Difference-in-Differences & two-way fixed effects

Nils Droste

2022 ClimBEco course

Causal Inference from observational data

Synopsis: Today, we will be looking into *the* classical research design for inferring causal effects from observational data (i.e. when experiments are unethical or infeasible), and its recent developments

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- quasi / natural experiments
  - Difference-in-Differences
  - (two-way) fixed-effects regressions
  - staggered treatment

Definition

Quasi- / natural experiment:

A setting where a subpopulation is treated with an intervention of sorts that occurs due to non-random assignment processes (outside of the researchers influence if its called *natural*).

Recall potential outcome approximation

We may choose to infer an average treatment effect (ATE) I: i.e. $I = \{A, B...\}$ by comparing the average outcomes of treated individuals a from A with the one of untreated individuals $b \in B$:

$$E\{\Delta Y_i\} \approx E\{Y_a(1)\} - E\{Y_b(0)\} \quad (1)$$

For such a case we can exploit *random chance* within sufficiently large samples to make these groups comparable.

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For such a case we can exploit *random chance* within sufficiently large samples to make these groups comparable.

But what if we do not have a random assignment (and there may be a selection-bias and / or substantial differences between groups)?

a famous case

By comparing a treated with an untreated group over 2+ periods, we can control for (time-constant) differences between groups.

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A classic example is the Card and Krueger (AER, 1994), comparing fast-food worker employment in Pennsylvania (PA) and New Jersey (NJ) before and after a minimum wage raise in NJ in 1992.

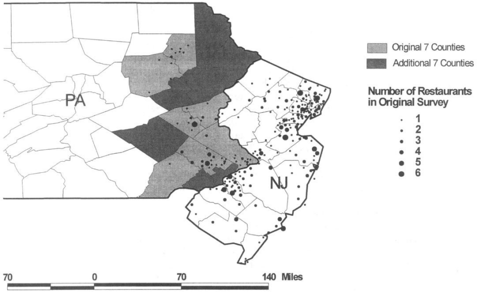
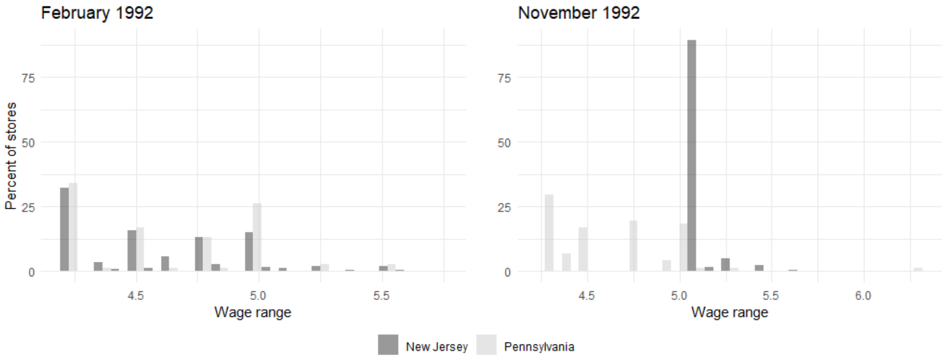


FIGURE 1. AREAS OF NEW JERSEY AND PENNSYLVANIA COVERED BY ORIGINAL SURVEY AND BLS DATA

Image source: Card and Krueger 2000

results

Here is a (tweaked) version of Card and Krueger 1994, Figure 1.



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potential outcome notation

Here, we are interested in the average treatment effect on the treated (ATT)

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In particular, the parallel-trends assumption:

$$E[Y_{it}(0) - Y_{it-1}(0)|D = 1] = E[Y_{it}(0) - Y_{it-1}(0)|D = 0] \quad (3)$$

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because then, we can assume

$$ATT = E[Y_t - Y_{t-1}|D = 1] - E[Y_t - Y_{t-1}|D = 0] \tag{4}$$

Directed acyclic graphs

Difference-in-Differences (DID)

