



LUND
UNIVERSITY

Differences-in-Differences & two-way fixed effects

Nils Droste

2021 ClimBEco course



Causal Inference from observational data

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Synopsis: Today, we will be looking into *the* classical research design for inferring causal effects from observational data (i.e. when experiments are unethical or infeasible), and its recent developments



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- quasi / natural experiments



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 - Difference-in-Differences
 - (two-way) fixed-effects regressions



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 - Difference-in-Differences
 - (two-way) fixed-effects regressions
 - staggered treatment



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Quasi- / natural experiment:

A setting where a subpopulation is treated with an intervention of sorts that occurs due to non-random assignment processes (outside of the researchers influence if its called *natural*).



Recall potential outcome approximation

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We may choose to infer an average treatment effect (ATE) I : i.e. $I = \{A, B\}$ by comparing the average outcomes of treated individuals a from A with the one of untreated individuals $b \in B$:

$$E\{\Delta Y_i\} \approx E\{Y_a(1)\} - E\{Y_b(0)\} \quad (1)$$

For such a case we can exploit *random chance* within sufficiently large samples to make these groups comparable.



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We may choose to infer an average treatment effect (ATE) τ : i.e. $\tau = \{A, B\}$ by comparing the average outcomes of treated individuals a from A with the one of untreated individuals $b \in B$:

$$E\{\Delta Y_i\} \approx E\{Y_a(1)\} - E\{Y_b(0)\} \quad (1)$$

For such a case we can exploit *random chance* within sufficiently large samples to make these groups comparable.

But what if we do not have a random assignment (and there may be a selection-bias and / or substantial differences between groups)?

a famous case

By comparing a treated with an untreated group over 2+ periods, we can control for (time-constant) differences between groups.

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a famous case

By comparing a treated with an untreated group over 2+ periods, we can control for (time-constant) differences between groups.

A classic example is the Card and Krueger (AER, 1994), comparing fast-food worker employment in Pennsylvania (PA) and New Jersey (NJ) before and after a minimum wage raise in NJ in 1992.

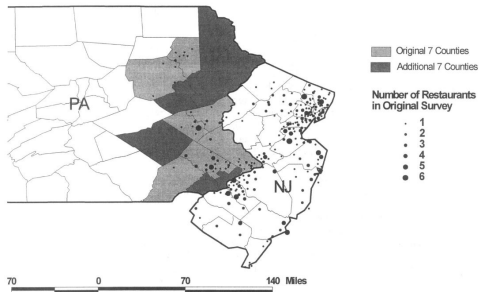


FIGURE 1. AREAS OF NEW JERSEY AND PENNSYLVANIA COVERED BY ORIGINAL SURVEY AND BLS DATA

Image source: Card and Krueger 2000



results

Here is a (tweaked) version of Card and Krueger 1994, Figure 1.

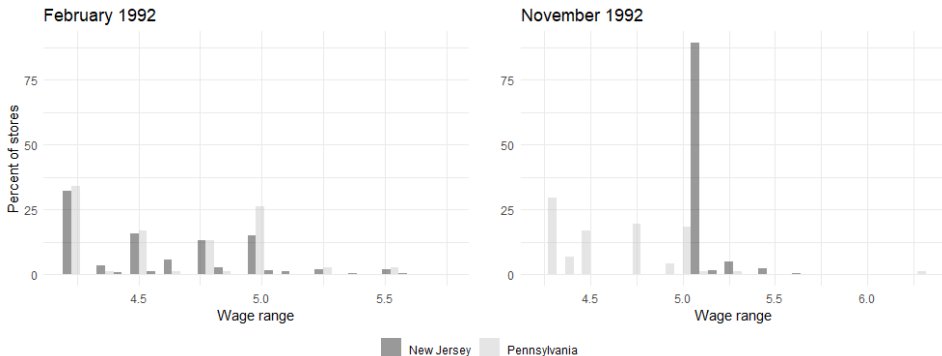


Image source: Card and Krueger 1994



potential outcome notation

Here, we are interested in the average treatment effect on the treated (ATT)

$$ATT = E[Y_i(1) - Y_i(0) | D = 1] \quad (2)$$

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potential outcome notation

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For this to be a consistent estimator, we will need a set of conditions to hold, some of which we can test, others we will need to assume.

