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Homework 2

I have not received unauthorized aid on this assignment. I understand the answers that I have submitted. The answers submitted have not been directly copied from another source, but instead are written in my own words.

1) Determine T(n) in terms of a Big-O notation for the following cases. Use expansion method:

a. $T(n) = T(n-2) + bn + c$
$T(1) = d$

T(n - 2) = T(n - 4) + b(n - 2) + b(n) + c + c
T(n - 4) = T(n - 6) + b(n - 4) + b(n – 2) + b(n) + c + c + c

T(n) = T(n – 2k) + ck + $\sum_{i=0}^{k-1} b(n-2i)$ T(1) = T(n – 2k) k = $\frac{n-1}{2}$
T(n) = T(1) + $\frac{c(n-1)}{2}$ + $\frac{bn(n-1)}{2}$ – 2b $\sum_{i=0}^{\frac{n-1}{2}} i$
= d + $\frac{c(n-1)}{2}$ + $\frac{bn(n-1)}{2}$ – 2b $\frac{(\frac{n-1}{2})(\frac{n-1}{2}+1)}{2}$
= O(n^2)

where b, c, and d are constants.

b. $T(n)=2T(n-1) + 1$
$T(0) = 0$ T(0) = T(n – k) n = k

T(n – 1) = 2[T(n – 1 – 1) + 1] + 1
T(n – 2) = 2[2[T(n – 2 – 1) + 1] + 1] + 1
T(n – 3) = 2[2[2[T(n – 3 – 1) + 1] + 1] + 1] + 1

T(n) = 2^k [T(n – k)] + $\sum_{i=0}^{k-1} 2^i$ = $\sum_{i=0}^{n-1} 2^i$ = $\frac{1-2^{n-1}}{1-2}$
= O(2^n)

2) Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that max($f(n)$, $g(n)$) = Θ($f(n)$ + $g(n)$).

From this we can derive that: f(n) ≤ max($f(n)$, $g(n)$)
g(n) ≤ max($f(n)$, $g(n)$)
$f(n)$ + $g(n)$ ≤ 2max($f(n)$, $g(n)$)
$\frac{1}{2}f(n)$ + $g(n)$ ≤ max($f(n)$, $g(n)$)

We can consider this the lower bound of the Big-Theta definition. The upper bound can come found considering a value of n where ≥ max($f(n)$, $g(n)$).

For a value $n \geq 1$: $\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n)) \leq f(n) + g(n)$

3) Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

If we consider a function that would be in $o(g(n)) \cap \omega(g(n))$, f(n):

For a value, such as $c > 0$, there is a constant n_0 where $0 \leq f(n) < cg(n)$ when $n \geq n_0$.
For a value, such as $c > 0$, there is a constant n_1 where $0 \leq cg(n) < f(n)$ when $n \geq n_1$.

If we assume that $n_1 > n_0$ then for all $n \geq n_1$, $cg(n) < f(n) < cg(n)$ which cannot be true. Thus, there is no function in $o(g(n)) \cap \omega(g(n))$.