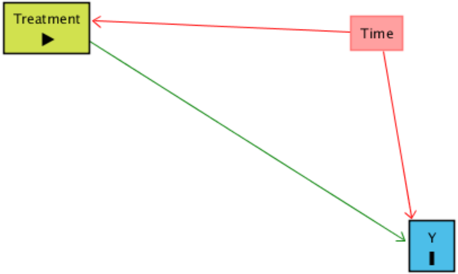

Image source: Huntington-Klein 2018

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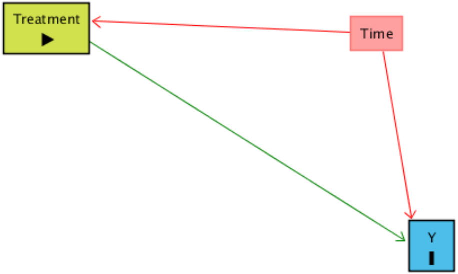

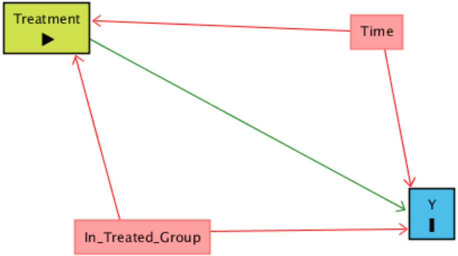

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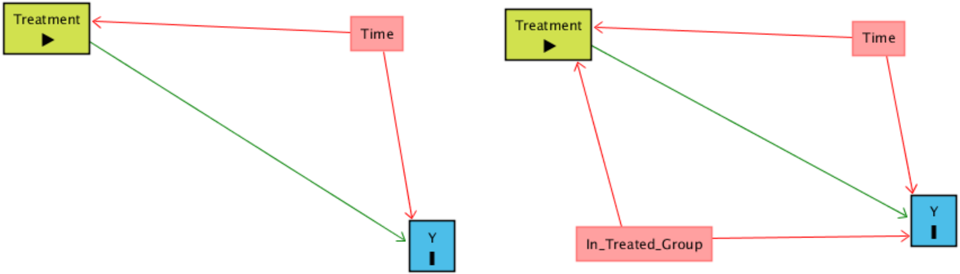
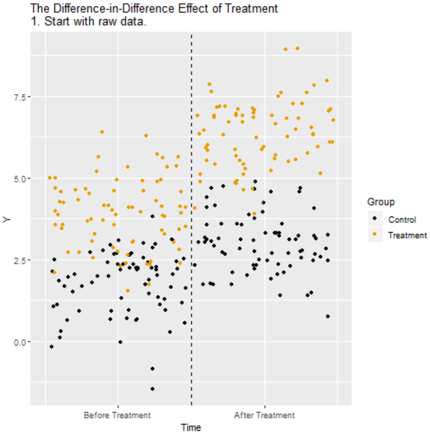


Image source: Huntington-Klein 2018

→ Accounting for differences between groups over time enables not just the estimation of time and group effects but also the differences-in-differences (i.e. the interaction of time and group effects).

Estimation



DID Estimator

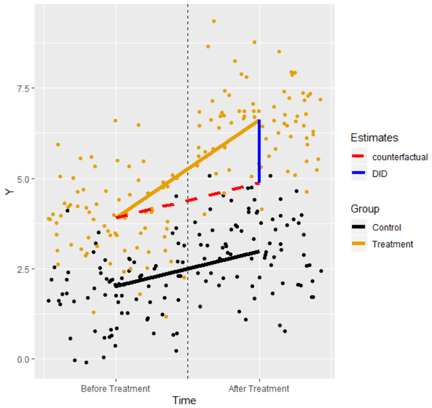
assume n individual units i, and $t = 2$ time periods, we can estimate the effect of a treatment ocurring at $P_{t=1}$, affecting the treated subpopulation $D_i = 1$

$$\begin{aligned} Y_i = \beta_0 + \beta_1 D_i + \beta_2 P_t \\ + \beta_3 D_i \times P_t + \varepsilon_i \end{aligned} \tag{5}$$

with D_i = Treatment, P_t = Period Dummy.

β_3 gives us an estimate of the diff-in-diff treatment effect.

