Aufgabe 1:

(a)

$$Y = [(A \wedge \overline{B}) \vee (\overline{C} \wedge D) \vee \overline{E} \vee F] \wedge G$$
$$\overline{Y} = [(\overline{A} \vee B) \wedge (C \vee \overline{D}) \wedge E \wedge \overline{F}] \vee \overline{G}$$

(b)

0000000:
$$\begin{aligned} Y &= [(0 \wedge \overline{0}) \vee (\overline{0} \wedge 0) \vee \overline{0} \vee 0] \wedge 0 \\ &= [(0 \wedge 1) \vee (1 \wedge 0) \vee 1 \vee 0] \wedge 0 \\ &= [(0) \vee (0) \vee 1 \vee 0] \wedge 0 \\ &= [1] \wedge 0 \\ &= 0 \end{aligned}$$
$$\begin{aligned} \overline{Y} &= [(\overline{0} \vee 0) \wedge (0 \vee \overline{0}) \wedge 0 \wedge \overline{0}] \vee \overline{0} \\ &= [(1 \vee 0) \wedge (0 \vee 1) \wedge 0 \wedge 1] \vee 1 \\ &= [(1) \wedge (1) \wedge 0 \wedge 1] \vee 1 \\ &= [0] \vee 1 \\ &= 1 \end{aligned}$$

1010101:
$$\begin{aligned} Y &= [(1 \wedge \overline{0}) \vee (\overline{1} \wedge 0) \vee \overline{1} \vee 0] \wedge 1 \\ &= [(1 \wedge 1) \vee (0 \wedge 0) \vee 0 \vee 0] \wedge 1 \\ &= [(1) \vee (0) \vee 0 \vee 0] \wedge 1 \\ &= [1] \wedge 1 \\ &= 1 \end{aligned}$$
$$\begin{aligned} \overline{Y} &= [(\overline{1} \vee 0) \wedge (1 \vee \overline{0}) \wedge 1 \wedge \overline{0}] \vee \overline{1} \\ &= [(0 \vee 0) \wedge (1 \vee 1) \wedge 1 \wedge 1] \vee 0 \\ &= [(0) \wedge (1) \wedge 1 \wedge 1] \vee 0 \\ &= [0] \vee 0 \\ &= 0 \end{aligned}$$

0101010:
$$\begin{aligned} Y &= [(0 \wedge \overline{1}) \vee (\overline{0} \wedge 1) \vee \overline{0} \vee 1] \wedge 0 \\ &= [(0 \wedge 0) \vee (1 \wedge 1) \vee 1 \vee 1] \wedge 0 \\ &= [(0) \vee (1) \vee 1 \vee 1] \wedge 0 \\ &= [1] \wedge 0 \\ &= 0 \end{aligned}$$
$$\begin{aligned} \overline{Y} &= [(\overline{0} \vee 1) \wedge (0 \vee \overline{1}) \wedge 0 \wedge \overline{1}] \vee \overline{0} \\ &= [(1 \vee 1) \wedge (0 \vee 0) \wedge 0 \wedge 0] \vee 1 \\ &= [(1) \wedge (0) \wedge 0 \wedge 0] \vee 1 \\ &= [0] \vee 1 \\ &= 1 \end{aligned}$$

1111111:
$$\begin{aligned} Y &= [(1 \wedge \overline{1}) \vee (\overline{1} \wedge 1) \vee \overline{1} \vee 1] \wedge 1 \\ &= [(1 \wedge 0) \vee (0 \wedge 1) \vee 0 \vee 1] \wedge 1 \\ &= [(0) \vee (0) \vee 0 \vee 1] \wedge 1 \\ &= [1] \wedge 1 \\ &= 1 \end{aligned}$$
$$\begin{aligned} \overline{Y} &= [(\overline{1} \vee 1) \wedge (1 \vee \overline{1}) \wedge 1 \wedge \overline{1}] \vee \overline{1} \\ &= [(0 \vee 1) \wedge (1 \vee 0) \wedge 1 \wedge 0] \vee 0 \\ &= [(1) \wedge (1) \wedge 1 \wedge 0] \vee 0 \\ &= [0] \vee 0 \\ &= 0 \end{aligned}$$

