Project 1 - Problem 4

2. september 2022

We have the following discretized expression:

$$\left[\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + O(h^2) = f_i\right] \tag{1}$$

and we define $v_i = \frac{u_{i+1}-2u_i+u_{i-1}}{h^2} \approx f_i$ and $\vec{v}$ to be the vector containing all v_i.
With this in mind we can set up a matrix equation with a tri-diagonal matrix, $\boldsymbol{A}\vec{v} = \vec{g}$, as

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} \tag{2}$$

We can now by compute $\boldsymbol{A}\vec{v}$ and compare the each row to equation 1. We can multiply equation 1 by h^2 on both sides and define the right-hand side to be g_i defining $\vec{g}$ appropriately we get the following:

$$\begin{bmatrix} 2v_1 & -v_2 & 0 & 0 \\ -v_1 & 2v_2 & -v_3 & 0 \\ 0 & -v_2 & 2v_3 & -v_4 \\ 0 & 0 & -v_3 & 2v_4 \end{bmatrix} = \begin{bmatrix} g_1 \equiv h^2 f_1 \\ g_2 \equiv h^2 f_2 \\ g_3 \equiv h^2 f_3 \\ g_4 \equiv h^2 f_4 \end{bmatrix} \tag{3}$$

So we can represent the discretized equation of the second derrivative as a matrix equation.