Centre for Mathematical Sciences, Mathematics, LTH, September 2021

Image Analysis, Assignment 2

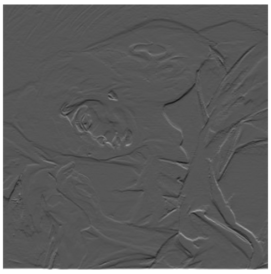
1 Filtering

Shown below is an image and the results of convolving this image with six different filters. Combine each convolved image with the correct filter.

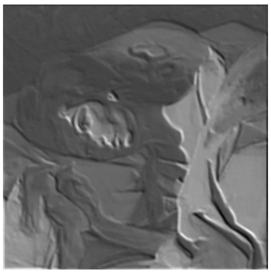
$$f_1 = \begin{bmatrix} -1 & 1 \end{bmatrix} \qquad f_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad f_3 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$f_4 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad f_5 = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

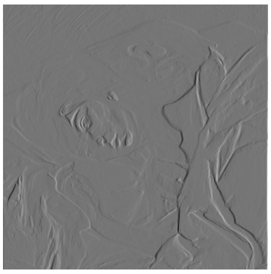
Original



A



B



C

D

E

For the report: Write down your answers *with a short motivation.*

2 Interpolation

Assume a one-dimensional image (or signal)

$$f = \begin{bmatrix} 3 & 4 & 7 & 4 & 3 & 5 & 6 \end{bmatrix}, \quad \text{(at points } i = 1, \ldots, 7).$$

a) Explain what linear interpolation means. Sketch the linear interpolation $F_{lin}(x)$ of f for $1 \leq x \leq 7$. Is $F_{lin}(x)$ continous? Is $F_{lin}(x)$ differentiable?
b) The interpolation can be expressed using the following equation

$$F_g(x) = \sum_{i=1}^{7} g(x-i)f(i), \tag{1}$$

where the function $g(x)$ is different for different types of interpolation. Which function $g(x)$ correspond to linear interpolation?

c) Let $g(x)$ be defined as

$$g(x) = \begin{cases} |x|^3 - 2|x|^2 + 1 & \text{if } |x| \leq 1, \\ -|x|^3 + 5|x|^2 - 8|x| + 4 & \text{if } 1 < |x| \leq 2, \\ 0 & \text{if } |x| > 2. \end{cases}$$

Plot $g(x), -3 \leq x \leq 3$. Plot the interpolation $F_g(x)$ of the function f using this $g(x)$ for $1 \leq x \leq 7$ (e.g. using Matlab for the calculations). Is $F_g(x)$ continous? Is $F_g(x)$ differentiable? (here you don't have to show whether this is true or not rigorously, but you need to have an argument that is better than "you can see it in the graph").

For the report: Provide answers to all questions, *with complete solutions and motivations.*

3 Classification using Nearest Neighbour and Bayes theorem

As output from an imaging system we get a measurement that depends on what we are seeing. For three different classes of objects we get the following measurements.
Class 1:

0.4003, 0.3988, 0.3998, 0.3997, 0.4010, 0.3995, 0.3991

Class 2:

0.2554, 0.3139, 0.2627, 0.3802, 0.3287, 0.3160, 0.2924

Class 3:

0.5632, 0.7687, 0.0524, 0.7586, 0.4243, 0.5005, 0.6769

3.1 Nearest Neighbours

Use nearest neighbour classification. Assume that the first four measurements in each class are used for training and the last three for testing. How many measurements will be correctly classified ?

3.2 Gaussian distributions

Assume that the measurements have normal distribution with known parameters (expected value m_i and variance σ^2). For class 1: $m_1 = 0.4, \sigma_1 = 0.01$. For class 2: $m_2 = 0.3, \sigma_2 = 0.05$. For class 3: $m_3 = 0.5, \sigma_3 = 0.2$. All classes are as likely to occur. What is the maximum a posteriori classification of a measured x? How many measurements are correctly classified? In this case you can try to classify all seven samples in each class, since no measurements were used for training. Hint: Matlab has a function called 'normpdf'.

> For the report: Provide a complete solution, i.e. in addition to your results, for at least one example, provide your calculations for both nearest neighbour classification and the Gaussian classification.

