\Delta y \; + \; \Gamma\left(\frac{-(\phi_P - \phi_N)}{\Delta y}\right)\Delta x$$

- $[\rho x(\phi_P - \phi_w) + \rho\Delta x\phi_P]\Delta y + [\rho x(\phi_P - \phi_w) - \rho\Delta y\phi_N]\Delta x - \Gamma\left(-\frac{(\phi_p - \phi_w)}{\Delta x}\right)\Delta y - \Gamma\left(\frac{-(\phi_P - \phi_N)}{\Delta y}\right)\Delta x = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\boldsymbol{\left(-\frac{\Gamma}{\Delta x} - \rho x\right)\Delta y\phi_W + \left[\left(\rho x + \rho\Delta x + \frac{\Gamma}{\Delta x}\right)\Delta y + \left(\rho y + \frac{\Gamma}{\Delta y}\right)\Delta x\right]\phi_P + \left(-\rho y - \rho\Delta y - \frac{\Gamma}{\Delta y}\right)\Delta x\phi_N = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y+\Delta y^2)}{4}}$

CDS APPROCH:

For interior node

$$\int_S \rho\phi v \cdot n\, dS = \int_S \Gamma\nabla\phi \cdot n\, dS + \int_\Omega q\phi\, d\Omega$$

$$\int_S \rho\phi v \cdot n\, dS = \rho x(\phi_e - \phi_w)\Delta y + \rho y(\phi_s - \phi_n)\Delta x + \rho\Delta x\Delta y(\phi_e - \phi_n)$$

$$= \rho x\Delta y\left(\frac{\phi_E + \phi_P}{2} - \frac{\phi_P + \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_S + \phi_P}{2} - \frac{\phi_P + \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E + \phi_P}{2}\right) - \rho\Delta x\Delta y\left(\frac{\phi_E + \phi_P}{2}\right)$$

$$\int_S \rho\phi v \cdot n\, dS = \rho x\Delta y\left(\frac{\phi_E - \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_S - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{d\phi}{dx}|_{\{e\}} - \frac{d\phi}{dx}|_{\{w\}}\right) + \Gamma\Delta x\left(\frac{d\phi}{dy}|_{\{n\}} - \frac{d\phi}{dy}|_{\{s\}}\right)$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_E + \phi_W - 2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)$$

$$\int_\Omega q\phi\, d\Omega = \frac{(2x * \Delta x + \Delta x^2)(2y * \Delta y + \Delta y^2)}{4}$$

- $\rho x\Delta y\left(\frac{\phi_E - \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_S - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_E + \phi_W - 2\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$
- $\boldsymbol{\left(\frac{\rho\Delta y x}{2} + \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_E + \left(\frac{-\rho\Delta y x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W + \left(\frac{\rho y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_S + \left(\frac{2\Gamma\Delta y}{\Delta x} + \frac{2\Gamma\Delta x}{\Delta y}\right)\phi_P + \left(-\frac{\rho y\Delta x}{2} - \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_N = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}}$

APPLYING BOUNDARY CONDITION AT DIFFERENT SIDES:

WEST SIDE:

$\phi_W = 1 - y$

$$\int_S \rho\phi v \cdot n\, dS = \rho x\Delta y\left(\frac{\phi_E + \phi_P}{2} - 1 + y\right) + \rho y\Delta x\left(\frac{\phi_S - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_E + 2 - 2y - 3\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S + \phi_N - 2\phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E+\phi_P}{2}-1+y\right) + \rho y\Delta x\left(\frac{\phi_S-\phi_N}{2}\right)+\rho\Delta x\Delta y\left(\frac{\phi_E-\phi_N}{2}\right)-\Gamma\Delta y\left(\frac{\phi_E+2-2y-3\phi_P}{\Delta x}\right)+\Gamma\Delta x\left(\frac{\phi_S+\phi_N-2\phi_P}{\Delta y}\right)=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_E+\left(\frac{\rho y\Delta x}{2}+\frac{\Gamma\Delta x}{\Delta y}\right)\phi_S+\left(\frac{\rho\Delta yx}{2}+\frac{3\Gamma\Delta y}{\Delta x}+\frac{2\Gamma\Delta x}{\Delta y}\right)\phi_P+\left(-\frac{\rho y\Delta x}{2}-\frac{\rho\Delta y\Delta x}{2}+\frac{\Gamma\Delta x}{\Delta y}\right)\phi_N=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}+\rho\Delta yx(1-y)+\frac{2\Gamma\Delta y(1-y)}{\Delta x}}$

