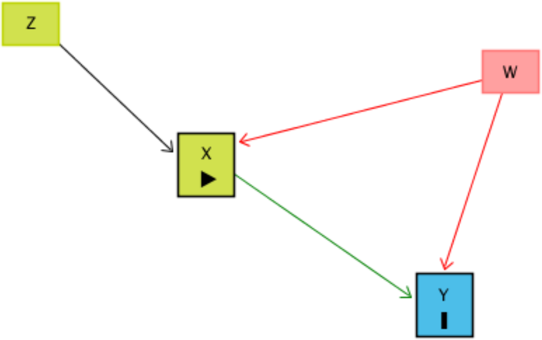
Instrumental Variables and regression discontinuity designs

Nils Droste

2022 ClimBEco course

Instrumental Variable (IV)



Using exogeneous variation in instrument to close back-door. Image source: Huntington-Klein 2018

Instrumental Variables

Introduction

Instrumental Variables

intuition

formalities

technicalities

Regression Discontinuity

intuition

notation

examples

An exemplary study

The Impact of the Women's March on the U.S. House Election*

Magdalena Larreboure   Felipe González

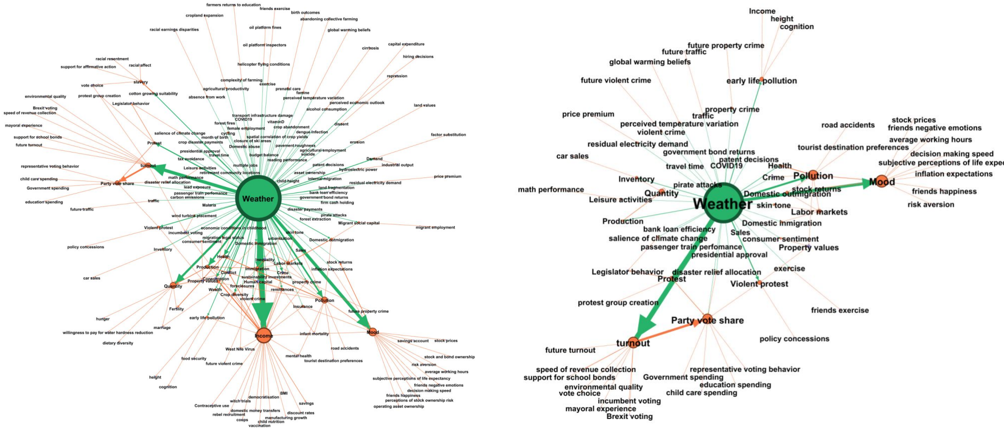
April 10, 2021

Three million people participated in the Women's March against discrimination in 2017, the largest single-day protest in U.S. history. We show that protesters in the March increased political preferences for women and people from ethnic minorities in the following federal election, the 2018 House of Representatives Election. Using daily weather shocks as exogenous drivers of attendance at the March, we show that protesters increased turnout at the Election and the vote shares obtained by minorities, particularly women, irrespective of their party affiliation. We conclude that protests can help to empower historically underrepresented groups through changes in local political preferences.

Larreboure2021

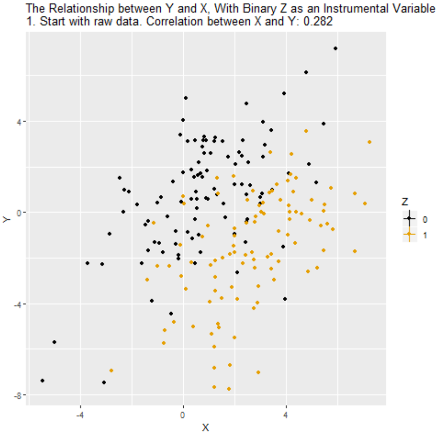
Instrumental Variables

But there may be plenty of causal pathways in reality



Weather IV and (in)dep vars, in general (left) and with temporal variations (right). Source: **Mellon2020**

Instrumental Variables



Two-Stage Least Square (2SLS) estimator

1. stage: regress Z on X:

$$X_i = \alpha_1 + \beta_1 Z_i + \gamma_1 W_i + \varepsilon_{1,i} \tag{1}$$

and predict the variation in X explained by Z: $\widehat{X} = \beta_1 Z_i$.