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For this to be a consistent estimator, we will need a set of conditions to hold, some of which we can test, others we will need to assume.

In particular, the parallel-trends assumption:

$$E[Y_{it}(0) - Y_{it-1}(0) | D = 1] = E[Y_{it}(0) - Y_{it-1}(0) | D = 0] \quad (3)$$

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In particular, the parallel-trends assumption:

$$E[Y_{it}(0) - Y_{it-1}(0) | D = 1] = E[Y_{it}(0) - Y_{it-1}(0) | D = 0] \quad (3)$$

because then, we can assume

$$ATT = E[Y_t - Y_{t-1} | D = 1] - E[Y_t - Y_{t-1} | D = 0] \quad (4)$$

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Directed acyclic graphs

Difference-in-Differences (DID)

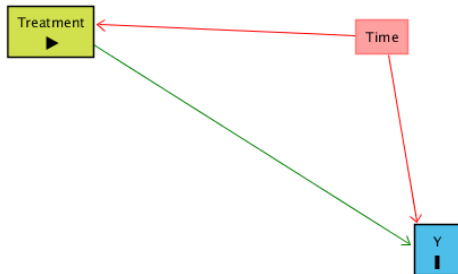


Image source: [Huntington-Klein 2018](#)

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Difference-in-Differences (DID)

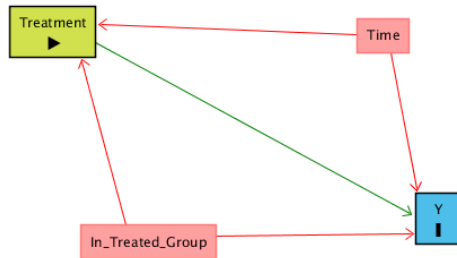
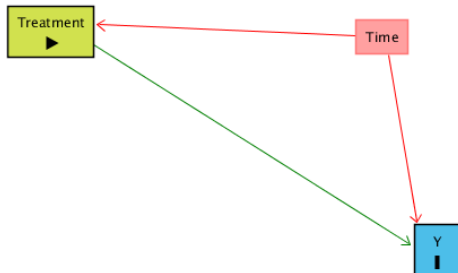


Image source: [Huntington-Klein 2018](#)



Directed acyclic graphs

Difference-in-Differences (DID)

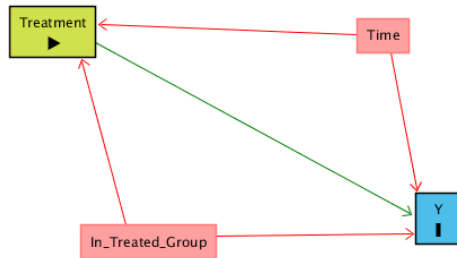
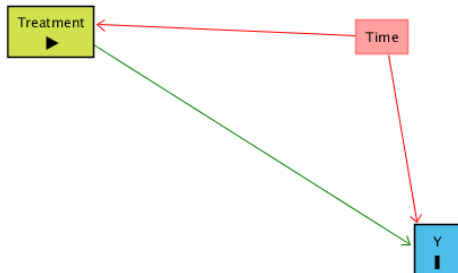


Image source: [Huntington-Klein 2018](#)

→ Accounting for differences between groups over time enables not just the estimation of time and group effects but also the differences-in-differences (i.e. the interaction of time and group effects).

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DID Estimator

assume n individual units i ,
and $t = 2$ time periods,
we can estimate the effect of a
treatment occurring at $P_{t=1}$,
affecting the treated
subpopulation $D_i = 1$

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 P_t + \beta_3 D_i \times P_t + \varepsilon_i \quad (5)$$

with $D_i = \text{Treatment}$,
 $P_t = \text{Period Dummy}$.

β_3 gives us an estimate of the
diff-in-diff treatment effect.

