Simulation of Complex Systems
Projects

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Summer term

Application 1: Two-Compartment Model

→ The two–compartment model has been a standard example in parameter optimization since many years:

$$\begin{aligned} y_1' &= -(p_3+p_1)\cdot y_1 + p_2\cdot y_2 \\ y_2' &= p_3\cdot y_1 - p_2\cdot y_2 \end{aligned}$$

with the initial conditions $y_1(0)=1,\ y_2(0)=0$

→ The reported optimal parameter values are $[0.2327, 0.2328]\times[1.9254, 1.9254]\times[0.1450, 0.1451]$ and $[1.9254, 1.9254]\times[0.2327, 0.2328]\times[0.1450, 0.1451]$ for the data from the corresponding literature

→ For this cooperative system, the brackets are

$$\begin{aligned} \underline{y}_1' &= -(\overline{p}_3+\overline{p}_1)\cdot \underline{y}_1 + \underline{p}_2\cdot \underline{y}_2 \qquad & \overline{y}_1' &= -(\underline{p}_3+\underline{p}_1)\cdot \overline{y}_1 + \overline{p}_2\cdot \overline{y}_2 \\ \underline{y}_2' &= \underline{p}_3\cdot \underline{y}_1 - \overline{p}_2\cdot \underline{y}_2 & \overline{y}_2' &= \overline{p}_3\cdot \overline{y}_1 - \underline{p}_2\cdot \overline{y}_2 \end{aligned}$$

with the same initial conditions.

Application 2: Optimization in MIMO Systems

A frequency flat MIMO link consisting of n_T transmitting and n_R receiving antennas can be described using the widely used linear stochastic model

$$\vec{y} = H\cdot\vec{c} + \vec{n}, \qquad \vec{y},\vec{n}\in\mathbb{C}^{n_\mathsf{R}},\ \vec{c}\in\mathbb{C}^{n_\mathsf{T}},\ H\in\mathbb{C}^{n_\mathsf{R}\times n_\mathsf{T}}\ , \qquad (1)$$

where $\vec{y}$ is the received data vector, $\vec{c}$ is the transmitted signal vector, $\vec{n}$ is the additive white Gaussian noise at the receiver side with the zero mean and the variance σ^2 in both real and imaginary parts, and H is the channel matrix.

Optimization in MIMO Systems (2)

Working with this model can be divided into the following steps. First, the system configuration has to be established, in particular, the number n_T, n_R of transmitting and receiving antennas. The second step is channel estimation producing a channel matrix H which characterizes the link. Having obtained H, we can transform the MIMO link into a number of independent, weighted, frequency flat single-input single-output links using, for example, singular value decomposition (SVD). This technique is employed to eliminate the interference between different antennas' data streams, which is true only for an *ideal* H and its SVD . The above mentioned weights (λ_l) are not necessarily equal and there is usually a need for power allocation. The goal of this final step is to optimize the link, for example, using Lagrange optimization and such criteria as the bit error rate (BER).

MIMO Systems: Power Allocation

The BER should be as small as possible for the whole link. One possibility to quantify BER in the general form for a transmission layer l using the complement error function erfc$(\cdot)$ is

$$P_b^{(l)} = f(M_l) \cdot \mathrm{erfc}\left(g(M_l, \lambda_l, \sigma^2, P_s^{(l)})\right) \quad . \tag{2}$$

Here, f, g are positive-valued functions depending on the number of bits per symbol M_l, the noise variance, the available transmit power per layer $P_s^{(l)}$ and the singular value λ_l corresponding to the considered layer. Usually, this singular value is not given exactly but is influenced by a (numerical or/and measurement) error ε_l of known magnitude, which can be represented by the interval $[\,\lambda_l - \varepsilon_l, \lambda_l + \varepsilon_l\,]$. Moreover, the noise variance might not be known exactly but similarly defined as an interval. That is, not only stochastic but also bounded uncertainty can be present, the major uncertainty source of the latter type being due to λ_l.

Minimizing the Overall BER for the MIMO Link

If the number of activated layers L is fixed (not all layers should necessarily be activated), the power allocation technique assigns more power to the layers with small weights depending on the number of transmitted bits per layer. Small λ_l lead to large values of $P_b^{(l)}$ since erfc$(\cdot)$ is monotonically decreasing with λ_l. The objective function is

$$\begin{aligned} J(\pi_1 \ldots \pi_L, \mu) \quad &= \quad \underbrace{\frac{2}{\sum\limits_{l=1}^{L} \log_2 M_l} \sum_{l=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) \cdot \mathrm{erfc}\left(\frac{\pi_l \lambda_l}{2\sigma} \sqrt{\frac{3 \cdot P_s}{L(M_l - 1)}}\right)}_{=: P_b \quad \text{(the overall BER)}} \\ &+ \quad \mu \left(\sum_{l=1}^{L} \pi_l^2 - L\right) \longrightarrow \min \ \text{w.r.t } \pi_i \quad . \end{aligned} \tag{3}$$

$\pi_1 > 0, \ldots, \pi_L > 0$ are the power allocation parameters with which we modify the weights λ_l, μ is the Lagrange multiplier, P_s is the overall available transmit power.

