

Machine learning, Bayes and & structural models

Nils Droste

2021 ClimBEco course



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References

Today, we will talk about

- machine learning techniques
- structural equation models
- Bayesian inference

and their relation to causal inference



"There is no hierarchy in causal inference"



Moritz Poll2021

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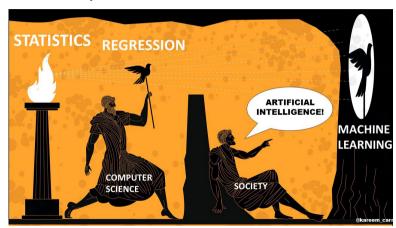
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Conclusion

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"There is no hierarchy in causal inference"





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estimation
matching

Bayesian inference

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Conclusion

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"There is no hierarchy in causal inference"

Level (Symbol)	Typical Activity	Typical Questions	Examples
1. Association $P(y x)$	Seeing	What is? How would seeing X change my belief inY?	What does a symptom tell me about a disease? What does a survey tell us about the election results?
2. Intervention $P(y do(x), z)$	Doing, Intervening	What if? What if I do X?	What if I take aspirin, will my headache be cured? What if we ban cigarettes?
3. Counterfactuals $P(y_x x',y')$	Imagining, Retrospection	Why? Was it X that caused Y? What if I had acted differently?	Was it the aspirin that stopped my headache? Would Kennedy be alive had Oswald not shot him? What if I had not been smoking the past two years?

The causal hierarchy. Source: Pearl 2019

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Conclusion

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Recall, the conditional average treatment effect (cf. Athey and Imbens 2016)

$$CATE = E(Y_i(1) - Y_i(0)|X_i = x)$$
(1)

→ we could look at average differences between treated and untreated groups within partitions of the observed



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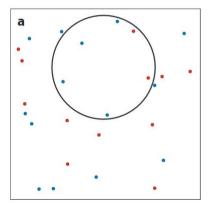
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why structural

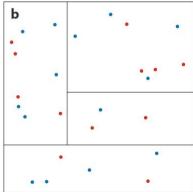
Conclusion

further reads

References

We could use covariates X_i to identify clusters within the observed, e.g. by recursive binary splitting







Source: Athey and Imbens 2019

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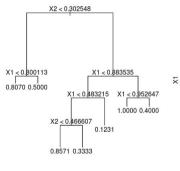
Structural Models

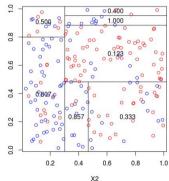
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datacamp 2021



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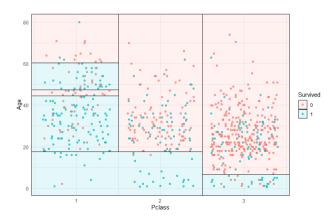
Structural Models

why structural

further reads

References

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Structural Models

Conclusion

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References

We can use classification and regression trees to estimate

$$\hat{\tau}(x) = \frac{1}{|\{i: D_i = 1, X_i \in L\}|} \sum_{\{i: D_i = 1, X_i \in L\}} Y_i - \frac{1}{|\{i: D_i = 0, X_i \in L\}|} \sum_{\{i: D_i = 0, X_i \in L\}} Y_i$$
(2)

by "recursively splitting the feature space ... into a set of leaves L" (Wager and Athey 2018)



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causal trees estimation

Ravesian inference

Structural Models

Conclusion

further reads

References

We can use classification and regression trees to estimate

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by "recursively splitting the feature space ... into a set of leaves L" (Wager and Athey 2018)

and repeat to build an ensemble of trees \rightarrow random forests. This particularly well suited for heterogeneous treatment effects. Athey and Imbens (2016) suggest an *honest* cross validation procedure for robust causal inference, see generalized random forests R package.



honest crossvalidation

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Structural Models

Conclusion

further reads

References

This recursive partioning uses multiple optimizations

- within sample (e.g. MSE, possibly with penalties) \rightarrow danger of overfitting to sample
- \blacksquare minimizing prediction error on unseen portions of the data to reduce out-of-sample bias \rightarrow (n-fold) crossvalidation



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Structural Models

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References

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If we do this honestly

- we can compute average treatment effects in the splits
- it promises a valid CATE estimation
- it allows to estimate heterogenous treatment effects



heterogenous treatment effects

Appendix A. Regression tree explanation of CATE

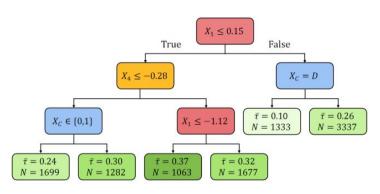


Figure 6: Visualization of a regression tree fit to the imputed CATE values based on the full covariate set.