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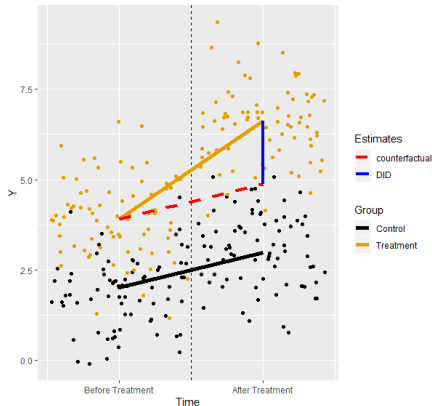
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regression

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To estimate an example ATT, we can use the Card and Krueger (1994) data

```
> did_model <- lm(emptytot ~ time + treated + time:treated,  
                  data = card_krueger_1994_mod)  
  
> summary(did_model)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	23.331	1.072	21.767	<2e-16 ***
time	-2.166	1.516	-1.429	0.1535
treated	-2.892	1.194	-2.423	0.0156 *
time:treated	2.754	1.688	1.631	0.1033

Note: heteroskedasticity and autocorrelation robust standard errors should be computed

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A difference-in-differences approach allows us to

- compare a treatment group with an untreated quasi-counterfactual
- even under conditions of a non-random assignment
- assuming that the groups behave comparably enough



Panel Data

Let us suppose a situation with repeated measurements for multiple individual(s) (units), i.e. cross-sectional time-series, or panel data.

An example panel data structure

individual (i)	time (t)	Y_i	X_i	D_{it}
A	1	0.8	0.3	0
A	2	0.7	0.2	0
A	3	0.5	0.2	1
B	1	1.2	0.4	0
B	2	1.1	0.5	0
B	3	0.9	0.6	1
...



Fixed-Effects

An exemplary study



Available online at www.sciencedirect.com



Ecological Economics 55 (2005) 527–538

**ECOLOGICAL
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ANALYSIS

Environmental pressure group strength and air pollution: An empirical analysis

Seth Binder, Eric Neumayer*

*Department of Geography and Environment and Center for Environmental Policy and Governance (CEPG),
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Received 7 December 2003; received in revised form 22 October 2004; accepted 14 December 2004

Available online 24 February 2005



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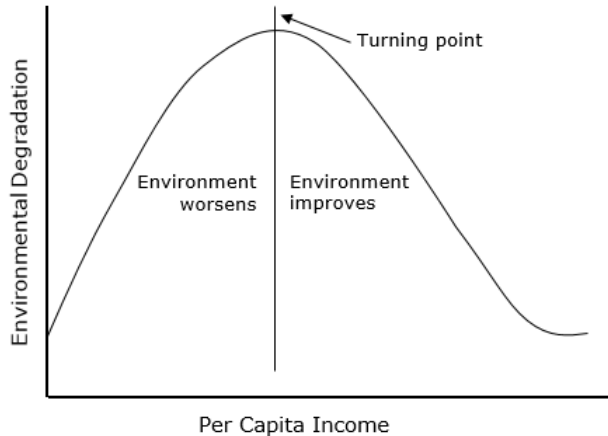
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An exemplary study



Theoretical underpinning for Binder and Neumayer 2005: Environmental Kuznets Curve. Image source:

[Wikipedia](#)

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Replicated results from table 1, model 1 Binder and Neumayer 2005

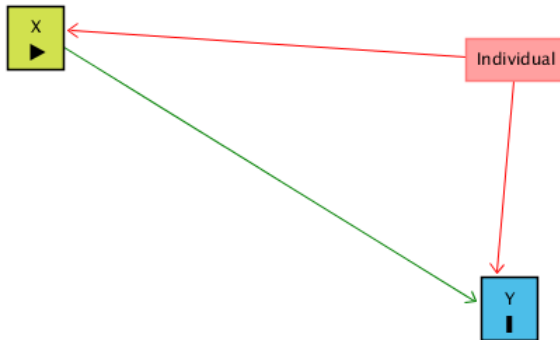
Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-57.55012	16.60671	-3.465	0.000548	***
lnengopc	-0.51121	0.11878	-4.304	1.82e-05	***
lnenergy	1.00887	0.60455	1.669	0.095425	.
lngdp	13.81819	4.17975	3.306	0.000975	***
