

GRAPH VISUALIZATION

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GRAPH VISUALIZATION
DATAVIS FALL 2025

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1. INTRODUCTION

2. VISUAL ENCODING

3. BASIC CHART TYPES

4. INTERACTION

5. VISUALIZATION DESIGN

6. DATA PREPROCESSING

7. RECAP 1st Half

8. MULTIVARIATE DATAVIS

9. TEMPORAL DATAVIS

10. GEOSPATIAL DATAVIS

11. GRAPH DATAVIS

12. 3D DATAVIS

13. VISUAL ANALYTICS

14. RECAP 2nd Half

Basics

Visualization
Building Blocks
& Processes

Visualization
Techniques

Visualization
Applications



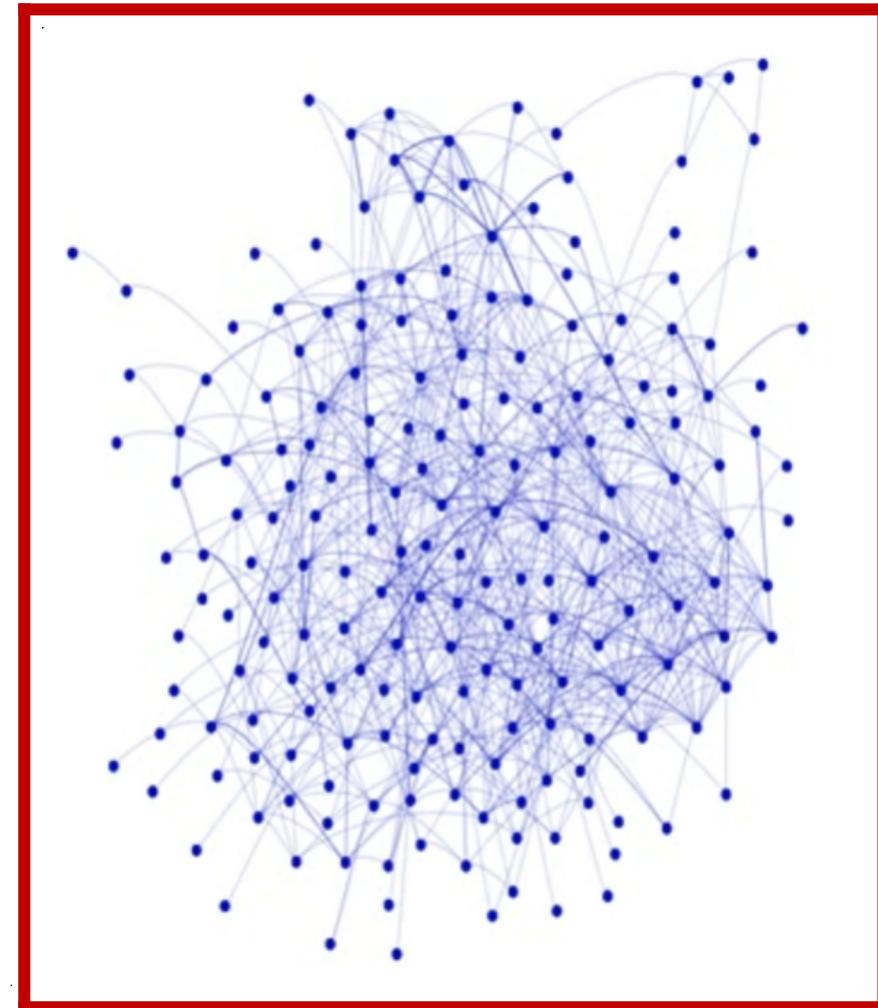
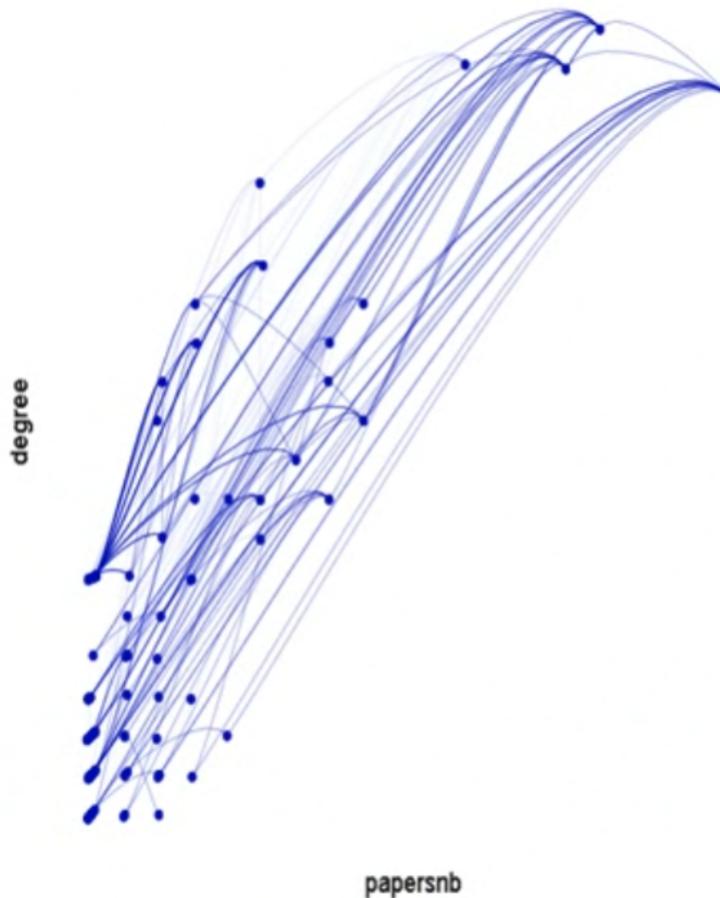
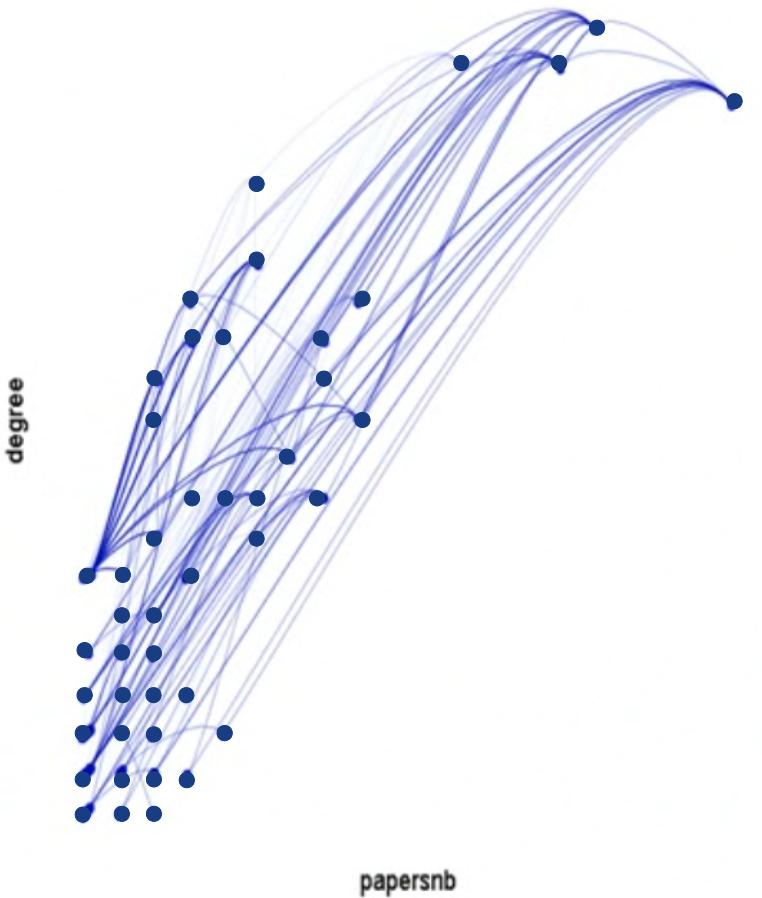
WHY TALK ABOUT GRAPH VISUALIZATION?

Graphs / networks put two data items into relation to each other.

- Social networks
- Computer networks
- Call graphs of a software
- Traffic networks
- ...

If that relation is hierarchical, we also speak of a tree structure.

DATA VALUES VS. DATA RELATION



FUNDAMENTAL TERMS

Data item => **Node**

Data relation => **Edge**

of Data items related to a given data item X => **Degree of X**

Numerical attributes of nodes and edges => **Weights**

Graph w/o cycles => **Tree**

Dedicated node of a tree => **Root**

Data item A (closer to the root) having an edge to B (further from the root)

=> A is the **Parent**, B is the **Child**

Nodes having the same parent => **Siblings**

Nodes having no children => **Leaves**

Average # of children in a tree => **Branching factor**



OVERVIEW

1. Node Link Diagrams

- Force-Directed Layout Mechanism & Extensions

2. Matrix Visualizations

- Matrices and Hierarchical Matrices
- Ordering Matrices

3. Implicit Tree Visualization

- TreeMap, Icicle Plot, Sunburst

4. Visualization of PO-Sets

- The Sugiyama Layout Framework

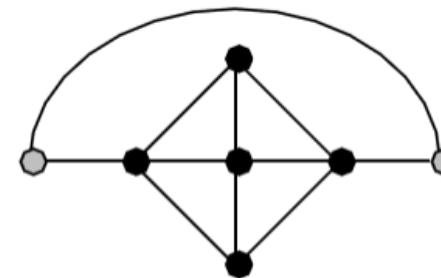
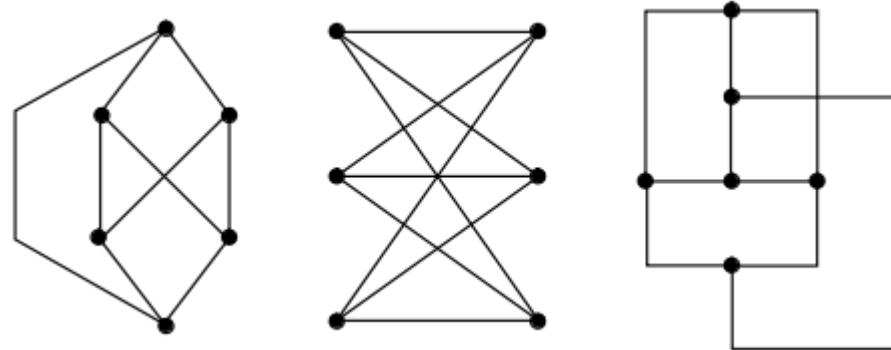


NODE-LINK DIAGRAMS

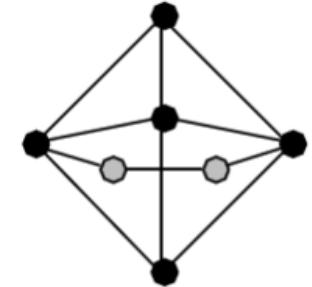


WHAT MAKES A GOOD NODE-LINK-DIAG.?

- Draw edges straight, polyline, orthogonally,...
- Minimize Edge Crossings
- Minimize Edge Bends
- Minimize Aspect Ratio
- Minimize Drawing Area
- Maximize Symmetries
- Maintain Uniform Edge Length
- ...

