Statistical Learning in Marketing

Individual Assignment 1

Faculty of Economics and Business

Methodological Approach

Managerial Question: To what extent does advertising (both Ixmør's and its competitors') drive Ixmør's sales growth over time compared to other factors such as price, and how significant is the lagged effect of advertising, given that its impact is usually not immediate and can take about one quarter to materialize?

Methodology Approach

Dynamic system analyses are used explore the dynamic interactions between Ixmor's sales, advertising, competitor advertising, and pricing over time. The approach is designed to answer how these factors influence sales performance both in the short term and over multiple periods, allowing us to analyze immediate versus lagged effects while also accounting for seasonality.

1. General Data Analysis

The initial analysis involved importing and inspecting the dataset, **visualizing key variables** over time, and **fitting linear regression** models to assess trends. **Combination plots** explored relationships between paired variables, while **binary bar plots** compared the presence of Ixmør's advertising to competitors. These steps provided insights into trends before moving on to further analysis.

2. Granger Causality Tests

The Granger causality analysis tested whether past values of one variable could predict another, focusing on Ixmør's sales, advertising, competitor advertising, and pricing. **Lag lengths of up to 15 weeks** (one quarter) were examined, with a **10% significance threshold** to identify meaningful causal links. This helped determine which factors significantly influenced sales, ensuring only impactful relationships were considered for further analysis.

3. Unit Root Tests

Unit root tests (ADF, PP, and KPSS) assessed the stationarity of key variables, with the **Phillips-Perron (PP) test prioritized** for its robustness. Based on theoretical expectations, **type 1 non-stationarity was assumed unlikely**. Results confirmed all **series were stationary,** making **cointegration tests unnecessary. Scatter plots visually confirmed** these findings, providing a solid foundation for VAR modeling.

4. Vector Autoregressive

A **VAR** model was developed to explore interactions among the 5 **(endogenous)** variables. **Lag selection** using AIC and BIC indicated an optimal lag of 1. Quarterly dummies (Qrtr1, Qrtr2, Qrtr3) were included as **exogenous controls** for seasonality, with Qrtr4 excluded to **avoid the dummy variable trap**. The model identified significant interactions at lag 1, forming the basis for further analysis through Impulse Response Functions (IRFs) and Forecast Error Variance Decomposition (FEVD).

5. Impulse Response Functions

Impulse Response Functions (IRFs) were generated **using the VAR model** to analyze normal and cumulative effects of shocks to the endogenous variables over 1**2 periods**. A total of **50 graphs** (5 normal and 5 cumulative per variable) were produced. A **68% confidence interval**, representing **1 standard deviation**, was used instead of the stricter 95%, enabling more practical insights into the dynamic effects of both Ixmør's and competitors' advertising and pricing on sales performance.

6. Forecast Error Variance

FEVD was conducted **using the VAR model** to determine the contribution of shocks from LnSales, LnAdvertising, LnCompAdvertising, LnPrice, and LnCompPrice to the forecast error variance over **12 periods.** This analysis identified the relative importance of each factor—such as advertising or pricing—in driving sales variability, providing insight into the proportional impact of Ixmør's and competitors' actions on sales.

Evaluation of the Managerial Question (½)

Advertising Variables	The **Granger Causality tests (**Appendix 2.4, Table 1) confirm the Managers suggestion that past values of **Advertising** (p = 0.0971) and **Competitor's Advertising** (p = 0.0513) granger cause the Sales of Ixmor, with the most significant granger effect being at 1 lag. It is also important to note that Sales granger cause Advertising (p = 0.0330) and Competitor's Advertising (p = 0.0121) at lags 1 and 2 respectively. This suggests that not only does past Advertising from Ixmor and the competitor affect Ixmor's sales, but Ixmor's past sales also impact the Advertising of Ixmor and its competitor.
Price Variables	However, relying solely on advertising could be detrimental. The Granger Causality tests (Appendix 2.4, Table 1) also provide strong evidence that **price variables** are significant determinants of Ixmor's sales. Ixmor's price (p = 0.0200) and Competitor's Price (p = 0.0185) significantly Granger-cause Ixmor's Sales, with effects observed at lags 1 and 11, respectively. The 11-week lag for Competitor's Price suggests a delayed yet substantial influence on Ixmør's sales, indicating that competitors' pricing strategies require careful monitoring over time. Ixmør's Sales also Granger-cause Competitor's Price (p = 0.0149) at lag 1, highlighting the dynamic interaction between sales performance and competitive pricing decisions. This implies that pricing plays a bidirectional role which makes considering pricing dynamics necessary along with advertising strategies for effective decision-making.
Stationary or Evolving Series?	Before applying the VAR model, it was essential to confirm the stationarity of key variables through **Unit Root tests** (Appendix 2.5, Tables 2-6). Given the nature of the variables, a Type 1 scenario (random walk without drift) was highly unlikely in practice, as evidenced by graphs in Appendix 2.1, which show that none of the series are centered around zero, suggesting the presence of a drift component. Therefore, scenarios of Type 2 or 3 were assumed to be more realistic. Using the Phillips-Perron test at a 0.10 significance level, all five variables were confirmed to be stationary, ensuring reliable and consistent relationships for further analysis in the VAR model. Since all variables are stationary, their statistical properties (e.g., mean, variance) remain constant over time. When the variables are stationary, cointegration test are not necessary. To be sure, scatter plots (Appendix 2.6, Figures 11-19) were used to confirm these results. This consistency implies that the relationships identified between the variables in the model are stable, which makes future predictions more reliable. For managers, this means that strategic actions such as advertising investments or pricing changes are likely to have predictable and consistent effects, simplifying planning and decision-making.
VAR Model	The **VAR model** helps assess the dynamic relationships between Ixmor's sales, advertising, competitor advertising, pricing, and competitor pricing. Including all variables allows an analysis of the ripple effect that different managerial actions create across the marketing mix. A lag of 1 was found to be optimal (Appendix 2.7, Table 7). Quarter dummies were included as exogenous variables for seasonality, while the five time-series variables are endogenous. Type was set to "const" since all variables in the model were stationary. The Granger Causality test indicated that Advertising, Competitor Advertising, Price, and Competitor Price all have predictive power for sales. However, the VAR model (Appendix 2.7, Table 8) shows that only LnSales, LnPrice, and LnCompPrice significantly impact LnSales, suggesting that advertising has predictive potential but may not have a direct effect when all factors are considered. LnPrice negatively affects LnSales (Estimate = -0.4035, p = 0.0101), whereas LnCompPrice positively affects LnSales (Estimate = 0.4236, p = 0.0218). Managers should consider placing more emphasis on pricing since it has a more direct impact on sales—specifically, avoiding price hikes to maintain competitiveness while leveraging competitor price increases to capture market share. The LnPrice model (Appendix 2.7, Table 9) reveals that Ixmor's pricing is primarily influenced by its own past prices (Estimate = 0.7321, p < 2e-16), highlighting price inertia, while competitor price has a marginal negative effect (Estimate = -0.1306, p = 0.0781). None of the advertising variables significantly influence Ixmor's pricing, emphasizing that pricing is largely independent of advertising strategies. Similarly, the LnCompPrice model shows that competitor pricing is driven by its own previous prices (Estimate = 0.7862, p < 2e-16) and also reacts to Ixmor's past sales (Estimate = 0.0480, p = 0.0054), indicating that competitors directly adjust pricing based on Ixmor's sales performance, but less on past pricing or advertising.

Evaluation of the Managerial Question (2/2)

VAR Model (Continued)	Furthermore, the quarter dummies significantly affect LnAdvertising (Appendix 2.7, Table 8), showing advertising spend peaks in Q3 (Estimate = 3.01876, p = 0.00027). Unlike other variables, LnSales and LnAdvertising are significant in the LnAdvertising model, indicating Ixmor's advertising is influenced mainly by its own past sales and advertising rather than competitors. The insignificant constant suggests there is no fixed baseline for advertising, emphasizing responsiveness to performance trends. Finally, the LnCompAdvertising model (Appendix 2.7, Table 9) shows that the only influential variable is past Competitor Advertising (Estimate = 0.31643, p = 7.07e-06).
Impulse Response Function	To explore interactions between marketing mix variables, **Impulse response functions (IRFs)** were used to assess how shocks to one variable impact the others over time. Orthogonal one-standard deviation shocks were applied to all five variables, generating a total of 50 IRF graphs (25 immediate and 25 cumulative: Appendix 2.8, Figures 20 - 69). Since all variables are stationary (Appendix 2.5, Tables 2-6), the effects of these shocks diminish over time. The immediate IRFs are used to show the direct magnitude of effects at certain lags, while the cumulative IRFs show the overall accumulated impact of the effect. The IRFs for each variable in response to a shock to itself display a consistent pattern: an immediate peak effect that diminishes over time, reflecting a typical response to self-shocks where the impact is strongest initially and then gradually reverts as the system stabilizes. The following results are crucial to the analysis: The effect of a one-standard-deviation shock to LnAdvertising on LnSales reaches a peak impact of approximately 1.5% increase at lag 3. (Table 21) The effect of a one-standard-deviation shock to LnCompAdvertising on LnSales reaches a peak impact of approximately 1.7% decrease at lag 2. (Table 22) The effect of a one-standard-deviation shock to LnPrice on LnSales reaches a peak impact of approximately 2.3% decrease at lag 2. (Table 23) The effect of a one-standard-deviation shock to LnCompPrice on LnSales reaches a peak impact of approximately 2.3% increase at lag 4. (Table 24) These findings align well with the earlier VAR model results. The VAR summary highlighted that only LnSales, LnPrice, and LnCompPrice significantly impact LnSales, with LnPrice negatively affecting LnSales and LnCompPrice having a positive effect. The IRF analysis extends these findings by showing that a shock in LnPrice decreases sales, while a shock in LnCompPrice increases sales, supporting the direct effects observed in the VAR model. In contrast, while advertising variables did not directly impact sales in the VAR model, their predictive power was evident from the Granger Causality results, and the IRFs illustrate that advertising shocks do have short-term effects on sales. However, the magnitude of these effects is smaller than the Price related variables. The rest of the IRF graphs also agree with the discussed VAR findings. See Appendix 2.7, Tables 25-69 for further details.
Forecast Error Variance Decomposition	The **FEVD graph** (Appendix 2.9, Figure 70) shows that the majority of all forecast error variances are primarily due own past shocks. This indicates that, over time, each variable's performance is largely explained by its own history rather than by external influences from other variables. In accordance with the VAR and IRF results, LnSales is impacted to a greater extent by Price related variables when compared to Advertising related variables. Notably, LnPrice and LnCompPrice are also significantly affected by LnSales.
Overall Manager Advice	Pricing has a more direct effect on Ixmor's sales than advertising. While both Advertising and Price granger cause LnSales, the VAR model shows only LnPrice and LnCompPrice significantly impact sales. The greater impact of Ixmor's Pricing and Competitor Pricing is also supported by the IRF and FEVD analyses. Prioritizing pricing strategies—such as avoiding price increases and leveraging competitor price hikes—is crucial for Sales growth. Sales are influenced by their history and pricing, but advertising should not be ignored, especially considering seasonality and other potential factors.

Appendices

Appendix 1: R Code

Appendix **1** - Lines 1-50

```
1 #Work Directory and libraries ----
2 setwd("C:/Users/nilsd/OneDrive/Documents/Statistical Learning in Marketing/Individual Assignments")
3
4 #Libraries
5 library(tidyverse)
6 library(lmtest)
7 library(aTSA)
8 library(urca)
9 library(strucchange)
10 library(vars)
11 library(gplots)
12 library(RColorBrewer)
13 library(lmtest)
14
15 #Importing Dataset  ----
16 Ixmor.df <- read_csv("4766946_timeseries.csv")
17
18 #Data overview ----
19 str(Ixmor.df)
20 summary(Ixmor.df)
21 sum(is.na(Ixmor.df))
22
23 # Data Overview ----
24
25 ## Individual Line Plots ----
26
27 ### Weeks and LnSales ----
28 plot(Ixmor.df$Week, Ixmor.df$LnSales, type="l", col="red", lwd=5,
29      xlab="Week", ylab="Log of Ixmor Sales", main="Ixmor Sales over time (weeks)")
30 model_LnSales <- lm(Ixmor.df$LnSales ~ Ixmor.df$Week)
31 abline(model_LnSales, col = "blue", lwd = 2)
32 summary(model_LnSales)
33 #trend line shows LnSales is increasing over time, and this model is statistically significant (p = 2.09e-06)
34
35 ### Weeks and Ixmor LnAdvertising ----
36 plot(Ixmor.df$Week, Ixmor.df$LnAdvertising, type="l", col="red", lwd=5,
37      xlab="Week", ylab="Log of Ixmor Advertising", main="Ixmor Advertising over time (weeks)")
38 model_LnAdvertising <- lm(Ixmor.df$LnAdvertising ~ Ixmor.df$Week)
39 summary(model_LnAdvertising)
40 abline(model_LnAdvertising, col = "blue", lwd =2)
41 #trendline shows LnAdvertising is increasing over time, and this model is statistically significant (p = 0.00707)
42
43 ### Weeks and LnCompAdvertising ----
44 plot(Ixmor.df$Week, Ixmor.df$LnCompAdvertising, type="l", col="red", lwd=5,
45      xlab="Week", ylab="Log of Competitor Advertising", main="Competitor Adveritisng over time (weeks)")
46 model_LnCompAdvertising <- lm(Ixmor.df$LnCompAdvertising ~ Ixmor.df$Week)
47 abline(model_LnCompAdvertising, col = "blue", lwd =2)
48 summary(model_LnCompAdvertising)
49 #trendline shows LnCompAdvertising is decreasing over time, but this model is statistically insignificant (p = 0.311)
50
```

