Title of the document

Your name(s) here
(Dated: September 2, 2022)

List a link to your github repository here!

PROBLEM 1

The one-dimensional Poisson equation can be written as

$$-\frac{d^2u}{dx^2} = f(x) \tag{1}$$

where $f(x)$, the source term, is known. We assume a setup such that the source term is $f(x) = 100e^{-10x}$, $x \in [0, 1]$, and the boundary conditions are $u(0) = 0$ and $u(1) = 0$.

We want to check analytically that an exact solution to (1) can be given by

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{2}$$

We differentiate $u(x)$ twice and find that

$$\begin{aligned} \frac{d^2}{dx^2}u(x) &= \left(1 - (1 - e^{-10})x - e^{-10x}\right) \\ &= \frac{d}{dx}\left(1 - e^{-10} + 10e^{-10x}\right) \\ &= -100e^{-10x} \\ &= -f(x) \end{aligned} \tag{3}$$

as we wanted. In addition, we check whether the boundary conditions are fulfilled:

$$\begin{aligned} u(0) &= \left(1 - e^{0}\right) = 1 - 1 \\ &= 0 \\ u(1) &= 1 - 1 + e^{-10} - e^{-10} \\ &= 0 \end{aligned} \tag{4}$$

This means $u(x)$ in (2) is a solution to our specific setup.