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Source: Johansson 2019

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Structural Models

Conclusion

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References

Bayesian Additive Regression Trees (BART), e.g. Hill 2011

- estimate two models
 - one on the treatment assignment process
 - a second on the treatment effect (aka response surface)



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Structural Models

Conclusion

further reads

References

Bayesian Additive Regression Trees (BART), e.g. Hill 2011

- estimate two models
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- seems to perform weel in comparison to linear regression, propensity-score matching, etc. (Hill 2011)

For a suite of latest and different approaches read up on C. Carvalho et al. 2019



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Conclusion

further reads

References

Bayesian Additive Regression Trees (BART), e.g. Hill 2011

- estimate two models
 - one on the treatment assignment process
 - a second on the treatment effect (aka response surface)
- seems to perform weel in comparison to linear regression, propensity-score matching, etc. (Hill 2011)
- "BART is not constrained by prior theories" (ibid.)
 - not sure this is unproblematic



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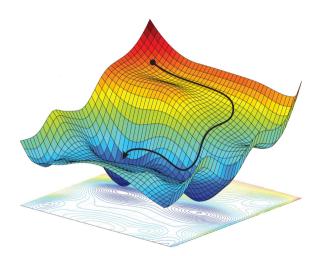
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Bayesian inference

Structural Models
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Conclusion

further reads







Instead of ordinary least squares

$$\hat{beta}^{ols} = \arg\min_{\beta} \sum_{i=1}^{N} (Y - \beta X_i)^2$$
 (3)

we can also use machine learning techniques (aka sequential updates / iterative estimation / optimization of parameter values) to estimate

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Structural Models

Conclusion

further reads



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$$\hat{beta}^{ml} = \arg\min_{\beta} \sum_{i=1}^{N} (Y - \beta X_i)^2 + \lambda (\|\beta\|_q)^{1/q}$$
 (4)

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Bayesian inference

Structural Models

Conclusion

further reads



Instead of ordinary least squares

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with a regularizing penalty term that results in sparse models, q=1 for Lasso, q=2 ridge regression, and $q\to 0$ for best subset regression, or hybrids such as elastic nets (cf. Athey and Imbens 2019) and posterior choice of λ by (n-fold) cross-validation.

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Bayesian inference

Structural Models

Conclusion

further reads



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causal trees
estimation

Bayesian inference

Structural Models

Conclusion

further reads

References

Or we use neural networks, where we transform

$$Z_{ik}^{(1)} = \sum_{j=1}^{K} \beta_{kj}(1) X_{ij} \quad \text{for} \quad k = i, ..., K_1$$
 (5)

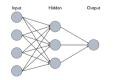
with K covariates (aka features) and latent (aka unobserved / hidden node) variable $Z_i k$, e.g. into a recitified linear function $g(z) = z 1_{z>0}$.



Or we use neural networks, where we transform

$$Z_{ik}^{(1)} = \sum_{j=1}^{K} \beta_{kj}(1) X_{ij}$$
 for $k = i, ..., K_1$ (5)

with K covariates (aka features) and latent (aka unobserved / hidden node) variable $Z_j k$, e.g. into a recitified linear function $g(z) = z 1_{z>0}$. which allows us a to formulate a neural network



$$Y_i = \sum_{k=1}^{\kappa_1} \beta k(2) g[Z_{ik}^2] + \varepsilon_i \qquad (6)$$

with a single layer K_1 , and a non-linear transformation $g(\cdot)$ (cf. Athey and Imbens 2019; Farrell, Liang and Misra 2021).

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causal trees
estimation

Bayesian inference

Structural Models

Conclusion

further reads



matching or synthetic control

ausal trees

matching

Bayesian interence

Structural Models

Conclusion

further reads

References

Or, we can use machine learning methods for matching or synthetic control (Doudchenko and Imbens 2020)

e.g to estimate weights w and intercept μ for simulating counterfactual

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i=1}^{N} w_i Y_{i,T}^{obs}$$

$$(7)$$

by

$$(\hat{\mu}^{ols}, \hat{w}^{ols}) = \arg\min_{\mu, w} \sum_{s=1}^{T_0} \left(Y_{0, T_0 - s + 1}^{obs} - \mu - \sum_{i=1}^{N} w_i \cdot Y_{0, T_0 - s + 1}^{obs} \right)^2 + \underbrace{\lambda \left(\frac{1 - \alpha}{2} \| w \|_2^2 + \alpha \| w \|_1 \right)}_{\text{penalty function}}$$
(8)



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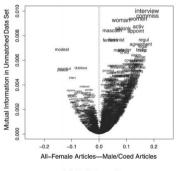
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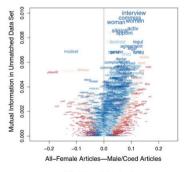
Conclusion

further reads

References

We can also used unsupervised machine learning, such as e.g. latent dirichlet allocation, to use text as data (Gentzkow, Kelly and Taddy 2019) for matching a treated with an untreated population (aka matching on similar content)





(a) Full Data Set

(b) Topic Matched

Topical Inverse regression matching (TIRM). Source: Roberts, Stewart and Nielsen 2020



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Bayesian inference

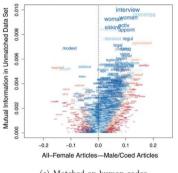
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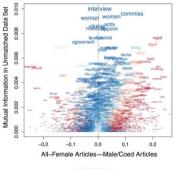
Conclusion

further reads

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(c) Matched on human codes

(d) TIRM

Topical Inverse regression matching (TIRM). Source: Roberts, Stewart and Nielsen 2020



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Bayesian inference

Structural Models
why structural

Conclusion

further read

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$
(9)



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Bayesian inference

Structural Models

Conclusion

further reads

References

Now that we talked about sequential parameter optimization, model ensembles (aka model averaging), and posteriors, ...



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Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

Now that we talked about sequential parameter optimization, model ensembles (aka model averaging), and posteriors, ...

we are close to talk about a Bayesian approach. Suppose we formulate econometric model



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Bayesian inference

Structural Models

Conclusion

further reads

References

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$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \quad \propto \quad p(y|\theta)p(\theta)$$
 (10)



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Bayesian inference

Structural Models
why structural

Conclusion

further reads

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 (10)

with *prior* parameter probability $p(\theta)$ and the *posterior* probability (aka conditional probability) $p(y|\theta)$), the likelihood of observables given the parameter.



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Structural Models

Conclusion

further reads

References

This framework can be used to estimate literally all types of models that we have been discussing, e.g.



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causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

This framework can be used to estimate literally all types of models that we have been discussing, e.g.

■ Bayesian linear regression (and thus Bayesian 2SLS)



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causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

This framework can be used to estimate literally all types of models that we have been discussing, e.g.

- Bayesian linear regression (and thus Bayesian 2SLS)
- Panel data (Ning, Ghosal and Thomas 2019)



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causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

This framework can be used to estimate literally all types of models that we have been discussing, e.g.

- Bayesian linear regression (and thus Bayesian 2SLS)
- Panel data (Ning, Ghosal and Thomas 2019)
- Regression discontinuity (Hinne, Van Gerven and Ambrogioni 2020)
- Ridge and Lasso regressions have their Bayesian interpretation already (cf. Athey and Imbens 2019)



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causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

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- Regression discontinuity (Hinne, Van Gerven and Ambrogioni 2020)
- Ridge and Lasso regressions have their Bayesian interpretation already (cf. Athey and Imbens 2019)
- Simulating a synthetic control by Bayesian structural time series analysis (Brodersen et al. 2015)
- Bayesian random forests (Hill 2011; Hahn, Murray and C. M. Carvalho 2020)

It has the merit of being explicit of prior beliefs about distributions (or probalities of parameters) and thus uncertainties



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causal trees estimation matching

Bayesian inference

Structural Models
why structural

Conclusio

further reads







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Bayesian inference

Structural Models
why structural

Conclusion

further reade

References



Black Box Models. Source: Zhao and Hastie 2021



We need a theory of change

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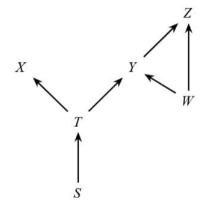
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Bayesian inference

Structural Models
why structural

Conclusion

further reads







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causal trees
estimation

Bavesian inference

Structural Models

Conclusion

further reads

"[G]raphical representations of structural causal models do not require the learner - whether artificial or human - to *impose any distributional or functional-form restrictions* on the underlying causal mechanisms under study.