Appendix **1** - Lines 51-100

```r
51 ### Weeks and LnPrice ----
52 plot(Ixmor.df$Week, Ixmor.df$LnPrice, type="l", col="red", lwd=5,
53      xlab="Week", ylab="Log of Ixmor Price", main="Ixmor Price over time (weeks)")
54 model_LnPrice <- lm(Ixmor.df$LnPrice ~ Ixmor.df$Week)
55 abline(model_LnPrice, col = "blue", lwd = 2)
56 summary(model_LnPrice)
57 #trendline shows LnPrice is decreasing over time, and this model is statistically significant (p = .00309)
58 
59 ### Weeks and LnCompPrice ----
60 plot(Ixmor.df$Week, Ixmor.df$LnCompPrice, type="l", col="red", lwd=5,
61      xlab="Week", ylab="Log of Ixmor CompPrice", main="Ixmor CompPrice over time (weeks)")
62 model_LnCompPrice <- lm(Ixmor.df$LnCompPrice ~ Ixmor.df$Week)
63 abline(model_LnCompPrice, col = "blue", lwd = 2)
64 summary(model_LnCompPrice)
65 #trendline shows LnCompPrice is increasing over time, and this model is significant (p < 2e-16)
66 
67 # LnSales, LnPrice, LnCompPrice, and LnAdvertising are significantly correlated to Weeks, while LnCompAdvertising is not.
68 
69 ## Combination Line Plots ----
70 
71 ### LnAdvertising and LnCompAdvertising ----
72 ggplot(data = Ixmor.df, aes(x = Week)) +
73   geom_line(aes(y = LnAdvertising, color = "LnAdvertising")) +
74   geom_line(aes(y = LnCompAdvertising, color = "LnCompAdvertising")) +
75   scale_color_manual(values = c("LnAdvertising" = "red", "LnCompAdvertising" = "blue")) +
76   labs(x = "Week", y = "LnAdvertising", color = "Legend") +
77   theme_minimal()
78 
79 ### LnPrice and LnCompPrice ----
80 ggplot(data = Ixmor.df, aes(x = Week)) +
81   geom_line(aes(y = LnPrice, color = "LnPrice")) +
82   geom_line(aes(y = LnCompPrice, color = "LnCompPrice")) +
83   scale_color_manual(values = c("LnPrice" = "red", "LnCompPrice" = "blue")) +
84   labs(x = "Week", y = "LnPrice and LnCompPrice", color = "Legend") +
85   theme_minimal()
86 
87 ## Bar Plots ----
88 
89 ### Advertising Bar Plots ----
90 barplot(Ixmor.df$LnAdvertising, main = "LnAdvertising", col = "red")
91 barplot(Ixmor.df$LnCompAdvertising, main = "LnCompAdvertising", col = "blue")
92 
93 ### Binary  stacked Bar Plot of LnAdvertising and LnCompAdvertising ----
94 LnAdvertising_binary <- ifelse(Ixmor.df$LnAdvertising > 0, 1, 0)
95 LnCompAdvertising_binary <- ifelse(Ixmor.df$LnCompAdvertising > 0, 1, 0)
96 barhelp.data <- data.matrix(cbind(LnAdvertising_binary, LnCompAdvertising_binary))
97 barhelp2.data <- t(barhelp.data)
98 barplot(barhelp2.data, main = "Presence of Advertising (red) and CompAdvertising (blue)", col=c("red", "blue"), xlab="weeks")
99 
100 # Granger Causality Tests----
```

Appendix **1** - Lines 100-150

```r

## Granger Causality for loop formula ----
find_lowest_pvalue <- function(dependent_var, independent_var, max_lag, data) {
  lowest_pvalue <- 1
  best_lag <- 0
  for (lag in 1:max_lag) {
    test_result <- grangertest(as.formula(paste(dependent_var, "~", independent_var)), order = lag, data = data)
    p_value <- test_result$`Pr(>F)`[2]
    if (p_value < 0.1 && p_value < lowest_pvalue) {
      lowest_pvalue <- p_value
      best_lag <- lag
    }
  }
  if (best_lag == 0) {
    message("No significant Granger causality found for any lag.")
  } else {
    message(paste("Lowest significant p-value:", lowest_pvalue, "at lag", best_lag))
  }
  return(list(best_lag = best_lag, lowest_pvalue = lowest_pvalue))
}

Ixmor.df <- as.data.frame(Ixmor.df)

# Lags (weeks) up to 15 are used, to account for atleast 1 quarter

### LnSales ~ LnAdvertising ----
LnSales_LnAdvertising <- find_lowest_pvalue("LnSales", "LnAdvertising", max_lag = 15, data = Ixmor.df)
# Lowest significant p-value: 0.0971242074723915 at lag 1

### LnSales ~ LnCompAdvertising ----
LnSales_LnCompAdvertising <- find_lowest_pvalue("LnSales", "LnCompAdvertising", max_lag = 15, data = Ixmor.df)
# Lowest significant p-value: 0.0512949190692117 at lag 1

### LnSales ~ LnPrice ----
LnSales_LnPrice <- find_lowest_pvalue("LnSales", "LnPrice", max_lag = 15, data = Ixmor.df)
# Lowest significant p-value: 0.0200102944389257 at lag 1

### LnSales ~ LnCompPrice ----
LnSales_LnCompPrice <- find_lowest_pvalue("LnSales", "LnCompPrice", max_lag = 15, data = Ixmor.df)
# Lowest significant p-value: 0.0184872116614598 at lag 11

### LnAdvertising ~ LnSales ----
LnAdvertising_LnSales <- find_lowest_pvalue("LnAdvertising", "LnSales", max_lag = 15, data = Ixmor.df)
# Lowest significant p-value: 0.0329780008824494 at lag 1

### LnAdvertising ~ LnCompAdvertising ----
LnAdvertising_LnCompAdvertising <- find_lowest_pvalue("LnAdvertising", "LnCompAdvertising", max_lag = 15, data = Ixmor.df)
# No significant Granger causality found for any lag.

### LnAdvertising ~ LnPrice ----
```

Appendix **1** - Lines 151-200

```r
151 LnAdvertising_LnPrice <- find_lowest_pvalue("LnAdvertising", "LnPrice", max_lag = 15, data = Ixmor.df)
152 # No significant Granger causality found for any lag.
153
154 ### LnAdvertising ~ LnCompPrice ----
155 LnAdvertising_LnCompPrice <- find_lowest_pvalue("LnAdvertising", "LnCompPrice", max_lag = 15, data = Ixmor.df)
156 # No significant Granger causality found for any lag.
157
158 ### LnCompAdvertising ~ LnSales ----
159 LnCompAdvertising_LnSales <- find_lowest_pvalue("LnCompAdvertising", "LnSales", max_lag = 15, data = Ixmor.df)
160 # Lowest significant p-value: 0.0120526738145634 at lag 2
161
162 ### LnCompAdvertising ~ LnAdvertising ----
163 LnCompAdvertising_LnAdvertising <- find_lowest_pvalue("LnCompAdvertising", "LnAdvertising", max_lag = 15, data = Ixmor.df)
164 #No significant Granger causality found for any lag.
165
166 ### LnCompAdvertising ~ LnPrice ----
167 LnCompAdvertising_LnPrice <- find_lowest_pvalue("LnCompAdvertising", "LnPrice", max_lag = 15, data = Ixmor.df)
168 # Lowest significant p-value: 0.000136296112651133 at lag 13
169
170 ### LnCompAdvertising ~ LnCompPrice ----
171 LnCompAdvertising_LnCompPrice <- find_lowest_pvalue("LnCompAdvertising", "LnCompPrice", max_lag = 15, data = Ixmor.df)
172 # No significant Granger causality found for any lag.
173
174 ### LnPrice ~ LnSales ----
175 LnPrice_LnSales <- find_lowest_pvalue("LnPrice", "LnSales", max_lag = 15, data = Ixmor.df)
176 #No significant Granger causality found for any lag.
177
178 ### LnPrice ~ LnAdvertising ----
179 LnPrice_LnAdvertising <- find_lowest_pvalue("LnPrice", "LnAdvertising", max_lag = 15, data = Ixmor.df)
180 #No significant Granger causality found for any lag.
181
182 ### LnPrice ~ LnCompAdvertising ----
183 LnPrice_LnCompAdvertising <- find_lowest_pvalue("LnPrice", "LnCompAdvertising", max_lag = 15, data = Ixmor.df)
184 # No significant Granger causality found for any lag.
185
186 ### LnPrice ~ LnCompPrice ----
187 LnPrice_LnCompPrice <- find_lowest_pvalue("LnPrice", "LnCompPrice", max_lag = 15, data = Ixmor.df)
188 #Lowest significant p-value: 0.00712815411410155 at lag 11
189
190 ### LnCompPrice ~ LnSales ----
191 LnCompPrice_LnSales <- find_lowest_pvalue("LnCompPrice", "LnSales", max_lag = 15, data = Ixmor.df)
192 # Lowest significant p-value: 0.0149253242555046 at lag 1
193
194 ### LnCompPrice ~ LnAdvertising ----
195 LnCompPrice_LnAdvertising <- find_lowest_pvalue("LnCompPrice", "LnAdvertising", max_lag = 15, data = Ixmor.df)
196 # No significant Granger causality found for any lag.
197
198 ### LnCompPrice ~ LnCompAdvertising ----
199 LnCompPrice_LnCompAdvertising <- find_lowest_pvalue("LnCompPrice", "LnCompAdvertising", max_lag = 15, data = Ixmor.df)
200 # No significant Granger causality found for any lag.
```

Appendix **1** - Lines 201-250

```
### LnCompPrice ~ LnPrice ----
LnCompPrice_LnPrice <- find_lowest_pvalue("LnCompPrice", "LnPrice", max_lag = 15, data = Ixmor.df)
# No significant Granger causality found for any lag.

# LnAdvertising, LnCompAdvertising, and LnPrice are granger causing LnSales at 1 lag.
# LnCompPrice is granger causing LnSales at 11 lags.
# LnSales is granger causing LnAdvertising at 1 lag.
# LnSales is granger causing LnCompAdvertising at 2 lags.
# LnPrice is granger causing LnCompAdvertising at 13 lags.
# LnCompPrice is granger causing LnPrice at 11 lags
# LnSales is granger causing LnCompPrice at 1 lag

# Testing whether variables are stationary or evolving ----
# # PP test is most important. Theoretical presumption is that variables are type 2 or 3

##LnSales ----
adf.test(Ixmor.df$LnSales, nlag = NULL, output = TRUE)
## Series is non-stationary for type 1, but stationary for type 2 and 3
pp.test(Ixmor.df$LnSales, output = TRUE)
## Series is non-stationary for type 1, but stationary for type 2 & 3
kpss.test(Ixmor.df$LnSales, output = TRUE)
## Series is non-stationary for for type 2, but stationary for type 1 & 3

# Decision: LnSales is stationary

##LnAdvertising ----
adf.test(Ixmor.df$LnAdvertising, nlag = NULL, output = TRUE)
## Series is non-stationary for type 1, but stationary for type 2 and 3
pp.test(Ixmor.df$LnAdvertising, output = TRUE)
## Series is stationary for type 1, 2, and 3. (Type 1 is at .0696 significance)
kpss.test(Ixmor.df$LnAdvertising, output = TRUE)
## Series is non-stationary for type 1, but non-stationary for type 2 and 3

## Decision: LnAdvertising is stationary

##LnCompAdvertising ----
adf.test(Ixmor.df$LnAdvertising, nlag = NULL, output = TRUE)
## Series is stationary for lags up to 3 (10% significance), but non-stationary afterwards for type 1. Series is stationary for type 2 and 3.
pp.test(Ixmor.df$LnAdvertising, output = TRUE)
## Series is non-stationary for type 1, and stationary for type 2 and 3.
kpss.test(Ixmor.df$LnAdvertising, output = TRUE)
## Series is non-stationary for type 1, and stationary for type 2 and 3.

# Decision: LnCompAdvertising is stationary.

##LnPrice ----
adf.test(Ixmor.df$LnPrice, nlag = NULL, output = TRUE)
## Series is non-stationary for type 1, but stationary for type 2 and 3.
pp.test(Ixmor.df$LnPrice, output = TRUE)
```

Appendix **1** - Lines 251-300

```
251 ## Series is non-stationary for type 1, but stationary for type 2 and 3.
252 kpss.test(Ixmor.df$LnPrice, output = TRUE)
253 ## Series is stationary for all 3 types.
254
255 # Decision: LnPrice is stationary
256
257 ##LnCompPrice ----
258 adf.test(Ixmor.df$LnCompPrice, nlag = NULL, output = TRUE)
259 ## Series is non-stationary for type 1, but stationary for type 2 and 3.
260 pp.test(Ixmor.df$LnCompPrice, output = TRUE)
261 ## Series is non-stationary for type 1, but stationary for type 2 and 3.
262 kpss.test(Ixmor.df$LnCompPrice, output = TRUE)
263 ## Series is stationary for type 1, but non-stationary for type 2 and 3.
264
265 # Decision: LnCompPrice is stationary
266
267 ## Scatterplots to confirm there is no cointegration ----
268
269 ### LnSales & LnAdvertising ----
270 ggplot(data = Ixmor.df, aes(x = LnSales, y = LnAdvertising)) +
271   geom_point() +
272   labs(x = "LnSales", y = "LnAdvertising", title = "Scatter Plot of LnSales vs LnAdvertising")
273 ## Scatter plot does not show a clear line, confirms no cointegration.
274
275 ### LnSales & LnCompAdvertising ----
276 ggplot(data = Ixmor.df, aes(x = LnSales, y = LnCompAdvertising)) +
277   geom_point() +
278   labs(x = "LnSales", y = "LnCompAdvertising", title = "Scatter Plot of LnSales vs LnCompAdvertising")
279 ## Scatter plot does not show a clear line, confirms no cointegration.
280
281 ### LnSales & LnPrice ----
282 ggplot(data = Ixmor.df, aes(x = LnSales, y = LnPrice)) +
283   geom_point() +
284   labs(x = "LnSales", y = "LnPrice", title = "Scatter Plot of LnSales vs LnPrice")
285 ## Scatter plot does not show a clear line, confirms no cointegration.
286
287 ### LnSales & LnCompPrice ----
288 ggplot(data = Ixmor.df, aes(x = LnSales, y = LnCompPrice)) +
289   geom_point() +
290   labs(x = "LnSales", y = "LnCompPrice", title = "Scatter Plot of LnSales vs LnCompPrice")
291 ## Scatter plot does not show a clear line, confirms no cointegration.
292
293 ### LnAdvertising & LnCompAdvertising ----
294 ggplot(data = Ixmor.df, aes(x = LnAdvertising, y = LnCompAdvertising)) +
295   geom_point() +
296   labs(x = "LnAdvertising", y = "LnCompAdvertising", title = "Scatter Plot of LnAdvertising vs LnCompAdvertising")
297 ## Scatter plot does not show a clear line, confirms no cointegration.
298
299 ### LnAdvertising & LnPrice ----
300 ggplot(data = Ixmor.df, aes(x = LnAdvertising, y = LnPrice)) +
```