Aufgabe 2:

$$Y = (\overline{A} \wedge (\overline{B} \vee \overline{C})) \vee (D \wedge \overline{E} \wedge F)$$

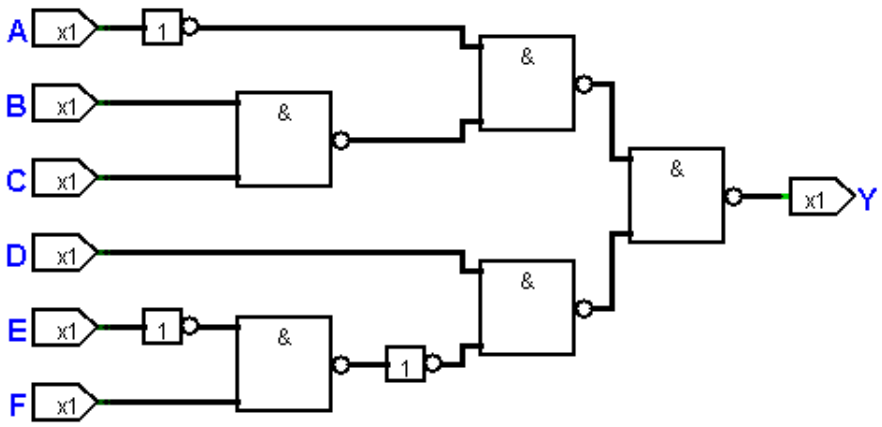
(a)

$$\begin{aligned}
Y &= (\overline{A} \wedge (\overline{B} \vee \overline{C})) \vee (D \wedge \overline{E} \wedge F) \\
&= (\overline{A} \wedge \overline{(B \wedge C)}) \vee \overline{\overline{(D \wedge \overline{E} \wedge F)}} \\
&= \overline{\overline{(\overline{A} \wedge \overline{(B \wedge C)})}} \vee \overline{\overline{(D \wedge \overline{E} \wedge F)}} \\
&= \overline{\overline{(\overline{A} \wedge \overline{(B \wedge C)})} \wedge \overline{(D \wedge \overline{E} \wedge F)}} \\
&= \overline{\overline{\overline{A} \wedge \overline{B \wedge C}} \wedge \overline{D \wedge \overline{E} \wedge F}}
\end{aligned}$$

(b)

$$\begin{aligned}
Y &= \overline{\overline{\overline{A} \wedge \overline{B \wedge C}} \wedge \overline{D \wedge \overline{E} \wedge F}} \\
&= \overline{\overline{(\overline{A})} \wedge (\overline{B \wedge C}) \wedge (D \wedge \overline{\overline{(\overline{E} \wedge F)}})}
\end{aligned}$$

(c)



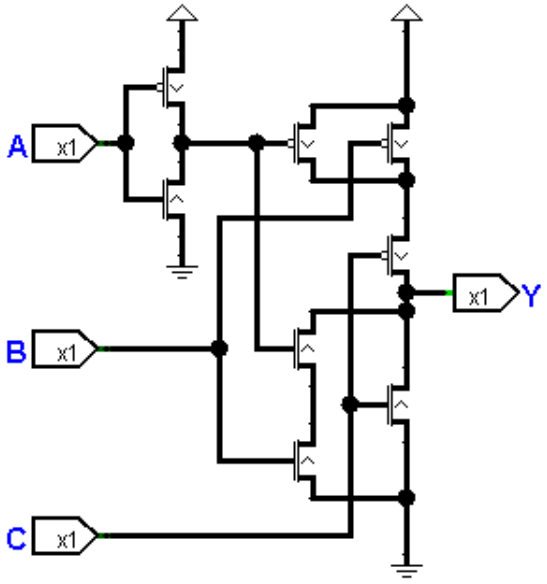
Aufgabe 3:

$$Y = (A \vee \overline{B}) \wedge \overline{C}$$

$$\begin{aligned} h(A,B,C) &= (A \vee \overline{B}) \wedge \overline{C} \\ &= (\overline{\overline{A}} \vee \overline{B}) \wedge \overline{C} \end{aligned}$$

$$\begin{aligned} g(A,B,C) &= \overline{h(A,B,C)} \\ &= (\overline{A} \wedge B) \vee C \end{aligned}$$

nach Shannonsches Gesetz:



Aufgabe 4:

$$
\begin{aligned}
F &= (F1 \vee F2 \vee F3) \wedge F4 \\
F1 &= \overline{A} \wedge B \\
F2 &= A \wedge B \\
F3 &= A \wedge \overline{B} \wedge \overline{C} \\
F4 &= \overline{A} \vee C \vee A
\end{aligned}
$$

$$
\begin{aligned}
F1 \vee F2 &= (\overline{A} \wedge B) \vee (A \wedge B) \\
&= B
\end{aligned}
$$

$$
\begin{aligned}
F4 &= \overline{A} \vee C \vee A \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \qquad F &= (F1 \vee F2 \vee F3) \wedge F4 \\
&= B \vee F3 \\
&= B \vee (A \wedge \overline{B} \wedge \overline{C}) \\
&= (B \vee A) \wedge (B \vee \overline{B}) \wedge (B \vee \overline{C}) \\
&= (B \vee A) \wedge (B \vee \overline{C}) \\
&= B \vee (A \wedge \overline{C})
\end{aligned}
$$

A	B	C	$\overline{B}$	$\overline{C}$	$F3 = A \wedge \overline{B} \wedge \overline{C}$	$F = B \vee F3$	$F = B \vee (A \wedge \overline{C})$
0	0	0	1	1	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	1
1	0	0	1	1	1	1	1
1	0	1	1	0	0	0	0
1	1	0	0	1	0	1	1
1	1	1	0	0	0	1	1

Aufgabe 5:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(a)

folgende Minterme sind hier relevant:

$$m_1 = \overline{A} \wedge \overline{B} \wedge C$$
$$m_2 = \overline{A} \wedge B \wedge \overline{C}$$
$$m_7 = A \wedge B \wedge C$$

(b)

Die DNF ist dann die Disjunktion der Minterme:

$$\begin{aligned} Y &= m_1 \vee m_2 \vee m_7 \\ &= (\overline{A} \wedge \overline{B} \wedge C) \vee (\overline{A} \wedge B \wedge \overline{C}) \vee (A \wedge B \wedge C) \end{aligned}$$

(c)

folgende Maxterme sind hier relevant:

$$M_0 = \overline{A} \vee \overline{B} \vee \overline{C}$$
$$M_3 = \overline{A} \vee B \vee C$$
$$M_4 = A \vee \overline{B} \vee \overline{C}$$
$$M_5 = A \vee \overline{B} \vee C$$
$$M_6 = A \vee B \vee \overline{C}$$

Die KNF ist dann die Konjunktion der Maxterme:

$$\begin{aligned} Y &= M_0 \wedge M_3 \wedge M_4 \wedge M_5 \wedge M_6 \\ &= (\overline{A} \vee \overline{B} \vee \overline{C}) \wedge (\overline{A} \vee B \vee C) \wedge (A \vee \overline{B} \vee \overline{C}) \wedge (A \vee \overline{B} \vee C) \wedge (A \vee B \vee \overline{C}) \end{aligned}$$

(d)

A	B	C	Y	m_1	m_2	m_7
0	0	0	0	0	0	0
0	0	1	1	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
1	1	1	1	0	0	1

(e)

A	B	C	Y	M_0	M_3	M_4	M_5	M_6
0	0	0	0	0	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	0	1	0	1	1	1
1	0	0	0	1	1	0	1	1
1	0	1	0	1	1	1	0	1
1	1	0	0	1	1	1	1	0
1	1	1	1	1	1	1	1	1