Estimation



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β_3 gives us an estimate of the diff-in-diff treatment effect.

regression

To estimate an example ATT, we can use the Card and Krueger (1994) data

```
> did_model <- lm(emptot ~ time + treated + time:treated,
                  data = card_krueger_1994_mod)
> summary(did_model)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    23.331      1.072  21.767   <2e-16 ***
time           -2.166      1.516  -1.429   0.1535
treated        -2.892      1.194  -2.423   0.0156 *
time:treated    2.754      1.688   1.631   0.1033
```

Note: heteroskedasticity and autocorrelation robust standard errors should be computed

intermediate summary

A difference-in-differences approach allows us to

- compare a treatment group with an untreated quasi-counterfactual
- even under conditions of a non-random assignment
- assuming that the groups behave comparably enough

Panel Data

Let us suppose a situation with repeated measurements for multiple individual(s) (units), i.e. cross-sectional time-series, or panel data.

An example panel data structure

individual (i)	**time (t)**	$\mathbf{Y}_{it}$	$\mathbf{X}_{it}$	$\mathbf{D}_{it}$
A	1	0.8	0.3	0
A	2	0.7	0.2	0
A	3	0.5	0.2	1
B	1	1.2	0.4	0
B	2	1.1	0.5	0
B	3	0.9	0.6	1
...	...	...	...	...

Fixed-Effects

An exemplary study

ELSEVIER
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Ecological Economics 55 (2005) 527–538
ECOLOGICAL ECONOMICS
www.elsevier.com/locate/ecolecon

ANALYSIS

Environmental pressure group strength and air pollution: An empirical analysis

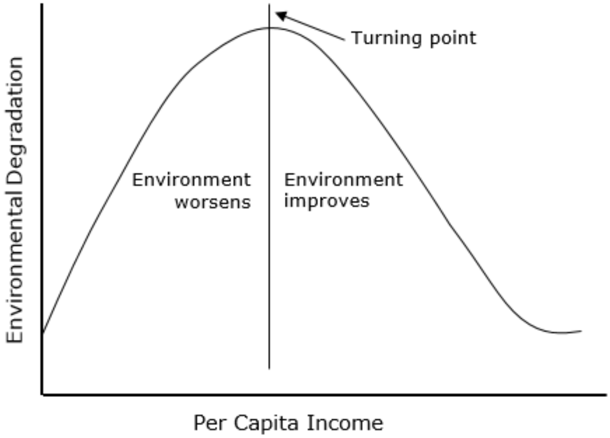
Seth Binder, Eric Neumayer*

Department of Geography and Environment and Center for Environmental Policy and Governance (CEPG), London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK

Received 7 December 2003; received in revised form 22 October 2004; accepted 14 December 2004
Available online 24 February 2005

Fixed-Effects

An exemplary study



Theoretical underpinning for Binder and Neumayer 2005: Environmental Kuznets Curve. Image source: Wikipedia

Fixed-Effects

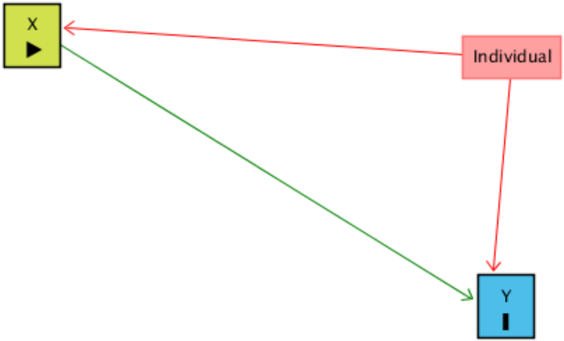
Replicated results from table 2, model 1 Binder and Neumayer 2005