4 Classification

A scene is generated by some random mechanism. By studying a large number of noise free realizations of the scene we have that (for the three possible cases):

$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \quad \text{has prob} \quad \frac{1}{4}; \qquad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad \text{has prob} \quad \frac{1}{2}; \qquad \begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} \quad \text{has prob} \quad \frac{1}{4}$$

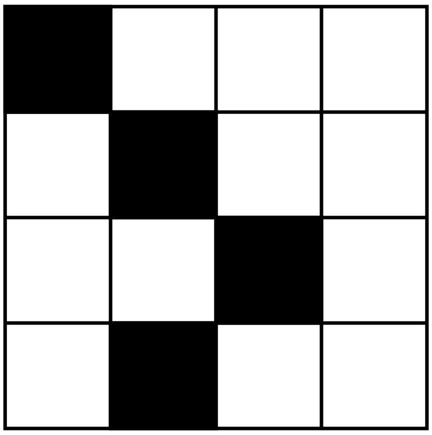
(a priori probabilities). One of these images has been distorted by noise in the sense that the value of a pixel has been changed with the probability ϵ, i.e. Prob (observing 0 | the correct value is 1) $= P$ (observing 1 | the correct value is 0) $= \epsilon$. Assume that noise in different pixels is independent. Now consider the image

$$\begin{matrix} 0 & 1 \\ 1 & 1 \end{matrix}$$

> For the report: Using Bayes theorem calculate a MAP (maximum a posteriori) estimation of the scene if (a) $\epsilon = 5\%$ and (b) if $\epsilon = 50\%$. Show the answers in percentages or percentage decimals

5 Classification

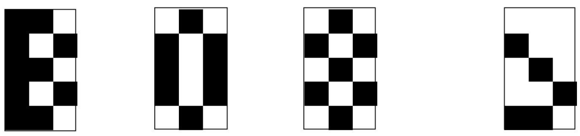
Consider a binary 4×4 image of a scene with a vertical line. In the *correct* image all pixels would be white except one vertical row with black pixels. Unfortunately the camera used is far from perfect. Errors in different pixels are independent with p(white | line)=p(black | not line)=ϵ and consequently, Prob(black | line)=Prob(white | no line)=$1 - \epsilon$. Assume the a priori probability for the line to be located in column 1 or 4 is 0.3 (each) and the the a priori probability that the line is in column 2 or 3 is 0.2 (each). Calculate the maximum a posteriori estimation of the following image when $\epsilon = 0.2$.



For the report provide the a priori probabilities of the four images. Calculate the a posteriori probabilities for the four images. Which is the most probable image. Show the answers in percentages or percentage decimals

6 Classification

An image (with 5×3 pixels) has probability 0.25 of being a 'B' (image ω_1), 0.4 of being a '0' (image ω_2) and 0.35 of being an '8' (image ω_3). In the final image there is also measurement noise. The probability of error is bigger for white pixels. An truly white pixel has a probability 0.35 of being wrongly measured as black in x. An truly black pixel has a probability 0.25 of being wrongly measured as white in x. How should the image x below be classified using maximum a posteriori probabilities (MAP) in the classification criterion.



'B' '0' '8' x

Figure 1: Mean images for three different classes, and an observed image x.

For the report provide the a priori probabilities of the three images. Calculate the a posteriori probabilities for the three images. Which is the most probable image? Show the answers in percentages or percentage decimals.

7 The OCR system - part 2 - Feature extraction

During all of the four assignments you are gradually going to build and test a small system for ocr (optical character recognition). During this second assignment your task is to write one function `x = segment2features(B)` in matlab. And to test this function on a few images using a benchmark script `inl2_test_and_benchmark`.

On the assignment page on Canvas there is a data file available. By downloading and unpacking the file you will obtain a folder `ocr_project` and in this folder there are two folders `datasets matlab` In the datasets folder there is for now only one folder 'short1', which contains a few test examples and ground-truth both for segmentation and for recognition. Later on we will add more folders with additional (and more challenging images).

Given a binary image B (thus only with zeros and ones), study the region of pixels that are equal to one. Use your imagination to define *at least, possibly more than* six different features (numbers), that can be used to classify which letter the region corresponds to. It is good if the feature (the value) does not depend on which position the region has. In other words, if the regions is shifted in the x or y direction, then the same features will be obtained.

Write a function in matlab that, given such and image matrix B as input, returns a feature vector

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} .$$

One suitable function name is

```
function x = segment2features(B);
```

Suggestions: One could try using moments, histograms, the sum of the pixel values along a row or in a subregion, etc. You can also think about the filter bank idea from the lecture on features. Remember that it's difficult to know which features that work well, without testing. To remove the influence of the position one could first translate the image so that the centre of mass is in the origin. Normalizing features so that they all have approximately the same range of values is also often a good idea (e.g. use values between 0 and 1 or -1 and 1).

In the zip-file from the Canvas page, you can find both images and an unfinished routine `inl2_stub.m` that can be used to try things out. Try both `im2segment` (from Handin 1) and then `segment2features` on such images. Do you get similar feature vectors for the same character? Do you get different feature vectors for different characters?

When you are finished you can also try running `inl2_test_and_benchmark`. In the script there is a simple visualization part that might be used to get a better feeling for the features.

For the report describe and motivate your features, supply both code (e.g. matlab code) and a printout the results of using your algorithm, i.e. supply examples of input data (e.g. as images) and the corresponding feature vectors, both for different characters and for different images of the same character.