1. Aufgabe

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{1}{7} - 0,2 \\ &= \frac{35}{70} + \frac{10}{70} - \frac{14}{70} \\ &= \frac{31}{70} \end{aligned}$$

$$\begin{aligned} x_2 &= \frac{\sqrt{75}}{5\sqrt{3}} \\ \Leftrightarrow \qquad {x_2}^2 &= \frac{\sqrt{75}}{5\sqrt{3}} \frac{\sqrt{75}}{5\sqrt{3}} \\ &= \frac{75}{25 \cdot 3} \\ &= 1 \end{aligned}$$

$$\begin{aligned} a &= \frac{\frac{1}{8} + \frac{1}{6}}{\frac{3}{4} + \frac{1}{5}} \\ &= \frac{\frac{6}{48} + \frac{8}{48}}{\frac{15}{20} + \frac{4}{20}} \\ &= \frac{\frac{14}{48}}{\frac{19}{20}} \\ &= \frac{14}{48} \cdot \frac{20}{19} \\ &= \frac{280}{912} \\ &= \frac{35}{114} \end{aligned}$$

$$\begin{aligned} b &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} z &= \sqrt{10201} \\ &= 101 \end{aligned}$$

$$\begin{aligned} y &= \sqrt{2}(\sqrt{8} + \sqrt{72} + \sqrt{18}) \\ &= \sqrt{2}(\sqrt{2^2 \cdot 2} + \sqrt{2^2 \cdot 18} - \sqrt{18}) \\ &= \sqrt{2}(2\sqrt{2} + 2\sqrt{18} - \sqrt{18}) \\ &= \sqrt{2}(2\sqrt{2} + \sqrt{18}) \\ &= 4 + \sqrt{2} \cdot \sqrt{18} \\ &= 4 + \sqrt{2} \cdot \sqrt{2 \cdot 9} \\ &= 4 + \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{9} \\ &= 4 + 2 \cdot 3 \\ &= 10 \end{aligned}$$

2. Aufgabe

(1)

$$\begin{aligned} \frac{a^2b + 2ab^2 + b^3}{a^2b - b^3} &= \frac{\cancel{b} \cdot (a^2 + 2ab + b^2)}{\cancel{b} \cdot (a^2 - b^2)} \\ &= \frac{(a+b)^{\cancel{2}}}{\cancel{(a+b)}(a-b)} \\ &= \frac{a+b}{a-b} \end{aligned}$$

(2)

$$\begin{aligned}
\frac{x-y}{\sqrt{x}+\sqrt{y}} &= \frac{x-y}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} \\
&= \frac{\cancel{(x-y)}\,(\sqrt{x}-\sqrt{y})}{\cancel{\sqrt{x}^2-\sqrt{y}^2}} \\
&= \sqrt{x}-\sqrt{y}
\end{aligned}$$

(3)

$$\begin{aligned}
\frac{a\sqrt{b}-b\sqrt{a}}{\sqrt{a}-\sqrt{b}} &= \frac{\sqrt{a^2b}-\sqrt{b^2a}}{\sqrt{a}-\sqrt{b}} \\
&= \frac{\sqrt{ab}\sqrt{a}-\sqrt{ab}\sqrt{b}}{\sqrt{a}-\sqrt{b}} \\
&= \frac{\sqrt{ab}\,\cancel{(\sqrt{a}-\sqrt{b})}}{\cancel{\sqrt{a}-\sqrt{b}}} \\
&= \sqrt{ab}
\end{aligned}$$

(4)

$$\begin{aligned}
\frac{a^2+ay}{a^2-2ay+y^2} \cdot \frac{a^2-y^2}{ay} \cdot \frac{y^2-ay}{(a+y)^2} &= \frac{a^4y^2 - a^5y - a^2y^4 + a^3y^3 + a^3y^3 - a^4y^2 - ay^5 + a^2y^4}{(a-y)^2 \cdot ay \cdot (a+y)^2} \\
&= \frac{-a^5y + 2a^3y^3 - ay^5}{ay(a+y)(a-y)(a+y)(a-y)} \\
&= \frac{-\cancel{ay}(a^4-2a^2y^2+y^4)}{\cancel{ay}(a^2-y^2)(a^2-y^2)} \\
&= -\frac{a^4-2a^2y^2+y^4}{a^4-2a^2y^2+y^4} \\
&= -1
\end{aligned}$$

