Vestibular_1_MathDescription

May 6, 2016

1 The Vestibular System - Part 1: Mathematical Description

- 3 DOF Translations -> Otoliths (Utricle, Saccule)
- 3 DOF Rotations -> 3 Semicircular Canals

<img src="data/images/labyrinth_labelled.png", width=350px>

1.1 Semicircular Canals

The three semicircular canals (SCCs) on each side of the head transduce angular velocity. To describe this transduction mathematically, each canal can be described by a vector perpendicular to the surface spanned by the canal $\vec{n}$.

1.1.1 Orientation of all 3 Canals, on the Right Side of the Head

In [1]:
```
# Import the required packages
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt

# Make the display easy to read
% precision 2
```

Out[1]: '%.2f'

In [2]:
```
# Enter the information for the canalas into a Python dictionary.
# The data are taken from Blanks RH, Curthoys IS, and Markham CH. Planar relationships of
# the semicircular canals in man. Acta Otolaryngol  80:185-196, 1975.
Canals = {'info': 'The matrix rows describe horizontal, anterior, and posterior canal orientati
 'right': np.array(
        [[0.365,  0.158, -0.905],
         [0.652,  0.753, -0.017],
         [0.757, -0.561,  0.320]]),
 'left': np.array(
        [[-0.365,  0.158,  0.905],
         [-0.652,  0.753,  0.017],
         [-0.757, -0.561, -0.320]])}

# Normalize these vectors (just a small correction):
for side in ['right', 'left']:
    Canals[side] = (Canals[side].T / np.sqrt(np.sum(Canals[side]**2, axis=1))).T

# Show the results for the right SCCs:
print(Canals['info'])
print(Canals['right'])
```

```
The matrix rows describe horizontal, anterior, and posterior canal orientation
[[ 0.37  0.16 -0.92]
 [ 0.65  0.76 -0.02]
 [ 0.76 -0.56  0.32]]
```

2 Example: Stimulation of the Right Horizontal Canal, by Rotation to the Right, with 100 deg/2

$$stim = \vec{\omega} \cdot \vec{n} \tag{1}$$

$\vec{\omega}$ ... relativ to the head!
<img src="data/images/SCC_stimulation_with_omega_.jpg", width=150px>

```
In [3]: omega = np.r_[0, 0, -100]
        stim = omega @ Canals['right'][0]

        # Before Python 3.5:
        # stim = np.dot(omega, Canals['right'][0])

        print('The angular velocity sensed by the right horizontal canal is {0:3.1f} deg/s.'.format(sti
```

```
The angular velocity sensed by the right horizontal canal is 91.5 deg/s.
```

<img src="data/images/matlab_logo_s.jpg", width=50px, align='left'>Matlab Equivalent Commands:

```
% Define the semicircular canals
Canals(1).side = 'right';
Canals(1).rows = {'horizontal canal'; 'anterior canal'; 'posterior canal'};
Canals(1).orientation = [   .365     .158   -.905
    .652     .753   -.017
    .757    -.561    .320];
Canals(2).side = 'left';
Canals(2).rows = {'hor'; 'ant'; 'post'};
Canals(2).orientation = [-.365    .158   .905
    -.652     .753   .017
    -.757    -.561  -.320];

% Normalize the canal-vectors (only a tiny correction):
for ii = 1:2
    for jj = 1:3
        Canals(ii).orientation(jj,:) = ...
            Canals(ii).orientation(jj,:) / norm( Canals(ii).orientation(jj,:) );
    end
end

% Calculate the stimulation
omega = [0, 0, -100];
stim = omega * Canals(1).orientation(1,:).T;
```

2.1 Otoliths

The description of the otoliths is more involved, since each haircell on the otolith membrane senses the local strength of the gravito-inertal force. The direction of maximum sensitivity can for each haircell be described by a Sensitivity Vector $\vec{n}$.

<img src="data/images/UtriShapeHair.jpg", width=600px>

$$\overrightarrow{GIF} = \vec{g} + \frac{d^2\vec{x}}{dt^2} \tag{2}$$

$$stim \approx \overrightarrow{GIF} \cdot \vec{n} \tag{3}$$

- For the utricle, the sensitivity vectors lie approx. in the horizontal plane
- For the saccule, the sensitivity vectors lie approx. in the vertical plane

<img src="data/images/Forward.png", align='left'> Vestibular_2_SCC_Transduction.ipynb

Author: Thomas Haslwanter
Date: April 4, 2016
Ver: 1.0

In [ ]: