Homework 1 – CPSC 4100-01 , Winter 2017

"I have not received unauthorized aid on this assignment. I understand the answers that I have submitted. The answers submitted have not been directly copied from another source, but instead are written in my own words."

1) Select a data structure that you have seen previously, and discuss its strengths and limitations. [CLRS 1.1-3]

Linked Lists: A strength of a linked list is that insertion is rather fast and simple. A new node is simply attached to the front of the list or added between two existing nodes and the pointers are updated easily. A weakness however is that searching/deleting a specific value contained in the list is rather slow and requires a traversal of the entire list from beginning to end worst case. This is because there is no way to index a linked list.

2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n2$ steps, while merge sort runs in $64n\log n$ steps. For which values of n does insertion sort beat merge sort? [CLRS 1.2-2]

$8n^2 = 64n\log_2 n$ = about 44. When $n < 44$ insertion sort is faster than merge sort. Example, when $n = 43$, insertion sort: 14792 and merge sort: 14933. When $n = 45$, insertion sort: 16200 and merge sort: 15817.

3) Let Fi represents the i-th Fibonacci number ($F0$=$F1$=1). Prove that for all values of N≥3, the following equation is true using induction:

Theorem: $\Sigma_{i=1}^{N-2} (F_i) = F_N - 2$

Base Case: $N = 3$ $\quad \Sigma_1^{(3-2)} (F_i) = 1$ and $F_3 - 2 = 3 - 2 = 1$

Induction Hypothesis: Theorem is true for N

Induction Step: Prove $N + 1$

$\Sigma_{i=1}^{(N+1)-2} (F_i) = \Sigma_{i=1}^{N-1} (F_i)$. For $N = 3$: 3

$F_N - 2 + F_{N-1} = F_{N+1} - 2$. For $N = 3$: 3

4) Prove that the following equation is correct for all values of N≥1:

Theorem: Σ i=1 to N $(2i - 1) = N^2$

Base Case: $N = 1$ $\quad \Sigma$ i=1 to 1 $(2i - 1) = 2(1) - 1 = 1$ and $N^2 = 1^2 = 1$

Induction Hypothesis: Theorem is true for N

Induction Step: Prove $N + 1$

$\Sigma_{i=1}^{N+1} (2i - 1) = \Sigma_{i=1}^{N} (2i - 1) + (2(N+1) - 1)$

$= \Sigma$ i=1 to N $(2i - 1) + (2N + 1)$. For $N = 1$: 4

$N^2 + (2N + 1) = (N + 1)^2$. For $N = 1$: 4