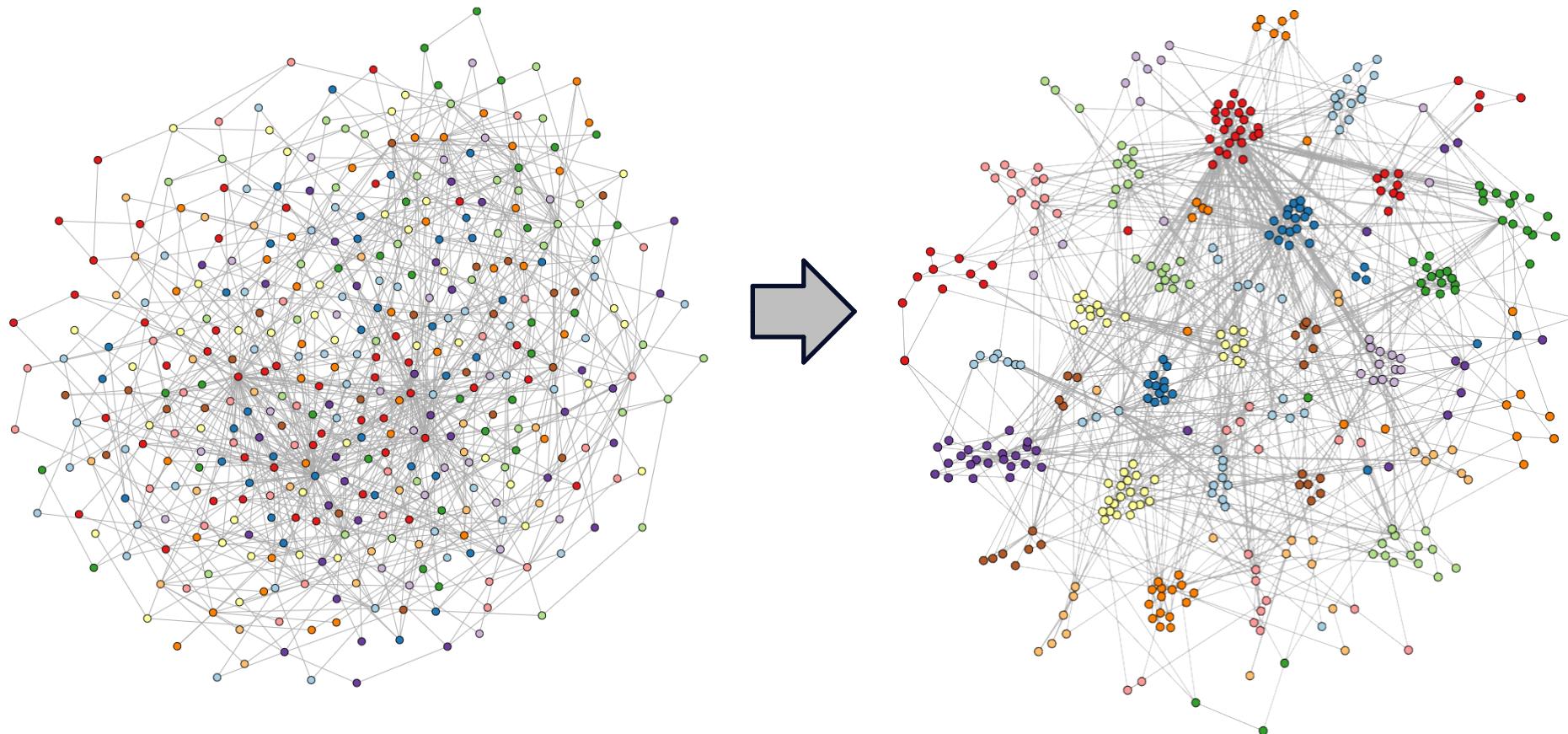


Minimum number
of edge crossings

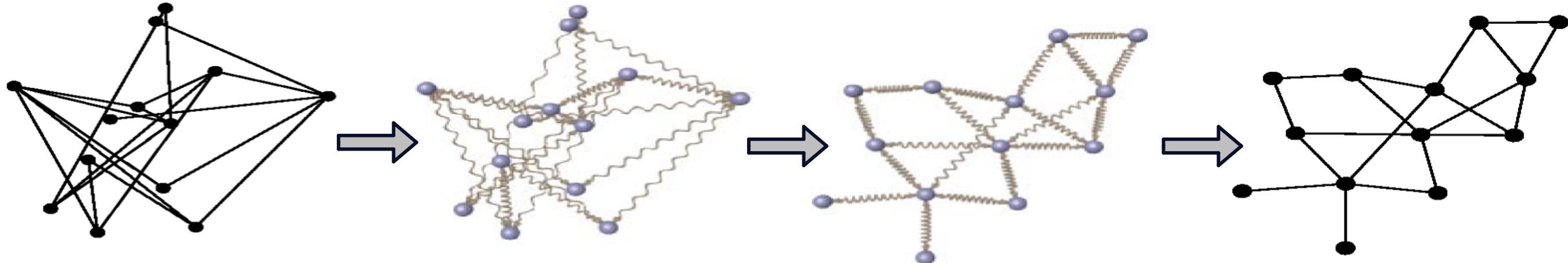


Uniform
edge length

NODE LINK LAYOUTS



FORCE-DIRECTED GRAPH LAYOUT



picture source: [Brandes 2001]

For each edge between nodes U and V:
attractive force (spring)



Hooke's Law

For each pair of nodes U and V:
repulsive force (expander)



Coulomb's Law

A FORCE-DIRECTED LAYOUT ALGORITHM

1. Initialize Node Positions (e.g., randomly or via pre-layout)

2. For a number of iterations do

3. For all node pairs (u,v) do

4. Computer Repulsive Force according to Coulomb's Law: $F_C = k_r(Q_1 * Q_2 / r^2)$

5. Move u and v apart from each other by a distance F_C

repulsion
constant

"charges"
of u and v

distance
between
 u and v

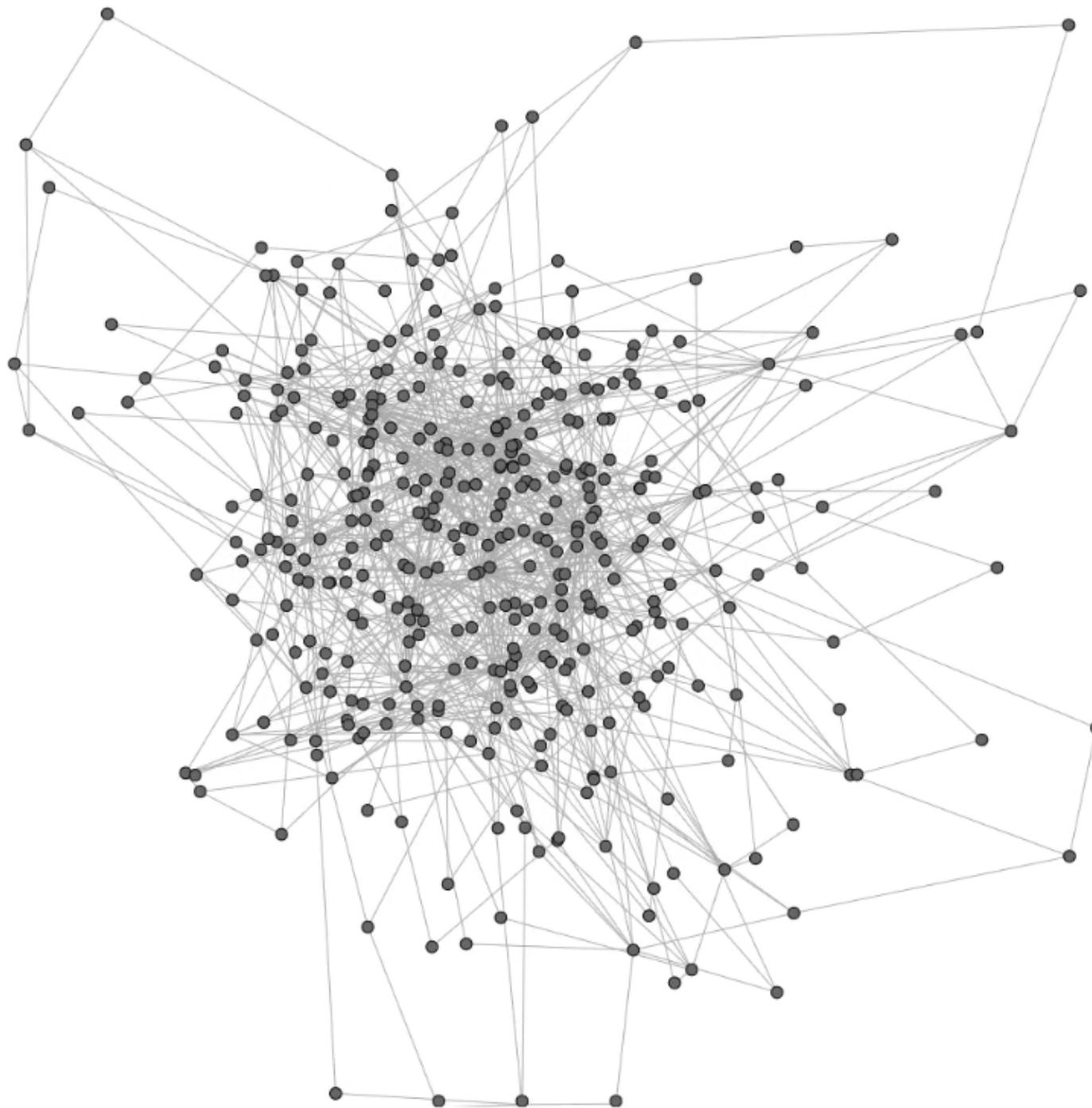
6. For all edges (u,v) do

7. Compute Attracting Force according to Hooke's Law: $F_H = -k_a * x$

spring
constant

difference
between the
distance (u,v)
and the length
of the spring

8. Move u and v closer to each other by F_H



...ADD SIMULATED ANNEALING TO THE MIX

1. Initialize Node Positions (e.g., randomly or via pre-layout)

Initialize velocity = 1

2. **While velocity > 0.0001 do**

3. For all node pairs (u,v) do

4. Computer Repulsive Force according to Coulomb's Law: $F_C = k_r(Q_1 * Q_2 / r^2)$

5. Move u and v apart from each other by a distance $F_C * \text{velocity}$

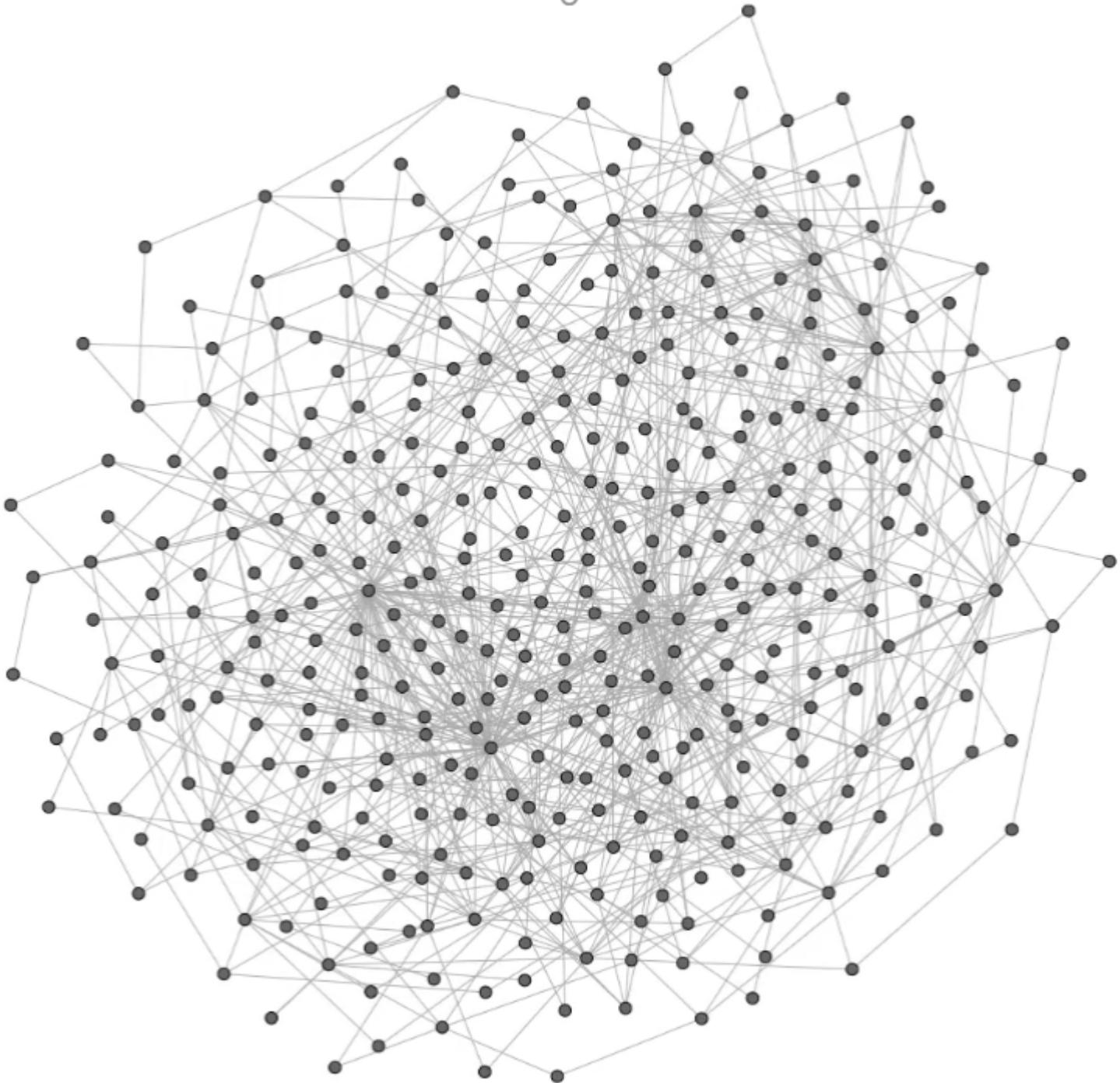
6. For all edges (u,v) do

7. Compute Attracting Force according to Hooke's Law: $F_H = -k_a * x$

8. Move u and v closer to each other by $F_H * \text{velocity}$

9. Velocity = velocity * 0.95

cooling rate



...ADD MORE FORCES INTO THE MIX

Given a pre-clustered graph =>

Pull nodes of the same cluster closer together:

For **all clusters (c)** in the graph do

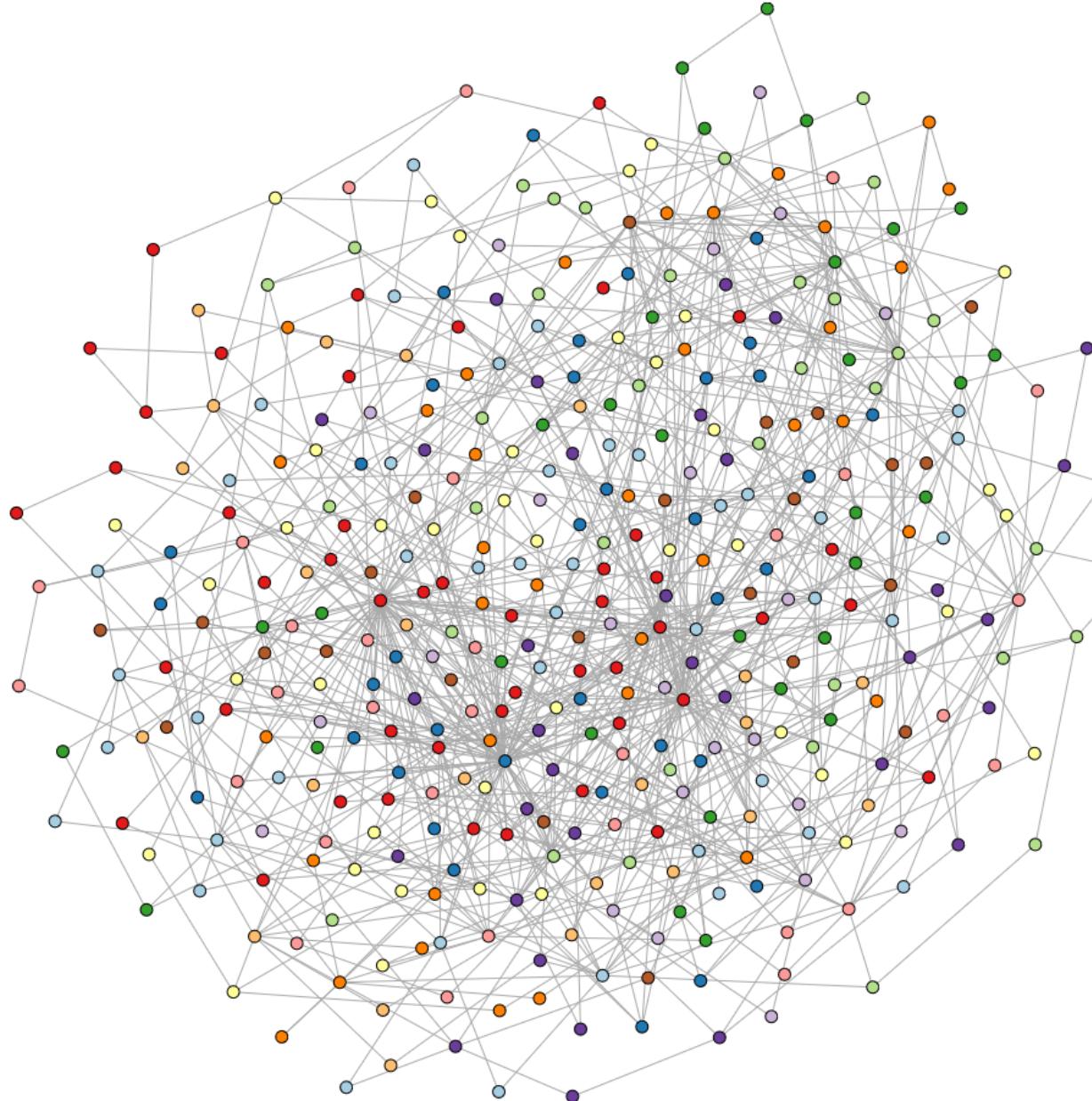
 For **all nodes u** in c do

 For **all nodes v** in c do

If $u \neq v$ do

 Compute additional attracting force $F_c = -k_c * x$

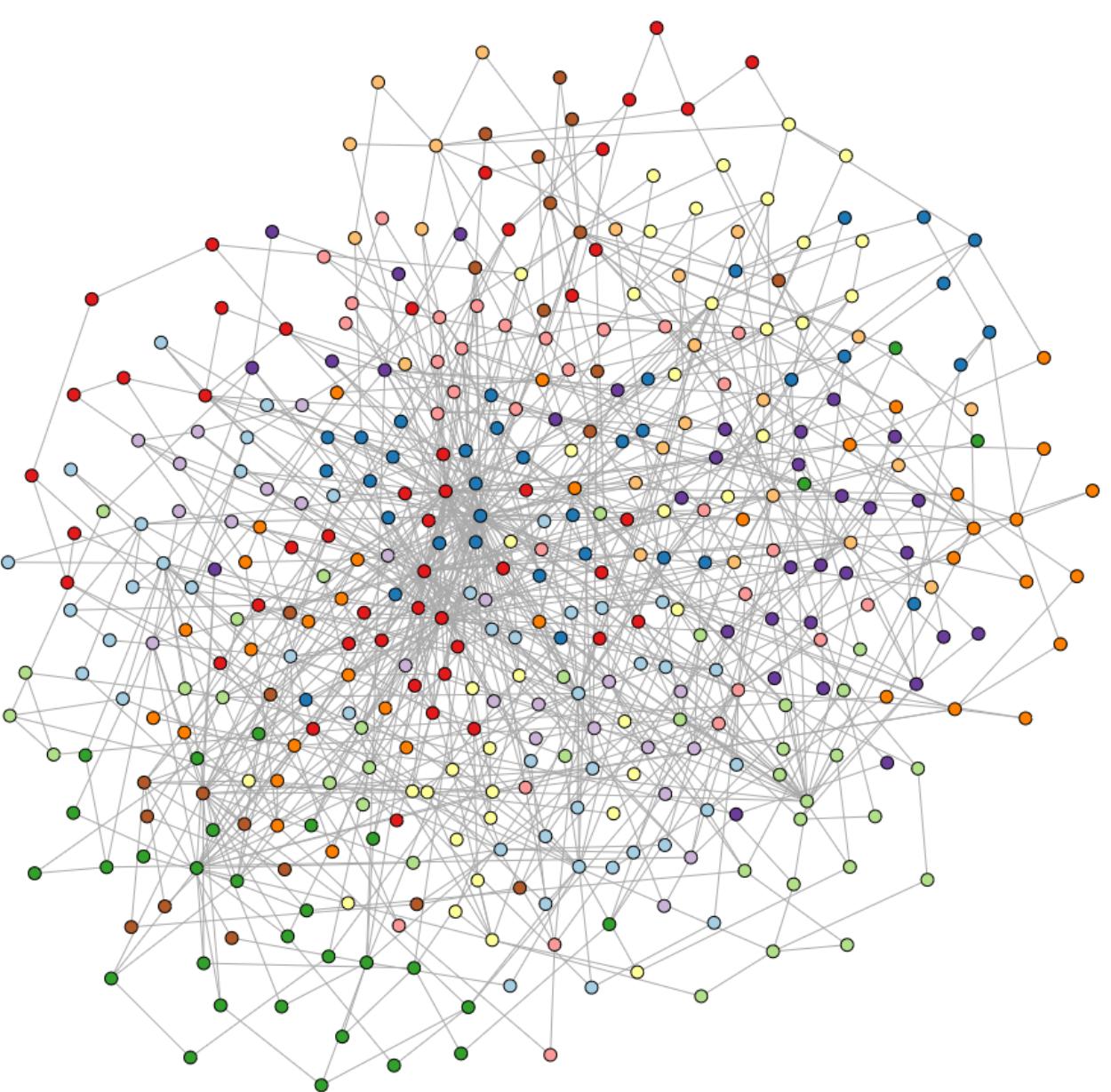
 Move u and v even closer to each other by $F_c * \text{velocity}$



