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Machine learning, Bayes and & structural models

Nils Droste

2021 ClimBEco course

introduction

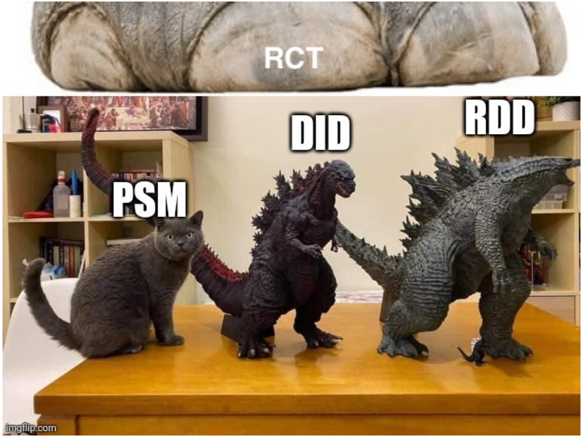
Today, we will talk about

- machine learning techniques
- structural equation models
- Bayesian inference

and their relation to causal inference

introduction

"There is no hierarchy in causal inference"



Moritz Poll2021

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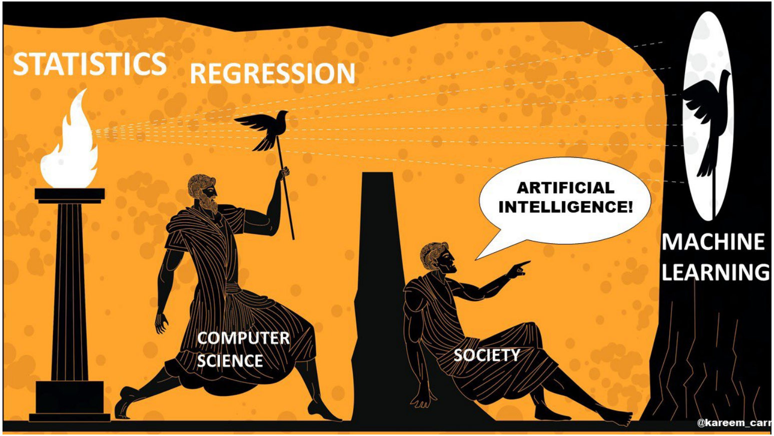


Kareem Carr 2021

introduction

"There is no hierarchy in causal inference"

Level (Symbol)	Typical Activity	Typical Questions	Examples
1. Association $P(y \mid x)$	Seeing	What is? How would seeing X change my belief inY?	What does a symptom tell me about a disease? What does a survey tell us about the election results?
2. Intervention $P(y \mid do(x), z)$	Doing, Intervening	What if? What if I do X?	What if I take aspirin, will my headache be cured? What if we ban cigarettes?
3. Counterfactuals $P(y_x \mid x', y')$	Imagining, Retrospection	Why? Was it X that caused Y? What if I had acted differently?	Was it the aspirin that stopped my headache? Would Kennedy be alive had Oswald not shot him? What if I had not been smoking the past two years?

The causal hierarchy. Source: Pearl 2019

causal trees

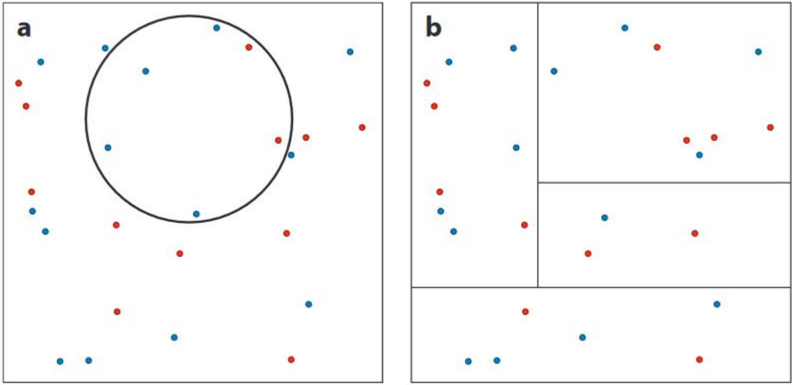
Recall, the conditional average treatment effect (cf. citeAthey2016)

$$CATE = E(Y_i(1) - Z - i(0)|X_i = x) \tag{1}$$

$\rightarrow$ we could look at average differences between treated and untreated groups within partitions of the observed

causal trees

We could use covariates X_i to identify clusters within the observed, e.g. by recursive binary splitting



