A review of the effect of the physical properties of a leaky faucet system on its chaotic behaviour

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PRO 1101: Introduction to Project Period

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Introduction

The concept of chaos is researched by scientists from various perspectives within the field of physics. An example of a model that exhibits chaotic behaviour is a leaking faucet system. Its dripping pattern can shift from periodic to irregular. Robert Shaw asks in his publication from 1984 if we can determine the degree of predictability of such a system (Shaw, 1984). In our study, we will provide an analysis of a dripping faucet system in terms of changes in physical properties. The research papers we used for our literature review covered the behaviour of a chaotic system in relation to applied physical properties. This is highly relevant for the leaky faucet system we are examining, because it requires knowledge about dripping dynamics in relation to chaotic behaviour.

There are plenty of articles talking about varying system properties separately, but since there have been few to no papers that combine all discussed physical properties, this literature review is significant. Also, this research contains relevant information for applications in medical, pharmaceutical but also everyday applications (Rubio-Rubio, et al., 2008). The concept of ink-jet printing, for example, is based on dripping behaviour. Researching this topic shows what is already known about the model but also what could be researched more.

Therefore, the aim of this paper is to provide a more complete view of the effects of relevant physical properties and to connect these pieces of information in a coherent manner. Furthermore, the paper will determine how the chaotic behaviour of a leaky faucet system is influenced when certain physical properties are changed. The hypothesised result would be to find links between the physical characteristics and the chaotic behaviour of a dripping faucet system. The properties researched were: surface tension (σ), viscosity (μ), pressure (P) and Temperature (T).

Methods

The method followed in this literature review started with orienting on the possible physical properties that could influence the chaotic behaviour. This was done by using systematic searching. A web search through the Web of Science was performed, using three keywords (in bold) taken out of the research question: “What is the influence of changing the **physical properties** of a **leaky faucet** system in regards to observed **chaotic behaviour** ?”

Flow rate

The flow rate in a dripping faucet experiment is the driving force of the system, which affects the time intervals between drops. Under normal conditions, a change in water flow rate impacts the periodic dripping of a leaking faucet and causes it to behave chaotically once the flow rate exceeds a certain threshold. When the water enters the jetting state, the chaotic behaviour ceases. The flow rate of water determines the amount of water passing through an area at any given time and is affected by the pressure, as well as the velocity of the water. Increasing the pressure of the water decreases the flow rate, whereas increasing the velocity of the water also increases its flow rate.

Various experiments have shown that water transitions from periodic to chaotic dripping behaviour at a certain flow rate until it eventually starts jetting at a higher rate. (Shaw, 1984; Dreyer & Hickey, 1991)

At lower regimes of flow rate, the dripping of water will occur periodically. Up to a certain point, the time intervals between the drops are similar with some minor variation due to unstable conditions, such as vibration or air currents (Shaw, 1984). When a critical mass of the water drop is reached, part of it separates, while another smaller part relaxes back into the faucet. As such, when the drops are created faster than the remaining portion can relax, the time gaps between the drops are affected by one another and vary, leading to different types of drop behaviour. Fig.1 is an example of the corresponding evolution in relation to the influx of water. In the figure, L_d is the length of the drop at break up, a property which can be used instead of time intervals between drops. The variables Oh and Bo will be discussed later on. The Weber number We is often used instead of the flow rate and is in this case described by $We = \frac{\rho v^2 R}{\sigma}$ and $v = \frac{Q}{\pi R^2}$, where σ is the surface tension, ρ is the density, R is the outer radius of the tube and Q is the flow rate. (Ambravaneswaran et al., 2004) Effectively, whenever the flow rate Q increases, the Weber number We does too.