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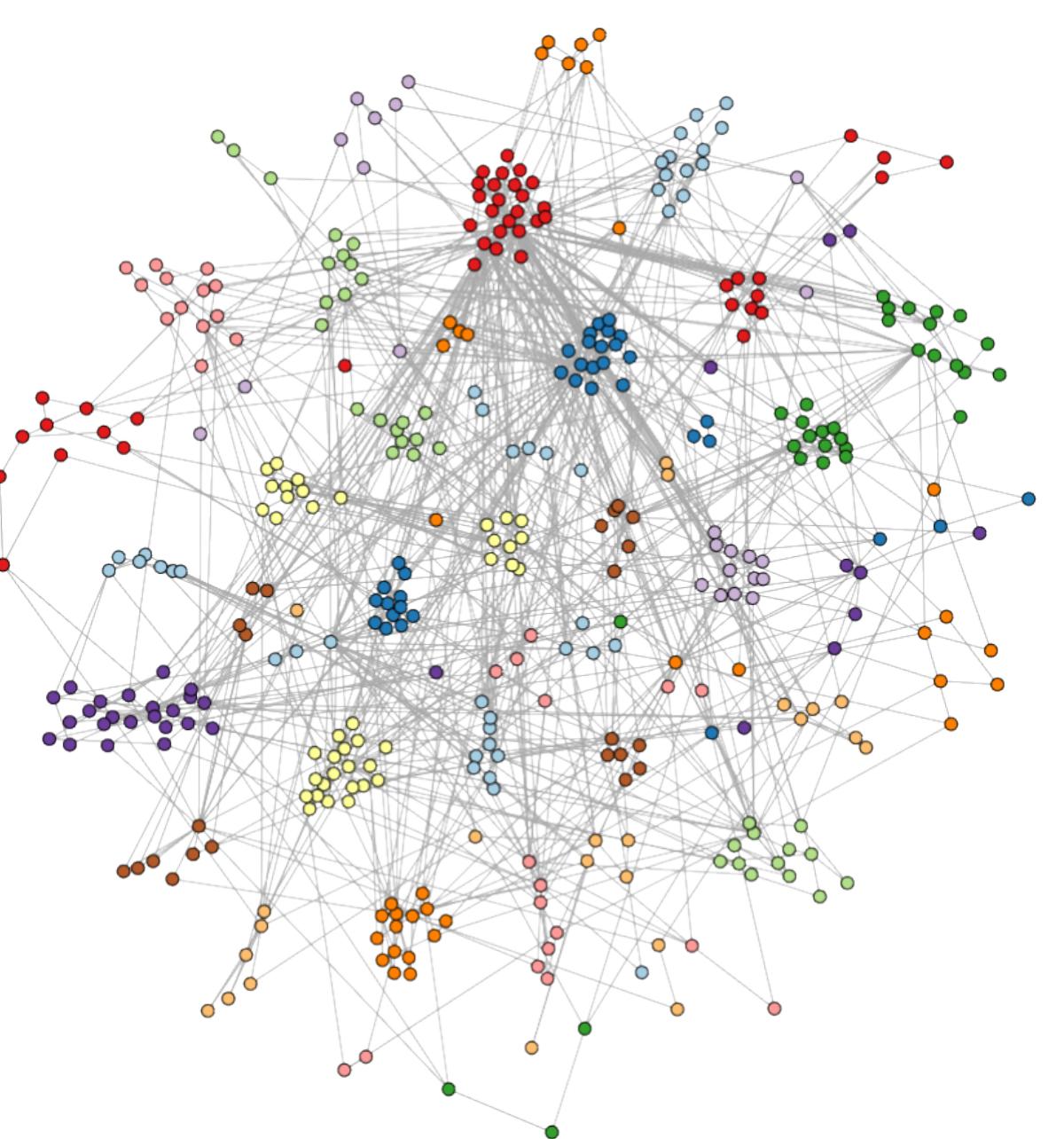
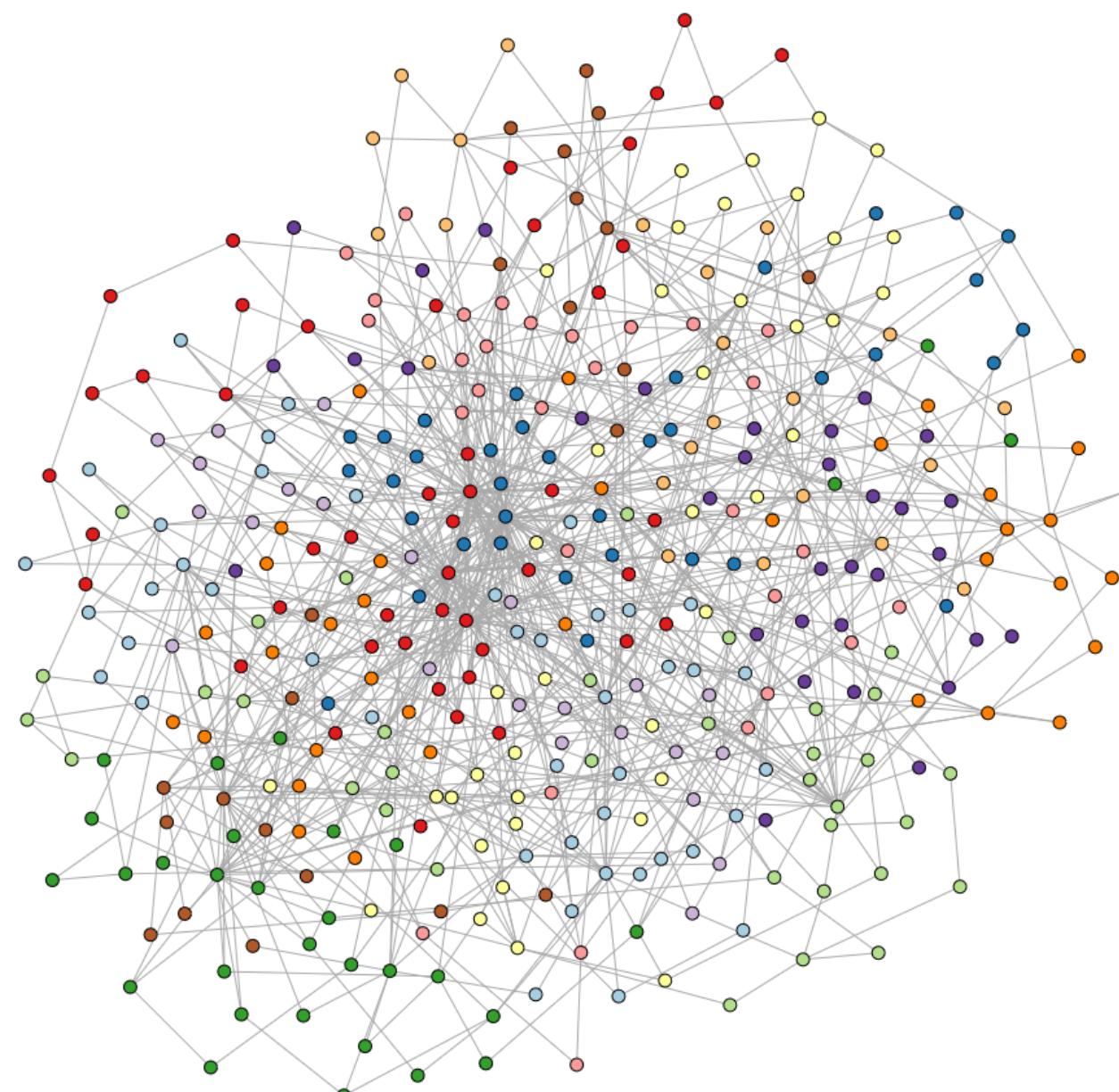


...AND ADJUST OTHER FORCES

Given a pre-clustered graph =>

Push nodes of different cluster further apart:

3. For all node pairs (u,v) do
4. Compute Repulsive Force according to Coulomb's Law: $F_C = k_r(Q_1 * Q_2 / r^2)$
5. **If $\text{cluster}(u) == \text{cluster}(v)$ do**
 6. Move u and v apart from each other by a distance $F_C * \text{velocity}$
- Else**
 7. Move u and v apart from each other by a distance $F_C * 10 * \text{velocity}$



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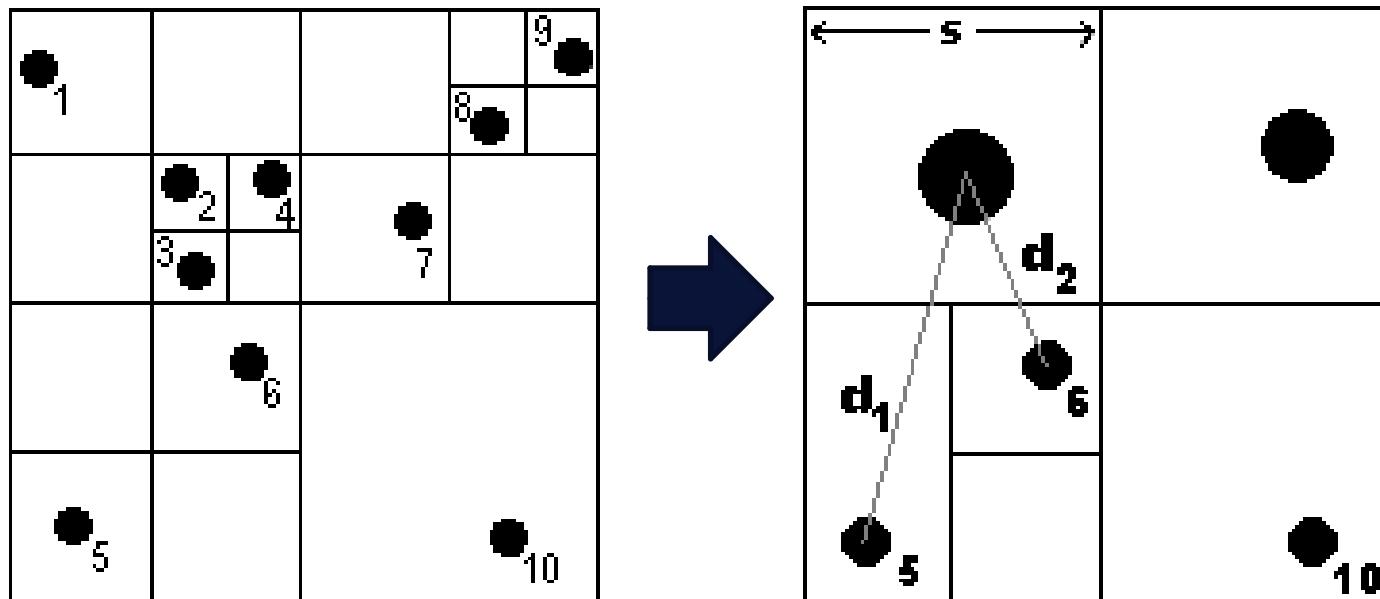
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ADDON: BARNES-HUT OPTIMIZATION

Speed-up by reducing the number of comparisons

-> use a quadtree data structure to approximate forces from further-away nodes in bulk [Barnes & Hut 1986, Quigley & Eades 2000]



picture source: [Quigley et al. 2000]

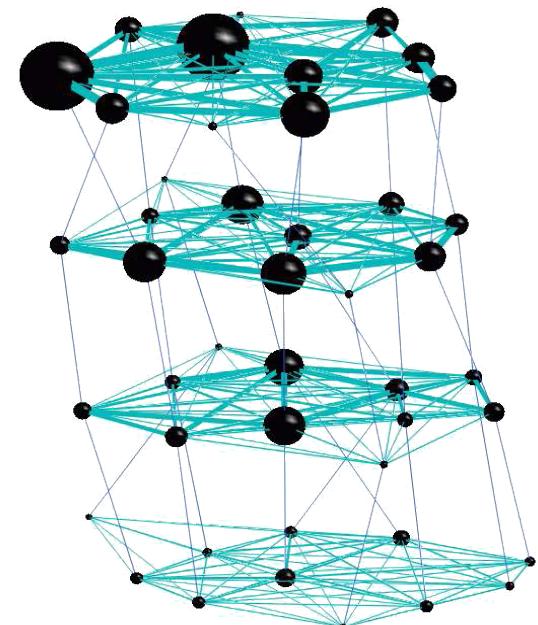


ADDON: STIFFNESS FOR STREAMING GRAPHS

Different approaches ensure layout stiffness to the displacement of nodes:

Inter-Timeslice Edges (TGRIP) [Erten et al. 2004]

-> nodes that existed before have an added attracting force to previous position



Pinning [Tal, Frishman 2007]

-> node's velocity \sim shortest path to last change

Aging [Gorochowski et al. 2012]

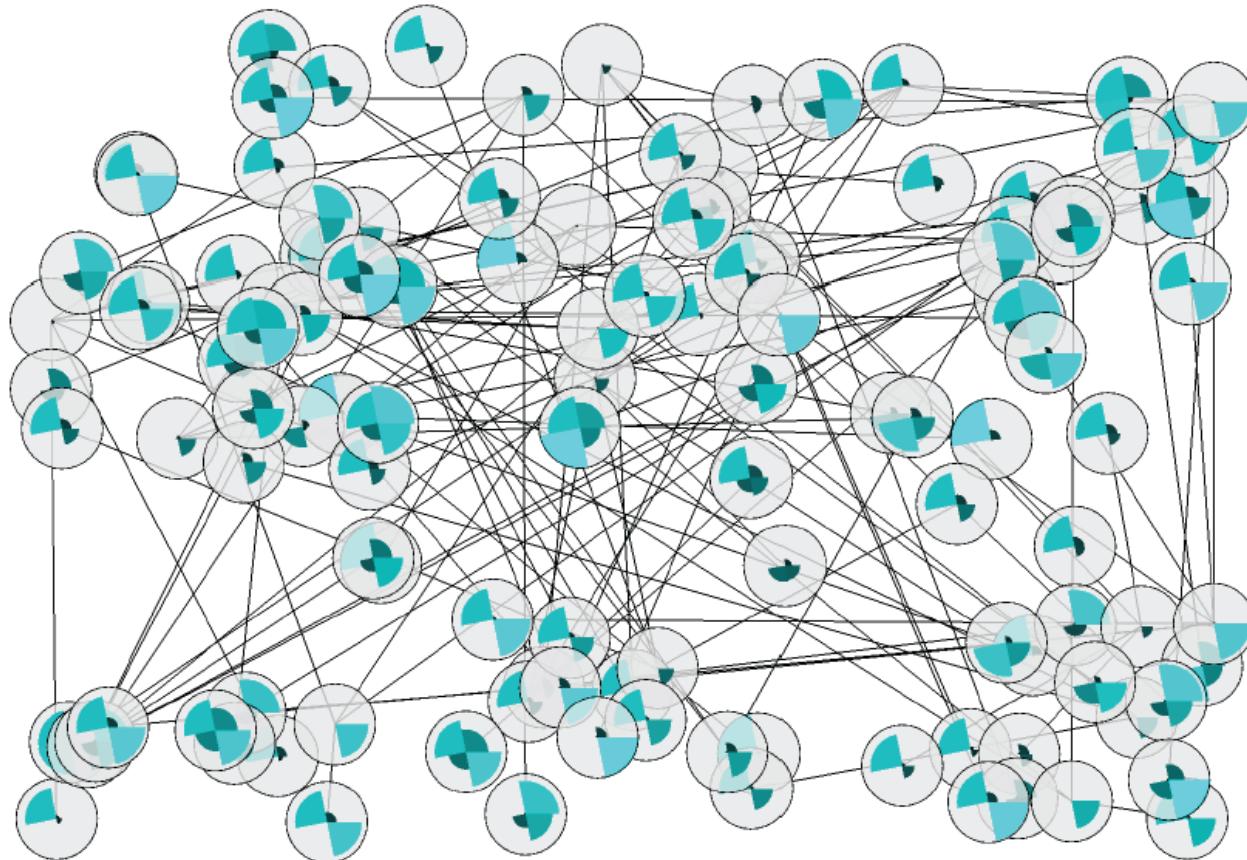
-> node's velocity \sim time steps since it was inserted