2. stage: plug in $\widehat{X}$ to estimate the variation in Y not explained by confounder W:

$$Y_i = \alpha_2 + \beta_2 \widehat{X}_i + \gamma_2 W_i + \varepsilon_{2,i} \tag{2}$$

conditions

There are important conditions to consider

- *relevance* of instrument for predicting $Y \rightarrow E((\widehat{X}_i|Z=1)-(\widehat{X}_i|Z=0)) \neq 0$, aka Z is correlated with X, and thus with Y.

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Now, let us see how to formulate this in the potential outcome notation.

For this let treatment or participation again be denoted by D, now as a function of the instrument $\rightarrow D_i(Z_i)$, the *intention to treat*.

notation

Step by step

- Imbens and Angrist (**Imbens1994**) formulate local average treatment effect (LATE)
  - for the *subpopulation* responding to instrument Z, that is those who participate $P(1)$ in treatment D

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Why?

Who's gonna be "treated"

Consider Angrist, Imbens and Rubin (**Angrist1996a**)

		$Z_i = 0$	
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  $\rightarrow$ assumes existence of only one of compliers or defiers, e.g. *no one* defies
- allows valid estimate of LATE, but may not always be realistic

Relaxing monotonicity assumption

de Chaisemartin (**DeChaisemartin2017**) shows IVs can be valid without strong monotonicity

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- ”*is satisfied if a subgroup of compliers accounts for the same percentage of the population as defiers and has the same LATE*”
- I believe this can be approached with matching, too. See Murray et al. (**Murray2021**) who suggest to estimate the intention to treat D(Z) with logistic regression, providing leeway for a propensity score or other matching approach (cf. **Hirano2003**; **Rosenbaum1984**).

Weak instruments

When instruments are only weakly correlated with treatment, reconsider

$$Y_i = \alpha_2 + \beta_2 D_i + \gamma_2 W_i + \varepsilon_i \quad (4)$$

$$D_i = \alpha_1 + \beta_1 Z_i + \gamma_1 W_i + \upsilon_i \quad (5)$$

A condition was relevance, i.e. $E((D_i|Z=1)-(D_i|Z=0)) \neq 0$, or $\mathrm{Cov}(Z_i, D_i|W_i) \neq 0$

- to estimate IV, $\widehat{\beta}_2 = \frac{\mathrm{Cov}(Y_i, Z_i)}{\mathrm{Cov}(Z_i, D_i|W_i)}$
- problematic when $\mathrm{Cov}(Z_i, D_i|W_i) \to 0$ as $\Delta\beta_2$ grows large even for small variations

Weak instruments

A range of techniques for robust parameter estimation in weak IV 2SLS

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$\rightarrow$ We will look into some (basic) testing in the seminar.

intermediate summary

Instrumental variables allow us to

- isolate a treatment effect by looking at the outcomes of exogeneously caused treatment variation
- it is considered a very robust causal inference, but assumptions are *somewhat* crucial
- mainly it is theory and reason that make a ”valid instrument”
- there is loads of tests, I do not think they alone suffice

A first thought experiment

Available online at www.sciencedirect.com
SCIENCE DIRECT
Ecological Economics 55 (2005) 527–538
ECOLOGICAL ECONOMICS
www.elsevier.com/locate/ecolecon

ANALYSIS

Environmental pressure group strength and air pollution: An empirical analysis

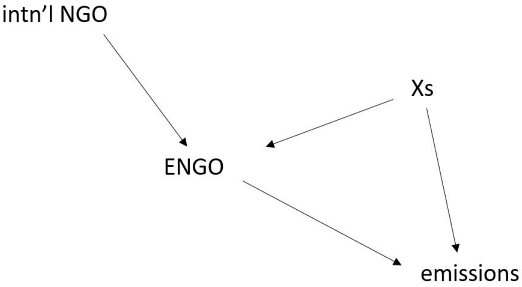
Seth Binder, Eric Neumayer*

Department of Geography and Environment and Center for Environmental Policy and Governance (CEPG), London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK

Received 7 December 2003; received in revised form 22 October 2004; accepted 14 December 2004
Available online 24 February 2005

A first thought experiment

Do you think this is a valid instrument?



reformulating **Binder2005**: a (partial DAG)

Regression discontinuity designs (RDD) – intuition

Suppose we believe there is an effect for which assignment is non-random, but the cut-off at which treatment is assigned is quasi-random (cf. **Thistlewaite2016**).