NORTH SIDE:

$$\phi_n=\phi_0, \frac{d\phi}{dx}|_{\{n\}}=\frac{2(\phi_0-\phi_P)}{\Delta y}$$

$$\int_S \rho\phi v\cdot n\,dS=\rho x\Delta y\left(\frac{\phi_E-\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P+\phi_S-2\phi_0}{2}\right)+\rho\Delta x\Delta y\left(\frac{\phi_E+\phi_P-2\phi_0}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\,dS=\Gamma\Delta y\left(\frac{\phi_E+\phi_W-2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S-3\phi_P+2\phi_0}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E-\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P+\phi_S-2\phi_0}{2}\right)+\rho\Delta x\Delta y\left(\frac{\phi_E+\phi_P-2\phi_0}{2}\right)-\Gamma\Delta y\left(\frac{\phi_E+\phi_W-2\phi_P}{\Delta x}\right)-\Gamma\Delta x\left(\frac{\phi_S-3\phi_0+2\phi_0}{\Delta y}\right)=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_E+\left(\frac{-\rho\Delta yx}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_W+\left(\frac{\rho y\Delta x}{2}+\frac{\Gamma\Delta x}{\Delta y}\right)\phi_S+\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}+\frac{2\Gamma\Delta y}{\Delta x}+\frac{3\Gamma\Delta x}{\Delta y}\right)\phi_P=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}}$

EAST SIDE:

$\frac{d\phi}{dx}|_{\{e\}}=0\,,\phi_E=\phi_P$

$$\int_S \rho\phi v\cdot n\,dS=\rho x\Delta y\left(\frac{\phi_P+\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_S-\phi_N}{2}\right)+\rho\Delta x\Delta y\left(\frac{\phi_P-\phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\,dS=\Gamma\Delta y\left(\frac{\phi_W-\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S+\phi_N-2\phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_P+\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_S-\phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_P-\phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_W-\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_S+\phi_N-2\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\boldsymbol{\left(\frac{\rho\Delta yx}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_W+\left(\frac{\rho y\Delta x}{2}+\frac{\Gamma\Delta x}{\Delta y}\right)\phi_S+\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}+\frac{\Gamma\Delta y}{\Delta x}+\frac{2\Gamma\Delta x}{\Delta y}\right)\phi_P+\left(-\frac{\rho y\Delta x}{2}-\frac{\rho\Delta y\Delta x}{2}-\frac{\Gamma\Delta x}{\Delta y}\right)\phi_N=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}}$

SOUTH SIDE:

FOR x = 0.2 to 0.8

$\phi_S = \phi_P + a*\Delta y/2\,\Gamma$

$$\int_S \rho\phi v\cdot n\,dS = \rho x\Delta y\left(\frac{\phi_E-\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P-\phi_N}{2}-\frac{a*\Delta y}{2\,\Gamma}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E-\phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\,dS = \Gamma\Delta y\left(\frac{\phi_E+\phi_W-2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_N-\phi_P}{\Delta y}-\frac{a}{\Gamma}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E-\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P-\phi_N}{2}-\frac{a*\Delta y}{2\,\Gamma}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E-\phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_E+\phi_W-2\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_N-\phi_P}{\Delta y}-\frac{a}{\Gamma}\right) = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$