[Erten et al 2004]



VISUALIZING NODE ATTRIBUTES

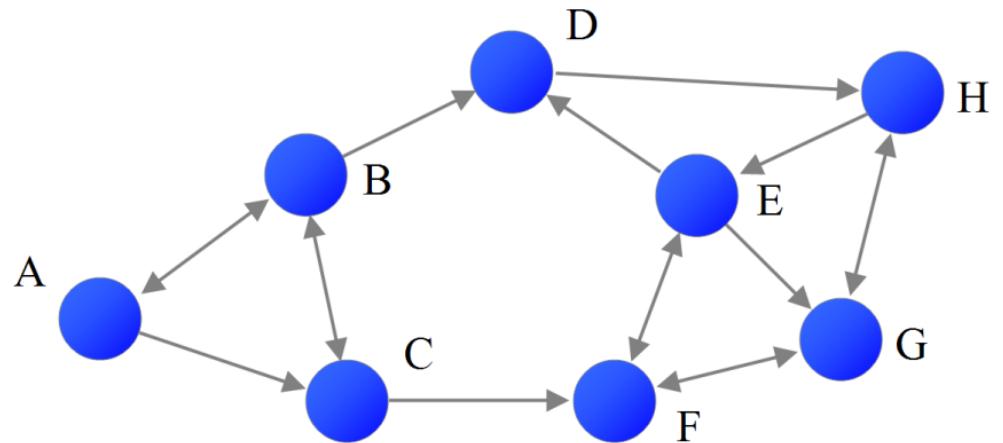
Using Glyphs for more complex Node Data:



MATRIX REPRESENTATIONS



MATRIX VISUALIZATION

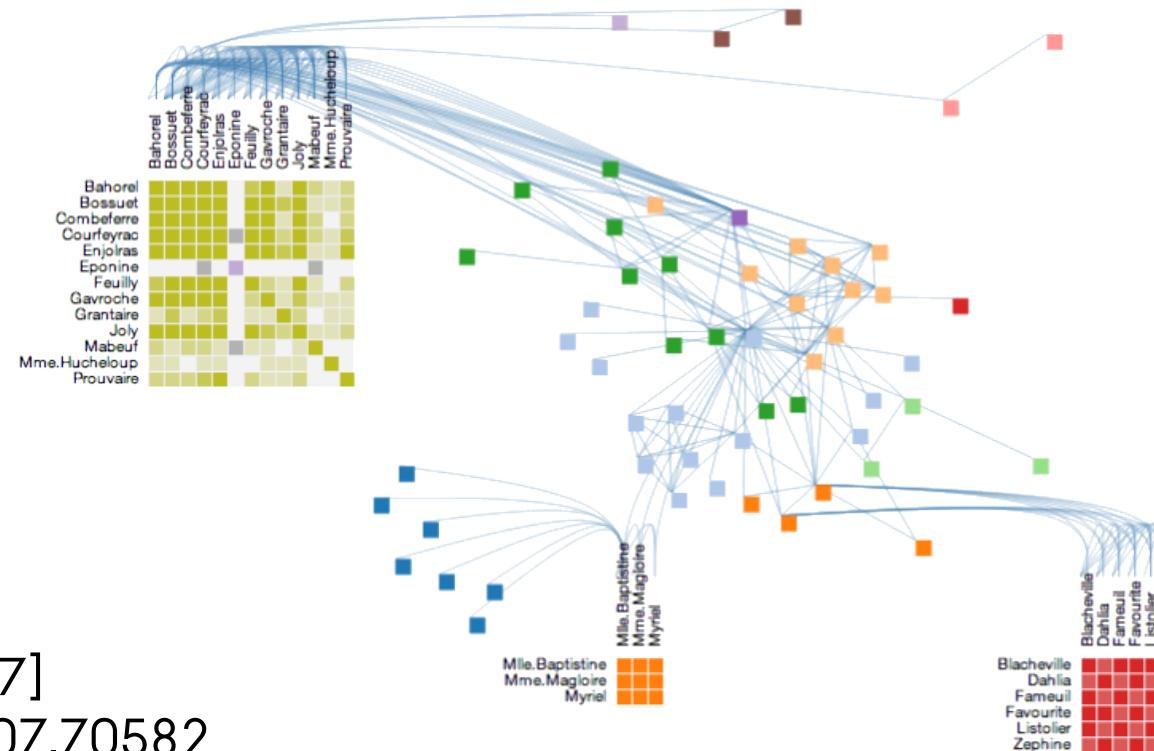


		TO							
		A	B	C	D	E	F	G	H
FROM	A								
	B								
	C								
	D								
	E								
	F								
	G								
	H								



WHY USE MATRIX VISUALIZATIONS?

1) Compact representation of dense subgraphs



[Henry et al. 2007]

DOI: 10.1109/TVCG.2007.70582



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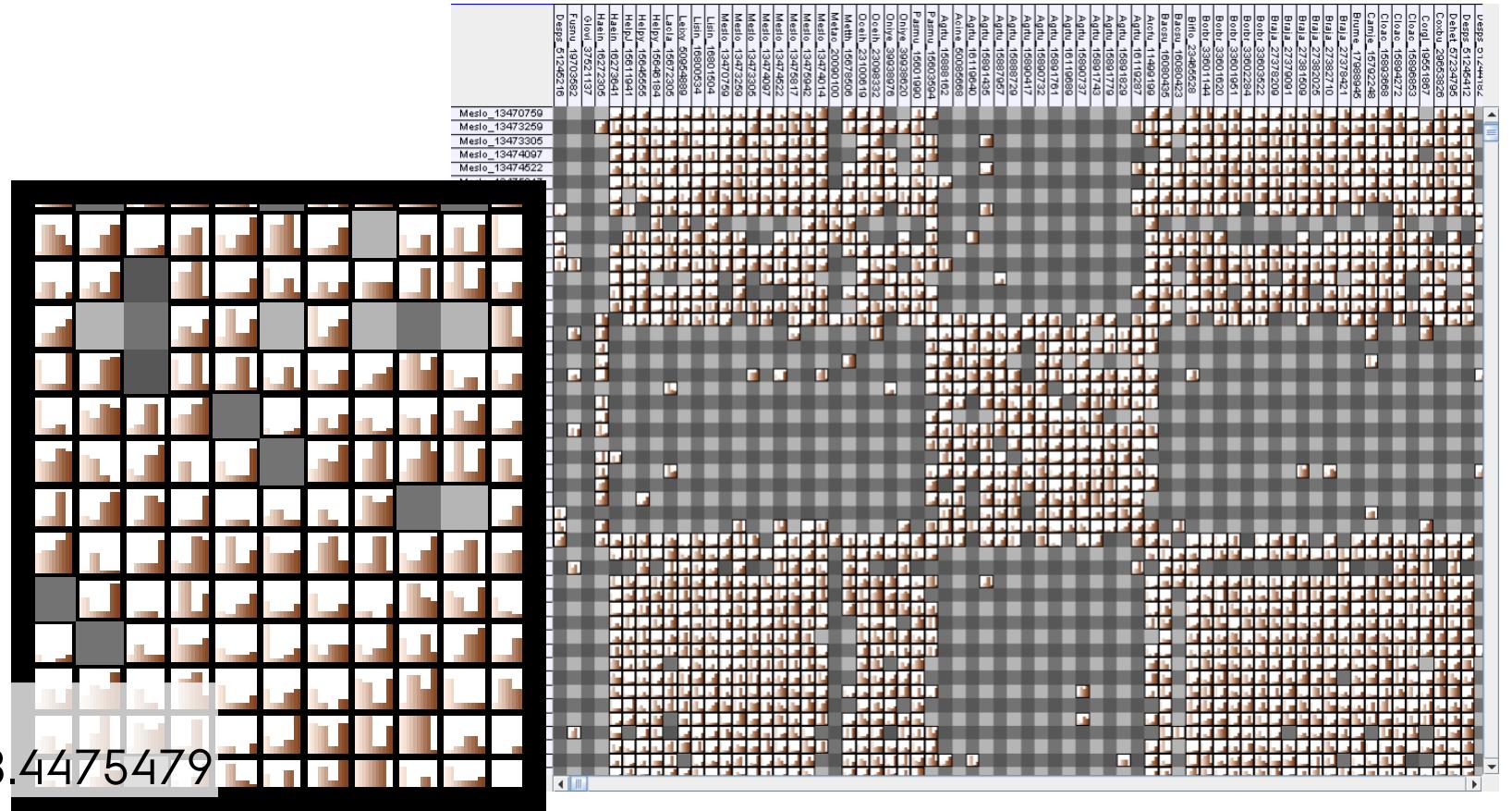
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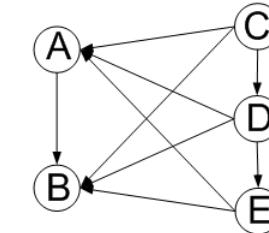
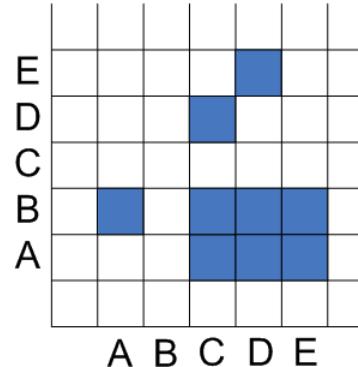
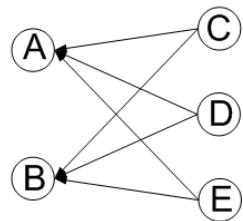
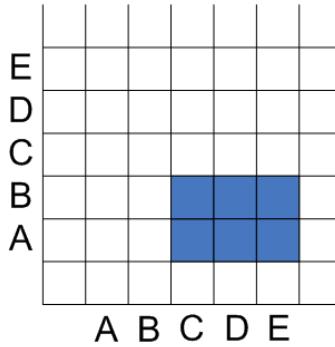
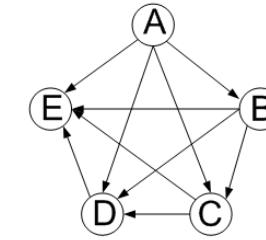
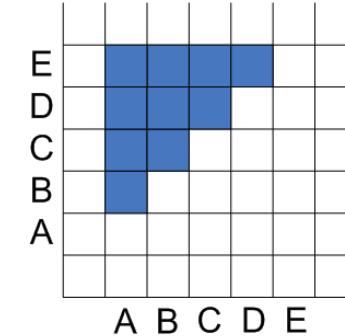
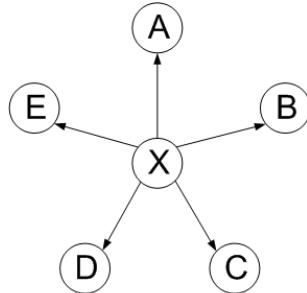
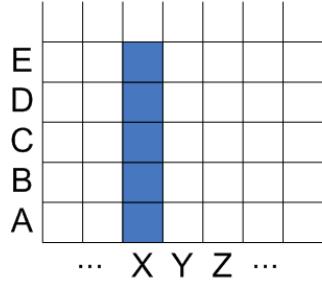
WHY USE MATRIX VISUALIZATIONS?