Appendix **1** - Lines 301-350

```
301   geom_point() +
302   labs(x = "LnAdvertising", y = "LnPrice", title = "Scatter Plot of LnAdvertising vs LnPrice")
303 ## Scatter plot does not show a clear line, confirms no cointegration.
304
305 ### LnAdvertising & LnCompPrice ----
306 ggplot(data = Ixmor.df, aes(x = LnAdvertising, y = LnCompPrice)) +
307   geom_point() +
308   labs(x = "LnAdvertising", y = "LnCompPrice", title = "Scatter Plot of LnAdvertising vs LnCompPrice")
309 ## Scatter plot does not show a clear line, confirms no cointegration.
310
311 ### LnCompAdvertising & LnPrice
312 ggplot(data = Ixmor.df, aes(x = LnCompAdvertising, y = LnPrice)) +
313   geom_point() +
314   labs(x = "LnCompdvertising", y = "LnPrice", title = "Scatter Plot of LnCompAdvertising vs LnPrice")
315 ## Scatter plot does not show a clear line, confirms no cointegration.
316
317 ### LnCompAdvertising & LnCompPrice
318 ggplot(data = Ixmor.df, aes(x = LnCompAdvertising, y = LnCompPrice)) +
319   geom_point() +
320   labs(x = "LnCompdvertising", y = "LnCompPrice", title = "Scatter Plot of LnCompAdvertising vs LCompPrice")
321 ## Scatter plot does not show a clear line, confirms no cointegration.
322
323 ### LnPrice & LnCompPrice
324 ggplot(data = Ixmor.df, aes(x = LnPrice, y = LnCompPrice)) +
325   geom_point() +
326   labs(x = "LnPrice", y = "LnCompPrice", title = "Scatter Plot of LnPrice vs LCompPrice")
327 ## Scatter plot does not show a clear line, confirms no cointegration.
328
329 # VAR ----
330
331 ## Endogenous Variables ----
332 Ixmor.endo <- Ixmor.df[, c("LnSales", "LnAdvertising", "LnCompAdvertising", "LnPrice", "LnCompPrice")]
333
334 ## Exogenous Control Variables ----
335 Ixmor.exo <- Ixmor.df[, c("Qrtr1", "Qrtr2", "Qrtr3")]
336
337 ## Lag selection ----
338 lag_selection <- VARselect(Ixmor.endo, lag.max = 13, type = "const")
339 print(lag_selection)
340 #all criteria suggest to use 1 lag.
341
342 ## VAR model ----
343 Ixmor_var <- VAR(Ixmor.endo, p=1, type = "const", exogen = Ixmor.exo)
344 summary(Ixmor_var)
345
346 # Results:
347 # LnSales is positively affected by LnSales and LnCompPrice but negatively affected by past LnPrice at lag 1.
348 # LnAdfvertising is positively affected by  LnAdvertising at lag 1. There are notable seasonal effects on LnAdvertising through the quarter dummies.
349 # LnCompAdvertising is positively affected by LnCompAdvertising at lag 1.
350 # LnPrice is positively affected by past LnPrice at lag 1.
```

Appendix **1** - Lines 351-400

```
351 # LnCompPrice is positively affected by LnSales, LnCompPrice at lag 1.
352 # LnAdvertising and LnCompAdveritisng have insignificant intercepts.
353
354 #Impulse Response function ----
355
356 ## LnSales ----
357 irf_sales <- irf(Ixmor_var, impulse = NULL, response = "LnSales", n.ahead = 12, ortho = TRUE, cumulative = FALSE, boot = TRUE, ci = 0.68, runs = 500)
358 plot(irf_sales)
359 irf_sales_cum <- irf(Ixmor_var, impulse = NULL, response = "LnSales", n.ahead = 12, ortho = TRUE, cumulative = TRUE, boot = TRUE, ci = 0.68, runs = 500)
360 plot(irf_sales_cum)
361
362 ## LnAdvertising ----
363 irf_Advertising <- irf(Ixmor_var, impulse = NULL, response = "LnAdvertising", n.ahead = 12, ortho = TRUE, cumulative = FALSE, boot = TRUE, ci = 0.68, runs = 500)
364 plot(irf_Advertising)
365 irf_Advertising_cum <- irf(Ixmor_var, impulse = NULL, response = "LnAdvertising", n.ahead = 12, ortho = TRUE, cumulative = TRUE, boot = TRUE, ci = 0.68, runs = 500)
366 plot(irf_Advertising_cum)
367
368 ## LnCompAdvertising ----
369 irf_CompAdvertising <- irf(Ixmor_var, impulse = NULL, response = "LnCompAdvertising", n.ahead = 12, ortho = TRUE, cumulative = FALSE, boot = TRUE, ci = 0.68, runs = 500)
370 plot(irf_CompAdvertising)
371 irf_CompAdvertising_cum <- irf(Ixmor_var, impulse = NULL, response = "LnCompAdvertising", n.ahead = 12, ortho = TRUE, cumulative = TRUE, boot = TRUE, ci = 0.68, runs = 500)
372 plot(irf_CompAdvertising_cum)
373
374 ## LnPrice ----
375 irf_Price <- irf(Ixmor_var, impulse = NULL, response = "LnPrice", n.ahead = 12, ortho = TRUE, cumulative = FALSE, boot = TRUE, ci = 0.68, runs = 500)
376 plot(irf_Price)
377 irf_Price_cum <- irf(Ixmor_var, impulse = NULL, response = "LnPrice", n.ahead = 12, ortho = TRUE, cumulative = TRUE, boot = TRUE, ci = 0.68, runs = 500)
378 plot(irf_Price_cum)
379
380 ## LnCompPrice ----
381 irf_CompPrice <- irf(Ixmor_var, impulse = NULL, response = "LnCompPrice", n.ahead = 12, ortho = TRUE, cumulative = FALSE, boot = TRUE, ci = 0.68, runs = 500)
382 plot(irf_CompPrice)
383 irf_CompPrice_cum <- irf(Ixmor_var, impulse = NULL, response = "LnCompPrice", n.ahead = 12, ortho = TRUE, cumulative = TRUE, boot = TRUE, ci = 0.68, runs = 500)
384 plot(irf_CompPrice_cum)
385
386 #FEVD ----
387 Ixmorfevd <-fevd(Ixmor_var,n.ahead = 12)
388 Ixmorfevd
389 barbasis1 <- Ixmorfevd[1]
390 barbasis2 = as.matrix(unlist(barbasis1),ncol =5, byrow = TRUE)
391
392 bartry = Reduce(rbind,Ixmorfevd)
393 bartry2 = t(bartry)
394 bartry2 = bartry2[,c(12,24,36, 48, 60)]
395
396 par(mar = c(5, 4, 4, 8) + 0.1)
397 barplot(bartry2,
398         col = brewer.pal(5, "YlOrRd"),
399         names.arg = c("LnSales", "LnAdv", "LnCompAdv", "LnPrice", "LnCompPrice"),
400         xlab = "Variables",
```

Appendix **1** - Lines 401-406

```
401          ylab = "Variance Decomposition")
402 legend("topright", inset = c(-0.3, 0),
403        legend = c("Shock to LnSales", "Shock to LnAdvertising", "Shock to LnCompAdvertising", "Shock to LnPrice", "Shock to LnCompPrice"),
404        fill = brewer.pal(5, "YlOrRd"),
405        title = "Contributing Shocks", xpd = TRUE)
406
```

Appendix 2: Tables & Figures

Appendix **2.1** - Individual Line Plots: LnSales, LnAdvertising, and LnCompAdvertising

Figure 1: Ixmor Sales over time

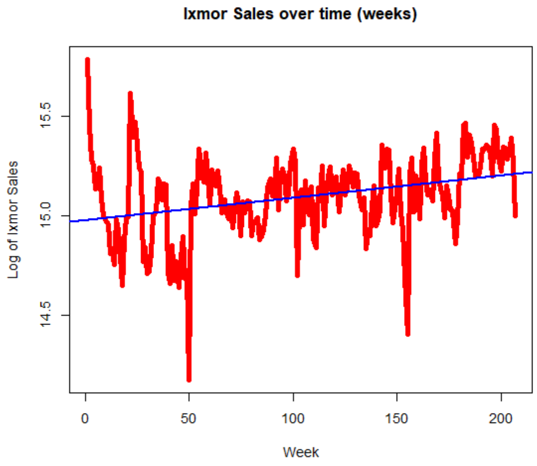


Figure 2: Ixmor Advertising over time

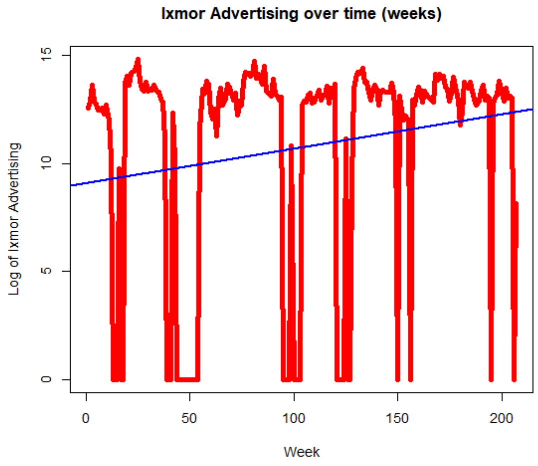
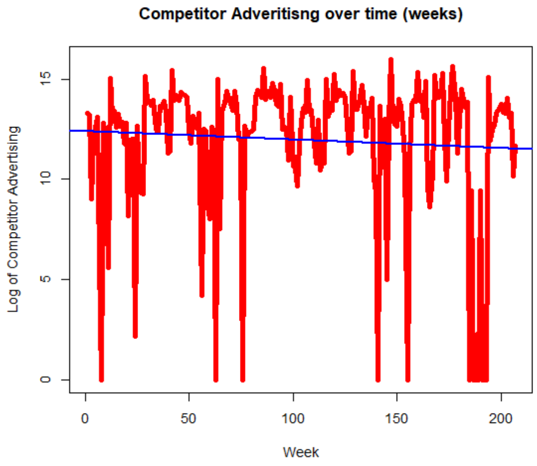


Figure 3: CompetitorAdvertising over time



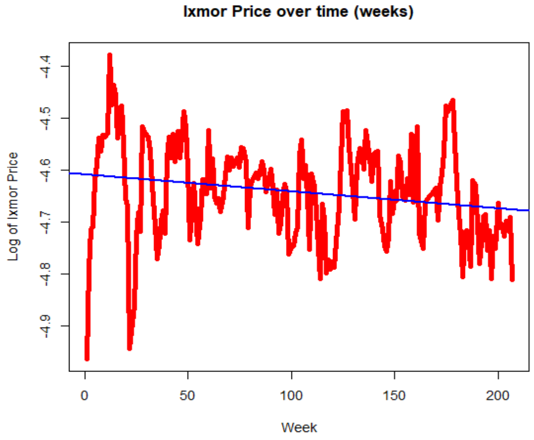
Trendline shows LnSales is increasing over time, and this model is statistically significant (p = 2.09e-06)

Trendline shows LnAdvertising is increasing over time, and this model is statistically significant (p = 0.00707)

Trendline shows LnCompAdvertising is decreasing over time, but this model is statistically insignificant (p = 0.311)

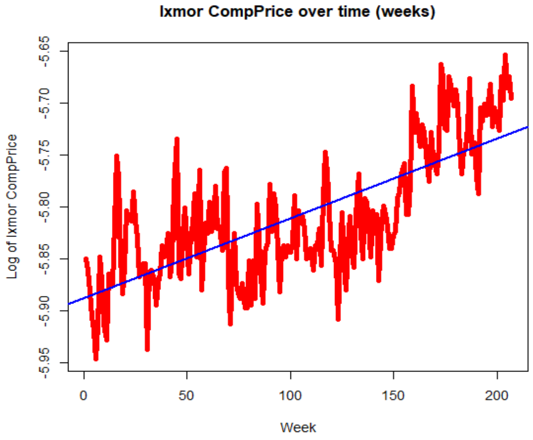
Appendix **2.1** - Individual Line Plots: LnPrice and LnCompPrice

Figure 4: Ixmor Price over time



Trendline shows LnPrice is decreasing over time, and this model is statistically significant (p = .00309)

Figure 5: Competitor Price over time



Trendline shows LnCompPrice is increasing over time, and this model is significant (p < 2e-16)

Appendix **2.2** - Combination Line Plots: LnPrice & LnCompPrice, and LnSales & LnCompSales