System Description

Consider a wireless MIMO link with $n_\mathrm{T} = n_\mathrm{R} = 4$ antennas

Desired throughput: 8 bit/s/Hz

The available transmit power: $P_s = 1$W

The singular values: $\lambda_1 = 3.698971$ $\lambda_2 = 2.107508$ $\lambda_3 = 1.348946$ $\lambda_4 = 0.366802$

The SNR: 10dB ($\sigma \approx 0.223607$)

Project 1

Consider the two-compartment model with $p_1 = 1.9255$, $p_2 \in [0.2, 0.3]$, $p_3 = 0.1451$

Task 1: Compute the analytical solution to the two-compartment model depending on the parameters

Task 2: Install a verified ODE solver library of your choice and compute the enclosure of the solution for the two-compartment model for all times $t \in [0, 16]$

Task 3: Compare your results

Project 2

Consider the two-compartment model with $p_1 \in [0.2, 0.3]$, $p_2 = 1.9255$, $p_3 = 0.1451$

Task 1: Install several verified ODE solver libraries of your choice (at least two) and compute the enclosure of the solution for the two-compartment model for all times $t \in [0, 16]$

Task 2: Compare your results

Project 3

Consider the two-compartment model with $p_1 \in [0.1, 0.4]$, $p_2 = 1.9255$, $p_3 = 0.1452$

Task 1: Show that the two-compartment model is a cooperative system

Task 2: Compute the enclosure of the solution to the two-compartment model using the bracketing approach for $t \in [0, 16]$ (using a non-verified solver; e.g., from BOOST)

Project 4

Consider the two-compartment model with $p_1 = 1.9255$, $p_2 \in [0.1, 0.4]$, $p_3 = 0.1452$

Task 1: Install a verified ODE solver library of your choice and compute the enclosure of the solution for the two-compartment model using it for $t \in [0, 16]$

Task 2: Compute the enclosure of the solution to the two-compartment model using the bracketing approach for $t \in [0, 16]$ (using a non-verified solver; e.g., from BOOST)

Task 3: Compare your results

Project 5

BER optimization for MIMO with $L = 2$ active layers

Suppose that $L = 2$, $M_1 = M_2 = M = 16$

Task 1: Minimize the BER analytically on a piece of paper as far as possible using Lagrange multipliers method

Task 2: Minimize the BER using global optimization functionality of C-XSC

Vary your SNR between 10 – 20 dB and plot both normal and optimized BER for both tasks

Compare your results

Project 6

BER optimization for MIMO with $L = 2$ active layers

Suppose that $L = 2,\ M_1 = 64,\ M_2 = 4$

Task 1: Minimize the BER analytically on a piece of paper as far as possible using Lagrange multipliers method

Task 2: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC

Vary your SNR between 10 – 20 dB and plot both normal and optimized BER for both tasks

Compare your results

Project 7

BER optimization for MIMO with $L = 2$ active layers

Suppose that $L = 2,\ M_1 = 32,\ M_2 = 8$

Task 1: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC

Task 2: Minimize the BER using global optimization functionality of C-XSC

Vary your SNR between 10 – 20 dB and plot both normal and optimized BER for both tasks

Compare your results

Project 8

BER optimization for MIMO with $L = 3$ active layers

Suppose that $L = 3$

Task 1: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC for $M_1 = 64,\ M_2 = 2,\ M_3 = 2$

Task 2: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC for $M_1 = 32,\ M_2 = 4,\ M_3 = 2$

Vary your SNR between 10 – 20 dB and plot both normal and optimized BER for both tasks

Compare your results

Project 9

BER optimization for MIMO with $L = 3,\ L = 4$ active layers

Task 1: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC for three active layers $M_1 = 32,\ M_2 = 4,\ M_3 = 2\ (L = 3)$

Task 2: Minimize the BER using Lagrange multipliers method in combination with the nonlinear equations solver functionality of C-XSC for four active layers $M_1 = 32,\ M_2 = 2,\ M_3 = 2,\ M_4 = 2\ (L = 4)$

Vary your SNR between 10 – 20 dB and plot both normal and optimized BER for both tasks

Compare your results