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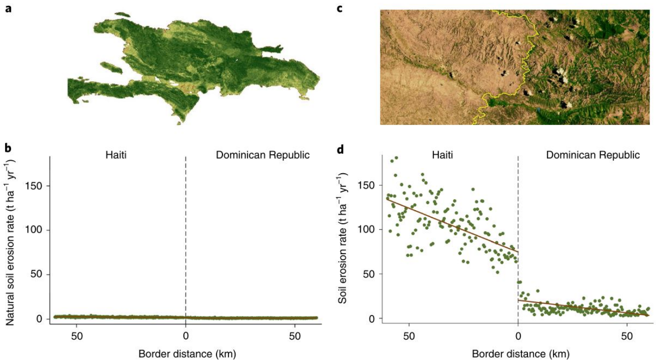
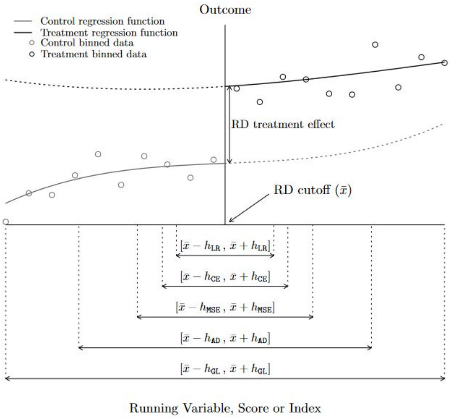


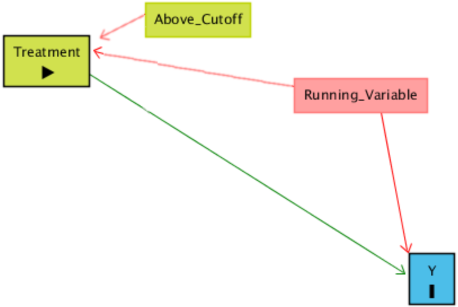
The border between Haiti and the Dominican Republic. Image source: **Wuepper2020**

concept



The RDD concept and the effect of bin size choice. Image source: **Cattaneo2016**

Regression-Discontinuity-Design (RDD)



Focussing on effects just around the cutoff value. Image source: Huntington-Klein 2018

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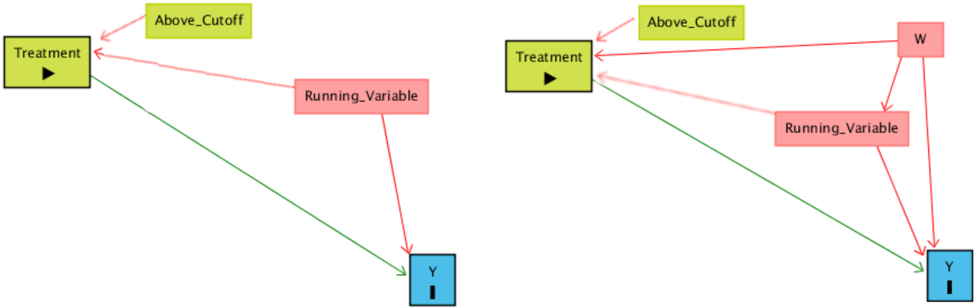

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→ Do you see the IV in RDD?

notation

Let us formulate in the potential outcomes notation. Suppose there is a outcome $(Y(1), Y(0))$ that depends on treatment D and covariate X.

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$$D_i = \begin{cases} 1 \text{if} x_i \geq \overline{x} \\ 0 \text{if} x_i < \overline{x} \end{cases} \tag{6}$$

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The identifying assumption is again that treatment assignment is

independent of outcomes $E(Y(1) - Y(0) \perp D|X = \overline{x})$.

Sharp RDD

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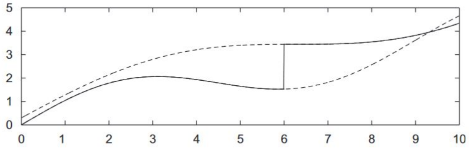

Imbens2008

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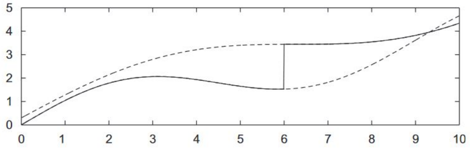

Imbens2008

- and measure $\tau_{srdd} = \lim_{x \to \overline{x}} E[Y(1)|X = x] - \lim_{x \leftarrow \overline{x}} E[Y(0)|X = x]$

