Problem 5

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PROBLEM 5

We let $\vec{v}^* = [v_1^*, v_2^*, ...v_m^*]$ denote the vector of length m that represents the complete solution of the discretized Poisson equation. The corresponding x values are contained in $\vec{x} = [x_1, x_2, ..., x_m]$, with length m. We let $\mathbf{A}$ be an $n \times n$ matrix.

a)

Writing out our matrix equation $\boldsymbol{A}\vec{v} = \vec{g}$, we get

$$\begin{bmatrix} 2 & -1 & 0 & ... & 0 \\ -1 & 2 & ... & ... & ... \\ 0 & ... & ... & -1 & 0 \\ ... & ... & -1 & 2 & -1 \\ 0 & ... & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_{n-1} \\ v_n \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ ... \\ g_{n-1} \\ g_n \end{bmatrix} \tag{1}$$

Further, by writing out the multiplication for the first element, we get

$$2v_1 - v_2 = g_1 \tag{2}$$

But we see, from the discretized version of the Poisson equation, **REFERER TIL den ultednigngng** that this corresponds to the *second* term in our solution $\vec{v}^*$, meaning the first element *after* the boundary term (which in our case is $u(0) = v_1^* = 0$). Including the boundary term, we get $2v_1 - v_2 = -v_0 + 2v_1 - v_2 = g_1$.

Further, we write out the last element of the matrix equation:

$$-v_{n-1} + 2v_n = g_n \tag{3}$$

Once again, we see that v_n corresponds to the second-to-last element of v^* (meaning the v_{m-1}^*), and the equation holds because the mth entry of $\vec{v}^*$ is $v_m^* = 0 = u(1)$.

In general, we find there is a correspondence $v_i \longleftrightarrow v_{i+1}^*$ for $i \in \{1, ..., n\}$.

As $\vec{v}$ has length n and $\vec{v}^*$ has length m, we must have that $m = n + 2$, where the additional two terms are due to the boundary terms.

b)

As $m = n+2$, we see that $\vec{v}$ only contains the 'middle' part of $\vec{v}^*$, meaning all elements excluding the two boundary terms.

For the right hand side, we have to remember that g_i corresponds to $f(x_{i+1})$.