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -57.55012   16.60671  -3.465 0.000548 ***
lnengopc      -0.51121    0.11878  -4.304 1.82e-05 ***
lnenergy       1.00887    0.60455   1.669 0.095425 .
lngdp         13.81819    4.17975   3.306 0.000975 ***
lngdpsq       -0.88657    0.27384  -3.238 0.001239 **
polity        -0.05079    0.03023  -1.680 0.093135 .
```

Note: This is not a fixed effects regression but an OLS

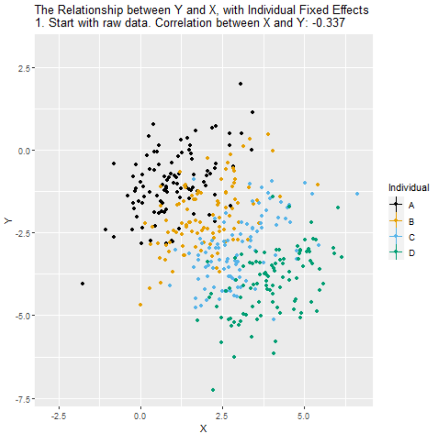
Panel Data

One way fixed-Effects (FE) structure

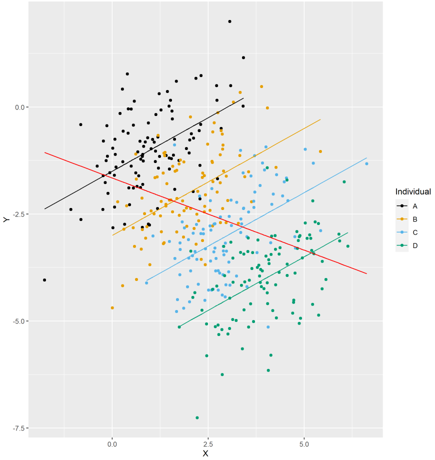


Accounting for individual fixed effects. Image source: Huntington-Klein 2018

Fixed-Effects



Fixed-Effects



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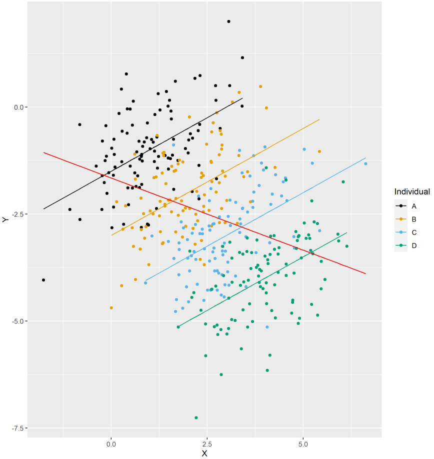


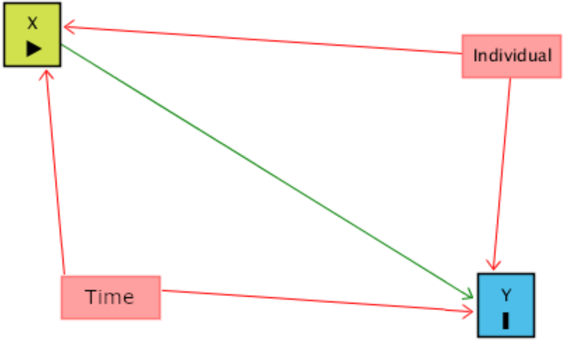
FE Estimator

$$Y_i = \alpha_i + \beta X_{it} + \varepsilon_{it} \quad (6)$$

with α_i = individual, time-invariant effect, and X_i = a variable of interest. The so called *within* transformation accounts for α_i through demeaning such that we can consistently estimate $\partial Y / \partial X$

Fixed-Effects

Two way fixed-effects



Accounting for individual and time (two-way) fixed effects. Image source: Huntington-Klein 2018

Estimator

The two-way fixed effects model can be formulated as

$$Y_{it} = \alpha_i + \theta_t + \tau D_{it} + \beta X_{it} + \varepsilon_i \tag{7}$$

where α_i individual and θ_t time fixed effects, respectively
Note that D here is a binary treatment indicator ($D_{it} = D_i \times P_t$),
possibly a vector of control variables X_i,
and error term ε_i, assumed to be normally distributed and centered around 0,
indepdendent of everything else.

time-trends

This allows to take both individual differences and shocks in time into account

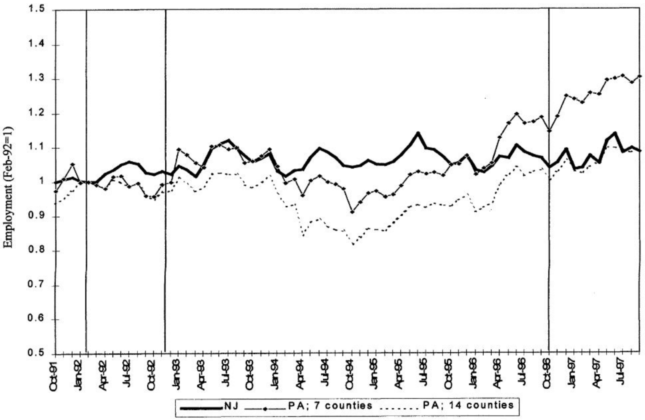


FIGURE 2. EMPLOYMENT IN NEW JERSEY AND PENNSYLVANIA FAST-FOOD RESTAURANTS, OCTOBER 1991 TO SEPTEMBER 1997

Note: Vertical lines indicate dates of original Card-Krueger survey and the October 1996 federal minimum-wage increase.
Source: Authors' calculations based on BLS ES-202 data.

Image source: Card and Krueger 2000

intermediate summary

A two-way fixed effects approach allows us to

- control for time-constant, unobserved heterogeneity between individuals
- control for common time shocks, that affect all individuals
- a multi-period 2WFE approach resembles DiD when
  - treatment is simultaneous