- $\boldsymbol{\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_E+\left(-\frac{\rho\Delta yx}{2}-\frac{\Gamma\Delta y}{\Delta x}\right)\phi_W+\left(\frac{\rho y\Delta x}{2}+\frac{\Gamma\Delta x}{\Delta y}\right)\phi_S+\left(\frac{\rho\Delta yx}{2}+\frac{\rho\Delta y\Delta x}{2}+\frac{\Gamma\Delta y}{\Delta x}\right)\phi_P+\left(-\frac{\rho y\Delta x}{2}-\frac{\rho\Delta y\Delta x}{2}-\frac{\Gamma\Delta x}{\Delta y}\right)\phi_N=\frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}+\frac{\rho\Delta y\Delta xya}{2\Gamma}-a\Delta x}$

For x= 0 to 0.2 and 0.8 to 1

$\frac{d\phi}{dx}|_{\{s\}} = 0\,,\ \phi_S = \phi_P$

$$\int_S \rho\phi v\cdot n\,dS = \rho x\Delta y\left(\frac{\phi_E-\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P-\phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E-\phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\,dS = \Gamma\Delta y\left(\frac{\phi_E+\phi_W-2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_N-\phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E - \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_E + \phi_W - 2\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_N - \phi_P}{\Delta y}\right) = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

- $\boldsymbol{\left(\frac{\rho\Delta yx}{2} + \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_E + \left(-\frac{\rho\Delta yx}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W + \left(\frac{\rho y\Delta x}{2} + \frac{\Gamma\Delta x}{\Delta y}\right)\phi_S + \left(\frac{\rho\Delta yx}{2} + + \frac{2\Gamma\Delta y}{\Delta x} + \frac{\Gamma\Delta x}{\Delta y}\right)\phi_P + \left(-\frac{\rho y\Delta x}{2} - \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_N = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}}$

Corner points:

Left top:

$\phi_n = \phi_0, \phi_p = \phi_0$

$$\frac{d\phi}{dy}|_{\{n\}} = \frac{-2\phi_P}{\Delta y} \quad \frac{d\phi}{dx}|_{\{w\}} = \frac{-2\phi_P}{\Delta x}$$

$$\int_S \rho\phi v\cdot n\, dS = \rho x\Delta y\left(\frac{\phi_E + \phi_P}{2}\right)(1 + \Delta x) + \rho y\Delta x\left(\frac{\phi_P + \phi_S}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_E - 2\phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S - 3\phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E + \phi_P}{2}\right)(1 + \Delta x) + \rho y\Delta x\left(\frac{\phi_P + \phi_S}{2}\right) - \Gamma\Delta y\left(\frac{\phi_E - 2\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_S - 3\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

- $\boldsymbol{\left(\frac{\rho\Delta yx}{2} + \frac{\rho x\Delta y\Delta x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_E + \left(-\frac{\rho\Delta yx}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W + \left(\frac{\rho y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_S + \left(\frac{\rho\Delta yx}{2} + \frac{\rho x\Delta y\Delta x}{2} + \frac{\rho y\Delta x}{2} + \frac{3\Gamma\Delta y}{\Delta x} + \frac{3\Gamma\Delta x}{\Delta y}\right)\phi_P = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}}$

Right top corner:

$$\phi_n = \phi_0, \phi_e = \phi_p$$

$$\frac{d\phi}{dy}|_{\{n\}} = \frac{-2\phi_P}{\Delta y}, \frac{d\phi}{dx}|_{\{e\}} = 0$$

$$\int_S \rho\phi v\cdot n\, dS = \rho x\Delta y\left(\frac{\phi_P + \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P + \phi_S}{2}\right) + \rho\Delta x\Delta y(\phi_P)$$

$$\int_S \Gamma\nabla\phi\cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_W - \phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_S - 3\phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_P+\phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P+\phi_S}{2}\right) + \rho\Delta x\Delta y(\phi_P) - \Gamma\Delta y\left(\frac{\phi_W-\phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_S-3\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(-\frac{\rho\Delta y x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W + \left(\frac{\rho y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_S + \left(\frac{\rho\Delta y x}{2} + \frac{\rho y\Delta x}{2} + \rho\Delta y\Delta x + \frac{\Gamma\Delta y}{\Delta x} + \frac{3\Gamma\Delta x}{\Delta y}\right)\phi_P = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}}$