Figure 6: Comparison of LnAdvertising and LnCompAdvertising

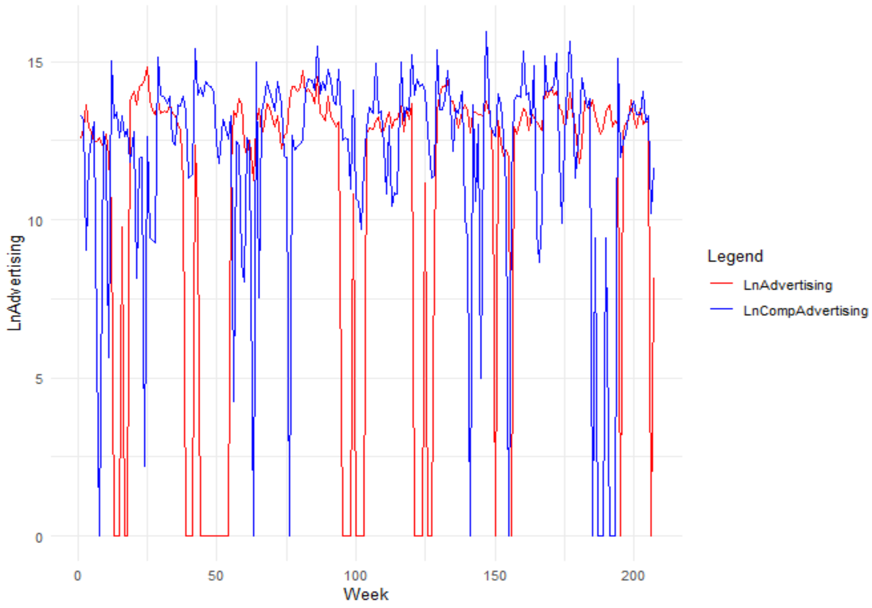
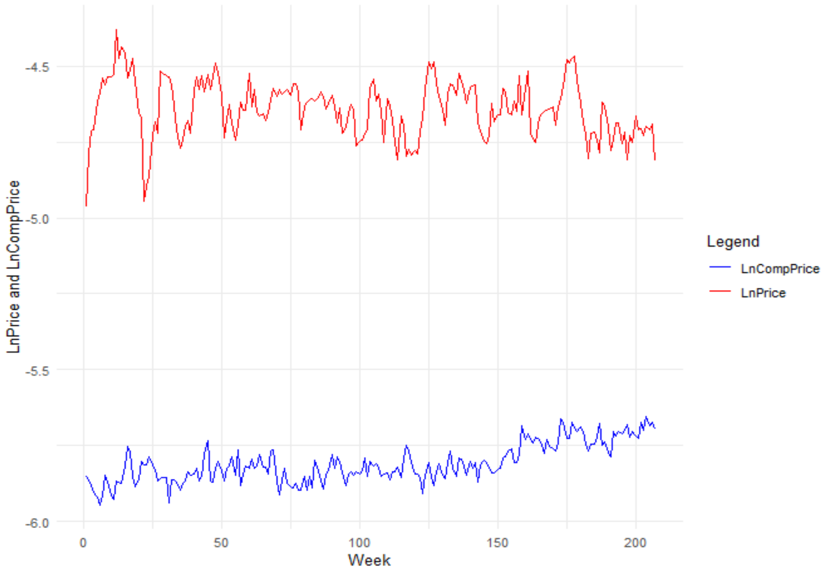


Figure 7: Comparison of LnPrice and LnCompPrice



Appendix **2.3** - BarPlots: LnAdvertising and LnCompAdvertising

Figure 8: BarPlot of LnAdvertising

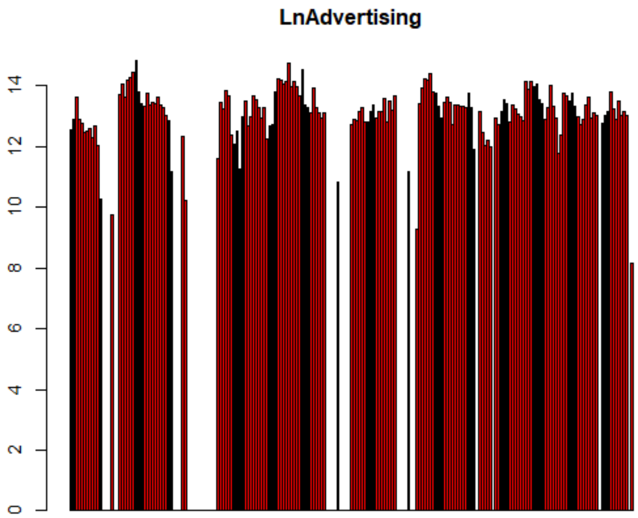
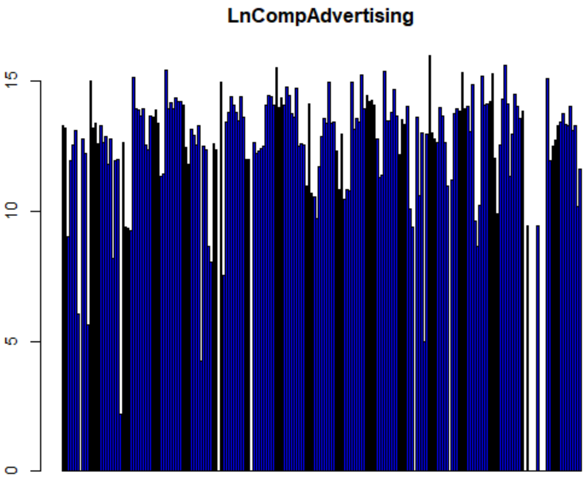
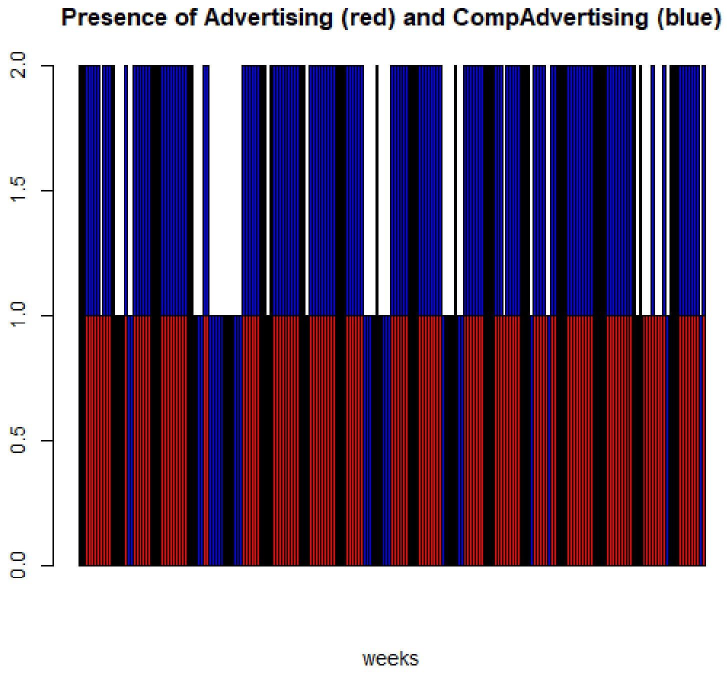


Figure 9: BarPlot of LnCompAdvertising



Appendix **2.3** - BarPlots: Binary Bar plot of Advertising comparison

Figure 10: Presence of Advertising



This binary bar plot shows the presence of advertising for Ixmor and the Competitor throughout weeks 1-207. It can be seen that at first only the Competitor advertises when Ixmor does not, and later on in the data set Ixmor advertises when the Competitor does not.

Appendix **2.4** - Granger Causality Results

Table 1: Granger Causality Results

Dependent Variable	Independent Variable	Significant?	Lowest p-value	Lag
LnSales	LnAdvertising	Yes	0.0971	1
LnSales	LnCompAdvertising	Yes	0.0513	1
LnSales	LnPrice	Yes	0.0200	1
LnSales	LnCompPrice	Yes	0.0185	11
LnAdvertising	LnSales	Yes	0.0330	1
LnAdvertising	LnCompAdvertising	No	-	-
LnAdvertising	LnPrice	No	-	-
LnAdvertising	LnCompPrice	No	-	-
LnCompAdvertising	LnSales	Yes	0.0121	2
LnCompAdvertising	LnAdvertising	No	-	-
LnCompAdvertising	LnPrice	Yes	0.0001	13
LnCompAdvertising	LnCompPrice	No	-	-
LnPrice	LnSales	No	-	-
LnPrice	LnAdvertising	No	-	-
LnPrice	LnCompAdvertising	No	-	-
LnPrice	LnCompPrice	Yes	0.0071	11
LnCompPrice	LnSales	Yes	0.0149	1
LnCompPrice	LnAdvertising	No	-	-
LnCompPrice	LnCompAdvertising	No	-	-
LnCompPrice	LnPrice	No	-	-

The LnSales variable is significantly granger-caused by all other variables at lag 1, except for LnCompPrice which granger causes LnSales at lag 11

LnAdvertising is granger-baused by LnSales at lag 1.

LnCompAdvertising is granger caused by LnSales and LnPrice at lags 2 and 13 respectively.

LnPrice is granger caused by LnCompPrice at lag 11.

LnCompPrice is granger caused by LnSales at lag 1.

Overall Interpretation: Most causal relationships occur at **1 or 2 lags**, suggesting a relatively short response time for the effects to materialize. However, there are also notable longer lags of **11 to 13 weeks** for certain variables, indicating a **substantial delay** in impact for specific predictors, particularly **competitor price (CompPrice)**, which affects **LnSales** after an **11-week lag**.

It is important for the manager to note that **LnSales** is significantly Granger-caused not only by advertising and competitor advertising but also by both **Ixmor's and competitors' pricing**, with a delayed impact from competitor pricing at **11 weeks**. This implies **high confidence** in the predictive influence of pricing on sales, and suggests that **pricing decisions**, especially those made by competitors, have a lasting and

Appendix **2.5** - Unit Root Tests: LnSales and LnAdvertising

Table 2: LnSales

```
> ##LnSales ----
> adf.test(Ixmor.df$LnSales, nlag = NULL, output = TRUE)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag    ADF p.value
[1,]   0 -0.403   0.528
[2,]   1 -0.304   0.556
[3,]   2 -0.235   0.576
[4,]   3 -0.164   0.596
[5,]   4 -0.121   0.609
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -7.13    0.01
[2,]   1 -5.92    0.01
[3,]   2 -4.85    0.01
[4,]   3 -5.03    0.01
[5,]   4 -4.43    0.01
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -7.86    0.01
[2,]   1 -6.68    0.01
[3,]   2 -5.61    0.01
[4,]   3 -5.88    0.01
[5,]   4 -5.32    0.01
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3
> pp.test(Ixmor.df$LnSales, output = TRUE)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag   Z_rho p.value
   4 -0.0599   0.677
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   4   -78    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   4 -87.4    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 & 3
> kpss.test(Ixmor.df$LnSales, output = TRUE)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag   stat p.value
   3 0.0573     0.1
-----
 Type 2: with drift no trend
 lag  stat p.value
   3 0.802    0.01
-----
 Type 1: with drift and trend
 lag   stat p.value
   3 0.0411     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
>
```

Result: Stationary

Explanation: In real-world scenarios, economic and marketing variables like sales, advertising, and prices are unlikely to exhibit **type 1 non-stationarity** (random walk without drift) due to the presence of a **drift** (intercept). Therefore, **type 1 results are disregarded** in this analysis.

The focus is instead on **type 2 (random walk with drift)** and **type 3 (trend stationary)** behaviors. The **Phillips-Perron (PP) test** is prioritized for its robustness against heteroscedasticity, providing a more reliable assessment of stationarity, which aligns better with the typical behavior of real-world economic variables.

Table 3: LnAdvertising

```
> ##LnAdvertising ----
> adf.test(Ixmor.df$LnAdvertising, nlag = NULL, output = TRUE)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag   ADF p.value
[1,]   0 -2.53  0.0125
[2,]   1 -2.03  0.0428
[3,]   2 -1.75  0.0800
[4,]   3 -1.62  0.0996
[5,]   4 -1.41  0.1746
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -6.15    0.01
[2,]   1 -5.02    0.01
[3,]   2 -4.37    0.01
[4,]   3 -4.17    0.01
[5,]   4 -3.66    0.01
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -6.26  0.0100
[2,]   1 -5.13  0.0100
[3,]   2 -4.49  0.0100
[4,]   3 -4.30  0.0100
[5,]   4 -3.80  0.0203
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3
> pp.test(Ixmor.df$LnAdvertising, output = TRUE)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag Z_rho p.value
   4 -7.07  0.0696
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   4 -61.7    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   4 -64.4    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
> ## Series is stationary for type 1, 2, and 3. (Type 1 is at .0696 significance)
> kpss.test(Ixmor.df$LnAdvertising, output = TRUE)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag stat p.value
   3 2.57  0.0145
-----
 Type 2: with drift no trend
 lag  stat p.value
   3 0.202     0.1
-----
 Type 1: with drift and trend
 lag   stat p.value
   3 0.0397     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
>
```