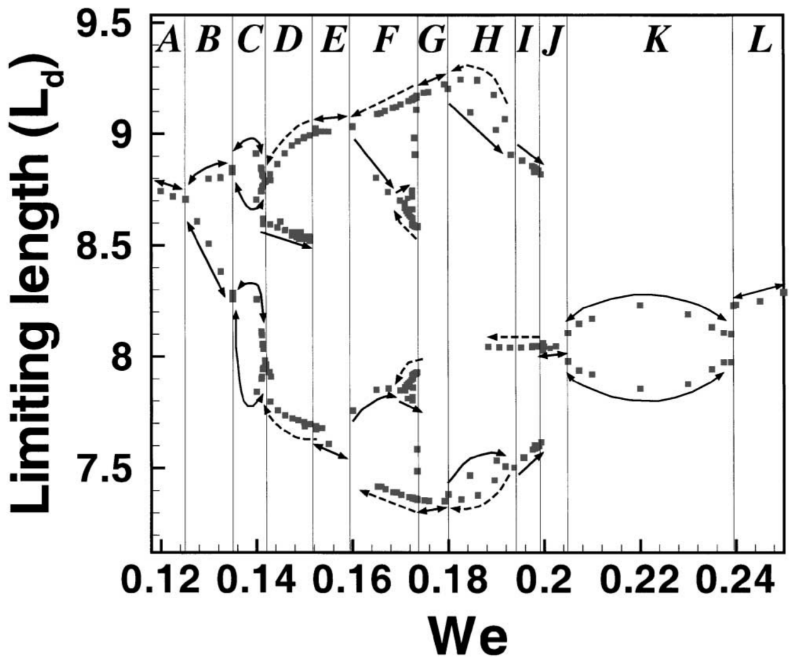

Fig.1: We ** vs.* L_d** Diagram demonstrating the evolution of the periodic time intervals, where Oh = 0.1, Bo = 0.5. From A-L: The system goes through different types of periodic patterns, the points (on the graph) show the values the pattern switches between for a specific We. As published by Ambravaneswaran et al. in 2000.*

**definitions above the figure*

The figure portrays periods with different kinds of patterns. In period A, the limiting length stays the same, while in period B, there are two values which the drops switch between. This specific transition is called a period-doubling bifurcation, as the slight change in We incites a new path to form. Other kinds of transitions, including distinct types of bifurcation can also be seen.

When the flow rate reaches a certain point, the dripping is altered from having a periodic to having a chaotic behaviour. While the system is in this chaotic state, the time intervals show no sign of any simple pattern. Slightly changing the flow rate results in a completely different structure, an indication of a chaotic system (Boccaletti et al., 2002). This is caused by the aforementioned oscillation of the leftover part of the drop, which affects the behaviour more drastically due to the increased drop rate. However, certain patterns can still be observed. The dripping in *Fig. 2* shows no regular structure at first, but after around the 500th time interval, the system shows an orderly response where the values on the graph start accumulating around one point, a strange non-chaotic attractor. An attractor can be seen as the entirety of conditions the system tends to develop towards as time advances. During the transition towards the limit $T \approx 0.65\ sec$, the right graph cannot be divided into different sub-functions describing the behaviour of the drop intervals and their effect on each other, as the pattern is irregular, a sign for a strange attractor. Nonetheless, the accumulation at $T \approx 0.65\ sec$ makes it non-chaotic. (Somarakis et al., 2008)

The left image represents a return map, a technique commonly employed to analyse the relationship between consecutive drop intervals, namely Tn and Tn+1, at a given flow rate. The right image shows the progression of the drop intervals, where n represents the n^{th} interval since the onset of the experiment.