2) For edge-centric displays



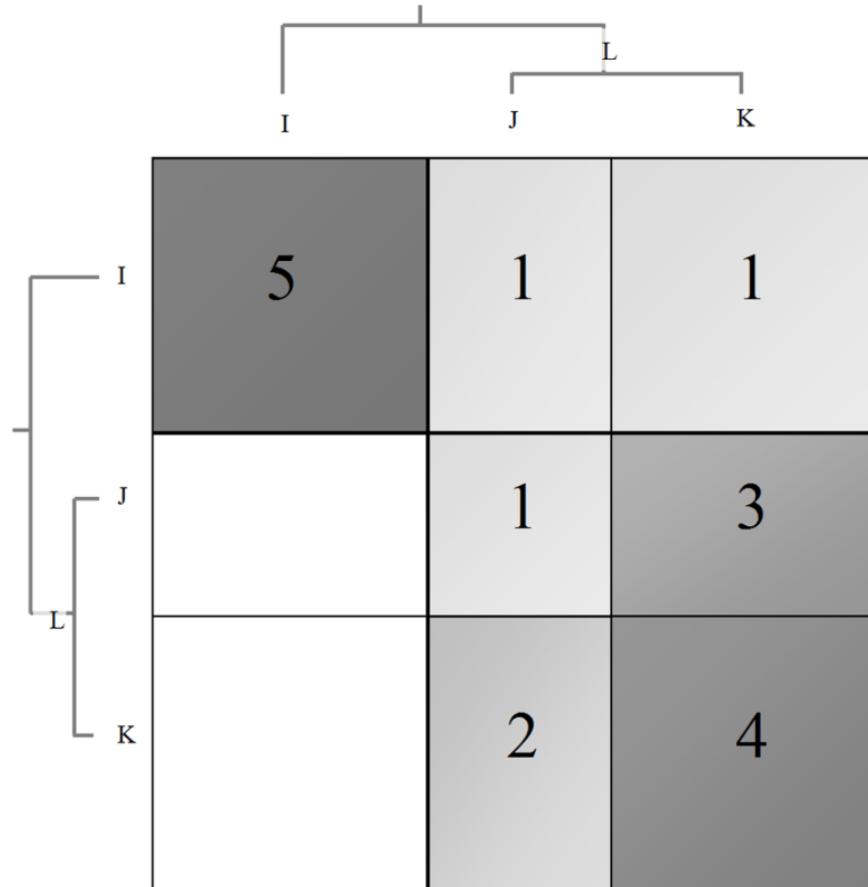
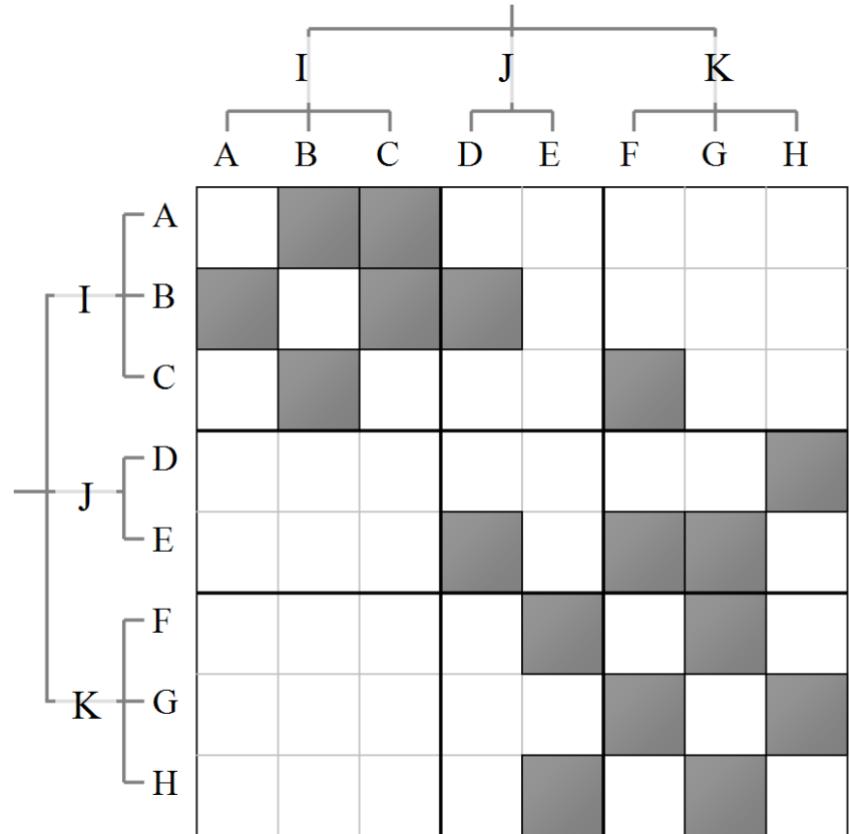
MATRIX VISUALIZATION

Examples [Shen & Ma 2007]:



HIERARCHICAL MATRIX VISUALIZATIONS

[v.Ham et al 2009]



EXPECTATION OF CONNECTING EDGES

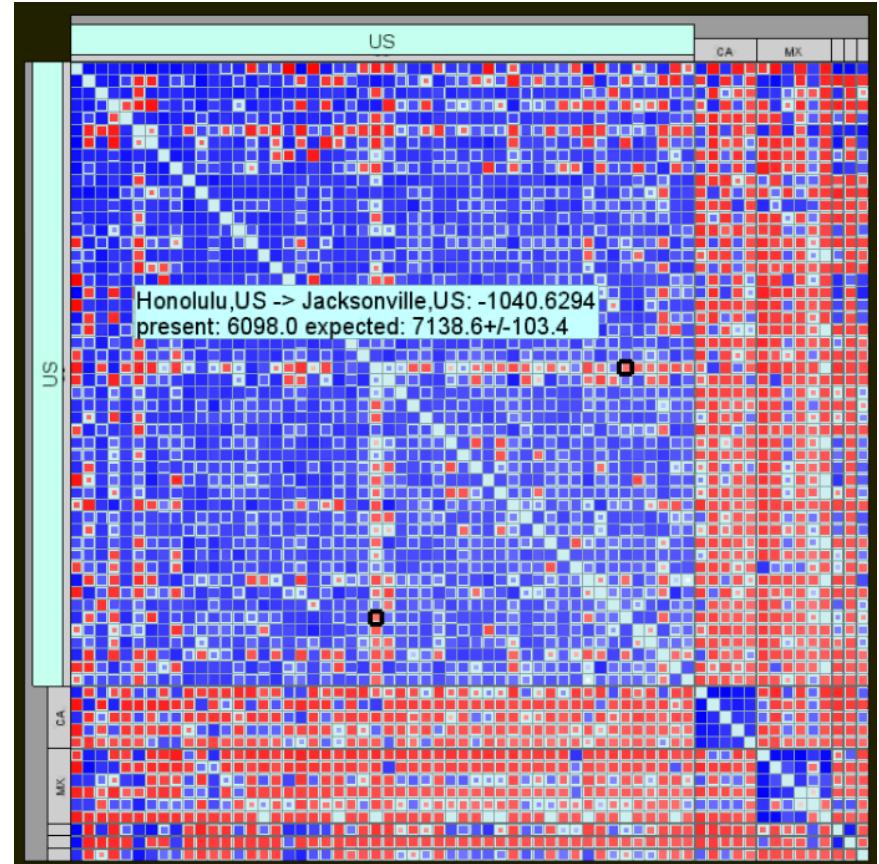
[v.Ham et al 2009]

$$P(X, Y) = \frac{X_{out}}{E} * \frac{Y_{in}}{E}$$

Probability of edges
starting in X and
ending in Y

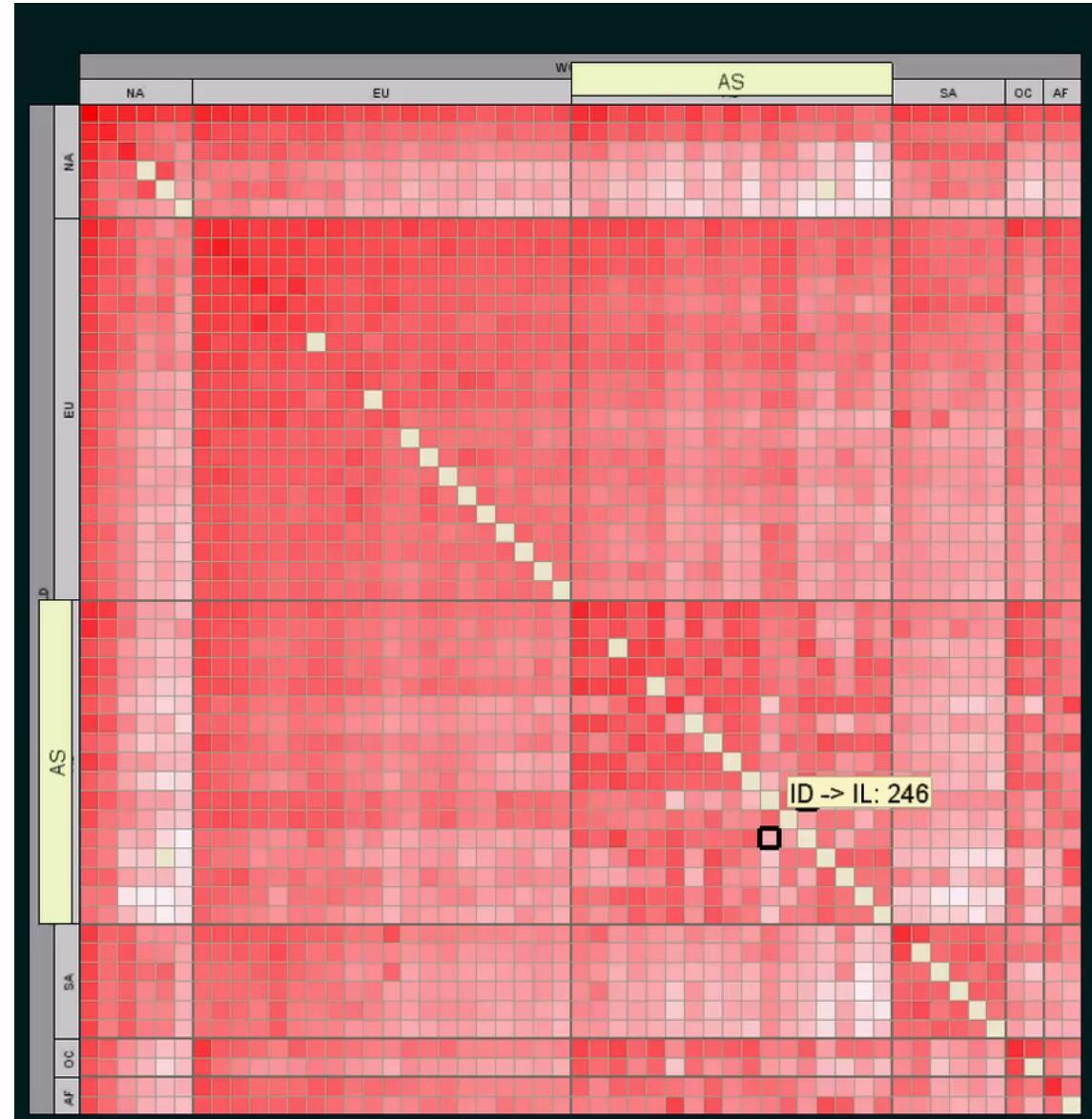
$$EXP(X, Y) = E * P(X, Y) = \frac{X_{out} \cdot Y_{in}}{E}$$

