



Simon Fraser University  
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# Deep Learning - Assignment 1

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## A.1

Least-squares error:

$$E(w) = \frac{1}{2}((x.w) - y)^2$$

$$\nabla E(w) = [\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, \dots] = \frac{\partial E}{\partial w_i} = \frac{1}{2}(2)(x.w - y)(x)$$

Cross Entropy:

$$E(w) = y \ln(p) + (1 - y) \ln(1 - p)$$

$$\text{chain rule: } \frac{\partial E}{\partial x} = \frac{\partial E}{\partial p} \times \frac{\partial p}{\partial x}$$

$$\frac{\partial E}{\partial p} = \frac{y}{p} - \frac{1-y}{1-p}$$

$$\frac{\partial p}{\partial x} = \frac{\partial \sigma(x)}{\partial x} = \sigma(x) \times (1 - \sigma(x))$$

$$\frac{\partial E}{\partial x} = \left(\frac{y}{p} - \frac{1-y}{1-p}\right) \times \sigma(x) \times (1 - \sigma(x))$$

## A.2

$$\begin{pmatrix} 0.5 & 1.5 \\ 3.5 & 2 \end{pmatrix} \begin{pmatrix} 0.5 & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 1.25 & 3 \end{pmatrix} = \begin{pmatrix} 0.25 + 4.5 & 0.5 + 3 \\ 1.75 + 6 & 3.5 + 4 \end{pmatrix} + \begin{pmatrix} 1.25 & 3 \\ 1.25 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 6.5 \\ 9 & 10.5 \end{pmatrix}$$

In broadcasting in  $w^*x+b$ ,  $b$  is usually give in one row and we copy the same row for each data point.

## A.3

### 1

$$a_c = \text{Sigmoid}(a_a.W_{ac} + a_b.W_{bc} + W_{0c})$$

$$a_d = \text{Sigmoid}(a_c.W_{cd} + W_{0d})$$

$$\Rightarrow a_x = \text{Sigmoid}(a_a.W_{ax} + a_b.W_{bx} + W_{0x})$$

$$\Delta[x] = -\text{sigmoid}'(a_x) \frac{\partial E(a_x, y_x)}{\partial a_x}$$

$$W_{xy} \leftarrow W_{xy} + \alpha \times a_x \times \Delta[y]$$

### 2

I perform the first iteration on data point 1 and the second iteration on data point 2:

$$a_c = \text{sigmoid}(w_{0c} + w_{ac}.a + w_{bc}.b) = \text{sigmoid}(0.2 + 0.2 \times 1 + 0.1 \times 0) = 0.598$$

$$a_d = \text{sigmoid}(w_{0d} + w_{cd}.c) = \text{sigmoid}(0.2 + 0.1 \times 0.598) = 0.612$$

$$\Delta[d] = (a_d - 1) * \text{sigmoid}'(w_{cd} \times a_c + w_{0d}) = (0.612 - 1) \times \text{sigmoid}'(0.460) = -0.388 \times 0.247 = -0.0958$$

$$\Delta[c] = \text{sigmoid}'(w_{ac} \times a_x + w_{0c}) \times (\Delta[d] \times w_{cd}) = \text{sigmoid}'(0.2 \times 1 + 0.2) \times (-0.0960 \times 0.1) = 0.247 \times (-0.00960) = -0.00237$$

$$w_{cd} \leftarrow w_{cd} - lr \times \Delta[d] \times a_c = 0.1 - 0.1 \times (-0.0958) \times 0.599 = 0.1 + 0.00572 = 0.1057$$

$$w_{0d} \leftarrow w_{0d} - lr \times \Delta[d] = 0.2 - 0.1 \times (-0.0958) = 0.2 + 0.00958 = 0.2095$$

$$w_{ac} \leftarrow w_{ac} - lr \times \Delta[c] \times a_x = 0.2 - 0.1 \times (-0.00237) \times 1 = 0.2 + 0.00024 = 0.2002$$

$$w_{0c} \leftarrow w_{0c} - lr \times \Delta[c] = 0.2 - 0.1 \times (-0.00237) = 0.2 + 0.000237 = 0.2002$$

Second iteration on data point 2:

$$a_c = \text{sigmoid}(w_{ac} \times a_x + w_{0c}) = \text{sigmoid}(0.2 \times 0 + 0.2) = \text{sigmoid}(0.2) = 0.545$$

$$a_d = \text{sigmoid}(w_{cd} \times a_c + w_{0d}) = \text{sigmoid}(0.1 \times 0.55 + 0.21) = \text{sigmoid}(0.055 + 0.21) = \text{sigmoid}(0.264) = 0.565 - \Delta[d] = (a_d - 0) \times \text{sigmoid}'(w_{cd} \times a_c + w_{0d}) = (0.565 - 0) \times \text{sigmoid}'(0.2645) = 0.565 \times 0.2467 = 0.1396$$

$$\Delta[c] = \text{sigmoid}'(w_{ac} \times a_x + w_{0c}) \times (\Delta[d] \times w_{cd}) = \text{sigmoid}'(0.2 \times 0 + 0.2) \times (0.1396 \times 0.1) = 0.2493 \times 0.01396 = 0.003475$$

$$w_{cd} \leftarrow w_{cd} - lr \times \Delta[d] \times a_c = 0.1057 - 0.1 \times 0.1396 \times 0.550 = 0.1057 - 0.0076 = 0.098$$

$$w_{0d} \leftarrow w_{0d} - lr \times \Delta[d] = 0.21 - 0.1 \times 0.1396 = 0.21 - 0.01396 = 0.1956$$

$$w_{ac} \leftarrow w_{ac} - lr \times \Delta[c] \times a_x = 0.2002 - 0.1 \times 0.00347 \times 0 = 0.2002 - 0 = 0.2002$$

$$w_{0c} \leftarrow w_{0c} - lr \times \Delta[c] = 0.200237073162 - 0.1 \times 0.00347 = 0.2002 - 0.000347 = 0.1999$$

Data	$a_c$	$\Delta[c]$	$a_d$	$\Delta[d]$	$W_{0c}$	$W_{ac}$	$W_{bc}$	$W_{cd}$	$W_{0d}$
$x_1$	0.598	-0.00237	0.613	-0.0958	0.2002	0.2002	0.1	0.1057	0.21
$x_2$	0.5498	0.00347	0.565	0.1396	0.1999	0.2002	0.1	0.098	0.1956