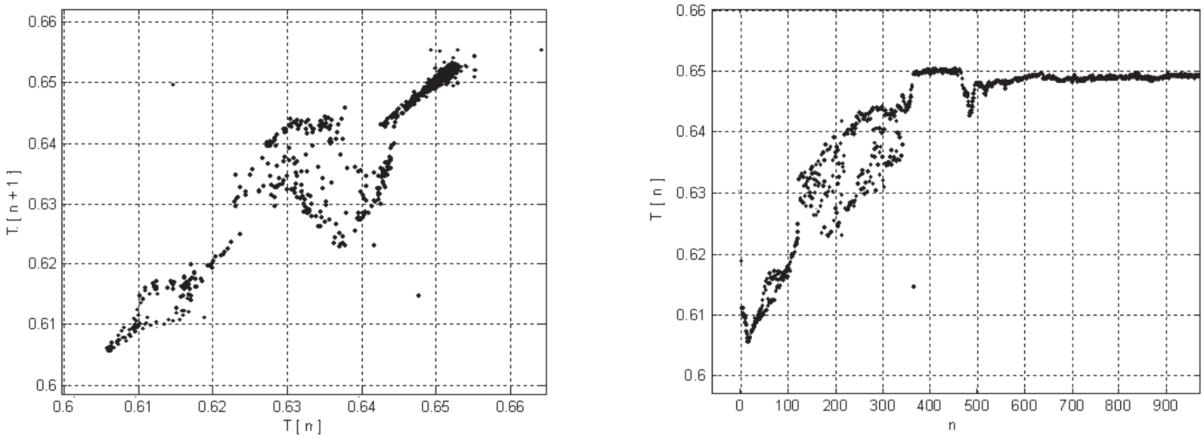

Fig. 2: *Left image: Return map illustrating the structure found by relating consecutive drop intervals. Right image: Time-Series documenting the change of time intervals between drops. After* $n \approx 500$*, the values only differ minimally. The dripping rate settles down to* $f = 1.54\ drops/sec$*. As published by Somarakis et al. in 2008.*

On the contrary, the dripping system shows even more chaotic behaviour at higher flow rates, as shown in *Fig. 3*. During this state, unlike *Fig. 2*, there are no specific spots on the map where the points evolve towards.

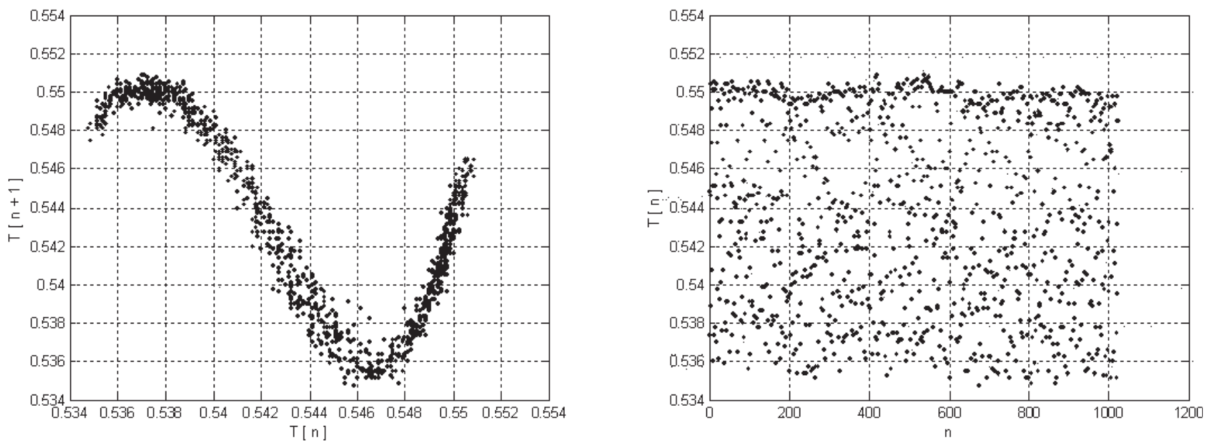

Fig.3: *Left image: return map illustrating the structure found by relating consecutive drop intervals. Right image: time-series graph documenting the change of time intervals between drops. No apparent shift in intervals can be studied. Average dripping rate is f=1.84 drops/sec. As published by Somarakis et al. in 2008*

A strange chaotic attractor can be observed, since no convergence towards one specific point can be seen, yet the values do not deviate much from the attractor and a general structure can be observed in the return map. Thus, it is not completely random.

At higher dripping rates, the strange attractors become more complex (Somarakis et al., 2008), until the water starts jetting.

Viscosity, surface tension, and temperature

Although most of the papers published regarding the topic do not consider viscosity, surface tension or temperature as separate variables, they do use several dimensionless numbers useful in fluid mechanics. Those are: the Weber number (We) as mentioned in the Flow Rate section, the Ohnesorge number (Oh), Kapitza number and the Bond number (Bo) as used by Ambravaneswaran et al. (2002) and Rubio-Rubio et al. (2018). Their expressions are visible in the table below, and as can be seen, they do contain the variables that are of interest (viscosity and surface tension). Temperature is not considered a separate variable, but it does affect both viscosity and surface tension to a great extent, which will be discussed in the later part of the paragraph.