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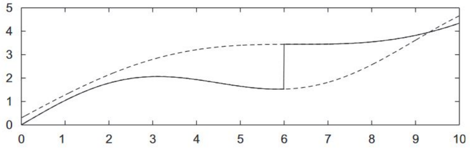

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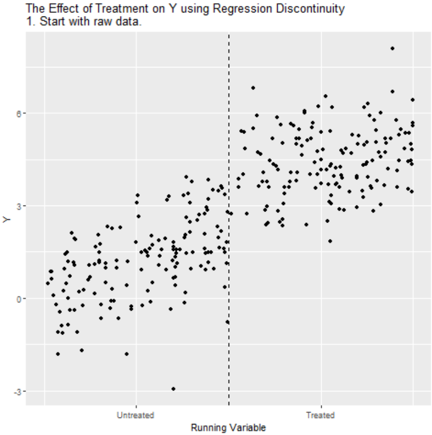
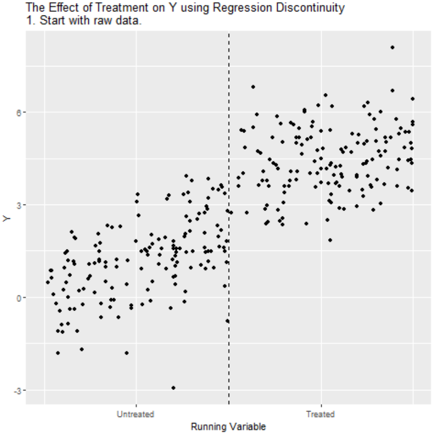
- and measure $\tau_{srdd} = \lim_{x \to \overline{x}} E[Y(1)|X = x] - \lim_{x \leftarrow \overline{x}} E[Y(0)|X = x]$
- D is not just correlated but a deterministic function of x (once we know x and $\overline{x}$, we know D)

estimator


The Effect of Treatment on Y using Regression Discontinuity
1. Start with raw data.
6
3
0
-3
Y
Untreated
Treated
Running Variable



RDD estimation

$$Y_i = \alpha_i + \beta X_{it} + \gamma t_i + \varepsilon_{it} \tag{7}$$

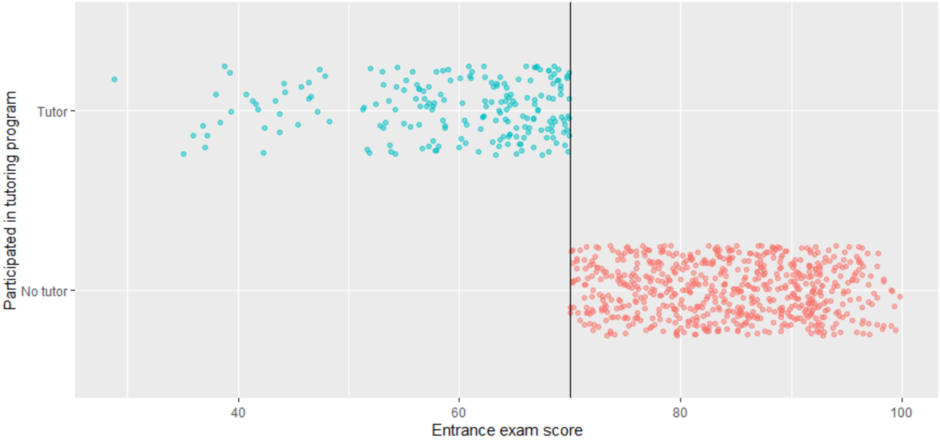
where t indicates treatment cutoff values $\overline{x}$:

$$t_i = \begin{cases} 1 \text{if} x_i \geq \overline{x} \\ 0 \text{if} x_i < \overline{x} \end{cases} \tag{8}$$

This would often include polynomial terms to allow for non-linear functional forms (but should not, cf. **Gelman2019**). Another typical approach is a local linear regression (which is displayed in the animation) or smoothing functions.

fuzzy RDD

Suppose the data did **not** look like this



Heiss 2020

fuzzy RDD

but rather looked like this

Heiss 2020

fuzzy RDD

So we need to evaluate

Heiss 2020

fuzzy RDD

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Heiss 2020

This is literally an IV setting where a different probability on two sides of the cutoff predicts participation.

