Lund University

Computer Vision

FMAN95

Assignment 3

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Contents

2 The Fundamental Matrix **1**

2.1 E1 1

2.2 E2 1

2.3 E3 1

2.4 CE1 2

2.5 E4 3

2.6 CE2 4

3 The Essential Matrix **5**

3.1 CE3 5

3.2 E6 6

3.3 CE4 7

2 The Fundamental Matrix

2.1 E1

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} A & t \end{bmatrix} \tag{1}$$

$$[\,t\,]_\times = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \qquad F = [\,t\,]_\times A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix} \tag{2}$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \ell = Fx = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} \sim \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \tag{3}$$

$$\ell^T x_1 = \ell^T x_2 = 0 \qquad \ell^T x_3 = -2 \neq 0 \tag{4}$$

So x_1 and x_2 could be projections but not x_3

2.2 E2

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} A & t \end{bmatrix} \tag{5}$$

$$\mathbb{C}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad 0 = P_2 \mathbb{C}_2 \;\Rightarrow\; \mathbb{C}_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$e_1 = P_1 \mathbb{C}_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \qquad e_2 = t = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \tag{7}$$

$$[\,t\,]_\times = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix} \qquad F = [\,t\,]_\times A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix} \tag{8}$$

Determinant of F is clearly 0 and the other statements holds.

2.3 E3

$$F = N_2^T \tilde{F} N_1 \tag{9}$$

2.4 CE1

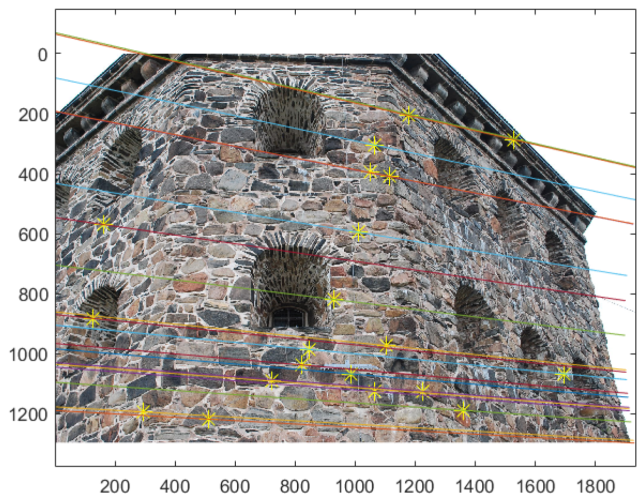

Figure 1: A few randomly selected points with corresponding epipolar lines

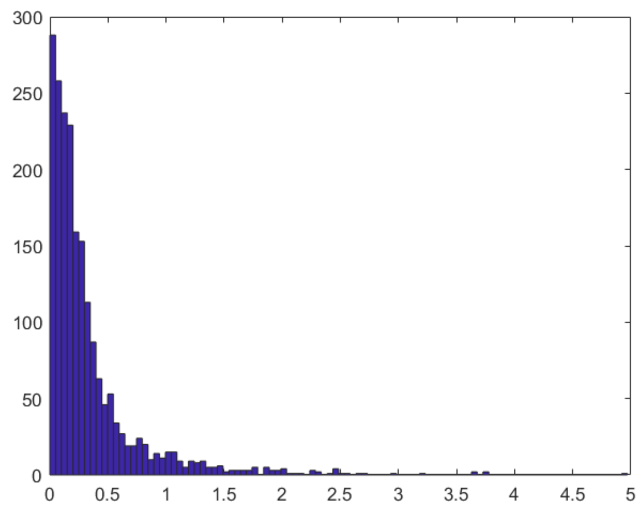

Figure 2: Histogram over distances of points from their epipolar lines

$$F = \begin{bmatrix} -0.0000 & -0.0000 & 0.0058 \\ 0.0000 & 0.0000 & -0.0269 \\ -0.0073 & 0.0265 & 1.0000 \end{bmatrix} \tag{10}$$

2.5 E4

$$F = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \mathbb{X}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad \mathbb{X}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} \tag{11}$$

$$F^T e_2 = 0 \qquad \Rightarrow \qquad e_2 = k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \Rightarrow \qquad P_2 = k \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 2 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Projections

$$P_1 : \qquad x_{1_1} = \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix} \qquad x_{1_2} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{13}$$

$$P_2 : \qquad x_{2_1} = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix} \qquad x_{2_2} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \tag{14}$$

Using this we confirm that $x_2^T F x_1 = 0$ for both points.