The approach remains fully popparametric, a characteristic it shares with the

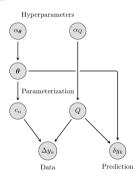
The approach remains fully nonparametric, a characteristic it shares with the potential outcomes framework. At the same time, however, crucial identification assumptions, such as ignorability, are derived from the properties of the underlying structural model, rather than being assumed to hold a priori.

Causal graphical models thus combine the accessibility and flexibility of potential outcomes with the preciseness and analytical rigor of structural econometrics" (Hünermund and Bareinboim 2021)



structural causal models (SCM)

Parameters of a probabilistic structural model can be estimated with a <u>Bayesian network approach</u> ...



Bayesian network. Source: Melendez et al. 2019

... partial least squares, or through <u>double debiased machine learning</u> (cf. citeChernozhukov2018,Jung2021)

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Bayesian inference

Structural Models
why structural

Conclusion

further reads

References



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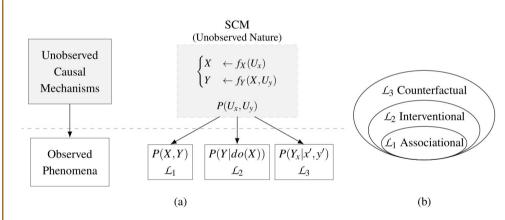
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Bayesian inference

Structural Models
why structural

Conclusion

further reads







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Bayesian inference

Structural Models

why structural

Conclusion

further reads





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estimation matching

Bayesian inference

Structural Models

Conclusion

further reads

References

To estimate a causal effect, we can compare the outcome to its counterfactual.

■ potential outcomes: $\tau = E\{Y_i(1)\} - E\{Y_i(0)\}$



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estimation matching

Bayesian interence

Structural Models

Conclusion

further reads

References

- potential outcomes: $\tau = E\{Y_i(1)\} E\{Y_i(0)\}$
 - RCTs
 - DID & 2WFE
 - Matching & synthetic controls
 - IV & RDD
 - causal trees
 - SCM



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estimation matching

Bayesian interence

Structural Models

Conclusion

further reads

References

- potential outcomes: $\tau = E\{Y_i(1)\} E\{Y_i(0)\}$
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- with frequentist, Bayesian, or machine learning methods



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Bayesian interence

Structural Models

Conclusion

further reads

References

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estimation matching

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

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- with frequentist, Bayesian, or machine learning methods
- it should be conceptually sound and methodologically robust
- I think the relation with theory should be healthy (inductively so or deductive)



sources

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Structural Models
why structural

Conclusion

further reads

References

Beyond the coursebooks, see

- James, Witten, Hastie and Tibshirani (2013)
 - introduction to statistical learning (<u>edX</u>)
- Boehmke and Greenwell (2020)
 - Hands-On Machine Learning with R
- Paul Goldsmith-Pinkham 2021
 - applied methods PhD course
- Nick Huntington-Klein 2021
 - The Effect: An Introduction to Research Design and Causality
- Peters, Janzing and Schölkopf 2017
 - Elements of causal inference: foundations and learning algorithms
- Nagarajan, Scutari and Lèbre 2013
 - Bayesian Networks in R



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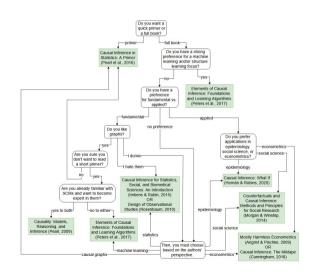
Bayesian inference

Structural Models

further reads

References





decision tree, source: Brady Neal 2020

27/30

References I

machine learning
causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References



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References II

machine learning
causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

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References III

machine learning
causal trees
estimation

Bayesian inference

Structural Models
why structural

Conclusion

further reads

References

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