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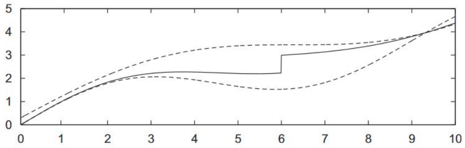

Imbens2008

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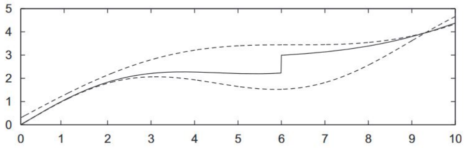

Imbens2008

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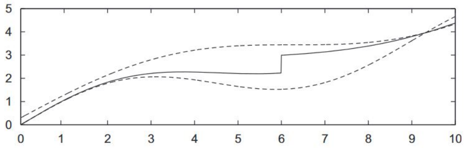

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- if unconfounded, $\tau_{frdd} = E[Y(1)|D = 1, X = \overline{x}] - E[Y(0)|D = 1, X = \overline{x}]$
- which we can estimate with 2SLS, predicting D in first stage, plugging estimates into second stage

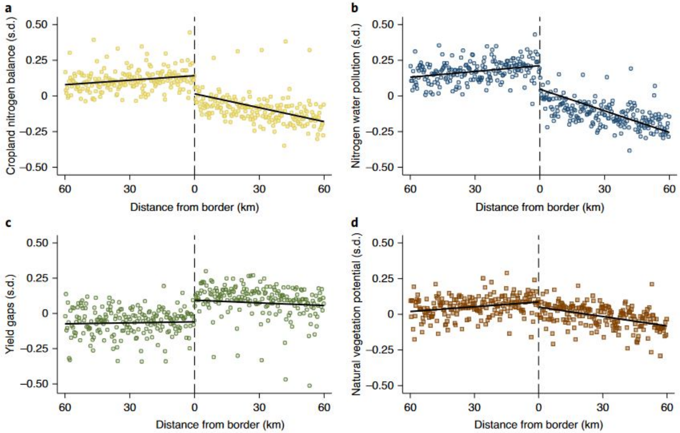
examples

There are discontiuities in space

Wuepper2020a

examples

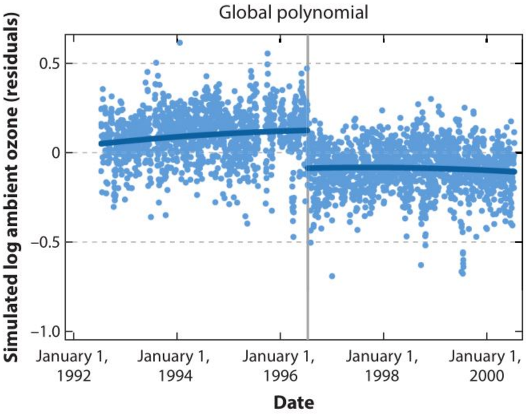
There are discontiuities in space



Regression discontiuities in covariates but not in vegetation potential, **Wuepper2020a**

examples

Time



Hausman2018

examples

Rules

The Causal Effect of Radical Right Success on Mainstream Parties' Policy Positions: A Regression Discontinuity Approach

TARIK ABOU-CHADI AND WERNER KRAUSE*

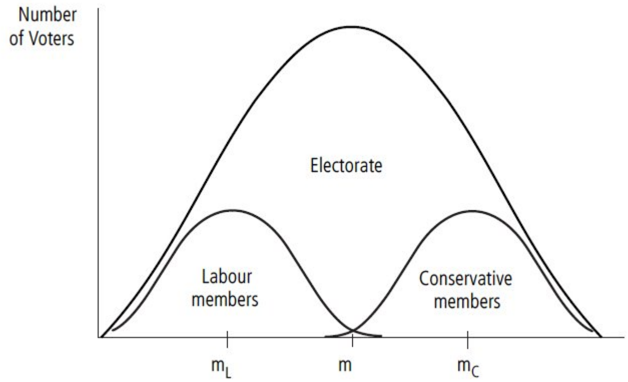
This article investigates how the success of radical right parties affects the policy positions of mainstream parties. We do this using a regression discontinuity approach that allows us to causally attribute mainstream parties' positional changes to radical right strength independent of public opinion as a potential confounder. Making use of exogenous variation created through differences in electoral thresholds, we empirically demonstrate that radical right success, indeed, causally affects mainstream parties' positions. This is true for mainstream left as well as mainstream right parties. These findings make an important contribution to the broader literature on party competition as they indicate that other parties' behavior and not only public opinion plays a crucial role in explaining parties' policy shift.