Source: Athey and Imbens 2019

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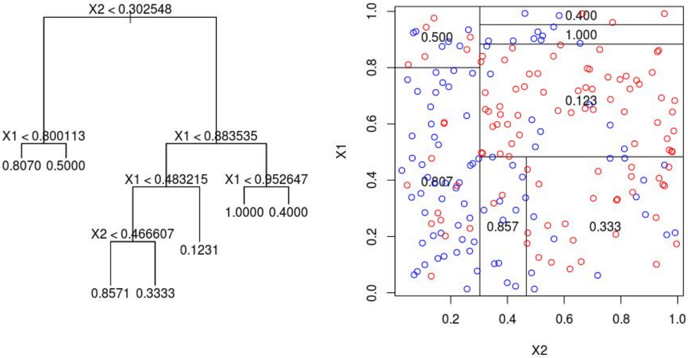

datacamp 2021

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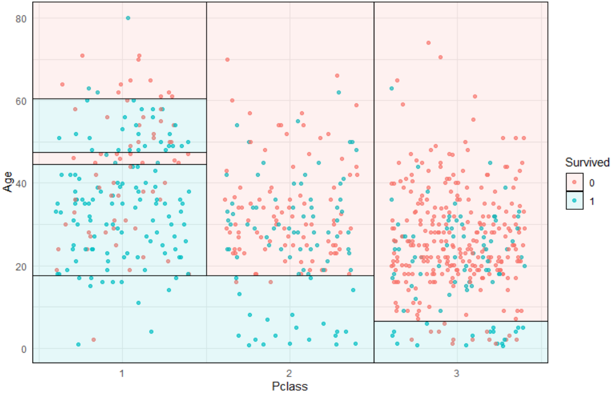

McDermott 2021

causal trees

We can use classification and regression trees to estimate

$$\hat{\tau}(x) = \frac{1}{|\{i : D_i = 1, X_i \in L\}|} \sum_{\{i:D_i=1,X_i \in L\}} Y_i - \frac{1}{|\{i : D_i = 0, X_i \in L\}|} \sum_{\{i:D_i=0,X_i \in L\}} Y_i \tag{2}$$

by ”recursively splitting the feature space ... into a set of leaves L” (Wager and Athey 2018)

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and repeat to build an ensemble of trees $\rightarrow$ random forests. This particularly well suited for heterogeneous treatment effects. Athey and Imbens (2016) suggest a sort of cross validation procedure for robust causal inference, see generalized random forests R package.

estimation techniques

Instead of ordinary least squares

$$b\hat{e}ta^{ols} = \arg\min_{\beta} \sum i = 1^N (Y - \beta X_i)^2 \tag{3}$$

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with a regularizing penalty term that results in sparse models, $q = 1$ for Lasso, $q = 2$ ridge regression, and $q \to 0$ for best subset regression, or hybrids such as elastic nets (cf. Athey and Imbens 2019) and posterior choice of λ by (n-fold) cross-validation.

estimation techniques

Or we use neural networks, where we transform

$$Z_{ik}^{(1)} = \sum j = 1^K \beta_{kj}(1) X_{ij} \quad \text{for} \quad k = i, ..., K_1 \tag{5}$$

with K covariates (aka features) and latent (aka unobserved / hidden node) variable $Z_j k$, e.g. into a recitified linear function $g(z) = z 1_{z>0}$.

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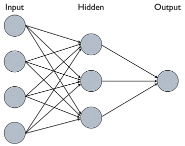


$$Y_i = \sum_{k=1}^{K_1} \beta k(2) g[Z_{ik}^2] + \varepsilon_i \tag{6}$$

with a single layer K_1, and a non-linear transformation $g(\cdot)$ (cf. Athey and Imbens 2019; Farrell, Liang and Misra 2021).