  - effects are homogeneous

Staggered treatment and other current developments

Over the last 6 or so years, DiD methdology has seen quite some development.

I will briefly introduce

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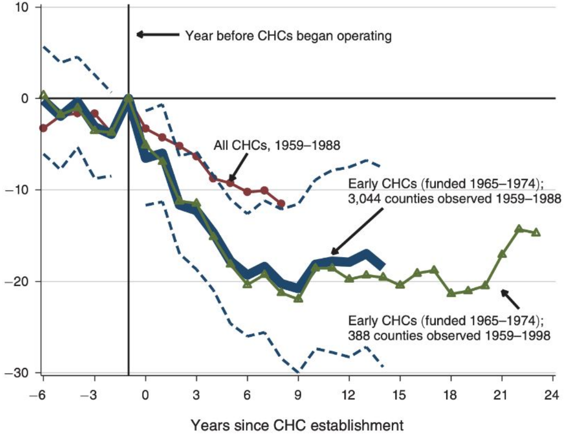
- non-parallel trend corrections
- heterogeneous treatment corrections

intuition

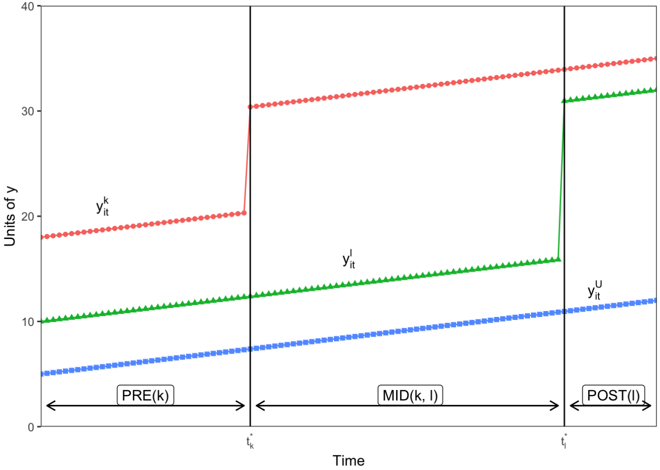
What if treatment happened at different times?



Reduction in mortality by Community Health Centers (CHC). Image source: Bailey and Goodman-Bacon 2015

intuition

We may run into time series length related weighting problems



DiD with variations in treatment timing. Image source: Goodman-Bacon 2018, as reproduced by A.C. Baker

intuition

Recall,

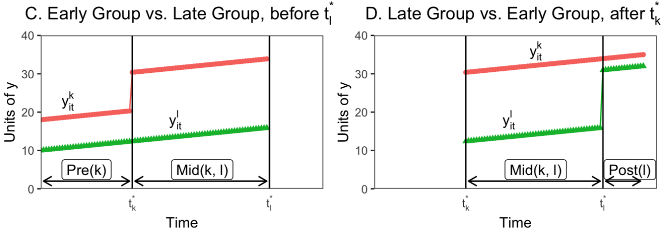
$$ATT = E[Y_t - Y_{t-1}|D = 1] - E[Y_t - Y_{t-1}|D = 0]$$

which is a comparison of mean differences between groups over time. Straightforward for 2 periods.

Now, we have multiple periods.

intuition

Picture a case with multiple periods, three groups, and thus four comparisons



DiD with variations in treatment timing. Image source: Goodman-Bacon 2018, as reproduced by A.C. Baker

notation

Turns out: 2WFE DiD estimator is a weighted average of these comparisons

Theorem 1. Difference-in-Differences Decomposition Theorem
Assume that the data contain $k = 1, \dots, K$ groups of units ordered by the time when they receive a binary treatment, $t_k^* \in (1, T]$. There may be one group, U, that never receives treatment. The OLS estimate, $\hat{\beta}^{DD}$, in a two-way fixed-effects model (2) is a weighted average of all possible two-by-two DD estimators.

$$\hat{\beta}^{DD} = \sum_{k \neq U} s_{kU} \hat{\beta}_{kU}^{2x2} + \sum_{k \neq U} \sum_{\ell > k} s_{k\ell} \left[\mu_{k\ell} \hat{\beta}_{k\ell}^{2x2,k} + (1 - \mu_{k\ell}) \hat{\beta}_{k\ell}^{2x2,\ell} \right] \quad (7)$$

Where the two-by-two DD estimators are:

$$\hat{\beta}_{kU}^{2x2} \equiv \left(\bar{y}_k^{POST(k)} - \bar{y}_k^{PRE(k)} \right) - \left(\bar{y}_U^{POST(j)} - \bar{y}_U^{PRE(j)} \right)$$

$$\hat{\beta}_{k\ell}^{2x2,k} \equiv \left(\bar{y}_k^{MID(k,\ell)} - \bar{y}_k^{PRE(k)} \right) - \left(\bar{y}_\ell^{MID(k,\ell)} - \bar{y}_\ell^{PRE(k)} \right)$$

$$\hat{\beta}_{k\ell}^{2x2,\ell} \equiv \left(\bar{y}_\ell^{POST(\ell)} - \bar{y}_\ell^{MID(k,\ell)} \right) - \left(\bar{y}_k^{POST(\ell)} - \bar{y}_k^{MID(k,\ell)} \right)$$