Bottom left corner:

$$\phi_w = 1 - y$$

$$\frac{d\phi}{dy}|_{\{w\}} = \frac{-2(\phi_P - 1 + y)}{\Delta x}, \frac{d\phi}{dx}|_{\{s\}} = 0$$

$$\int_S \rho\phi v\cdot n\, dS = \rho x\Delta y\left(\frac{\phi_E + \phi_P}{2} - 1 + y\right) + \rho y\Delta x\left(\frac{\phi_P - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right)$$

$$\int_S \Gamma\nabla\phi\cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_E - 3\phi_P + 1 - y}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_N - \phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_E+\phi_P}{2} - 1 + y\right) + \rho y\Delta x\left(\frac{\phi_P-\phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E-\phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_E-3\phi_P+1-y}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_N-\phi_P}{\Delta y}\right) = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4}$
- $\boldsymbol{\left(\frac{\rho\Delta y x}{2} + \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_E + \left(\frac{\rho\Delta y x}{2} + \frac{\rho y\Delta x}{2} + \frac{3\Gamma\Delta y}{\Delta x} + \frac{\Gamma\Delta x}{\Delta y}\right)\phi_P + \left(-\frac{\rho y\Delta x}{2} - \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_N = \frac{(2x*\Delta x+\Delta x^2)(2y*\Delta y+\Delta y^2)}{4} + \rho\Delta y x(1-y) + \frac{\Gamma\Delta y}{\Delta x}(1-y)}$

Bottom right corner:

$$\frac{d\phi}{dy}|_{\{w\}} = 0, \frac{d\phi}{dx}|_{\{s\}} = 0$$

$$\phi_s = \phi_p, \phi_e = \phi_p$$

$$\int_S \rho\phi v \cdot n\, dS = \rho x\Delta y\left(\frac{\phi_P - \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right)$$

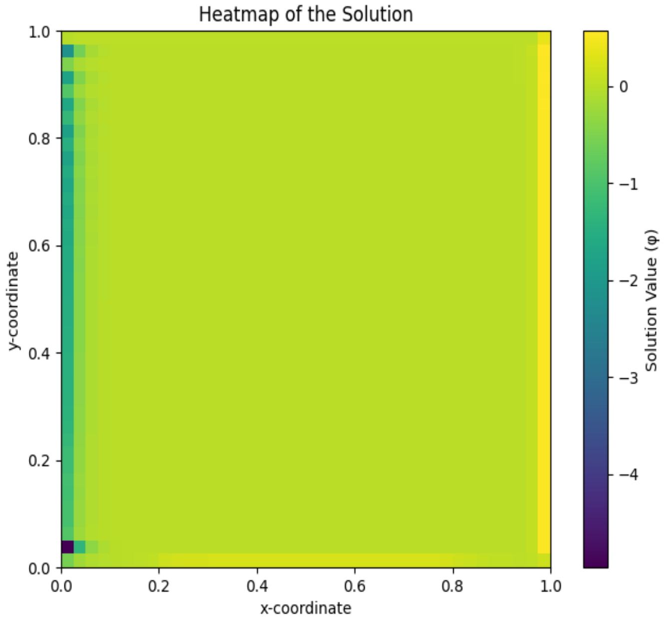
$$\int_S \Gamma\nabla\phi \cdot n\, dS = \Gamma\Delta y\left(\frac{\phi_W - \phi_P}{\Delta x}\right) + \Gamma\Delta x\left(\frac{\phi_N - \phi_P}{\Delta y}\right)$$