lngdpsq	-0.88657	0.27384	-3.238	0.001239	**
polity	-0.05079	0.03023	-1.680	0.093135	.

Note: This is not a fixed effects regression but an OLS

Panel Data

One way fixed-Effects (FE) structure



Accounting for individual fixed effects. Image source: [Huntington-Klein 2018](#)



Fixed-Effects

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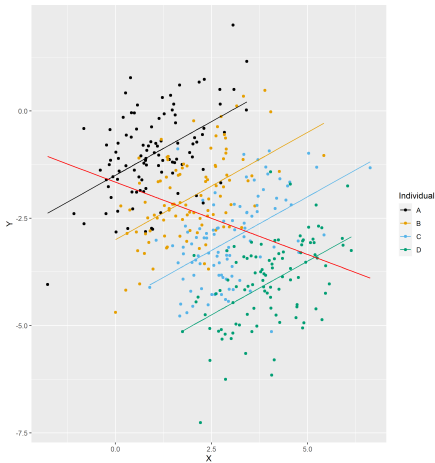
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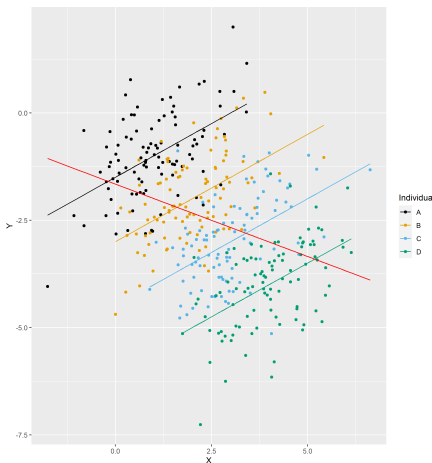
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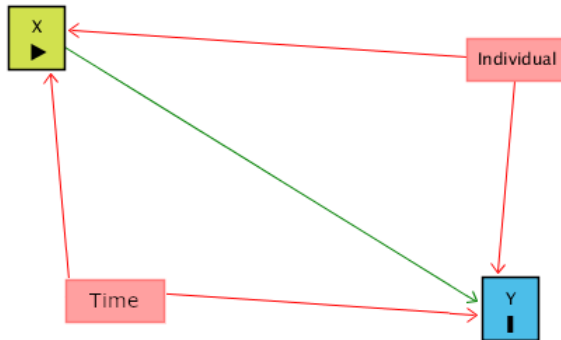
FE Estimator

$$Y_i = \alpha_i + \beta X_{it} + \varepsilon_{it} \quad (6)$$

with α_i = individual, time-invariant effect, and X_i = a variable of interest. The so called *within* transformation accounts for α_i through demeaning such that we can consistently estimate $\partial Y / \partial X$

Fixed-Effects

Two way fixed-effects



Accounting for individual and time (two-way) fixed effects. Image source: [Huntington-Klein 2018](#)



Estimator

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The two-way fixed effects model can be formulated as

$$Y_{it} = \alpha_i + \theta_t + \tau D_{it} + \beta X_{it} + \varepsilon_i \quad (7)$$

where α_i individual and θ_t time fixed effects, respectively

Note that D here is a binary treatment indicator ($D_{it} = D_i \times P_t$), possibly a vector of control variables X_i ,

and error term ε_i , assumed to be normally distributed and centered around 0, independent of everything else.

time-trends

This allows to take both individual differences and shocks in time into account

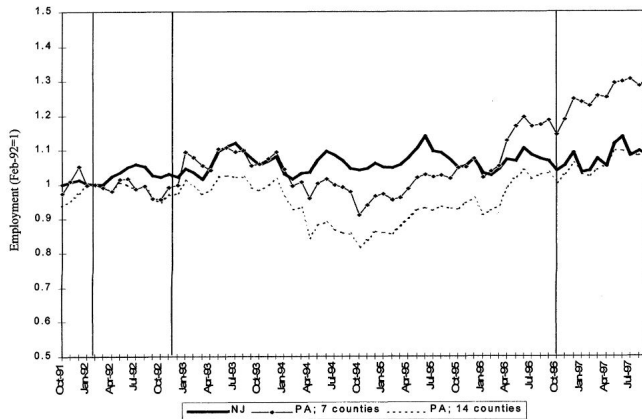


FIGURE 2. EMPLOYMENT IN NEW JERSEY AND PENNSYLVANIA FAST-FOOD RESTAURANTS, OCTOBER 1991 TO SEPTEMBER 1997

Note: Vertical lines indicate dates of original Card-Krueger survey and the October 1996 federal minimum-wage increase.

Source: Authors' calculations based on BLS ES-202 data.

Image source: Card and Krueger 2000



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A two-way fixed effects approach allows us to

- control for time-constant, unobserved heterogeneity between individuals
- control for common time shocks, that affect all individuals
- a multi-period 2WFE approach resembles DiD when
 - treatment is simultaneous
 - effects are homogeneous

Staggered treatment and other current developments

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Over the last 6 or so years, DiD methodology has seen quite some development.

I will briefly introduce