Keywords: radical right; party competition; immigration.

AbouChadi2018

examples

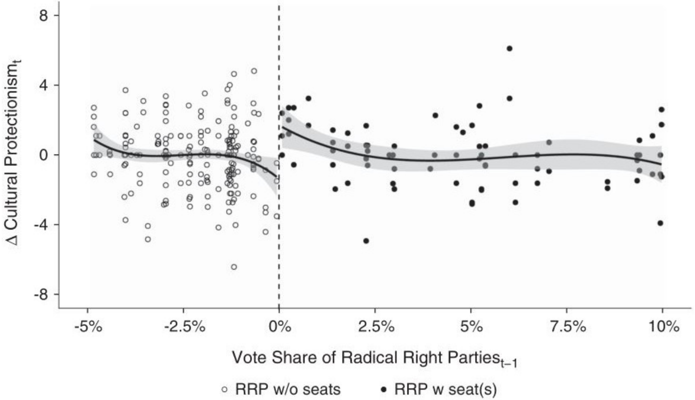
Rules



Hotelling-Downs Model of 2 Party Competition. Image Source: Daniel Corradi Stevens

examples

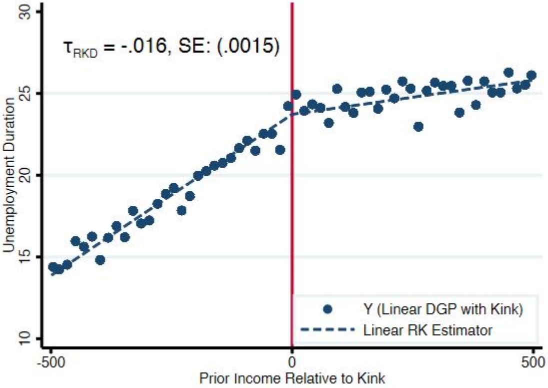
Rules



Mainstream party position on cultural position. Image source: **AbouChadi2018**

examples

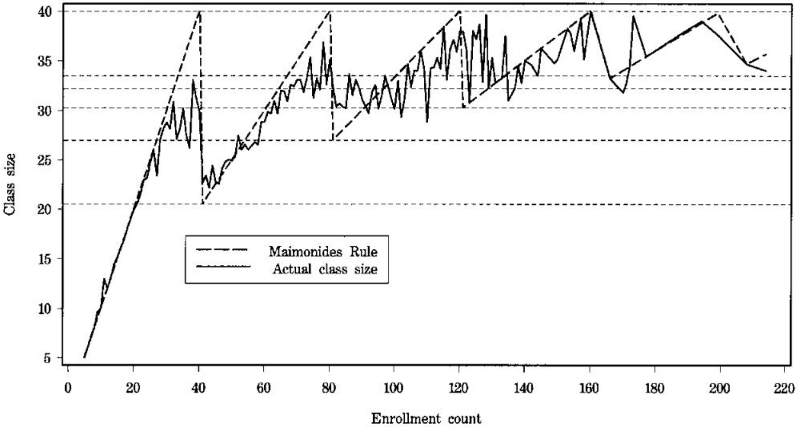
There can be kinks, aka slope shifts



Ganong2018

examples

Multiple breaks



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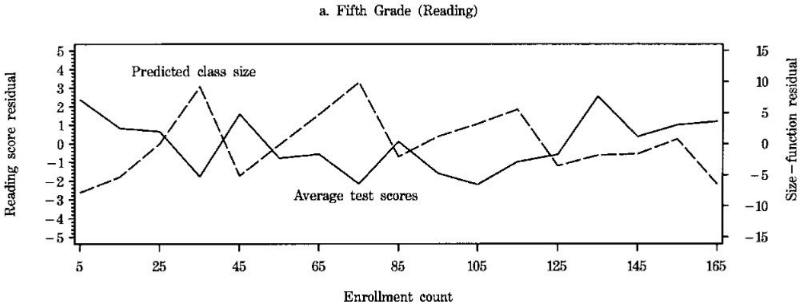
Multiple breaks



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software

available packages

- rdd
- rdrobust
- rdlocrand
- rddensity
- rdmulti
- rdpower

intermediate summary

Regression discontinuity designs

- identify a causal effect at a (quasi-)randomly occuring break point that introduces treatment
- are the youngest ”classical” causal inference methods and seen as favorable
- use breaks that can occur in space, time, institutions, etc. pp.

References I