- $\rho x\Delta y\left(\frac{\phi_P - \phi_W}{2}\right) + \rho y\Delta x\left(\frac{\phi_P - \phi_N}{2}\right) + \rho\Delta x\Delta y\left(\frac{\phi_E - \phi_N}{2}\right) - \Gamma\Delta y\left(\frac{\phi_W - \phi_P}{\Delta x}\right) - \Gamma\Delta x\left(\frac{\phi_N - \phi_P}{\Delta y}\right) = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}$

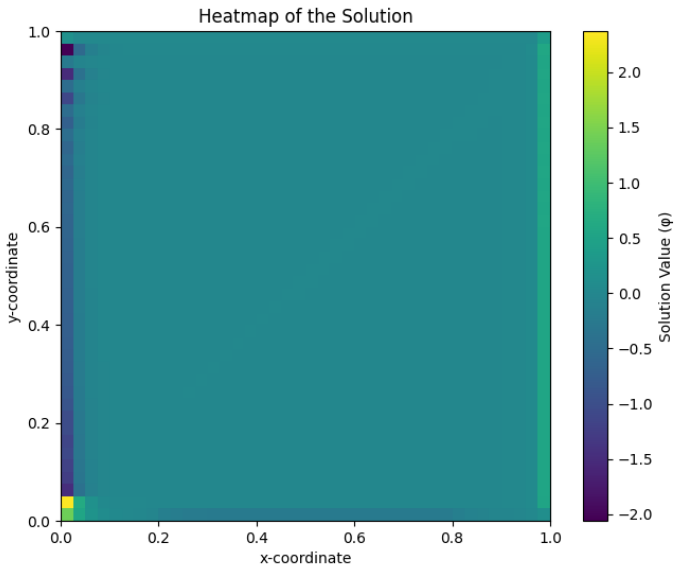
- $\boldsymbol{\left(\frac{\rho\Delta y x}{2} + \frac{\rho y\Delta x}{2} + \frac{\rho\Delta y\Delta x}{2} + \frac{3\Gamma\Delta y}{\Delta x} + \frac{\Gamma\Delta x}{\Delta y}\right)\phi_P + \left(-\frac{\rho y\Delta x}{2} - \frac{\rho\Delta y\Delta x}{2} - \frac{\Gamma\Delta x}{\Delta y}\right)\phi_N + \left(-\frac{\rho\Delta y x}{2} - \frac{\Gamma\Delta y}{\Delta x}\right)\phi_W = \frac{(2x*\Delta x + \Delta x^2)(2y*\Delta y + \Delta y^2)}{4}}$

HEAT MAP FOR DIFFERENT VALUES OF A:

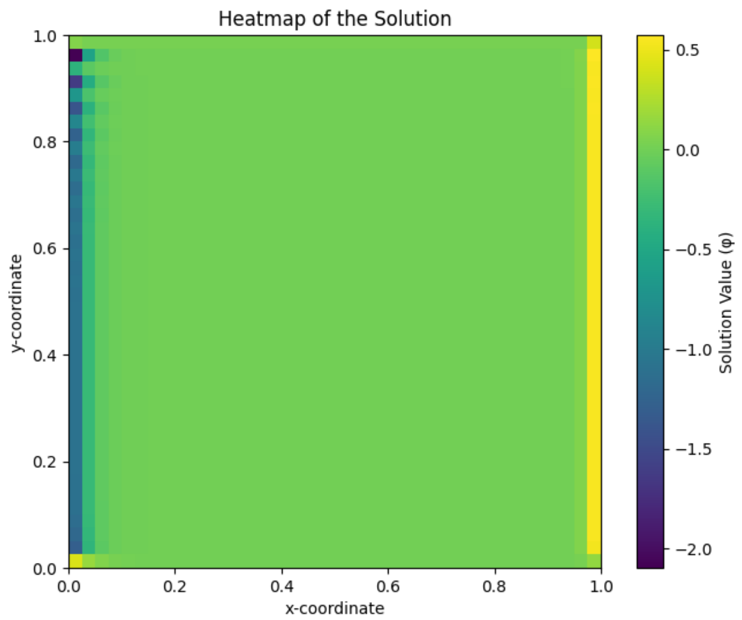
HEAT MAP FOR A = -0.1



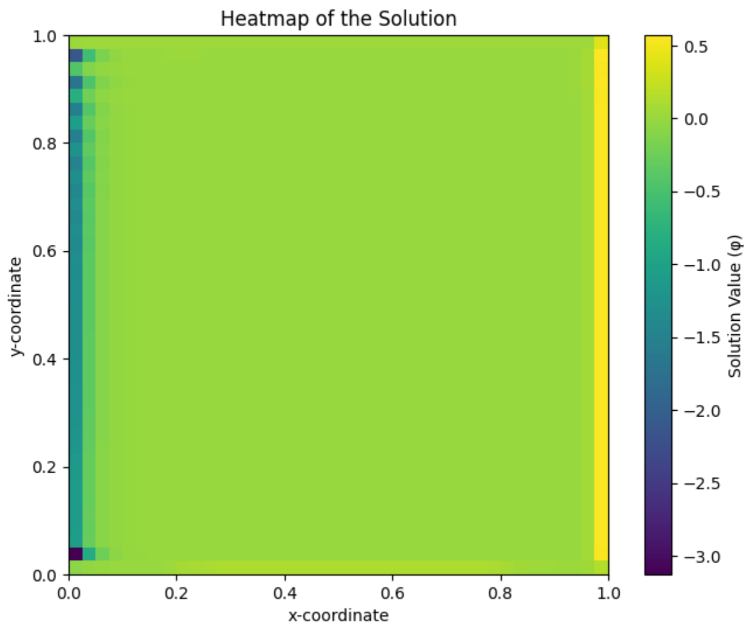
HEAT MAP FOR A =0.1



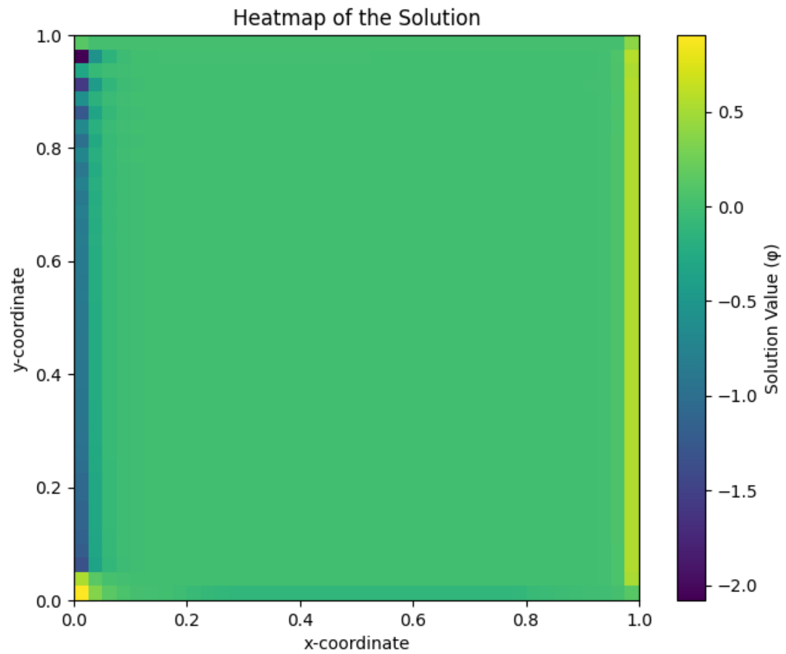
HEAT MAP FOR A = 0



HEAT MAP FOR A = -0.05



HEAT MAP FOR A = 0.05



Comment:

- As the value of 'a' increases from negative to positive i.e. −0.1 to 0.1, the value of ϕ increases in the inner domain.