Staggered treatment and other current developments

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I will briefly introduce

- staggered treatment / event-time studies



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I will briefly introduce

- staggered treatment / event-time studies
- non-parallel trend corrections



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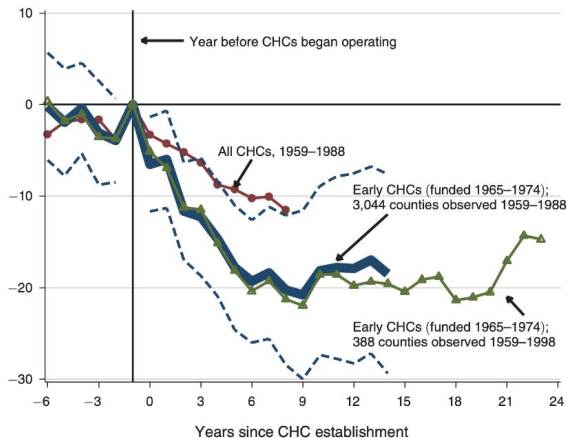
I will briefly introduce

- staggered treatment / event-time studies
- non-parallel trend corrections
- heterogeneous treatment corrections



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What if treatment happened at different times?

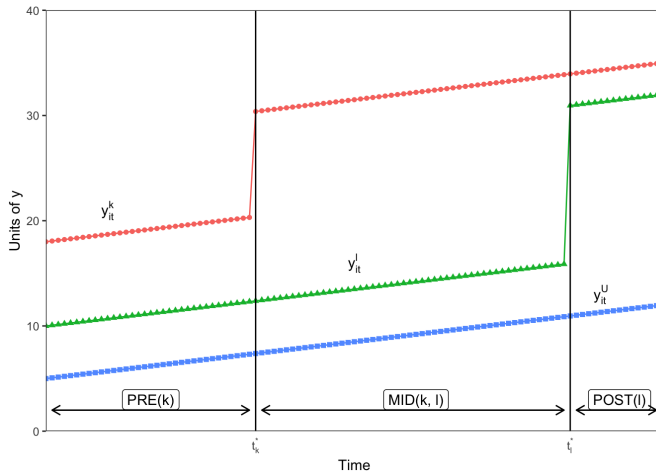


Reduction in mortality by Community Health Centers (CHC). Image source: Bailey and Goodman-Bacon 2015



intuition

We may run into time series length related weighting problems



DiD with variations in treatment timing. Image source: Goodman-Bacon 2018, as reproduced by [A.C. Baker](#)



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Recall,

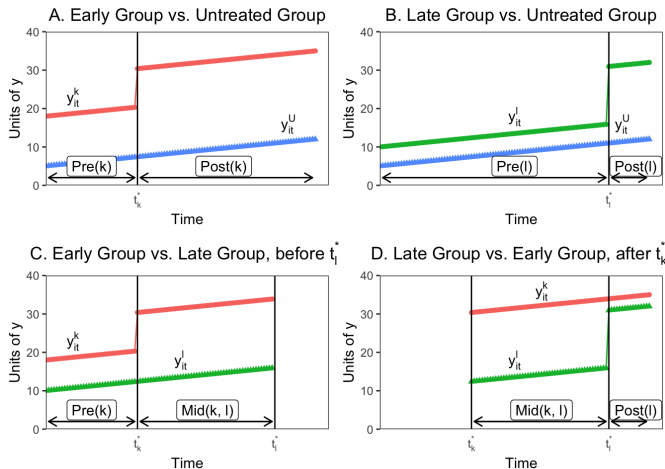
$$ATT = E[Y_t - Y_{t-1} | D = 1] - E[Y_t - Y_{t-1} | D = 0]$$

which is a comparison of mean differences between groups over time.
Straightforward for 2 periods.

Now, we have multiple periods.

intuition

Picture a case with multiple periods, three groups, and thus four comparisons



DiD with variations in treatment timing. Image source: Goodman-Bacon 2018, as reproduced by [A.C. Baker](#)



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Turns out: 2WFE DiD estimator is a weighted average of these comparisons

Theorem 1. Difference-in-Differences Decomposition Theorem

Assume that the data contain $k = 1, \dots, K$ groups of units ordered by the time when they receive a binary treatment, $\mathbf{t}_k^* \in (1, T]$. There may be one group U , that never receives treatment. The OLS estimate $\hat{\beta}^{DD}$, in a two-way fixed-effects model (2) is a weighted average of all possible two-by-two DD estimators.

$$\hat{\beta}^{DD} = \sum_{k \neq U} s_{kU} \hat{\beta}_{kU}^{2x2} + \sum_{k \neq U} \sum_{\ell > k} s_{k\ell} [\mu_{k\ell} \hat{\beta}_{k\ell}^{2x2,k} + (1 - \mu_{k\ell}) \hat{\beta}_{k\ell}^{2x2,\ell}] \quad (7)$$

Where the two-by-two DD estimators are:

$$\hat{\beta}_{kU}^{2x2} \equiv (\bar{y}_k^{POST(k)} - \bar{y}_k^{PRE(k)}) - (\bar{y}_U^{POST(j)} - \bar{y}_U^{PRE(j)})$$

$$\hat{\beta}_{k\ell}^{2x2,k} \equiv (\bar{y}_k^{MID(k,\ell)} - \bar{y}_k^{PRE(k)}) - (\bar{y}_\ell^{MID(k,\ell)} - \bar{y}_\ell^{PRE(k)})$$