Expectation of the number of edges
between X and Y out of all edges E



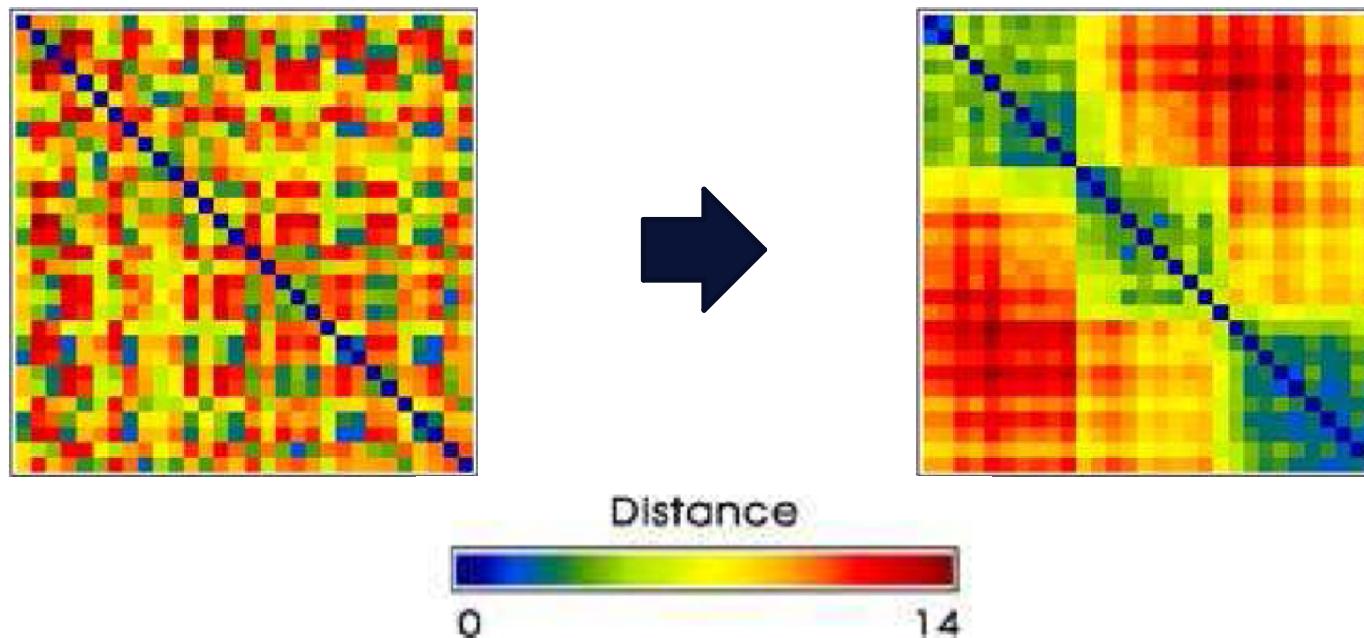
NAVIGATING HIERARCHICAL MATRICES

[v.Ham et al 2009]



ORDERING A MATRIX REPRESENTATION (1)

Aim: a linear order of rows and columns that makes the topological patterns clear(er)

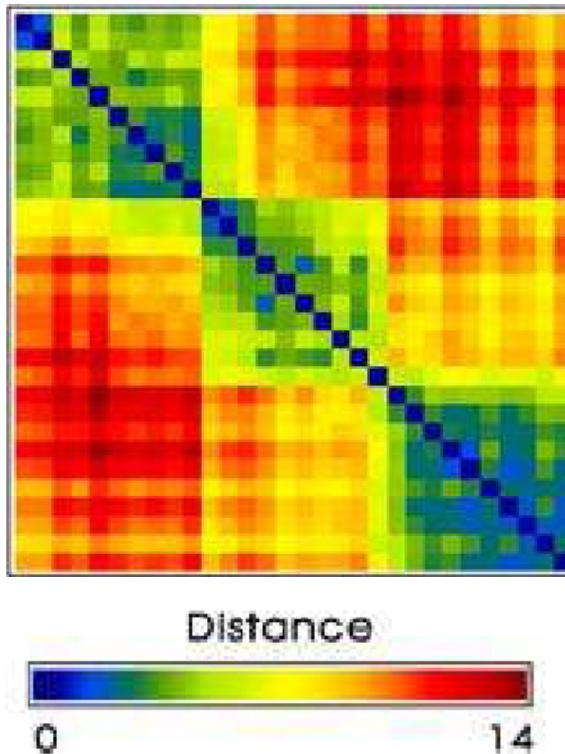


picture source: [Wu et al. 2008]



ORDERING A MATRIX REPRESENTATION (2)

Transform the matrix into a *Robinson Matrix*



Desired form: (near-)Robinson Matrix

“A MATRIX IS CALLED ROBINSON IF THE ELEMENTS IN ITS ROWS AND COLUMNS DO NOT INCREASE WHEN MOVING AWAY HORIZONTALLY OR VERTICALLY FROM ITS MAIN DIAGONAL.” (from N.Henry 2008)

There are multiple ways of computing this transformation.

picture source: [Wu et al. 2008]



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ORDERING A MATRIX REPRESENTATION (3)

Solution: Make it into an optimization problem

Possible cost functions [Wu et al. 2008]:

$$AR(i) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) + \sum_{i < j < k} I(d_{ij} > d_{ik}) \right]$$

$$AR(s) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) \cdot |d_{ij} - d_{ik}| + \sum_{i < j < k} I(d_{ij} > d_{ik}) \cdot |d_{ij} - d_{ik}| \right]$$

$$AR(w) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) |j - k| |d_{ij} - d_{ik}| + \sum_{i < j < k} I(d_{ij} > d_{ik}) |j - k| |d_{ij} - d_{ik}| \right]$$

of all Robinson violations

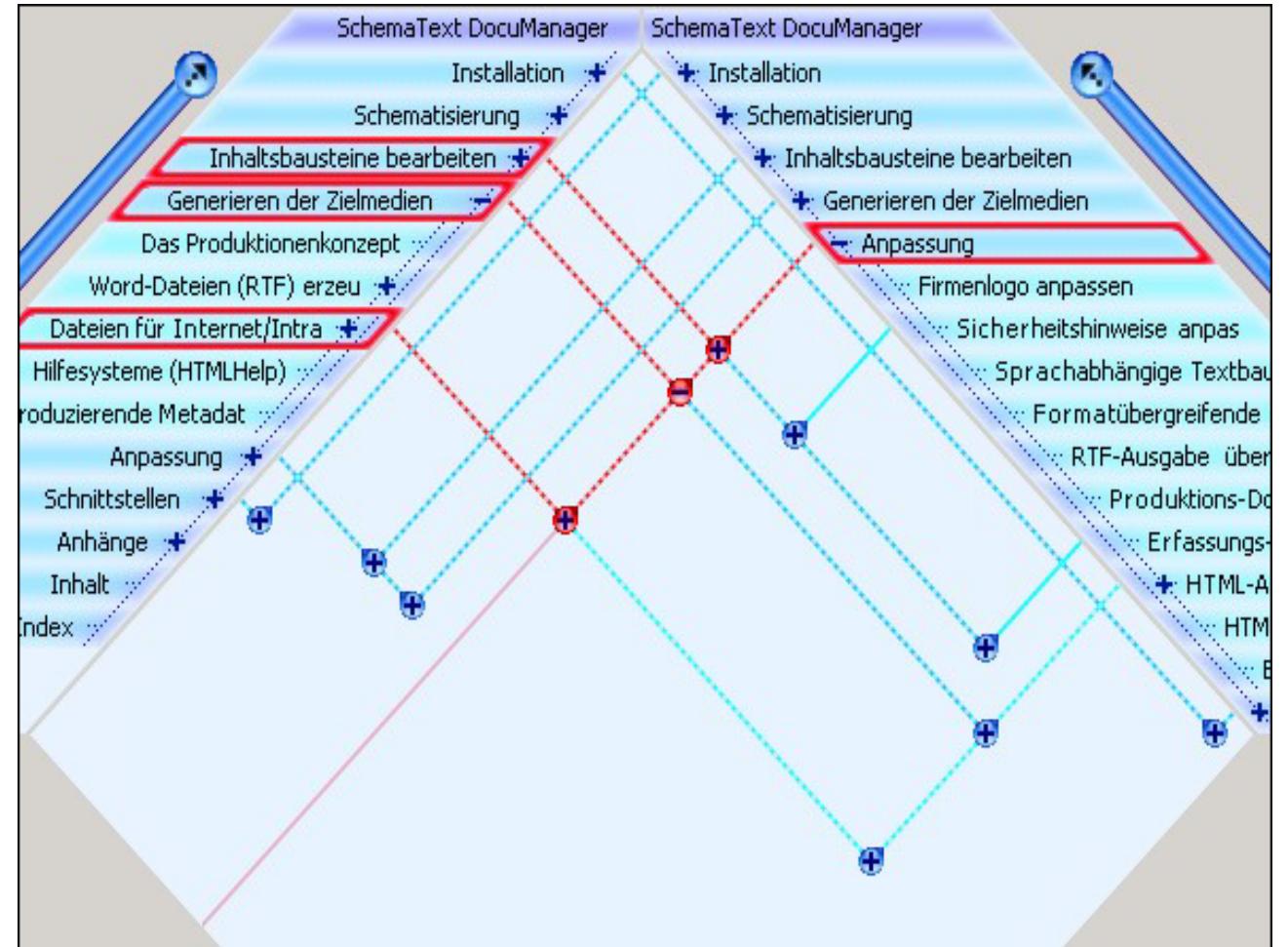
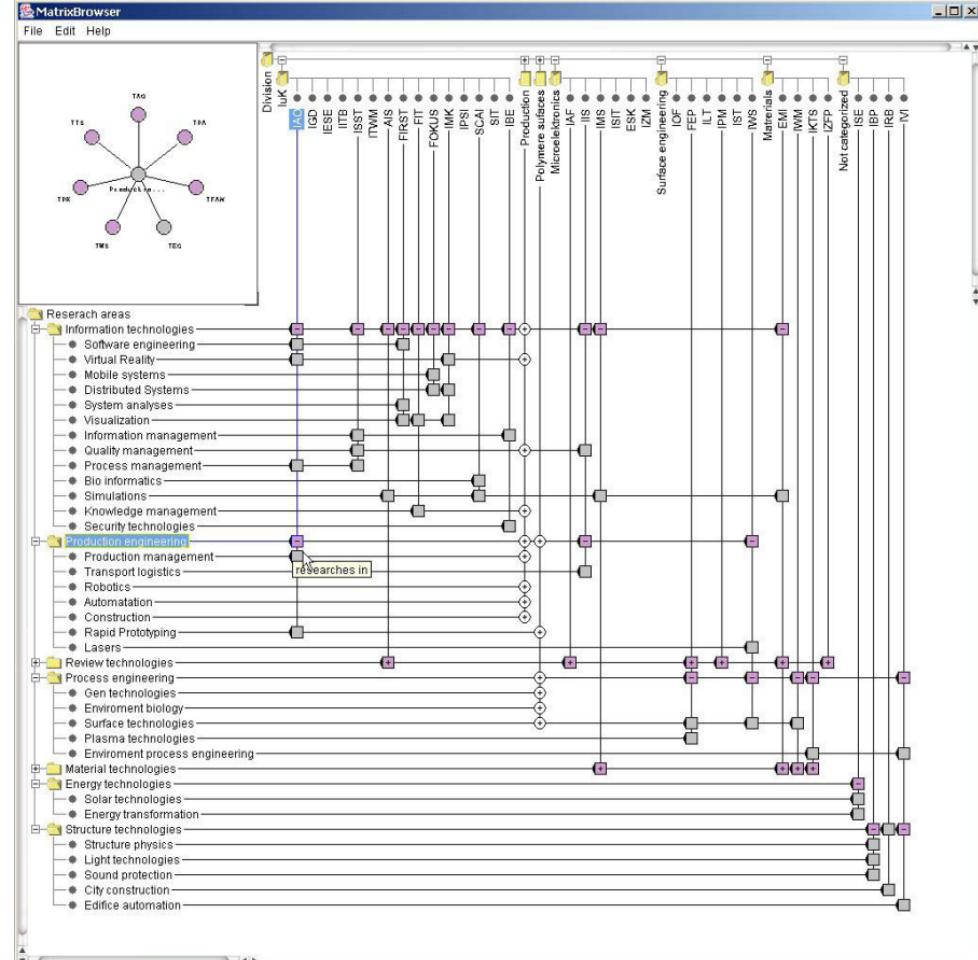
Σ of the deviations of all Robinson violations