the weights are:

$$s_{kU} = \frac{n_k n_U \bar{D}_k (1 - \bar{D}_k)}{\widehat{var}(\tilde{D}_{it})}$$

$$s_{k\ell} = \frac{n_k n_\ell (\bar{D}_k - \bar{D}_\ell)(1 - (\bar{D}_k - \bar{D}_\ell))}{\widehat{var}(\tilde{D}_{it})}$$

$$\mu_{k\ell} = \frac{1 - \bar{D}_k}{1 - (\bar{D}_k - \bar{D}_\ell)}$$

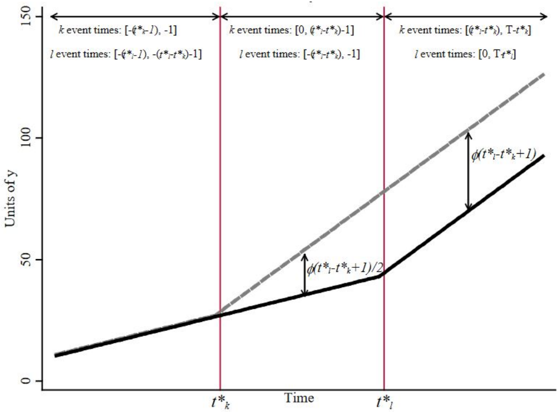
and $\sum_{k \neq U} s_{kU} + \sum_{k \neq U} \sum_{\ell > k} s_{k\ell} = 1$.

Proof: See appendix A.

Bacon decomposition theorem. Image source: Goodman-Bacon 2018, see also his tweet-thread

notation

Even worse, a positive but staggered slope change, can even change the sign of the DD estimate



Staggered treatment with linear slope change. Image source: Goodman-Bacon 2018, see also his tweet-thread

Proposed solutions I

Several works on the issue:

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- Athey and Imbens 2020
  - under random adoption dates, a weighted DID is unbiased
  - comparing treated with not-yet treated

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  - under random adoption dates, a weighted DID is unbiased
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- Goodman-Bacon 2018
  - a time-in-treatment weighted DiD, fixing the weights to gain balance
  - R package bacondecomp

Proposed solutions II

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Proposed solutions II

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- Callaway and Sant'Anna 2020
  - bootstrapped inference with pre-intervention conditioning on co-variates
  - R package did
- Sun and Abraham 2020
  - time-to-treatment (cohort) weighted approach, comparing to never-treated
  - implemented in fixest R package

Heterogeneous Treatment

Further works on DiD design / application issues:

- Chaisemartin and D'Haultfœuille 2020
  - allowing for treatment effects "heterogeneous across groups and over time periods"
  - R package DIDmultiplegt
- Imai and Kim 2020
  - identify some problematic negative weigthing in 2WFE
  - they propose weighting or matching Imai 2020

Non-parallel trends

However, all these approaches, assume (to some extent) parallel trends

- Jonathan Roth 2021
  - test for parallel-trends, R package pretends
  - robust inference strategy under non-parralel trends Rambachan and Roth 2020, R package HonestDiD

Summary

Summary of DID

Instantaneous (Static) Effects	**Dynamic Effects**
Potential Outcomes	
