Homework 3, CPSC 4100-01, Winter 2017

1) Regarding the celebrity problem that we discussed in the class, implement the modified algorithm with time complexity of $\Theta(n)$ in your favorite language. 10 points

```
string celebrityYesNo(int R[ ][ ], int rows, int columns, int n) {
        if (n == 0) return NULL;
        if (n == 1) return 1;
        int i = 0;
        int j = 1;
        while (j < columns) {
                if (R[i][j] == 0) j++;
                else if (R[i][j] == 1) {
                        i = j;
                        j++;
                }
        }
        return "Person " + (i + 1) + " is the celebrity.";
}
```

2) For the Maximum Consecutive Subsequence problem, improve the proposed $\Theta(n)$ algorithm such that it not only returns the maximum sum, but also returns the corresponding subsequence. No implementation is needed. 10 points

Once you have the max sum and you are at the end of the sequence, you move backtrack through the sequence and when the trailing sum equals the max sum you start recording the indexes. You keep recording until the trialing sum reaches 0 or you reach the beginning of the list. The last index you record is the first index in your subsequence and the first index you recorded is your last. Run time is 2n = Θ(n).

3) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures: 30 points

a) $f(n)=O(g(n))$ implies $g(n)=O(f(n))$

False: $f(n)=O(g(n))$ because f(n) grows at a slower rate than g(n). Therefore, $g(n)=\Theta(f(n))$

b) $f(n)=\Theta(f(n^2))$

False: $\lim_{n\to\infty}\left(\frac{n}{n^2}\right) = 0$ therefore because $f(n)=O(f(n^2))$ because they have the same growth rate. Also, if we look at $0 \le c_1 2n \le n \le c_2 2n$ we get $c_1 = \frac{1}{2}$ and $c_2 = 1$.

c) $f(n)=O(g(n))$ implies $\log f(n)=O(\log g(n))$ where $\log g(n)\geq 1$ and $f(n)\geq 1$ for all sufficiently large n.

True: If we assume f(n) = x and g(n) = x^2 for which $f(n)=O(g(n))$ is true, then if we take the $\lim_{x\to\infty}\left(\frac{log_2 x}{log_2 x^2}\right) = \frac{1}{2}$ which means that they grow at the same rate implying $\log f(n)=\Theta(\log g(n))$. This implies $\log f(n)=O(\log g(n))$ and $\log f(n)=\Omega(\log g(n))$.