Result: Stationary

Appendix **2.5** - Unit Root Tests: LnCompAdvertising

Table 4: Unit Root tests for LnCompAdvertising

```
> ##LnCompAdvertising ----
> adf.test(Ixmor.df$LnAdvertising, nlag = NULL, output = TRUE)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag   ADF p.value
[1,]   0 -2.53  0.0125
[2,]   1 -2.03  0.0428
[3,]   2 -1.75  0.0800
[4,]   3 -1.62  0.0996
[5,]   4 -1.41  0.1746
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -6.15    0.01
[2,]   1 -5.02    0.01
[3,]   2 -4.37    0.01
[4,]   3 -4.17    0.01
[5,]   4 -3.66    0.01
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -6.26  0.0100
[2,]   1 -5.13  0.0100
[3,]   2 -4.49  0.0100
[4,]   3 -4.30  0.0100
[5,]   4 -3.80  0.0203
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
> ## Series is stationary for lags up to 3 (10% significance), but non-stationary afterwards for type 1. Series is stationary for type 2 and 3.
> pp.test(Ixmor.df$LnAdvertising, output = TRUE)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag Z_rho p.value
   4 -7.07  0.0696
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   4 -61.7    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   4 -64.4    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, and stationary for type 2 and 3.
> kpss.test(Ixmor.df$LnAdvertising, output = TRUE)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag stat p.value
   3 2.57  0.0145
-----
 Type 2: with drift no trend
 lag  stat p.value
   3 0.202     0.1
-----
 Type 1: with drift and trend
 lag   stat p.value
   3 0.0397     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
```

Result: Stationary

Appendix **2.5** - Unit Root Tests: LnPrice and LnCompPrice

Table 5: Unit Root tests for LnPrice

```
> ##LnPrice ----
> adf.test(Ixmor.df$LnPrice, nlag = NULL, output = TRUE)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag      ADF p.value
[1,]   0 -0.27058   0.566
[2,]   1 -0.09773   0.616
[3,]   2 -0.00175   0.643
[4,]   3  0.01660   0.648
[5,]   4  0.11381   0.676
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -5.97    0.01
[2,]   1 -5.22    0.01
[3,]   2 -5.15    0.01
[4,]   3 -5.38    0.01
[5,]   4 -5.35    0.01
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -6.33    0.01
[2,]   1 -5.56    0.01
[3,]   2 -5.50    0.01
[4,]   3 -5.79    0.01
[5,]   4 -5.77    0.01
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3.
> pp.test(Ixmor.df$LnPrice, output = TRUE)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag   Z_rho p.value
   4 -0.0517   0.679
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   4 -64.4    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   4 -67.8    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3.
> kpss.test(Ixmor.df$LnPrice, output = TRUE)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag   stat p.value
   3 0.0724     0.1
-----
 Type 2: with drift no trend
 lag  stat p.value
   3 0.365  0.0924
-----
 Type 1: with drift and trend
 lag   stat p.value
   3 0.0488     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
```

Result: Stationary

Table 6: Unit Root tests for LnCompPrice

```
> ##LnCompPrice ----
> adf.test(Ixmor.df$LnCompPrice, nlag = NULL, output = TRUE)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag    ADF p.value
[1,]   0 -0.327   0.550
[2,]   1 -0.412   0.525
[3,]   2 -0.580   0.471
[4,]   3 -0.887   0.361
[5,]   4 -0.974   0.330
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -4.35   0.010
[2,]   1 -3.56   0.010
[3,]   2 -2.58   0.102
[4,]   3 -1.64   0.471
[5,]   4 -1.71   0.440
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -6.92  0.0100
[2,]   1 -6.02  0.0100
[3,]   2 -4.63  0.0100
[4,]   3 -3.22  0.0851
[5,]   4 -3.24  0.0830
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3.
> pp.test(Ixmor.df$LnCompPrice, output = TRUE)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag   Z_rho p.value
   4 -0.0278   0.684
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   4   -26    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   4 -75.3    0.01
---------------
Note: p-value = 0.01 means p.value <= 0.01
> ## Series is non-stationary for type 1, but stationary for type 2 and 3.
> kpss.test(Ixmor.df$LnCompPrice, output = TRUE)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag   stat p.value
   3 0.0394     0.1
-----
 Type 2: with drift no trend
 lag stat p.value
   3 1.78    0.01
-----
 Type 1: with drift and trend
 lag  stat p.value
   3 0.403    0.01
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
```

Result: Stationary

Appendix **2.6** - Scatter Plots to confirm Unit Root results

Explanation: These scatter plots are done to confirm that all variables are stationary, as this is crucial for the advanced analysis techniques such as VAR, IRF, and FEVD. It can be seen in the scatter plots that none of the graphs form a straight line, confirming the previous unit root tests results.

Figure 11: LnSales vs LnAdvertising

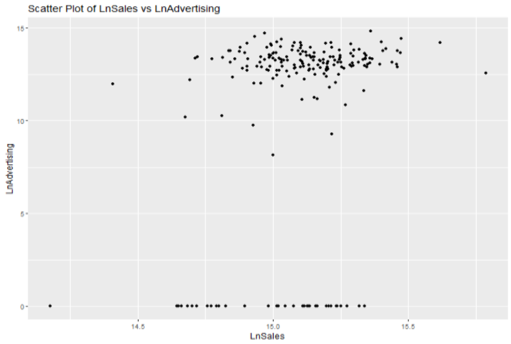


Figure 12: LnSales vs LnCompAdvertising

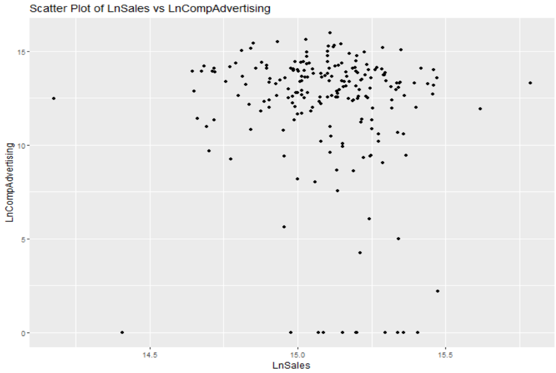


Figure 13: LnSales vs LnPrice

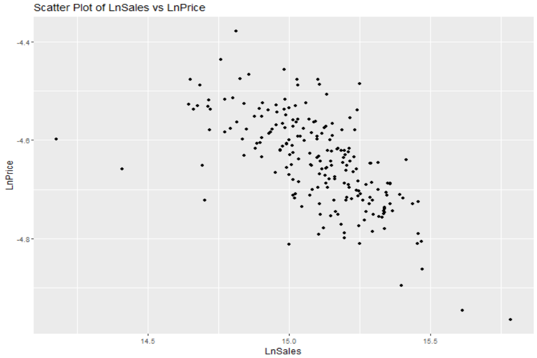


Figure 14: LnSales vs LnCompPrice

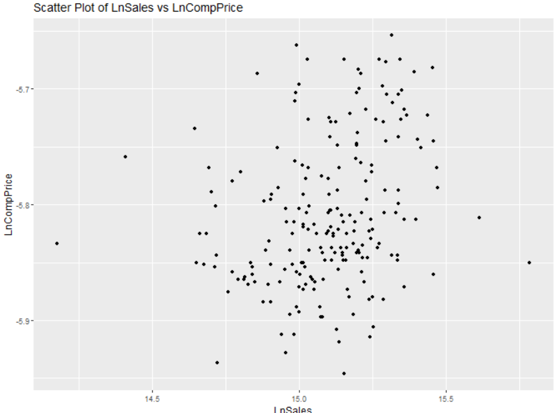
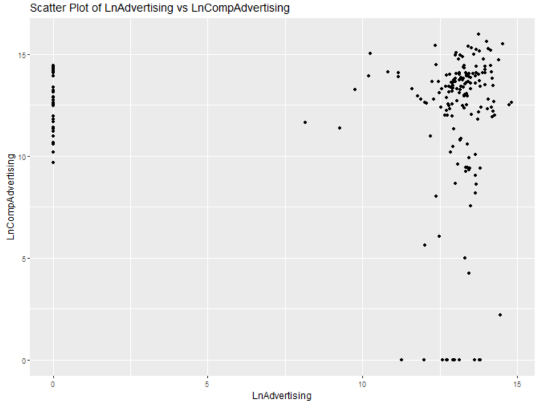


Figure 15: LnAdvertising vs LnCompAdvertising



Appendix **2.6** - Scatter Plots to confirm Unit Root results

Figure 15: LnAdvertising vs LnPrice

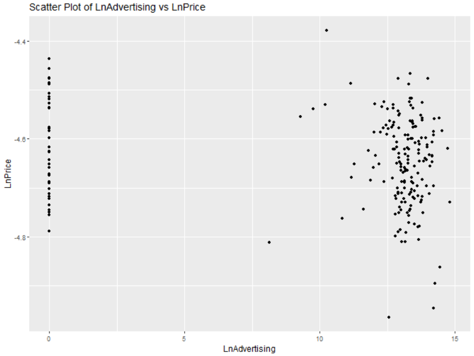


Figure 16: LnAdvertising vs LnCompPrice

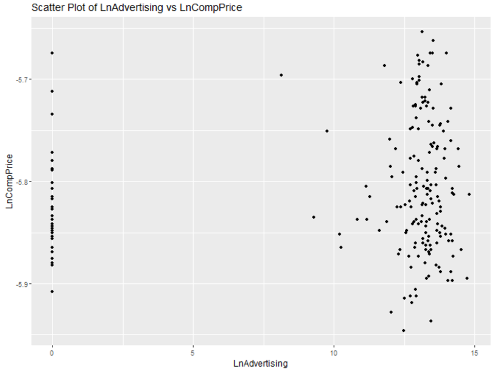


Figure 17: LnCompAdvertising vs LnPrice

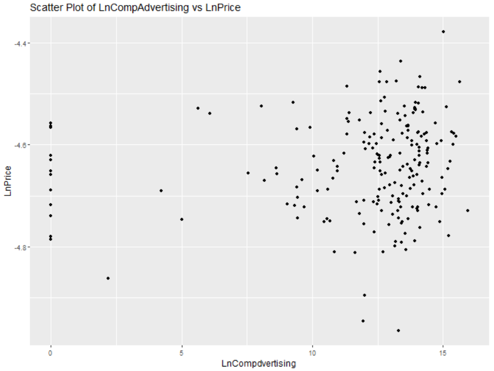


Figure 17: LnCompAdvertising vs LnCompPrice

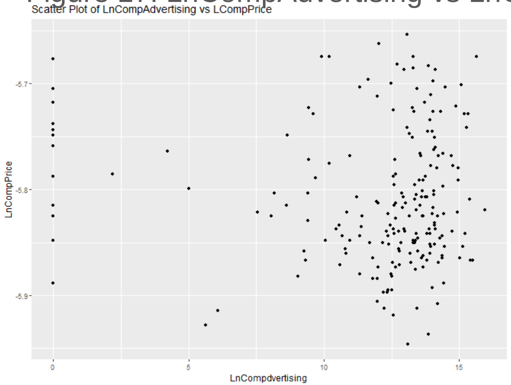
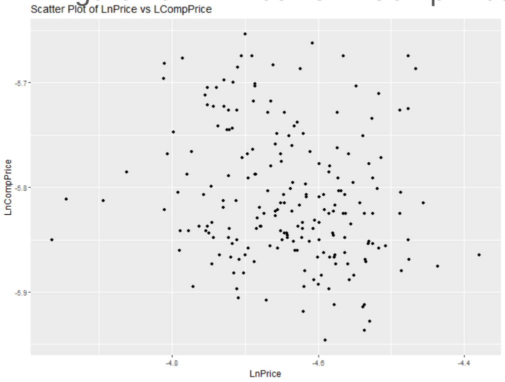


Figure 19: LnPrice vs LnCompPrice



Appendix **2.7** - VAR lag selection

Table 7: VAR Lag Selection

```
> ## Lag selection ----
> lag_selection <- VARselect(Ixmor.endo, lag.max = 10, type = "const")
> print(lag_selection)
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     1      1      1      1

$criteria
                   1             2             3             4             5             6             7             8             9            10
AIC(n) -1.107267e+01 -10.995665825 -1.092962e+01 -1.092902e+01 -1.085272e+01 -1.077882e+01 -1.073868e+01 -1.061339e+01 -1.051611e+01 -1.034998e+01
HQ(n)  -1.087027e+01 -10.624606968 -1.038990e+01 -1.022063e+01 -9.975668e+00 -9.733113e+00 -9.524304e+00 -9.230355e+00 -8.964412e+00 -8.629615e+00
SC(n)  -1.057269e+01 -10.079035342 -9.596343e+00 -9.179087e+00 -8.686135e+00 -8.195593e+00 -7.738796e+00 -7.196860e+00 -6.682930e+00 -6.100146e+00
FPE(n)  1.553254e-05   0.000016784  1.795161e-05  1.800336e-05  1.950242e-05  2.111263e-05  2.214456e-05  2.535493e-05  2.831186e-05  3.398130e-05
```

Explanation: The lag selection results suggest that the optimal lag length for the VAR model is 1, as indicated by all 4 criteria. This means that including only the previous period provides the best balance between model fit and simplicity. The max lag of 13 was chosen as the granger causality tests showed that for some variables lags of up to 13 were the most significant. This will ensure that the model can adequately capture both short-term and longer-term effects. The criteria values for longer lags (up to 13) show diminishing improvements, implying that additional lags do not enhance the model significantly and could unnecessarily complicate it. Therefore, a lag of 1 is sufficient to capture the key dynamics of the variables.

