1. Aufgabe

a)

$$\begin{aligned}
|3r-6| &= r+2\\
|3(r-2)| &= r+2\\
|3|\cdot|r-2| &= r+2\\
3\cdot|r-2| &= r+2
\end{aligned}$$

1. Fall: $r-2 \geq 0 \Leftrightarrow r \in [2,+\infty)$

$$\begin{aligned}
3\cdot|r-2| &= r+2 &&\\
3\cdot(r-2) &= r+2 &&\\
3r-6 &= r+2 && |-r+6\\
2r &= 8 && |:2\\
r &= 4 &&\\
\mathbb{L}_1 &= \{4\} &&
\end{aligned}$$

2. Fall: $r-2 < 0 \Leftrightarrow r \in (-\infty,2)$

$$\begin{aligned}
3\cdot|r-2| &= r+2 &&\\
-3\cdot(r-2) &= r+2 &&\\
-3r+6 &= r+2 && |-r-6\\
-4r &= -4 && |:(-4)\\
r &= 1 &&\\
\mathbb{L}_2 &= \{1\} &&
\end{aligned}$$

$$\Rightarrow \qquad \mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = \{4\} \cup \{1\} = \{1,4\}$$

b)

$$2 > |s-3|$$

1. Fall: $s-3 \geq 0 \Leftrightarrow s \in [3,+\infty)$

$$\begin{aligned}
2 &> |s-3| &&\\
2 &> s-3 && |+3\\
5 &> s &&\\
5 &> s &&\\
\mathbb{L}_1 &= [3,5) &&
\end{aligned}$$

2. Fall: $s-3 < 0 \Leftrightarrow s \in (-\infty,3)$

$$\begin{aligned}
2 &> |s-3| &&\\
2 &> -(s-3) && |\cdot(-1)\\
-2 &< s-3 && |+3\\
1 &< s &&\\
\mathbb{L}_2 &= (1,3) &&
\end{aligned}$$

$$\Rightarrow \qquad \mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = [3,5) \cup (1,3) = (1,5)$$

c)

$$\frac{1}{|t+1|} \geq 6 \Rightarrow t \neq -1$$

$$\begin{aligned} \frac{1}{|t+1|} &\geq 6 && | \cdot |t+1| \\ 1 &\geq 6 \cdot |t+1| && | : 6 \\ \frac{1}{6} &\geq |t+1| \end{aligned}$$

1. Fall: $t+1 > 0 \Leftrightarrow t \in (-1, +\infty)$

$$\begin{aligned} \frac{1}{6} &\geq |t+1| \\ \frac{1}{6} &\geq t+1 && | -1 \\ -\frac{5}{6} &\geq t \\ \mathbb{L}_1 &= (-\infty, -5/6] \cap (-1, +\infty) \\ &= (-1, -5/6] \end{aligned}$$

2. Fall: $t+1 < 0 \Leftrightarrow t \in (-\infty, -1)$

$$\begin{aligned} \frac{1}{6} &\geq |t+1| \\ \frac{1}{6} &\geq -(t+1) && | \cdot (-1) \\ -\frac{1}{6} &\leq t+1 && | -1 \\ -\frac{7}{6} &\leq t \\ \mathbb{L}_2 &= [-7/6, +\infty) \cap (-\infty, -1) \\ &= [-7/6, -1) \end{aligned}$$

$$\Rightarrow \qquad \begin{aligned} \mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 &= (-1, -5/6] \cup [-7/6, -1) \\ &= [-7/6, -1) \cup (-1, -5/6] \\ &= [-7/6, -5/6] \setminus \{-1\} \end{aligned}$$

2. Aufgabe

a)

Existenz neutraler Elemente:

$$\begin{aligned} a \cdot b &= 0 \\ &= 0 + 0 \cdot a \end{aligned}$$

3. Aufgabe

(1)

$\star$ ist kommutativ, denn hier ist in der Verknüpfungstabelle eine Spiegelsymmetrie über die Diagonale zu beobachten.

$\circ$ hingegen ist nicht kommutativ, denn z.B. $a \circ b \neq b \circ a$.

4. Aufgabe

(1)

Zu zeigen ist: a) $\forall a \in \mathbb{N}_0 : a \circ 0 = a$ und b) $\forall a \in \mathbb{N}_0 : a \circ a = 0$
a)

$$\begin{aligned} a \circ 0 &= |a - 0| \\ &= |a| - |0| \\ \text{da } a \in \mathbb{N}_0 \qquad &= a - 0 \\ &= a \end{aligned}$$

b)

$$\begin{aligned} a \circ a &= |a - a| \\ &= |0| \\ &= 0 \end{aligned}$$

5. Aufgabe

a)

Assoziativgesetz der Addition:

$$\begin{aligned} \overline{1} + (\overline{2} + \overline{4}) &= \overline{1} + \overline{1} \\ &= \overline{2} \\ (\overline{1} + \overline{2}) + \overline{4} &= \overline{3} + \overline{4} \\ &= \overline{2} \\ \Rightarrow \qquad \overline{1} + (\overline{2} + \overline{4}) &= (\overline{1} + \overline{2}) + \overline{4} \end{aligned}$$

Kommutativgesetz der Multiplikation:

$$\begin{aligned} \overline{2} \bullet \overline{4} &= \overline{3} \\ \overline{4} \bullet \overline{2} &= \overline{3} \\ \Rightarrow \qquad \overline{2} \bullet \overline{4} &= \overline{4} \bullet \overline{2} \end{aligned}$$

Distributivgesetz:

$$\begin{aligned}
\overline{2} \bullet (\overline{3} + \overline{4}) &= \overline{2} \bullet \overline{2} \\
&= \overline{4} \\
(\overline{2} \bullet \overline{3}) + (\overline{2} \bullet \overline{4}) &= \overline{1} + \overline{3} \\
&= \overline{4} \\
\Rightarrow \qquad \overline{2} \bullet (\overline{3} + \overline{4}) &= (\overline{2} \bullet \overline{3}) + (\overline{2} \bullet \overline{4})
\end{aligned}$$

6. Aufgabe

a)

$$\begin{aligned}
(\overline{12} + \overline{9})^2 &= (\overline{12 + 9})^2 \\
&= (\overline{21})^2 \\
&= (\overline{4})^2 \\
&= \overline{4^2} \\
&= \overline{16}
\end{aligned}$$

$$\begin{aligned}
\overline{12}^2 + \overline{2} \cdot \overline{12} \cdot \overline{9} + \overline{9}^2 &= \overline{12^2} + \overline{2 \cdot 9} \cdot \overline{12} + \overline{9^2} \\
&= \overline{144} + \overline{18} \cdot \overline{12} + \overline{81} \\
&= \overline{8} + \overline{1} \cdot \overline{12} + \overline{13} \\
&= \overline{8} + \overline{12} + \overline{13} \\
&= \overline{8 + 12 + 13} \\
&= \overline{33} \\
&= \overline{16}
\end{aligned}$$

$$\Rightarrow \qquad (\overline{12} + \overline{9})^2 = \overline{12}^2 + \overline{2} \cdot \overline{12} \cdot \overline{9} + \overline{9}^2$$