For camera center we solve the system

$$P_2 \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0 \quad \Rightarrow \quad \text{center} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \tag{15}$$

2.6 CE2

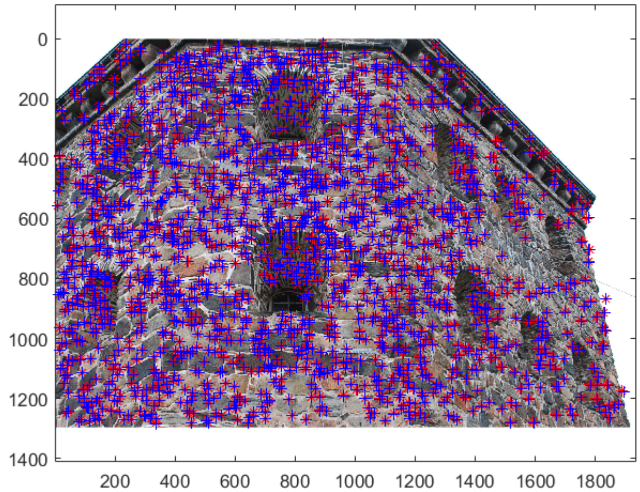

Figure 3: Points and projected points for camera 2

$$P_2 = \begin{bmatrix} -0.0016 & 0.0058 & 0.2166 & 0.9763 \\ 0.0071 & -0.0259 & -0.9762 & 0.2166 \\ 0.0000 & 0.0000 & -0.0276 & 0.0001 \end{bmatrix} \tag{16}$$

3 The Essential Matrix

3.1 CE3

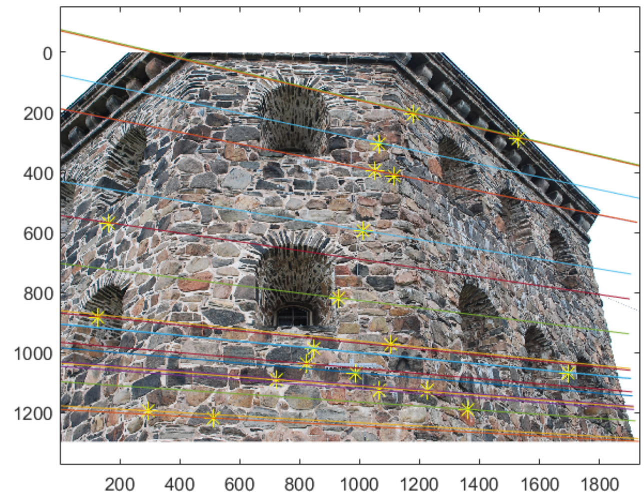

Figure 4: A few randomly selected points (same as CE1) with corresponding epipolar lines

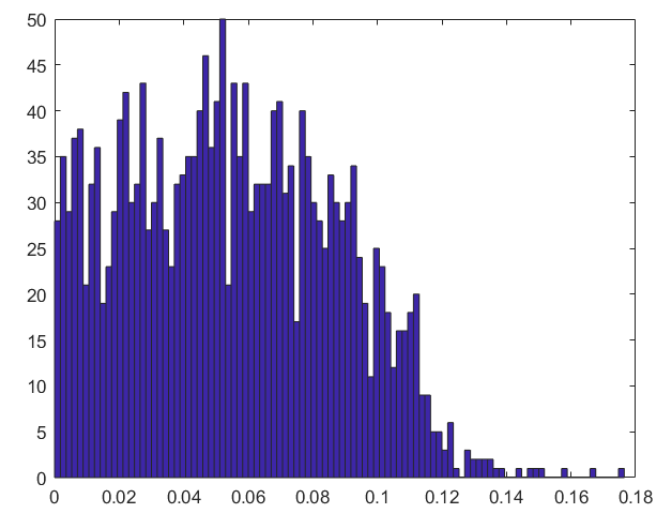

Figure 5: Histogram over distances of points from their epipolar lines

$$F = 10^3 \cdot \begin{bmatrix} -0.0089 & -1.0058 & 0.3771 \\ 1.2525 & 0.0784 & -2.4482 \\ -0.4728 & 2.5502 & 0.0010 \end{bmatrix} \quad \text{(Note the } 10^3\text{)} \tag{17}$$

3.2 E6

$$UV^T = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{bmatrix} \qquad det(UV^T) = 1 \tag{18}$$

$$E = U\text{diag}([1\ 1\ 0])V^T = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

With the four camera solutions

$$P_{2_a} = \frac{1}{\sqrt{(2)}} \begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -\sqrt{2} & 0 & 1 \end{bmatrix} \qquad P_{2_b} = \frac{1}{\sqrt{(2)}} \begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -\sqrt{2} & 0 & -1 \end{bmatrix} \tag{20}$$

$$P_{2_c} = \frac{1}{\sqrt{(2)}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -\sqrt{2} & 0 & 1 \end{bmatrix} \qquad P_{2_d} = \frac{1}{\sqrt{(2)}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -\sqrt{2} & 0 & -1 \end{bmatrix} \tag{21}$$

Which gives us

$$a: \begin{cases} s = -\frac{1}{\sqrt{2}} \\ \lambda = -\frac{1}{\sqrt{2}} \end{cases} \qquad b: \begin{cases} s = \frac{1}{\sqrt{2}} \\ \lambda = -\frac{1}{\sqrt{2}} \end{cases} \qquad c: \begin{cases} s = \frac{1}{\sqrt{2}} \\ \lambda = \frac{1}{\sqrt{2}} \end{cases} \qquad d: \begin{cases} s = -\frac{1}{\sqrt{2}} \\ \lambda = \frac{1}{\sqrt{2}} \end{cases} \tag{22}$$

3D point in front of both cameras for c.

3.3 CE4

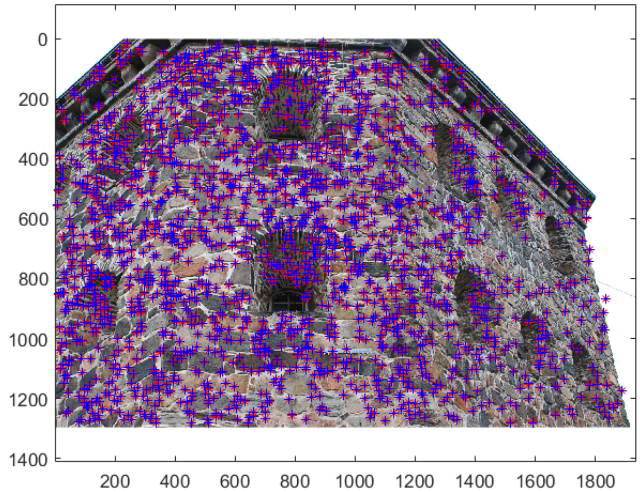

Figure 6: Points and projected points for camera 2

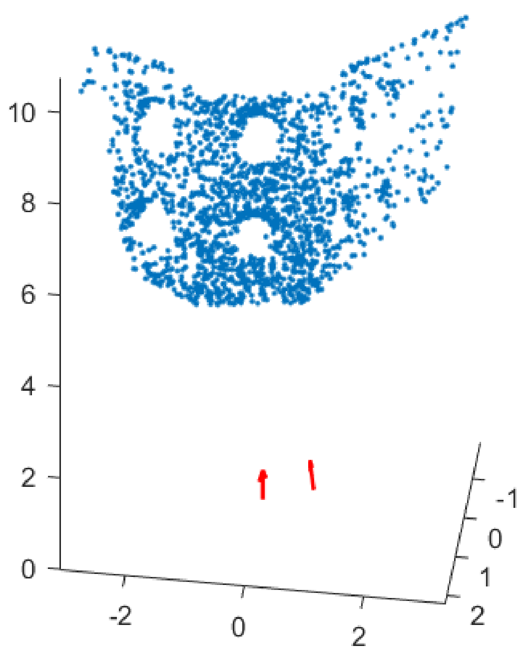

Figure 7: 3D-points and cameras