Appendix **2.7** - VAR model summary

Table 8: VAR Results LnSales and LnAdvertising

```
> summary(Ixmor_var)

VAR Estimation Results:
=========================
Endogenous variables: LnSales, LnAdvertising, LnCompAdvertising, LnPrice, LnCompPrice
Deterministic variables: const
Sample size: 206
Log Likelihood: -269.799
Roots of the characteristic polynomial:
0.8498 0.668 0.5214 0.5214 0.2817
Call:
VAR(y = Ixmor.endo, p = 1, type = "const", exogen = Ixmor.exo)


Estimation results for equation LnSales:
========================================
LnSales = LnSales.l1 + LnAdvertising.l1 + LnCompAdvertising.l1 + LnPrice.l1 + LnCompPrice.l1 + const + Qrtr1 + Qrtr2 + Qrtr3

                     Estimate Std. Error t value Pr(>|t|)
LnSales.l1           0.445129   0.071102   6.260 2.35e-09 ***
LnAdvertising.l1     0.002650   0.002401   1.104   0.2711
LnCompAdvertising.l1 -0.004332  0.003078  -1.407   0.1609
LnPrice.l1          -0.403515   0.155336  -2.598   0.0101 *
LnCompPrice.l1       0.423648   0.183247   2.312   0.0218 *
const                8.956900   1.480997   6.048 7.25e-09 ***
Qrtr1                0.050588   0.032854   1.540   0.1252
Qrtr2                0.041936   0.031344   1.338   0.1825
Qrtr3                0.022204   0.034563   0.642   0.5213
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1518 on 197 degrees of freedom
Multiple R-Squared: 0.4699,     Adjusted R-squared: 0.4484
F-statistic: 21.83 on 8 and 197 DF,  p-value: < 2.2e-16


Estimation results for equation LnAdvertising:
==============================================
LnAdvertising = LnSales.l1 + LnAdvertising.l1 + LnCompAdvertising.l1 + LnPrice.l1 + LnCompPrice.l1 + const + Qrtr1 + Qrtr2 + Qrtr3

                     Estimate Std. Error t value Pr(>|t|)
LnSales.l1            4.07895    1.67404   2.437  0.01572 *
LnAdvertising.l1      0.55900    0.05653   9.888  < 2e-16 ***
LnCompAdvertising.l1 -0.01335    0.07247  -0.184  0.85405
LnPrice.l1            4.42659    3.65729   1.210  0.22760
LnCompPrice.l1        4.12384    4.31443   0.956  0.34033
const               -13.96983   34.86916  -0.401  0.68912
Qrtr1                 2.51513    0.77354   3.251  0.00135 **
Qrtr2                 1.59956    0.73797   2.168  0.03140 *
Qrtr3                 3.01876    0.81377   3.710  0.00027 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 3.573 on 197 degrees of freedom
Multiple R-Squared: 0.531,      Adjusted R-squared: 0.512
F-statistic: 27.88 on 8 and 197 DF,  p-value: < 2.2e-16
```

LnSales is positively affected by LnSales and LnCompPrice but negatively affected by past LnPrice at lag 1.

LnAdfvertising is positively affected by LnAdvertising at lag 1. There are notable seasonal effects on LnAdvertising through the quarter dummies. The intercept is insignificant.

Table 9: VAR Results LnCompAdvertising and LnPrice

```
Estimation results for equation LnCompAdvertising:
==================================================
LnCompAdvertising = LnSales.l1 + LnAdvertising.l1 + LnCompAdvertising.l1 + LnPrice.l1 + LnCompPrice.l1 + const + Qrtr1 + Qrtr2 + Qrtr3

                     Estimate Std. Error t value Pr(>|t|)
LnSales.l1           -1.11852    1.58386  -0.706    0.481
LnAdvertising.l1     -0.01603    0.05349  -0.300    0.765
LnCompAdvertising.l1  0.31643    0.06856   4.615 7.07e-06 ***
LnPrice.l1            2.97403    3.46027   0.859    0.391
LnCompPrice.l1       -0.83541    4.08201  -0.205    0.838
const                34.43736   32.99075   1.044    0.298
Qrtr1                -0.26378    0.73187  -0.360    0.719
Qrtr2                 0.07732    0.69822   0.111    0.912
Qrtr3                -0.82700    0.76993  -1.074    0.284
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 3.381 on 197 degrees of freedom
Multiple R-Squared: 0.1578,     Adjusted R-squared: 0.1236
F-statistic: 4.613 on 8 and 197 DF,  p-value: 3.534e-05


Estimation results for equation LnPrice:
========================================
LnPrice = LnSales.l1 + LnAdvertising.l1 + LnCompAdvertising.l1 + LnPrice.l1 + LnCompPrice.l1 + const + Qrtr1 + Qrtr2 + Qrtr3

                       Estimate Std. Error t value Pr(>|t|)
LnSales.l1            0.0284975  0.0286047   0.996   0.3203
LnAdvertising.l1     -0.0015643  0.0009660  -1.619   0.1070
LnCompAdvertising.l1 -0.0003231  0.0012383  -0.261   0.7944
LnPrice.l1            0.7321209  0.0624929  11.715  < 2e-16 ***
LnCompPrice.l1       -0.1305754  0.0737215  -1.771   0.0781 .
const                -2.4254637  0.5958162  -4.071 6.78e-05 ***
Qrtr1                 0.0233123  0.0132176   1.764   0.0793 .
Qrtr2                 0.0187417  0.0126099   1.486   0.1388
Qrtr3                 0.0176065  0.0139051   1.266   0.2069
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.06106 on 197 degrees of freedom
Multiple R-Squared: 0.5736,     Adjusted R-squared: 0.5563
F-statistic: 33.13 on 8 and 197 DF,  p-value: < 2.2e-16
```

LnCompAdvertising is positively affected by LnCompAdvertising at lag 1. The intercept is insignificant.

LnPrice is positively affected by past LnPrice at lag 1.

Appendix **2.7** - VAR model summary

Table 8: VAR Results LnCompPrice & residual covariance & correlation matrix

```
Estimation results for equation LnCompPrice:
============================================
LnCompPrice = LnSales.l1 + LnAdvertising.l1 + LnCompAdvertising.l1 + LnPrice.l1 + LnCompPrice.l1 + const + Qrtr1 + Qrtr2 + Qrtr3

                       Estimate Std. Error t value Pr(>|t|)    
LnSales.l1            4.800e-02  1.707e-02   2.811  0.00543 ** 
LnAdvertising.l1      5.723e-05  5.766e-04   0.099  0.92104    
LnCompAdvertising.l1 -2.923e-04  7.391e-04  -0.396  0.69287    
LnPrice.l1            5.768e-02  3.730e-02   1.546  0.12363    
LnCompPrice.l1        7.862e-01  4.400e-02  17.868  < 2e-16 ***
const                -1.691e+00  3.556e-01  -4.755 3.82e-06 ***
Qrtr1                -5.829e-03  7.889e-03  -0.739  0.46085    
Qrtr2                -4.769e-03  7.526e-03  -0.634  0.52707    
Qrtr3                -6.262e-03  8.299e-03  -0.755  0.45142    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.03644 on 197 degrees of freedom
Multiple R-Squared: 0.6837,     Adjusted R-squared: 0.6709 
F-statistic: 53.23 on 8 and 197 DF,  p-value: < 2.2e-16 



Covariance matrix of residuals:
                    LnSales LnAdvertising LnCompAdvertising    LnPrice LnCompPrice
LnSales           2.303e-02      0.026732          0.019271 -4.822e-03  -5.039e-05
LnAdvertising     2.673e-02     12.768829          0.818129 -2.356e-02   7.169e-03
LnCompAdvertising 1.927e-02      0.818129         11.430169  7.437e-03  -4.502e-03
LnPrice          -4.822e-03     -0.023557          0.007437  3.728e-03  -1.429e-05
LnCompPrice      -5.039e-05      0.007169         -0.004502 -1.429e-05   1.328e-03

Correlation matrix of residuals:
                   LnSales LnAdvertising LnCompAdvertising   LnPrice LnCompPrice
LnSales           1.000000       0.04929           0.03756 -0.520338   -0.009111
LnAdvertising     0.049291       1.00000           0.06772 -0.107967    0.055049
LnCompAdvertising 0.037556       0.06772           1.00000  0.036025   -0.036541
LnPrice          -0.520338      -0.10797           0.03602  1.000000   -0.006423
LnCompPrice      -0.009111       0.05505          -0.03654 -0.006423    1.000000
```

LnCompPrice is positively affected by LnSales, LnCompPrice at lag 1.

Analysis of residuals:

There is a moderate negative correlation of -0.520 between the residuals of LnSales and LnPrice. This suggests that the residuals of LnSales and LnPrice tend to move in opposite directions. This could indicate that the model didn't successfully capture all dynamics between LnPrice and LnSales.

Most other off-diagonal correlations are relatively low, indicating weak relationships between residuals. Low off-diagonal correlations imply that the model has successfully captured most of the dynamic relationships between the variables, with little remaining unexplained correlation in the residuals.

Appendix **2.8** - IRF: Orthogonal Responses on LnSales

Figure 20: IRF of LnSales to a shock in LnSales

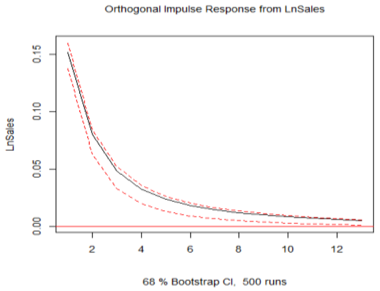


Figure 21: IRF of LnSales to a Shock in LnAdvertising

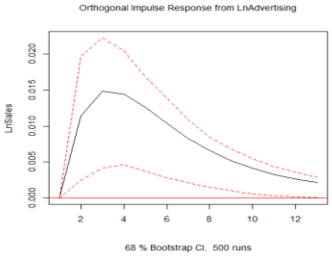


Figure 22: IRF of LnSales to a Shock in LnCompAdvertising

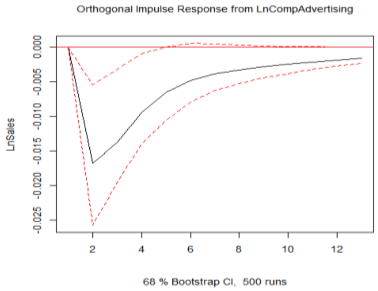


Figure 23: IRF of LnSales to a Shock in LnPrice

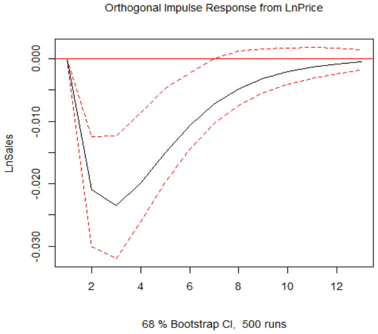
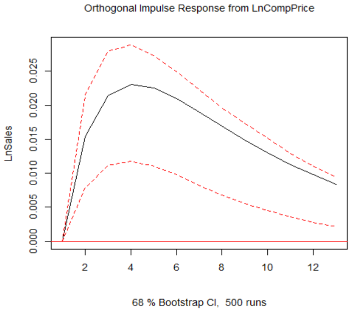


Figure 24: IRF of LnSales to a Shock in LnCompPrice



Advertising: Both Ixmor's and competitors' advertising significantly influence LnSales, but in opposite directions. Ixmør's advertising has a positive effect, while competitor advertising has a negative effect.

Pricing: Both Ixmor’s and competitors' pricing affect LnSales. Ixmør's price increases have a sustained negative impact, while competitor price increases lead to a positive effect on Ixmør’s sales, indicating price sensitivity among consumers.

Lag Effects: Most of the impacts peak around weeks 2-4, indicating relatively short-term effects of advertising and pricing changes. Competitor advertising has the most prolonged effect, while other effects tend to stabilize after a few weeks.

Appendix **2.8** - IRF: Cumulative Orthogonal Responses on LnSales

Figure 25: Cumulative IRF of LnSales to a shock in LnSales

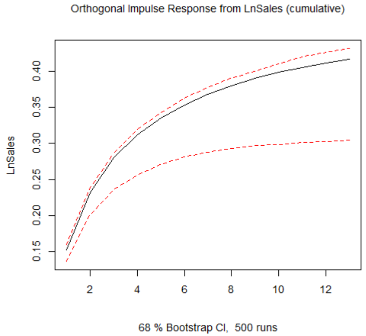


Figure 26: Cumulative IRF of LnSales to a Shock in LnAdvertising

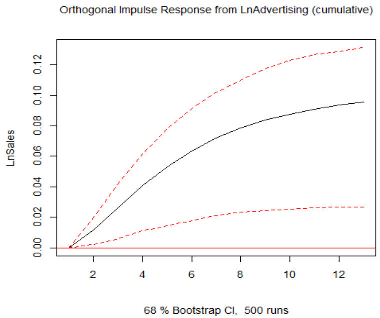


Figure 27: Cumulative IRF of LnSales to a Shock in LnCompAdvertising

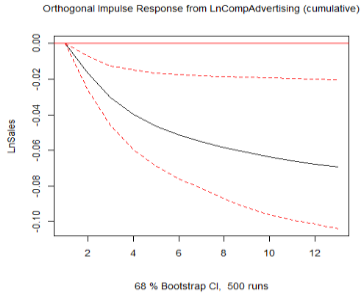


Figure 28: Cumulative IRF of LnSales to a Shock in LnPrice

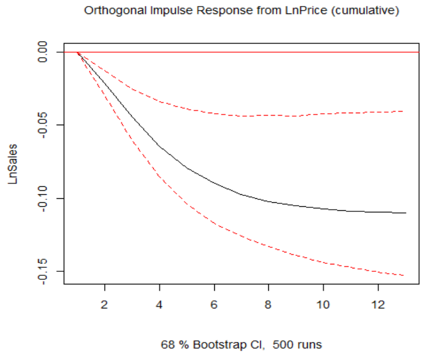
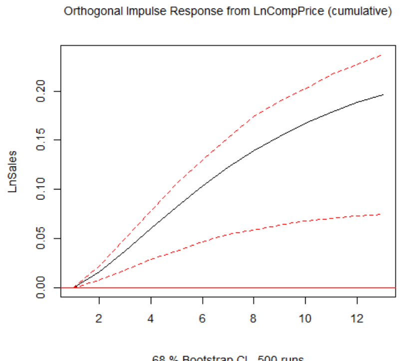


Figure 29: Cumulative IRF of LnSales to a Shock in LnCompPrice



Positive Effects: Shocks to LnSales (self-reinforcement), LnAdvertising, and LnCompPrice all have a positive cumulative effect on LnSales.
Negative Effects: Shocks to LnPrice (increasing Ixmor’s price) and LnCompAdvertising (increasing competitor advertising) have negative cumulative effects.
Long Term: Most effects start to plateau at around 10-12 lags, showing that the max lags used was enough.