$$\hat{\beta}_{k\ell}^{2x2,\ell} \equiv (\bar{y}_\ell^{POST(\ell)} - \bar{y}_\ell^{MID(k,\ell)}) - (\bar{y}_k^{POST(\ell)} - \bar{y}_k^{MID(k,\ell)})$$

the weights are:

$$s_{kU} = \frac{n_k n_U \bar{D}_k (1 - \bar{D}_k)}{\widehat{var}(\bar{D}_{it})}$$

$$s_{k\ell} = \frac{n_k n_\ell (\bar{D}_k - \bar{D}_\ell) (1 - (\bar{D}_k - \bar{D}_\ell))}{\widehat{var}(\bar{D}_{it})}$$

$$\mu_{k\ell} = \frac{1 - \bar{D}_k}{1 - (\bar{D}_k - \bar{D}_\ell)}$$

and $\sum_{k \neq U} s_{kU} + \sum_{k \neq U} \sum_{\ell > k} s_{k\ell} = 1$.

Proof: See appendix A.

Bacon decomposition theorem. Image source: Goodman-Bacon 2018, see also his [tweet-thread](#)

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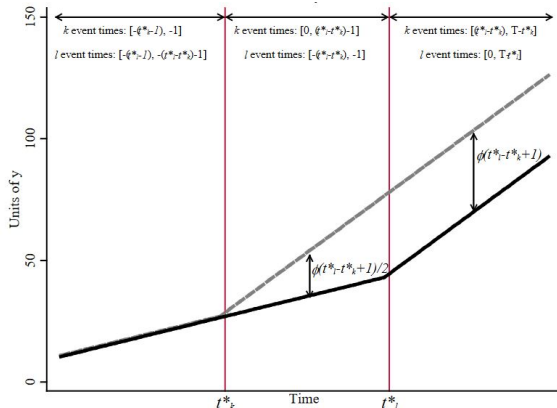
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notation

Even worse, a positive but staggered slope change, can even change the sign of the DD estimate



Staggered treatment with linear slope change. Image source: Goodman-Bacon 2018, see also his [tweet-thread](#)

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Several works on the issue:



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Several works on the issue:

- Athey and Imbens 2020
 - under random adoption dates, a weighted DID is unbiased
 - comparing treated with not-yet treated



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References



Several works on the issue:

- Athey and Imbens 2020
 - under random adoption dates, a weighted DID is unbiased
 - comparing treated with not-yet treated
- Goodman-Bacon 2018
 - a time-in-treatment weighted DiD, fixing the weights to gain balance
 - R package [bacondecomp](#)

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Several works on the issue:



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Several works on the issue:

- Callaway and Sant'Anna 2020
 - bootstrapped inference with pre-intervention conditioning on co-variates
 - R package did

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References



Several works on the issue:

- Callaway and Sant'Anna 2020
 - bootstrapped inference with pre-intervention conditioning on co-variates
 - R package did
- Sun and Abraham 2020
 - time-to-treatment (cohort) weighted approach, comparing to never-treated
 - implemented in fixest R package

Heterogeneous Treatment

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Further works on DiD design / application issues:

- Chaisemartin and D'Haultfœuille 2020
 - allowing for treatment effects "heterogeneous across groups and over time periods"