3. Aufgabe

$$39x - 4y = 3$$
$$13x + y = -4/3$$

$$\Rightarrow \quad \left(\begin{array}{cc|c} 39 & -4 & 3 \\ 13 & 1 & \frac{-4}{3} \end{array}\right) L_1 \leftarrow -3L_2$$
$$\Leftrightarrow \quad \left(\begin{array}{cc|c} 0 & -7 & 7 \\ 13 & 1 & \frac{-4}{3} \end{array}\right)$$

$$\Rightarrow \quad -7y = 7$$
$$\Leftrightarrow \quad \boxed{y = -1}$$

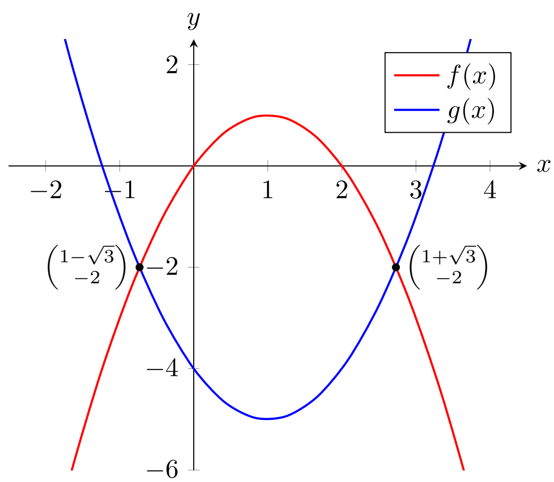
$$13x + y = -4/3$$
$$\Leftrightarrow \quad 13x - 1 = -4/3$$
$$\Leftrightarrow \quad 13x = -1/3$$
$$\Leftrightarrow \quad \boxed{x = -1/39}$$

4. Aufgabe

a)

$$
\begin{aligned}
& & f(x) &= g(x) \\
&\Leftrightarrow & 2x - x^2 &= x^2 - 2x - 4 \\
&\Leftrightarrow & 0 &= 2x^2 - 4x - 4 \\
&\Leftrightarrow & 0 &= x^2 - 2x - 2 \\
&\Rightarrow & x_{1,2} &= -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 + 2} \\
& & &= 1 \pm \sqrt{3}
\end{aligned}
$$

$f(x)$ und $g(x)$ schneiden sich also in den Punkten $x = 1 - \sqrt{3}$ und $x = 1 + \sqrt{3}$.



b)

Sei $f(x) = a \cdot x + b$ die Gerade, welche durch die Punkte $P_1(-1, 3)$ und $P_2(3, -1)$ läuft. Damit gilt:

$$\begin{aligned} & f(-1) = 3 \\ & f(3) = -1 \\ \Leftrightarrow \quad & a \cdot (-1) + b = 3 \\ & a \cdot 3 + b = -1 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & \left(\begin{array}{cc|c} -1 & 1 & 3 \\ 3 & 1 & -1 \end{array}\right) L_2 \leftarrow +3L_1 \\ \Leftrightarrow \quad & \left(\begin{array}{cc|c} -1 & 1 & 3 \\ 0 & 4 & 8 \end{array}\right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & 4b = 8 \\ \Leftrightarrow \quad & \boxed{b = 2} \end{aligned}$$

$$\begin{aligned} & -a + b = 3 \\ \Leftrightarrow \quad & -a + 2 = 3 \\ \Leftrightarrow \quad & -a = 1 \\ \Leftrightarrow \quad & \boxed{a = -1} \end{aligned}$$

$$\Rightarrow \quad \boxed{f(x) = 2 - x}$$

5. Aufgabe

(1)

$$
\begin{aligned}
& \sqrt{x+1} = x-1 \\
\Leftrightarrow\quad & 1 = \frac{x-1}{\sqrt{x+1}} \\
\Leftrightarrow\quad & 1 = \sqrt{\frac{(x-1)^2}{x+1}} \\
\overset{(1)}{\Leftrightarrow}\quad & \pm 1 = \frac{(x-1)^2}{x+1}
\end{aligned}
$$