Appendix **2.8** - IRF: Orthogonal Responses on LnAdvertising

Figure 30: IRF of LnAdvertising to a shock in LnSales

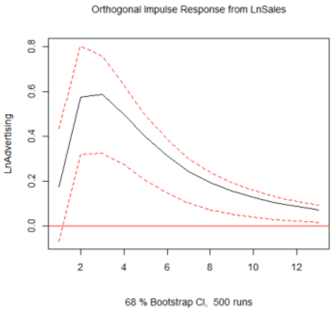


Figure 31: IRF of LnAdvertising to a Shock in LnAdvertising

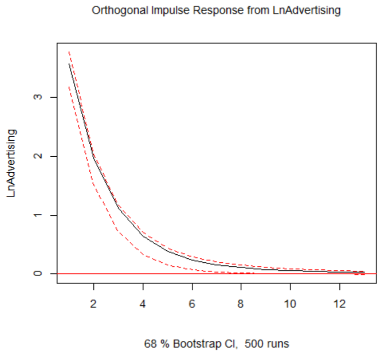


Figure 32: IRF of LnAdvertising to a Shock in LnCompAdvertising

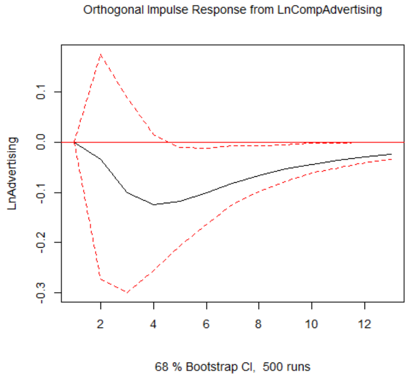


Figure 33: IRF of LnAdvertising to a Shock in LnPrice

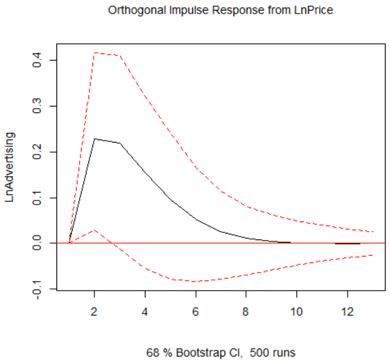
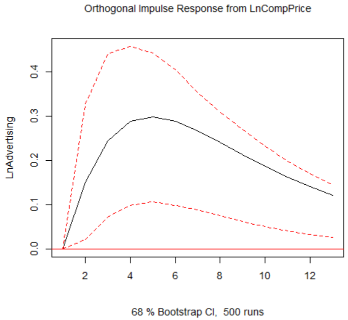


Figure 34: IRF of LnAdvertising to a Shock in LnCompPrice



LnSales: A shock to LnSales leads to an increase in LnAdvertising. The effect peaks quickly, around lag 3, and starts to decline afterward.
LnCompAdvertising: The response to is initially negative, strongest at lag 4.
LnPrice: Strong sharp positive effect at lag 2, levels out quickly
LnCompPrice: Gradual positive effect, strongest at lag 5-6

Appendix **2.8** - IRF: Cumulative Orthogonal Responses on LnAdvertising

Figure 35: CumulativeIRF of LnAdverising to a shock in LnSales

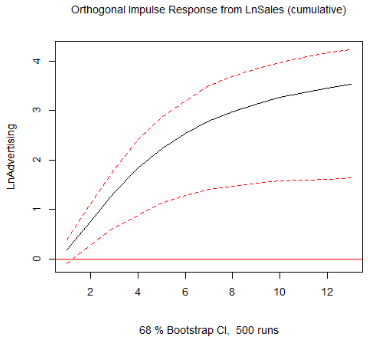


Figure 36: Cumulative IRF of LnAdvertising to a Shock in LnAdvertising

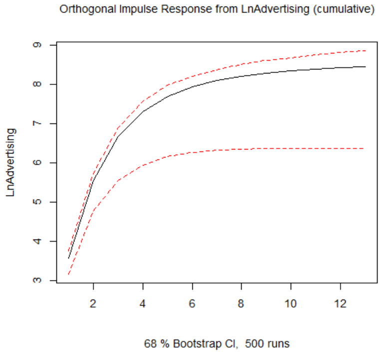


Figure 37: Cumulative IRF of LnAdvertising to a Shock in LnCompAdvertising

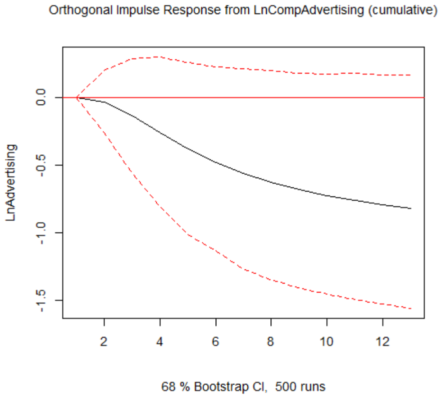


Figure 38: Cumulative IRF of LnAdvertising to a Shock in LnPrice

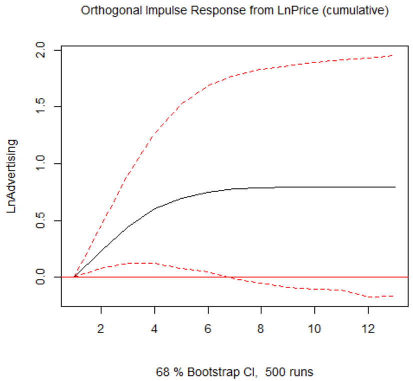
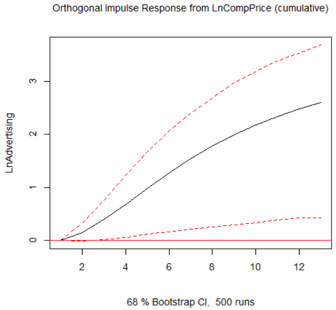


Figure 39: Cumulative IRF of LnAdvertising to a Shock in LnCompPrice



Positive Effects: Shocks to LnSales, LnAdvertising, LnPrice, and LnCompPrice have positive cumulative effects.
Negative Effects: Shocks to LnCompAdvertising have negative cumulative effects.
Long Term: Most effects start to plateau around 10-12 lags, with the exception of LnCompPrice.

Appendix **2.8** - IRF: Orthogonal Responses on LnCompAdvertising

Figure 40: IRF of LnCompAdvertising to a shock in LnSales

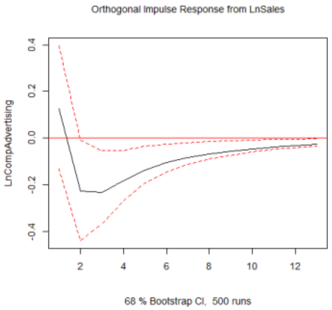


Figure 41: IRF of LnCompAdvertising to a Shock in LnAdvertising

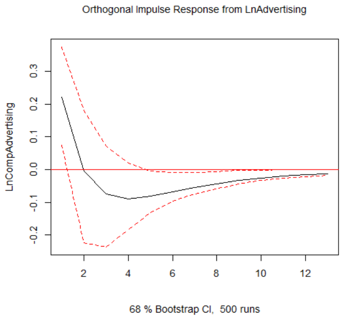


Figure 42: IRF of LnCompAdvertising to a Shock in LnCompAdvertising

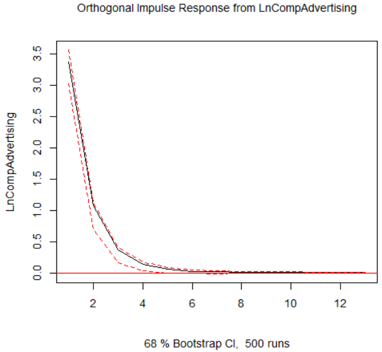


Figure 43: IRF of LnCompAdvertising to a Shock in LnPrice

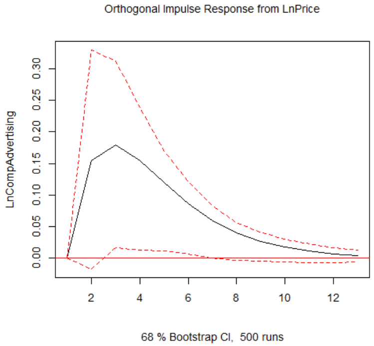
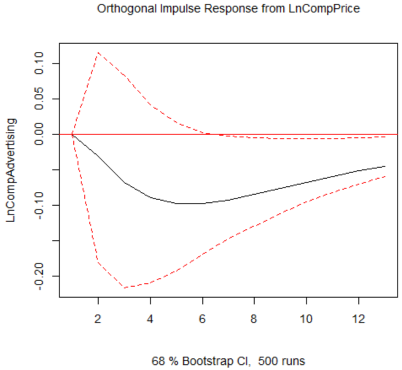


Figure 44: IRF of LnCompAdvertising to a Shock in LnCompPrice



Sales: LnSales influences LnCompAdvertising with a notable negative effect in the initial few weeks.
Advertising (Figure 41): LnAdvertising has a negative impact on LnCompAdvertising, with the effect being strongest immediately after the shock.
LnAdvertising: A shock in LnAdvertising initially has a positive effect, and at around 2 lags the effect becomes negative.
Price: LnPrice has a positive impact on LnCompAdvertising, with the effect peaking at around 4-5 lags. This indicates that a price change by Ixmor triggers a delayed but positive reaction in competitor advertising.
CompPrice: LnCompPrice shocks have a sustained negative impact on LnCompAdvertising, with a sharp response at the initial shock followed by stabilization.

Appendix **2.8** - IRF: Cumulative Orthogonal Responses on LnAdvertising

Figure 45: Cumulative IRF of LnCompAdvertising to a shock in LnSales

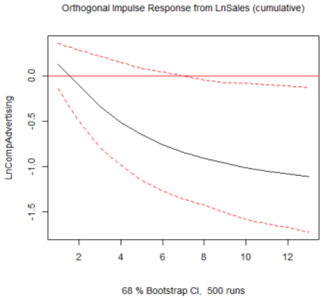


Figure 46: Cumulative IRF of LnCompAdvertising to a Shock in LnAdvertising

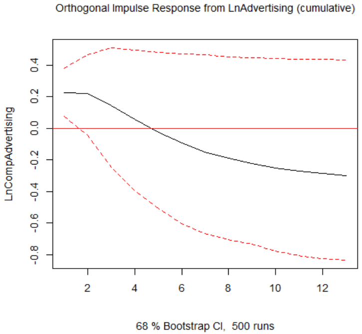


Figure 47: Cumulative IRF of LnCompAdvertising to a Shock in LnCompAdvertising

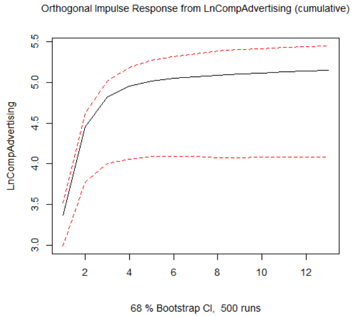


Figure 48: Cumulative IRF of LnCompAdvertising to a Shock in LnPrice

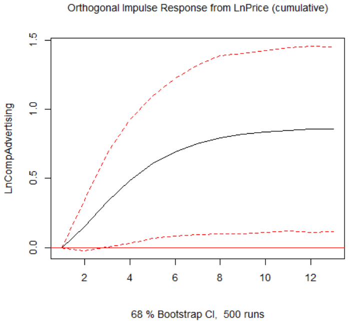
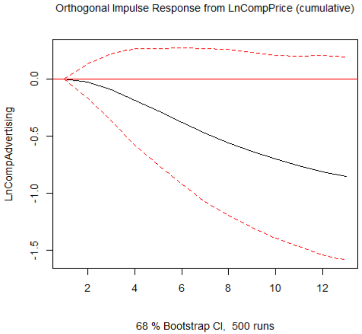


Figure 49: Cumulative IRF of LnCompAdvertising to a Shock in LnCompPrice



Positive Effects: Shocks to LnCompAdvertising and LnPrice have positive cumulative effects
Negative Effects: Shocks to LnAdvertising, LnSales, and LnCompPrice have negative cumulative effects.
Long Term: Most effects start to plateau around 10-12 lags.