 - R package DIDmultiplegt
- Imai and Kim 2020
 - identify some problematic negative weighting in 2WFE
 - they propose weighting or matching Imai 2020



Non-parallel trends

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However, all these approaches, assume (to some extent) parallel trends

- Jonathan Roth 2021
 - test for parallel-trends, R package pretends
 - robust inference strategy under non-parallel trends Rambachan and Roth 2020, R package HonestDiD



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Summary of DID

Instantaneous (Static) Effects <i>Potential Outcomes</i> $Y_{it}(d)$ for $d \in \{0, 1\}$	Dynamic Effects $Y_{it}(g)$ for $g \in \{1, \dots, T, \infty\}$
Homogeneous Effects <i>Target Parameter</i> $\tau = \mathbb{E}[Y_{it}(1) - Y_{it}(0) D_{it} = 1]$ <i>Estimators</i> $\hat{\tau}_{MM} = (\bar{Y}_{1,post} - \bar{Y}_{1,pre}) - (\bar{Y}_{0,post} - \bar{Y}_{0,pre})$ $\hat{\tau}_{OLS} : Y_{it} = \mu + \alpha D_i + \gamma P_t + \tau D_i P_t + \varepsilon_{it}$ $\hat{\tau}_{TWFE} : Y_{it} = \alpha_i + \gamma_t + \tau D_{it} + \varepsilon_{it}$ <i>Key references</i> Card and Krueger (1994) Bertrand, Duflo, and Mullainathan (2004)	$\tau_\ell = \mathbb{E}[Y_{ig+\ell}(g) - Y_{ig+\ell}(\infty) G_i = g]$ $\hat{\tau}_{\ell,DDID} : Y_{it} = \alpha_i + \gamma_t + \sum_{\ell \in \mathcal{L}} D_i P_t^\ell \tau_\ell + \varepsilon_{it}$ $\hat{\tau}_{\ell,ES} : Y_{it} = \alpha_i + \gamma_t + \sum_{\ell \in \mathcal{L}} D_{it}^\ell \tau_\ell + \varepsilon_{it}$ Ashenfelter and Card (1984) Jacobson, LaLonde, and Sullivan (1993)
Heterogeneous Effects <i>Target Parameter</i> $\tau_{gt} = \frac{1}{N_{gt}} \sum_{i=1}^{N_{gt}} [Y_{it}(1) - Y_{it}(0)]$ <i>Estimators</i> $DID_{c,t} = \sum_{i,t} [\rho_{it}^{h_1(c)} (Y_{it} - Y_{it-1}) - \rho_{it}^{h_2(c)} (Y_{it} - Y_{it-1})]$ $\hat{\tau}_{CdH} = \sum_{t=2}^T (\omega_{+,t} DID_{+,t} + \omega_{-,t} DID_{-,t})$ <i>Key references</i> de Chaisemartin and d'Haultfoeulle (2020) Goodman-Bacon (2021)	$\tau_t(g) = \mathbb{E}[Y_{it}(g) - Y_{it}(\infty) G_i = g]$ $\hat{\tau}_t(g) = (\bar{Y}_{g,t} - \bar{Y}_{\infty,t}) - (\bar{Y}_{g,g-1} - \bar{Y}_{\infty,g-1})$ $\hat{\tau}_{CS} = \frac{1}{\sum_t \sum_{g:g \leq t} N_g} \sum_t \sum_{g:g \leq t} \tau_t(g)$ Callaway and Sant'Anna (forthcoming) Sun and Abraham (2021)

Schönholzer 2021, cf. Roth and Sant'Anna 2021



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Re: DiD and (adjusted) 2WFE estimators

- compare treated individual (units) with a reference group