<table>
<tr><td>$Oh \equiv \mu/\sqrt{\rho R \sigma}$</td><td rowspan="3">$R$ is the injector size
Q is the injection flow rate
ρ is the density
σ is the surface tension
μ is the dynamic viscosity
g is the Earth's acceleration due to gravity</td></tr>
<tr><td>$Bo \equiv \rho R^2 g/\sigma$</td></tr>
<tr><td>$\Gamma \equiv 3v((\rho^3 g)/\sigma^3)^{1/4}$ (Kapitza number),
where $v = \mu/\rho$ (the kinematic viscosity)</td></tr>
</table>

Table 1: *Dimensionless numbers describing the leaky faucet system as used by Ambravaneswaran et al. (2002) and Rubio-Rubio et al. (2018)*

As stated earlier, the system's dynamics are dependent on all properties; thus, even the slightest change in combination can drastically alter its output. Subramani et al. (2006) experimented with different values of We at low bond values (Bo=0.33) and Oh=0.13. They found that when the ratio between the inertial and surface tension force is high (large We value), the system will become complex. As We increases from 0.119, it will increase its period of motion from period-1 dripping to a period-2 dripping and then to chaotic dripping. In their theoretical computations, they found that the system can produce more complex patterns, such as period-3 dripping, when the bond number is increased. As surface tension is inversely proportional to the bond number, the relative gravitational force is larger than the relative surface tension force, resulting in period-3 dripping.

A direct approach was taken by Wu & Schelly (1989) to investigate the effect of changing surface tension on the dripping dynamics of the system by using a surfactant, sodium dodecyl sulphate (SDS), to alter the surface tension of water. A range of 0 M to $1.2 \cdot 10^{-2}$ M was used, starting from $0.08 \cdot 10^{-2}$ M to surpass the critical micelle concentration. As the micelle concentration increases, it modifies the hydrogen bonds between the water molecules which in turn reduce the surface tension. They found that as surface tension is decreased, the graphical representation of the systems dynamics become more sharp and less scattered, whilst the water and SDS solutions will maintain similar skeleton structures, as seen in figure 4. published in (Wu & Schelly, 1989.)

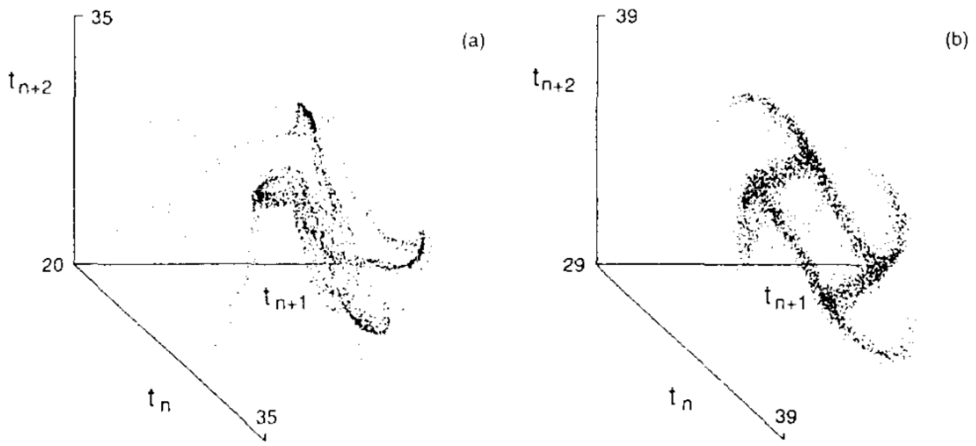

Fig. 4: *Comparison of dripping patterns for water and SDS solutions; (a) Water, (b)* $0.08 \cdot 10^{-2} M$ *SDS solution as published in Wu & Schelly (1989.)*