Appendix **2.8** - IRF: Orthogonal Responses on LnPrice

Figure 50: IRF of LnPrice to a shock in LnSales

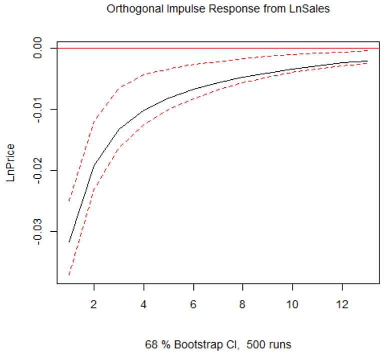


Figure 51: IRF of LnPrice to a Shock in LnAdvertising

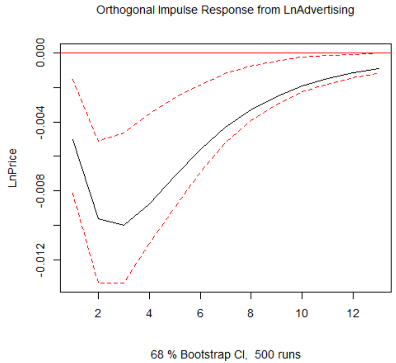


Figure 52: IRF of LnPrice to a Shock in LnCompAdvertising

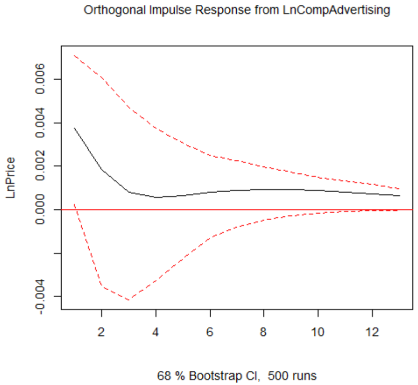


Figure 53: IRF of LnPrice to a Shock in LnPrice

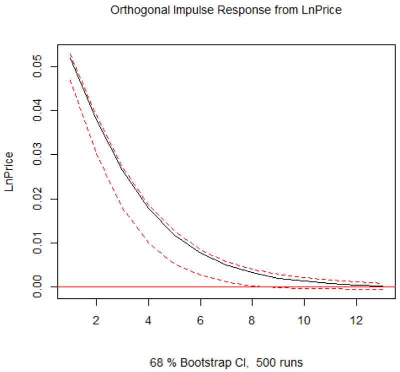
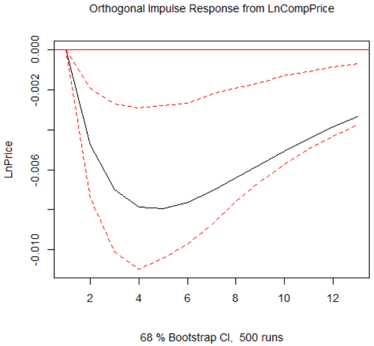


Figure 54: IRF of LnPrice to a Shock in LnCompPrice



LnSales: immediate negative effect, levels out slowly
LnAdvertising: negative effect strongest at lag 3
LnCompAdvertising: immediate positive effect that slowly levels out
LnPrice: Sharp positive initial effect. However, it is on the verge of statistical significance.
LnCompPrice: Delayed negative effect, strongest at lag 4-6

Appendix **2.8** - IRF: Cumulative Orthogonal Responses on LnPrice

Figure 55: Cumulative IRF of LnPrice to a shock in LnSales

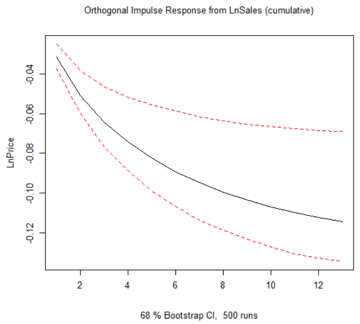


Figure 56: Cumulative IRF of LnPrice to a Shock in LnAdvertising

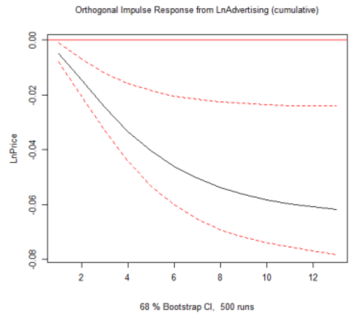


Figure 57: Cumulative IRF of LnPrice to a Shock in LnCompAdvertising

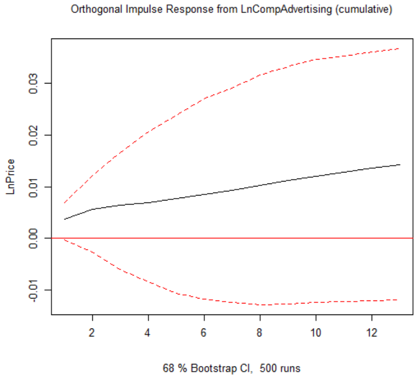


Figure 58: Cumulative IRF of LnPrice to a Shock in LnPrice

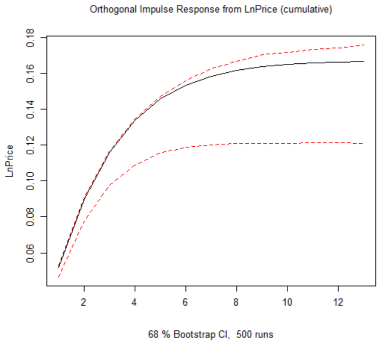
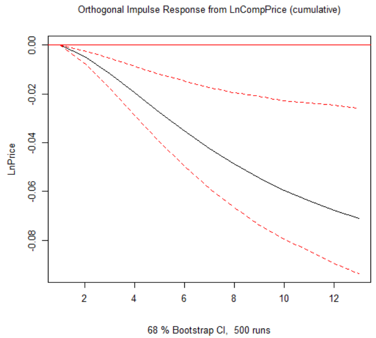


Figure 59: Cumulative IRF of LnPrice to a Shock in LnCompPrice



Positive effects: LnSales, LnCompAdvertising, and LnPrice have positive cumulative effects on LnPrice
Negative effects: LnAdvertising and LnCompPrice have cumulative negative effects on LnPrice

Appendix **2.8** - IRF: Orthogonal Responses on LnCompPrice

Figure 60: IRF of LnCompPrice to a shock in LnSales

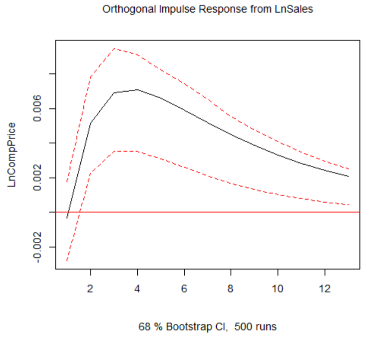


Figure 61: IRF of LnCompPrice to a Shock in LnAdvertising

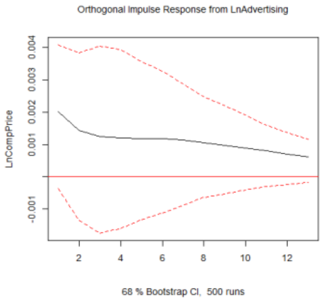


Figure 62: IRF of LnCompPrice to a Shock in LnCompAdvertising

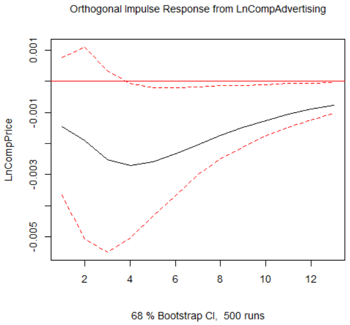


Figure 63: IRF of LnCompPrice to a Shock in LnPrice

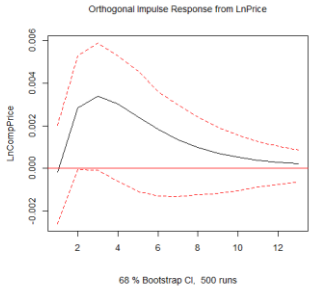
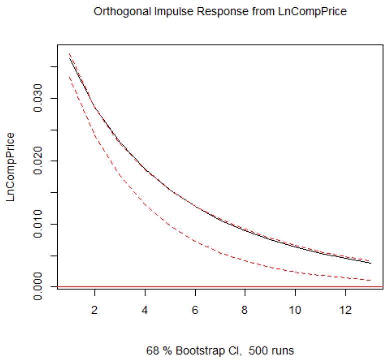


Figure 64: IRF of LnCompPrice to a Shock in LnCompPrice



LnSales: positive effect strongest at 4 lags
LnAdvertising: positive effect that dies out slowly
LnCompAdvertising: negative effect strongest at lag 4
LnPrice: positive effect strongest at lag 3-4
LnCompPrice: Immediate positive effect, however on the verge of significance.

Appendix **2.8** - IRF: Cumulative Orthogonal Responses on LnCompPrice

Figure 65: Cumulative IRF of LnCompPrice to a shock in LnSales

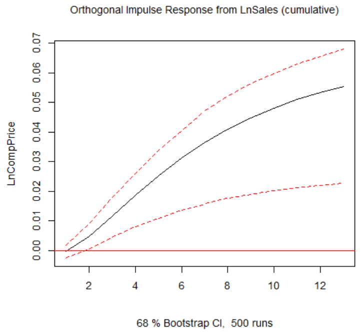


Figure 66: Cumulative IRF of LnComp Price to a Shock in LnAdvertising

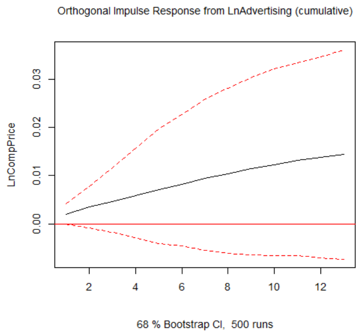


Figure 67: Cumulative IRF of LnComp Price to a Shock in LnCompAdvertising

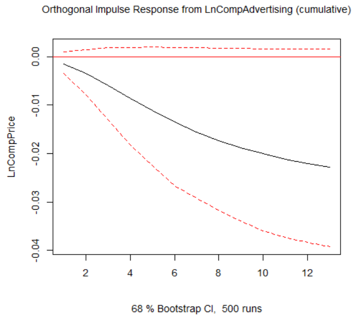


Figure 68: Cumulative IRF of LnComp Price to a Shock in LnPrice

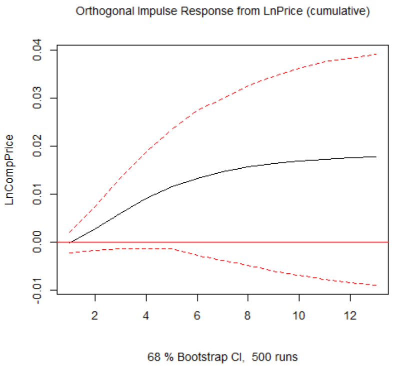
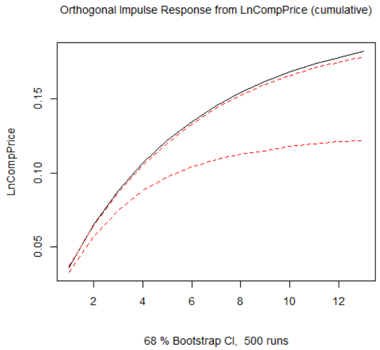


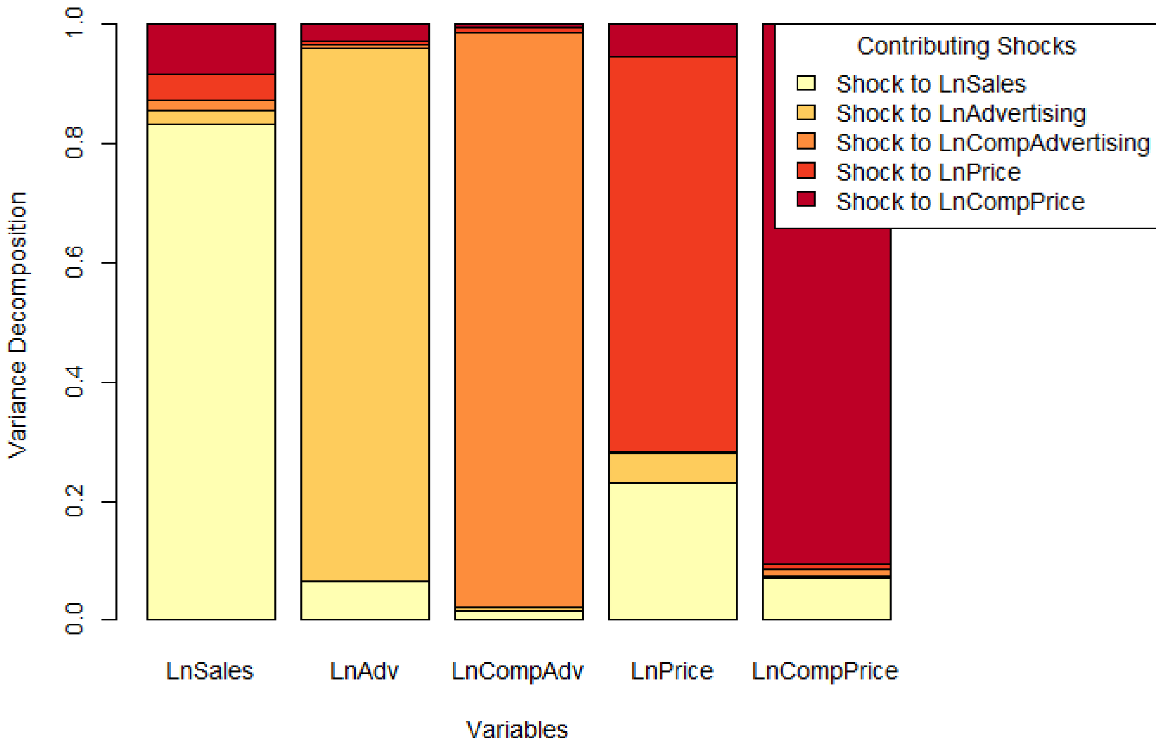
Figure 69: Cumulative IRF of LnComp Price to a Shock in LnCompPrice



Positive effects: LnSales, LnAdvertising, LnPrice, and LnCompPrice have positive cumulative effects
Negative effects: LnCompAdvertising has a negative cumulative effect

Appendix **2.9** - FEVD

Figure 70: Forecast Error Variance Decomposition



Explanation:
The vertical bars represent the decomposition of forecast variance for each of the variables.
Each color in the bar represents the proportion of variance explained by a shock to a different variable.

LnSales largely reacts to its own past values. However, past values LnCompPrice and LnPrice have the biggest impact on LnSales, while LnAdvertising and LnCompAdvertising have smaller impacts.

LnAdvertising also largely reacts to its own past values, with LnSales having the second largest impact.

LnCompAdvertising almost entirely reacts to its own past values.

LnPrice is the variable that is most influenced by other variables compared to the other 4 variables. It is mostly influenced by past values of itself, followed by past values of LnSales.

LnCompPrice is mostly influenced by past values of itself, with LnSales having some impact and the other 3 variables having minimal impact.