$Y_{it}(d)$ for $d \in \{0,1\}$	$Y_{it}(g)$ for $g \in \{1, \ldots, T, \infty\}$
Homogeneous Effects	
Target Parameter	
$\tau = \mathbb{E}\left[Y_{it}(1) - Y_{it}(0) \mid D_{it} = 1\right]$	$\tau_\ell = \mathbb{E}\left[Y_{ig+\ell}(g) - Y_{ig+\ell}(\infty) \mid G_i = g\right]$
Estimators	
$\widehat{\tau}_{\mathrm{MM}} = \left(\bar{Y}_{1,\mathrm{post}} - \bar{Y}_{1,\mathrm{pre}}\right) - \left(\bar{Y}_{0,\mathrm{post}} - \bar{Y}_{0,\mathrm{pre}}\right)$	$\widehat{\tau}_{\ell,\mathrm{DDID}} : Y_{it} = \alpha_i + \gamma_t + \sum_{\ell \in \mathcal{L}} D_i P_t^\ell \tau_\ell + \varepsilon_{it}$
$\widehat{\tau}_{\mathrm{OLS}} : Y_{it} = \mu + \alpha D_i + \gamma P_t + \tau D_i P_t + \varepsilon_{it}$	$\widehat{\tau}_{\ell,\mathrm{ES}} : Y_{it} = \alpha_i + \gamma_t + \sum_{\ell \in \mathcal{L}} D_{it}^\ell \tau_\ell + \varepsilon_{it}$
$\widehat{\tau}_{\mathrm{TWFE}} : Y_{it} = \alpha_i + \gamma_t + \tau D_{it} + \varepsilon_{it}$	
Key references	
Card and Krueger (1994)	Ashenfelter and Card (1984)
Bertrand, Duflo, and Mullainathan (2004)	Jacobson, LaLonde, and Sullivan (1993)
Heterogeneous Effects	
Target Parameter	
$\tau_{gt} = \frac{1}{N_{gt}} \sum_{i=1}^{N_{gt}} \left[Y_{it}(1) - Y_{it}(0)\right]$	$\tau_t(g) = \mathbb{E}\left[Y_{it}(g) - Y_{it}(\infty) \mid G_i = g\right]$
Estimators	
$\mathrm{DID}_{c,t} = \sum_{i,t} \left[\rho_{it}^{h_1(c)} \left(Y_{it} - Y_{it-1}\right) - \rho_{it}^{h_2(c)} \left(Y_{it} - Y_{it-1}\right)\right]$	$\widehat{\tau}_t(g) = \left(\bar{Y}_{g,t} - \bar{Y}_{\infty,t}\right) - \left(\bar{Y}_{g,g-1} - \bar{Y}_{\infty,g-1}\right)$
$\widehat{\tau}_{\mathrm{dCdH}} = \sum_{t=2}^{T} \left(\omega_{+,t} \mathrm{DID}_{+,t} + \omega_{-,t} \mathrm{DID}_{-,t}\right)$	$\widehat{\tau}_{\mathrm{CS}} = \frac{1}{\sum_t \sum_{g:g \le t} N_g} \sum_t \sum_{g:g \le t} \tau_t(g)$
Key references	
de Chaisemartin and d'Haultfoeuille (2020)	Callaway and Sant'Anna (forthcoming)
Goodman-Bacon (2021)	Sun and Abraham (2021)

Schönholzer 2022, cf. Roth and Sant'Anna 2021

Summary

Re: DiD and (adjusted) 2WFE estimators

- compare treated individual (units) with a reference group
- who we compare to whom deserves special attention
- software solutions available

References I

Athey, Susan and Guido W Imbens (2020). 'Design-based analysis in Difference-In-Differences settings with staggered adoption'. In: *Journal of Econometrics*. DOI: 10.1016/j.jeconom.2020.10.012.

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