Problem 9

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(Dated: September 8, 2022)

I. PROBLEM 9

A. Specialize the algorithm

We want to specialize the algorithm from Problem 6 for the special case where $\boldsymbol{A}$ is specified by the signature $(-1, 2, -1)$. This menas that $\vec{a}$ and $\vec{c}$ are vectors of length $n-1$, consisting only of the value -1, and $\vec{b}$ is an n-length vector filled with the value 2.

Substituting every a_i, c_i with -1 and b_i with 2, we obtain the following equations for

$$\begin{aligned} \tilde{b}_1 &= b_1 = 2 \\ \tilde{b}_i &= b_i - \frac{a_i}{\tilde{b}_{i-1}} c_{i-1} = 2 - \frac{1}{\tilde{b}_{i-1}} \end{aligned} \tag{1}$$

$$\begin{aligned} \tilde{g}_1 &= g_1 \\ \tilde{g}_i &= g_i - \frac{a_i}{\tilde{b}_{i-1}} \tilde{g}_{i-1} = g_i + \frac{\tilde{g}_{i-1}}{\tilde{b}_{i-1}} \end{aligned} \tag{2}$$

Similarly, for the backward substitution we get the following equations:

$$\begin{aligned} v_n &= \frac{\tilde{g}_n}{\tilde{b}_n} \\ v_i &= \frac{\tilde{g}_i - c_i v_{i+1}}{\tilde{b}_i} = \frac{\tilde{g}_i + v_{i+1}}{\tilde{b}_i} \end{aligned} \tag{3}$$

B. Count the number of FLOPs in specialized algorithm

We count the number of FLOPs required for the special algorithm.

When calculating every element of $\vec{\tilde{b}}$, we see from (1) that we perform 2 FLOPs (consisting of addition and division) for every $i \in [1, N-1]$. The initial element b_1 requires no FLOPs. Exactly the same holds for calculating $\vec{\tilde{g}}$ in (2).

However, for the backward substitution in (3), we see that in addition to the $2(n-1)$ FLOPs, we need one more for the last term v_n.

In total, we require

$$3 \cdot \left[2(n-1)\right] + 1 = 6n - 5 \text{ FLOPs} \tag{4}$$

C. Implement algorithm in code

Lastly, we write code that implements the algorithm. The code lies in Problem9.cpp.