There have been several approaches to studying the effect of varying viscosity on the behaviour of the leaky faucet system. One of them was done by first verifying the accuracy of 1-dimensional and 2-dimensional algorithms that simplify the 3D Navier-Stokes equations (Ambravaneswaran et al. 2002) and then comparing those simplified equations to experimental results with varying viscosities using water, glycerol-water mixture and silicone oils (Ambravaneswaran et al. 2004). It has been found that for $Bo = 0.5$, $Oh = 0.1$ and $0.12 < We < 0.26$, the drops display chaotic behaviour. In order for the system to not show any complex dripping, holding all the other variables constant, Oh (or analogically viscosity) has to be increased at least 5 fold.

Furthermore, the limitations of the simplified 1D model have been found: mainly the fact that they do not account for tube wall thickness and outlet type. Additionally, the simplified equations become rather unreliable at a low Oh number as We increases (Ambravaneswaran et al. 2004), and considering that Ohnesorge number is proportional to viscous forces it can be deduced that using the 1D model is inappropriate for low-viscosity liquids.

Rubio-Rubio et al. (2018) mention that depending on the liquid viscosity, the system may exhibit not only period-1 and chaotic dripping, but also other complex behaviours such as period-2 and period-3. They chose to use the $Kapitza$ number, which in contrast to the Ohnesorge number doesn't contain the injector size variable, meaning that variation in it would only be an implication of the variation in the fluid properties. Rubio-Rubio et al. (2018) showed that for $Bo = 1.0$, $We \approx 0.05$, and changes to viscosity ranging $Kapitza$ from 0.01 to 1.00, the system goes through chaotic dripping, then period-3 dripping, followed by period-2 dripping to finally achieve period-1 dripping. Considering that, their work is in accordance with Ambravaneswaran et al. (2004) and further confirms that decreasing the viscosity of a liquid leads to more complex dynamics in the system .

As seen, the dripping dynamics of the faucet are affected by both viscosity and surface tension. As temperature increases, both variables decrease and vice versa. With a change of ± 10℃, the average change of viscosity is $\mp 24\%$ and of surface tension is $\mp 2\%$ for simple liquids (Wu & Schelly, 1989). Yet they found that as temperature decreased, the graphical representations of the system's dynamics became less scattered and more detailed, due to the liquid's properties at higher viscosities, while the structural skeleton does not vary. It was also found that the flowrate increases as temperature increases due to low viscosity and surface tension allowing liquids to flow with less resistance, and if flow rate is reduced at higher temperatures a similar dripping pattern will appear as with lower temperature tests (Wu & Schelly, 1989).

Conclusion

The conducted literature review shows that certain physical properties do affect the chaotic behaviour of the leaky faucet system. Specific physical properties have key importance in many different fields. Studying the effect of flow rate of the liquid may be of interest in the area of ink-jet printing, where chaotic behaviour can disturb the precision of the printers, as ink possesses different physical characteristics than water. The same can be said about pharmaceutical applications, where measuring the exact dose of a drug is crucial. Given that certain drug solutions may possess different viscosity or surface tension than water, studying the effect of varying those characteristics on the precision of formed drops is of importance.

All in all, in accordance with chaos theory, even the smallest changes in physical properties of the liquid coming out of the faucet and the faucet itself, have profound implications on how the system behaves. Considering that, further research concerning the effect of manipulating those physical properties that affect the dripping dynamics is recommended, taking into account that those properties are all interconnected.

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Appendix: Organization plan

We divided the work in this project as following:

Introduction paragraph 1 and 2	Zofia Glabiak
Introduction paragraph 3	Lisa Robben
Method section	Lisa Robben
Surface tension, viscosity and temperature	Grzegorz Ratajski and Tom Kores Lesjak
Flow Rate and pressure	Nils Thiessen and Emre Berna
Nozzle characteristics and gravity (deleted for word count)	Lisa Robben and Shurently Recordino
Conclusion	Grzegorz Ratajski
Abundance of papers	Rhys Wittry and Ashley Heunen
Final editing and deleted topics	Dries Harts and Martin Maboge
Poster	Rhys Wittry and Zofia Glabiak