causal trees

Or, we can use machine learning methods for matching or synthetic control (Doudchenko and Imbens 2020)

e.g to estimate weights w and intercept μ for simulating counterfactual

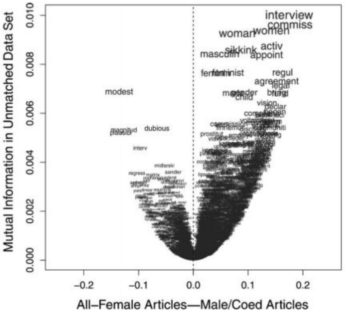
$$\hat{Y}_{t,post}(0) = \mu + \sum_{i=1}^{N} w_i Y_{i,T}^{obs} \tag{7}$$

by

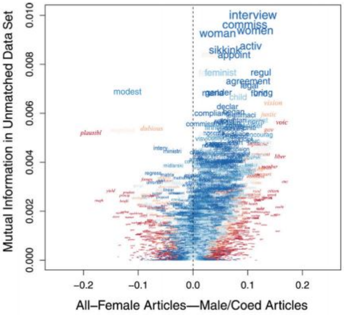
$$(\hat{\mu}^{ols}, \hat{w}^{ols}) = \arg\min_{\mu,w} \sum_{s=1}^{T_0} \left(Y_{0,T_0-s+1}^{obs} - \mu - \sum_{i=1}^{N} w_i \cdot Y_{0,T_0-s+1}^{obs} \right)^2 + \underbrace{\lambda \left(\frac{1-\alpha}{2} \|w\|_2^2 + \alpha \|w\|_1 \right)}_{penaltyfunction} \tag{8}$$

matching

We can also used unsupervised machine learning, such as e.g. latent dirichlet allocation, to use text as data (Gentzkow, Kelly and Taddy 2019) for matching a treated with an untreated population (aka matching on similar content)



(a) Full Data Set



(b) Topic Matched

Topical Inverse regression matching (TIRM). Source: Roberts, Stewart and Nielsen 2020

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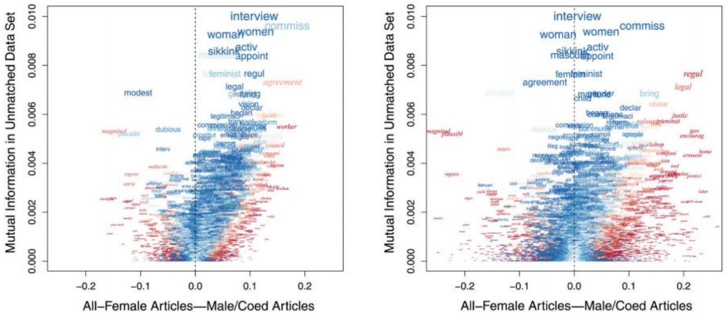

(c) Matched on human codes (d) TIRM

Topical Inverse regression matching (TIRM). Source: Roberts, Stewart and Nielsen 2020

Bayes theorem

$$\mathrm{p}(y|x) = \frac{\mathrm{p}(x|y)\mathrm{p}(y)}{\mathrm{p}(x)} \tag{9}$$

Bayesian inference

Now that we talked about sequential parameter optimization, model ensembles (aka model averaging), and posteriors, ...

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with *prior* parameter probability $\mathrm{p}(\theta)$ and the *posterior* probability (aka conditional probability) $\mathrm{p}(y|\theta)$), the likelihood of observables given the parameter.

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- Ridge and Lasso regressions have their Bayesian interpretation already (cf. Athey and Imbens 2019)
- Simulating a synthetic control by Bayesian structural time series analysis (Brodersen et al. 2015)
- Bayesian random forests (Hill 2011; Hahn, Murray and Carvalho 2020)

It has the merit of being explicit of prior beliefs about distributions (or probalities of parameters) and thus uncertainties

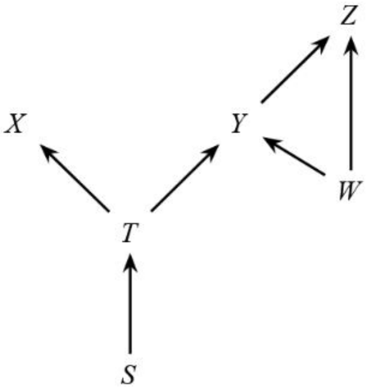
A far greater uncertainty

What do we really now about the world. Image source: Studio Binder 2020

A far greater uncertainty

Black Box Models. Source: Zhao and Hastie 2021

We need a theory of change



Directed Graph Model. Source: Stanford Encyclopedia of Philosophy 2018

structural models

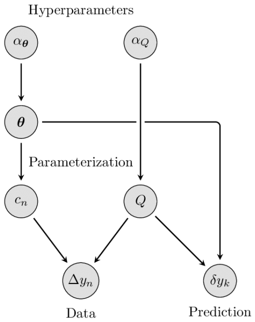
”[G]raphical representations of structural causal models do not requirethe learner - whether artificial or human - to impose any distributional or functional-form restrictions on the underlying causal mechanisms under study.

The approach remains fully nonparametric, a characteristic it shares with the potential outcomes framework. At the same time, however, crucial identification assumptions, suchas ignorability, are derived from the properties of the underlying structural model, rather than being assumed to hold a priori.

Causal graphical models thus combine the accessibility and flexibility of potential outcomes with the preciseness and analytical rigor of structural econometrics” (Hünermund and Bareinboim 2021)

structural causal models (SCM)

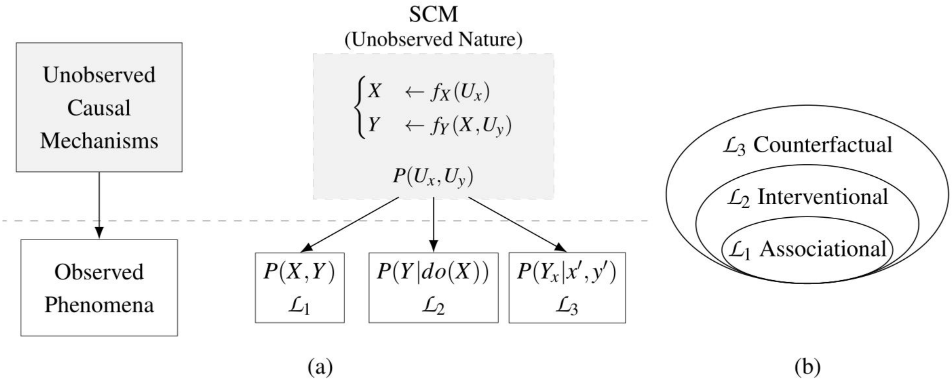
Parameters of a probabilistic structural model can be estimated with a Bayesian network approach ...



Bayesian network. Source: Melendez et al. 2019

... partial least squares, or through double debiased machine learning (cf. citeChernozhukov2018,Jung2021)

The do-calculus



The causal hierarchy. Source: Bareinboim et al. 2020

Wrapping it all up

To estimate a causal effect, we aim to compare the outcome to its counterfactual.

- potential outcomes: $\tau_{ATE} = E\{Y_i(1)\} - E\{Y_i(0)\}$

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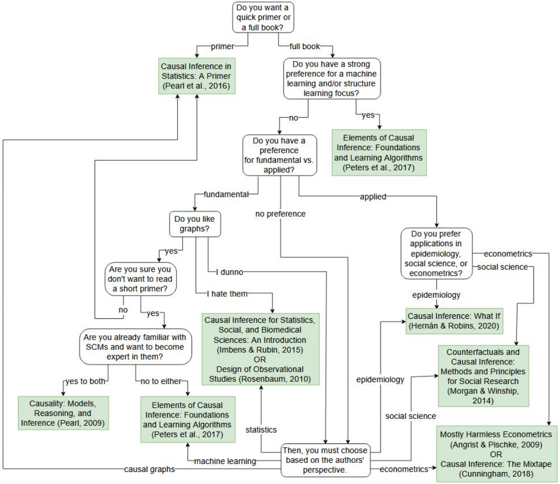
- with frequentist, Bayesian, or machine learning methods
- it should be conceptually sound and methodologically robust

sources

Beyond the coursebooks, see

- James, Witten, Hastie and Tibshirani (2013)
  - introduction to statistical learning (edX)
- Boehmke and Greenwell (2020)
  - Hands-On Machine Learning with R
- Paul Goldsmith-Pinkham 2021
  - applied methods PhD course
- Nick Huntington-Klein 2021
  - The Effect: An Introduction to Research Design and Causality
- Peters, Janzing and Schölkopf 2017
  - Elements of causal inference: foundations and learning algorithms
- Nagarajan, Scutari and Lèbre 2013
  - Bayesian Networks in R

deciding on literature



decision tree, source: Brady Neal 2020

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