$\Rightarrow x > -1$,(2)
da $\sqrt{x}$ für $x < 0$ undefiniert ist.

$$\sqrt{x} = 1 \Leftrightarrow x = \pm 1 \text{(1)}$$

Damit gibt es also zwei mögliche Lösungspfade: (1 und -1)

$$
\begin{aligned}
& \frac{(x-1)^2}{x+1} = 1 \\
\Leftrightarrow\quad & (x-1)^2 = x+1 \\
\Leftrightarrow\quad & x^2 - 2x + 1 = x + 1 \\
\Leftrightarrow\quad & x^2 - 2x = x \\
\Leftrightarrow\quad & x - 2 = 1 \\
\Leftrightarrow\quad & \boxed{x = 3}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{(x-1)^2}{x+1} = -1 \\
\Rightarrow\quad & (x-1)^2 < 0 \lor x + 1 < 0
\end{aligned}
$$

da $(x-1)^2$ immer positiv ist:

$$\Rightarrow\quad x + 1 < 0 \Leftrightarrow x < -1$$

Demnach müsste sowohl $x < -1$ und $x > -1$(2) gelten. Was einen Widerspruch darstellt.

Also hat die Aussage $\sqrt{x+1} = x - 1$ genau eine Lösung mit $x = 3$.

(2)

$$
\begin{aligned}
& & x^2 - 5x + 4 &= 0 \\
&\Rightarrow & x_{1,2} &= -\frac{-5}{2} \pm \sqrt{\left(\frac{-5}{2}\right)^2 - 4} \\
& & &= \frac{5}{2} \pm \sqrt{\frac{25}{4} - 4} \\
& & &= \frac{5}{2} \pm \sqrt{\frac{9}{4}} \\
& & &= \frac{5}{2} \pm \frac{3}{2} \\
&\Rightarrow & x &\in \{1, 4\}
\end{aligned}
$$

(3)

$$
\begin{aligned}
& & \frac{1}{5}x^2 - 2x + \frac{16}{5} &= 0 \\
&\Leftrightarrow & x^2 - 10x + 16 &= 0 \\
&\Rightarrow & x_{1,2} &= -\frac{-10}{2} \pm \sqrt{\left(\frac{-10}{2}\right)^2 - 16} \\
& & &= 5 \pm \sqrt{25 - 16} \\
& & &= 5 \pm 3 \\
&\Rightarrow & x &\in \{2, 8\}
\end{aligned}
$$

(4)

$$
\begin{aligned}
& & \frac{5x-3}{x+3} &= \frac{3x-2}{2x+2} \\
&\Leftrightarrow & (5x-3)\cdot(2x+2) &= (3x-2)\cdot(x+3) \\
&\Leftrightarrow & 10x^2+10x-6x-6 &= 3x^2+9x-2x-6 \\
&\Leftrightarrow & 10x^2+4x-6 &= 3x^2+7x-6 \\
&\Leftrightarrow & 7x^2-3x &= 0 \\
&\Leftrightarrow & x(7x-3) &= 0 \\
&\Leftrightarrow & x\left(x-\frac{3}{7}\right) &= 0 \\
&\Rightarrow & x &\in \left\{0, \frac{3}{7}\right\}
\end{aligned}
$$

(5)

$$
\begin{aligned}
& & x^6-6x^4+8x^2 &= 0 \\
&\Leftrightarrow & x^2\cdot(x^4-6x^2+8) &= 0 \\
&\Leftrightarrow & x=0 \vee x^4-6x^2+8 &= 0 \\
&\Leftrightarrow & x=0 \vee y^2-6y+8 &= 0 & & y := x^2 \\
&\Rightarrow & y_{1,2} &= -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2-8} \\
& & &= 3 \pm \sqrt{9-8} \\
& & &= 3 \pm 1 \\
&\Rightarrow & x=0 \vee y &\in \{2,4\} \\
&\Leftrightarrow & x=0 \vee x^2 &\in \{2,4\} \\
&\Leftrightarrow & x=0 \vee x &\in \left\{\sqrt{2}, -\sqrt{2}, 2, -2\right\} \\
&\Leftrightarrow & x &\in \left\{-2, -\sqrt{2}, 0, \sqrt{2}, 2\right\}
\end{aligned}
$$