- who we compare to whom deserves special attention
- software solutions available



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- Athey, Susan and Guido W Imbens (2020). 'Design-based analysis in Difference-In-Differences settings with staggered adoption'. In: *Journal of Econometrics*. DOI: 10.1016/j.jeconom.2020.10.012.
- Bailey, Martha J and Andrew Goodman-Bacon (2015). 'The war on poverty's experiment in public medicine: Community health centers and the mortality of older Americans'. In: *American Economic Review* 105.3, pp. 1067–1104. DOI: 10.1257/aer.20120070.
- Binder, Seth and Eric Neumayer (2005). 'Environmental pressure group strength and air pollution: An empirical analysis'. In: *Ecological Economics* 55.4, pp. 527–538. DOI: 10.1016/j.ecolecon.2004.12.009.
- Callaway, Brantly and Pedro H.C. Sant'Anna (2020). 'Difference-in-Differences with multiple time periods'. In: *Journal of Econometrics* xxxx, pp. 1–31. DOI: 10.1016/j.jeconom.2020.12.001.
- Card, David and Alan B Krueger (1994). 'Minimum wages and employment: a case study of the fast-food industry in New Jersey and Pennsylvania'. In: *American Economic Review* 84.4, pp. 772–793. DOI: 10.3386/w4509.
- (2000). 'Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Reply'. In: *American Economic Review* 90.5, pp. 1397–1420. DOI: 10.1257/aer.90.5.1397.
- Chaisemartin, Clément de and Xavier D'Haultfœuille (2020). 'Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects'. In: *American Economic Review* 110.9, pp. 2964–2996. DOI: 10.1257/aer.20181169.



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- Goodman-Bacon, Andrew (2018). 'Difference-in-Differences With Variation in Treatment Timing'. In: *National Bureau of Economic Research* 17.5, pp. 684–694. DOI: 10.3386/w25018.
- Imai, Kosuke and In Song Kim (2020). 'On the Use of Two-Way Fixed Effects Regression Models for Causal Inference with Panel Data'. In: *Political Analysis*, pp. 1–11. DOI: 10.1017/pan.2020.33.
- Rambachan, Ashesh and Jonathan Roth (2020). 'An Honest Approach to Parallel Trends'.
- Roth, Jonathan (2021). 'Pre-test with Caution: Event-study Estimates After Testing for Parallel Trends'.
- Roth, Jonathan and Pedro H.C. Sant'Anna (2021). 'Efficient Estimation for Staggered Rollout Designs'. In: pp. 1–52. arXiv: 2102.01291. URL: <http://arxiv.org/abs/2102.01291>.
- Sun, Liyang and Sarah Abraham (2020). 'Estimating dynamic treatment effects in event studies with heterogeneous treatment effects'. In: *Journal of Econometrics* xxxx, pp. 1–25. DOI: 10.1016/j.jeconom.2020.09.006.

