

2. Aufgabe

a)

$$\begin{aligned}
 8^2 - x^6 &= (2^3)^2 - x^6 \\
 &= 2^6 - x^6 \\
 &= (2^3)^2 - (x^3)^2 \\
 &= (2^3 - x^3) \cdot (2^3 + x^3) \\
 &= (2 - x) \cdot (2^2 - 2x + x^2) \cdot (2^3 + x^3) \\
 \Rightarrow p(x) &= (2^2 - 2x + x^2) \cdot (2^3 + x^3)
 \end{aligned}$$

3. Aufgabe

$$\begin{aligned}
 (10n + 5)^2 &= (10n)^2 + 2 \cdot 10n \cdot 5 + 5^2 \\
 &= 100n^2 + 100n + 25 \\
 &= 100 \cdot n(n + 1) + 25
 \end{aligned}$$

4. Aufgabe

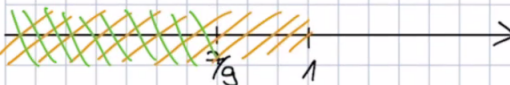
4 b) (*) $\frac{x+1}{2-2x} \leq 4 \quad | \cdot (2-2x) \quad \mathbb{D} = \mathbb{R} \setminus \{1\}$

1. Fall: $2-2x > 0$
 $\Leftrightarrow 2 > 2x$
 $\Leftrightarrow 1 > x$

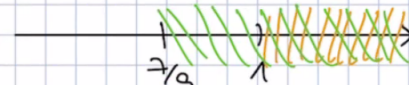
(*) $x+1 \leq 4 \cdot (2-2x)$
 $x+1 \leq 8-8x \quad | +8x; -1$
 $9x \leq 7$
 $x \leq \frac{7}{9}$

2. Fall: $2-2x < 0$
 $\Leftrightarrow 2 < 2x$
 $\Leftrightarrow 1 < x$

(*) $x+1 \geq 4 \cdot (2-2x)$
 $x+1 \geq 8-8x \quad | +8x; -1$
 $9x \geq 7$
 $x \geq \frac{7}{9}$



$\mathbb{L}_1 = (-\infty; \frac{7}{9}]$



$\mathbb{L}_2 = (1; \infty)$

$\Rightarrow \mathbb{L}_G = \mathbb{L}_1 \cup \mathbb{L}_2 = (-\infty; \frac{7}{9}] \cup (1; \infty)$

6. Aufgabe**A4**

$$\begin{aligned}A_4 &= 25r^2 - 40rs + 16s^2 + 49t^2 - 70tq + 25q^2 \\&= (5r - 4s)^2 + (7t - 5q)^2\end{aligned}$$

A6

$$\begin{aligned}A_6 &= 81^2 \\&= (80 + 1)^2 \\&= 80^2 + 2 \cdot 80 \cdot 1 + 1^2 \\&= 6400 + 160 + 1 \\&= 6561\end{aligned}$$

B1

$$\begin{aligned}B_1 &= \frac{39a^3 - 39a^2}{13a^2 - 13a} \\&= \frac{13a(3a^2 - 3a)}{13a(a - 1)} \\&= \frac{3a^2 - 3a}{a - 1} \\&= \frac{3a(a - 1)}{a - 1} \\&= 3a\end{aligned}$$

B2

$$\begin{aligned} B_2 &= \frac{15ab - 30b^2}{5a^2b - 20ab^2 + 20b^3} \\ &= \frac{15ab - 30b^2}{5b(a^2 - 4ab + 4b^2)} \\ &= \frac{15ab - 30b^2}{5b(a^2 - 2 \cdot a \cdot (2b) + (2b)^2)} \\ &= \frac{5b(3a - 6b)}{5b(a - 2b)^2} \\ &= \frac{3a - 6b}{(a - 2b)^2} \\ &= \frac{3(a - 2b)}{(a - 2b)^2} \\ &= \frac{3}{a - 2b} \end{aligned}$$

C1

$$\begin{aligned} C_1 &= (-a^5)^6 \cdot (-a^6)^{-5} \\ &= -a^{30} \cdot (-a^{-30}) \\ &= -a^{30} \cdot \frac{1}{-a^{30}} \\ &= 1 \end{aligned}$$

C2

$$\begin{aligned} C_2 &= \left(\frac{a}{3}\right)^2 \cdot \left(\frac{3}{a}\right)^{-5} \\ C_2 &= \left(\frac{a}{3}\right)^2 \cdot \left(\frac{a}{3}\right)^5 \\ C_2 &= \left(\frac{a}{3}\right)^{2+5} \\ C_2 &= \frac{a^7}{3^7} \\ C_2 &= \frac{a^7}{2187} \end{aligned}$$

d1

$$-7 = 3x^2 + 10x \quad | + 7$$

$$0 = 3x^2 + 10x + 7 \quad | : 3$$

$$0 = x^2 + \frac{10}{3}x + \frac{7}{3}$$

mittels pq-formel:

$$x_{1,2} = -\frac{\frac{10}{3}}{2} \pm \sqrt{\left(\frac{\frac{10}{3}}{2}\right)^2 - \frac{7}{3}}$$

$$= -\frac{10}{6} \pm \sqrt{\left(\frac{10}{6}\right)^2 - \frac{7}{3}}$$

$$= -\frac{10}{6} \pm \sqrt{\frac{100}{36} - \frac{7}{3}}$$

$$= -\frac{10}{6} \pm \sqrt{\frac{100}{36} - \frac{84}{36}}$$

$$= -\frac{10}{6} \pm \sqrt{\frac{16}{36}}$$

$$= -\frac{10}{6} \pm \frac{\sqrt{16}}{\sqrt{36}}$$

$$= -\frac{10}{6} \pm \frac{4}{6}$$

$$\begin{aligned} \Rightarrow \mathbb{L} &= \left\{ -\frac{10}{6} - \frac{4}{6}, -\frac{10}{6} + \frac{4}{6} \right\} \\ &= \left\{ -\frac{7}{3}, -1 \right\} \end{aligned}$$

d2

$$2x - 3 = 2x^2$$

$$\Leftrightarrow 0 = 2x^2 - 2x + 3 \quad | : 2$$

$$\Leftrightarrow 0 = x^2 - x + \frac{3}{2}$$

$$\Rightarrow \left(\frac{p}{2}\right)^2 - q < 0 \text{ mit } p = -1 \text{ und } q = \frac{3}{2}$$

$$\Rightarrow \mathbb{L} = \emptyset \text{ da die Diskriminante unter 0 liegt.}$$

d5

mittels pq-formel:

$$\begin{aligned}0 &= (x - 2)(x - 5) + 2 \\&= (x^2 - 5x - 2x + 10) + 2 \\&= x^2 - 7x + 12 \\x_{1,2} &= -\frac{-7}{2} \pm \sqrt{\left(\frac{-7}{2}\right)^2 - 12} \\&= \frac{7}{2} \pm \sqrt{\left(\frac{49}{4}\right) - 12} \\&= \frac{7}{2} \pm \sqrt{\frac{49}{4} - \frac{48}{4}} \\&= \frac{7}{2} \pm \sqrt{\frac{1}{4}} \\&= \frac{7}{2} \pm \frac{1}{2} \\&\Rightarrow \mathbb{L} = \left\{ \frac{6}{2}, \frac{8}{2} \right\} \\&= \{3, 4\}\end{aligned}$$