Σ of the weighted deviations of all Robinson violations
(weight = index difference)



NODE-LINK MATRICES

picture source: [Beinhauer & Ziegler 2003]



IMPLICIT TREE VISUALIZATIONS



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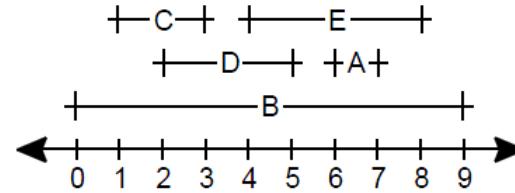
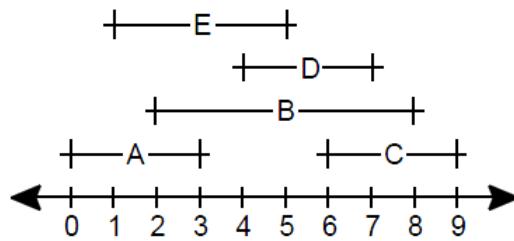
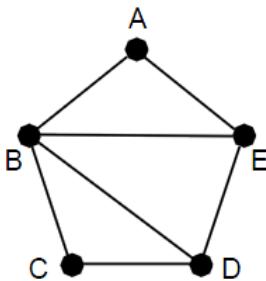
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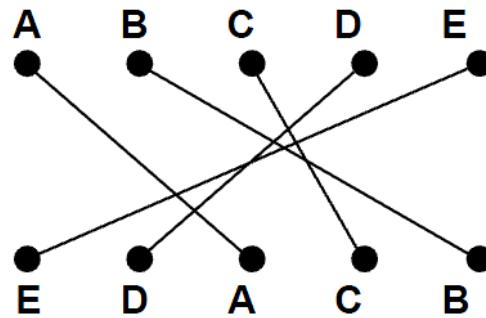
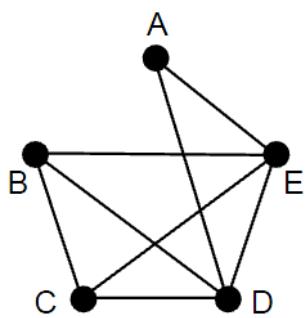


MAIN IDEA BEHIND IMPLICIT LAYOUTS

Representation of the edges through meaningful positioning of the nodes



Interval
Representation

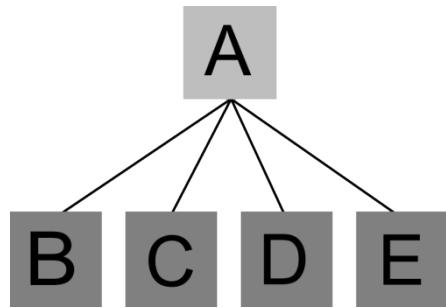


Permutation
Diagram

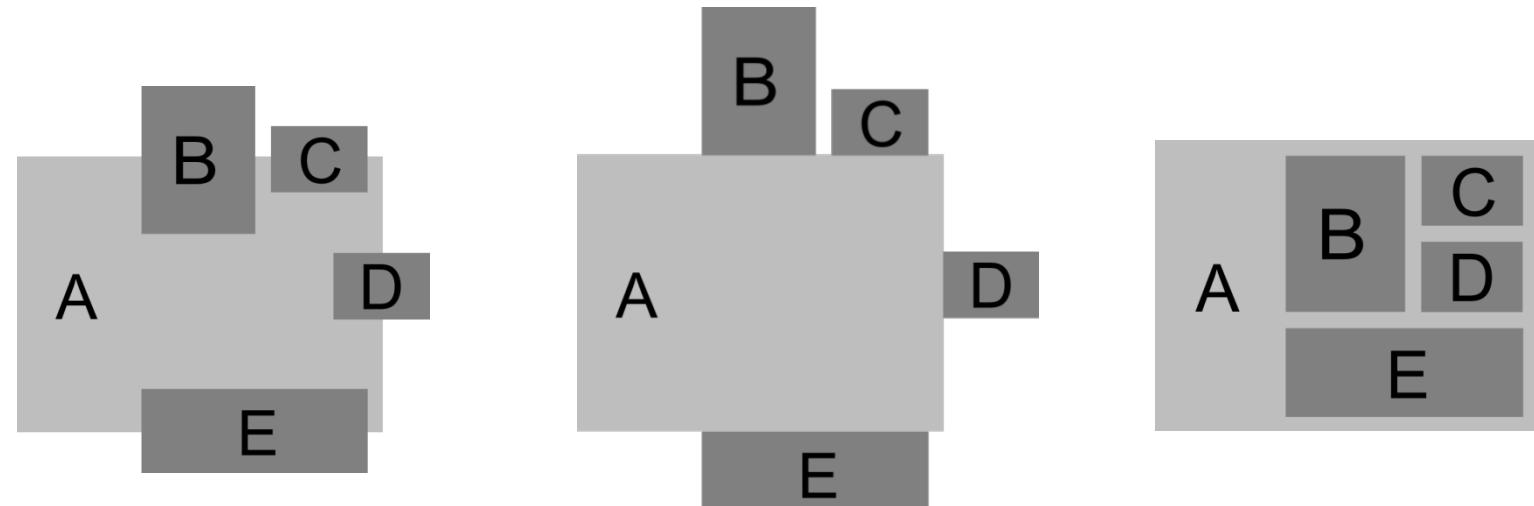


...SPECIFICALLY FOR TREES

Explicit = Node-Link-Diagram



Implicit = Positional Hierarchy

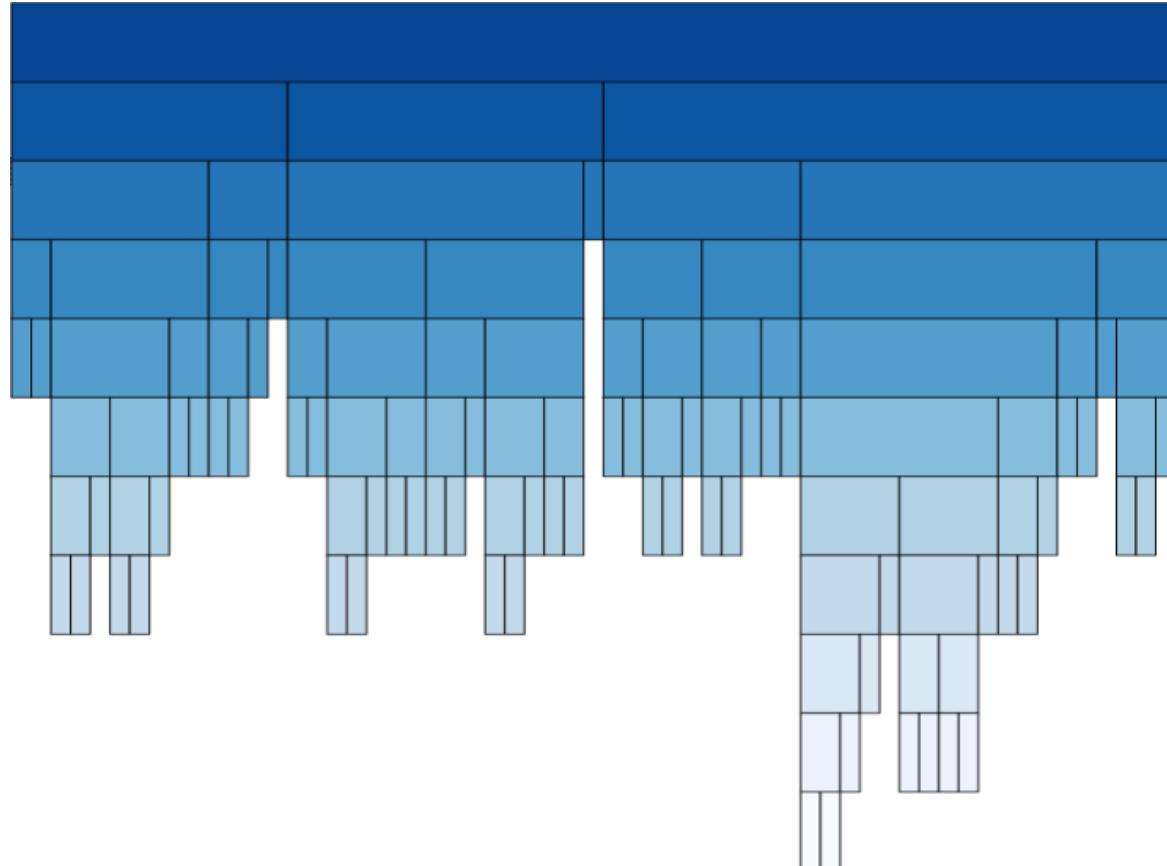


Overlap

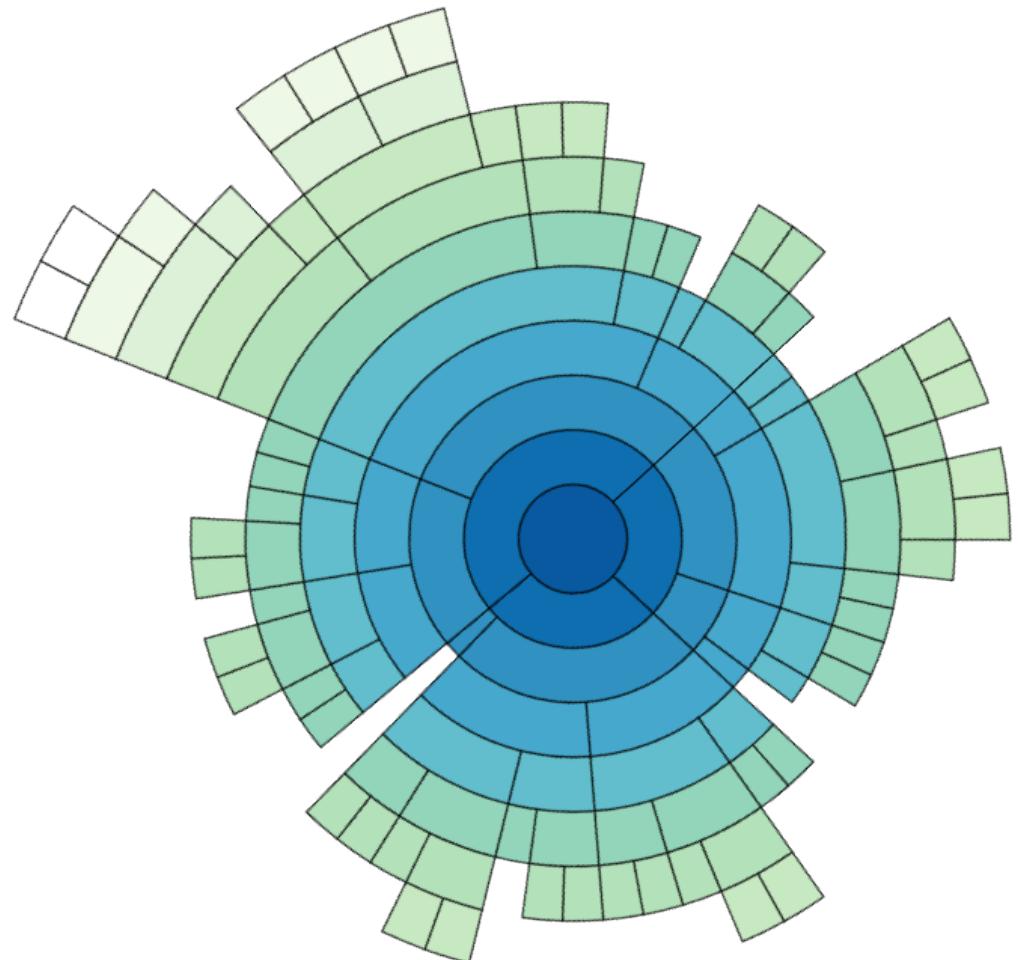
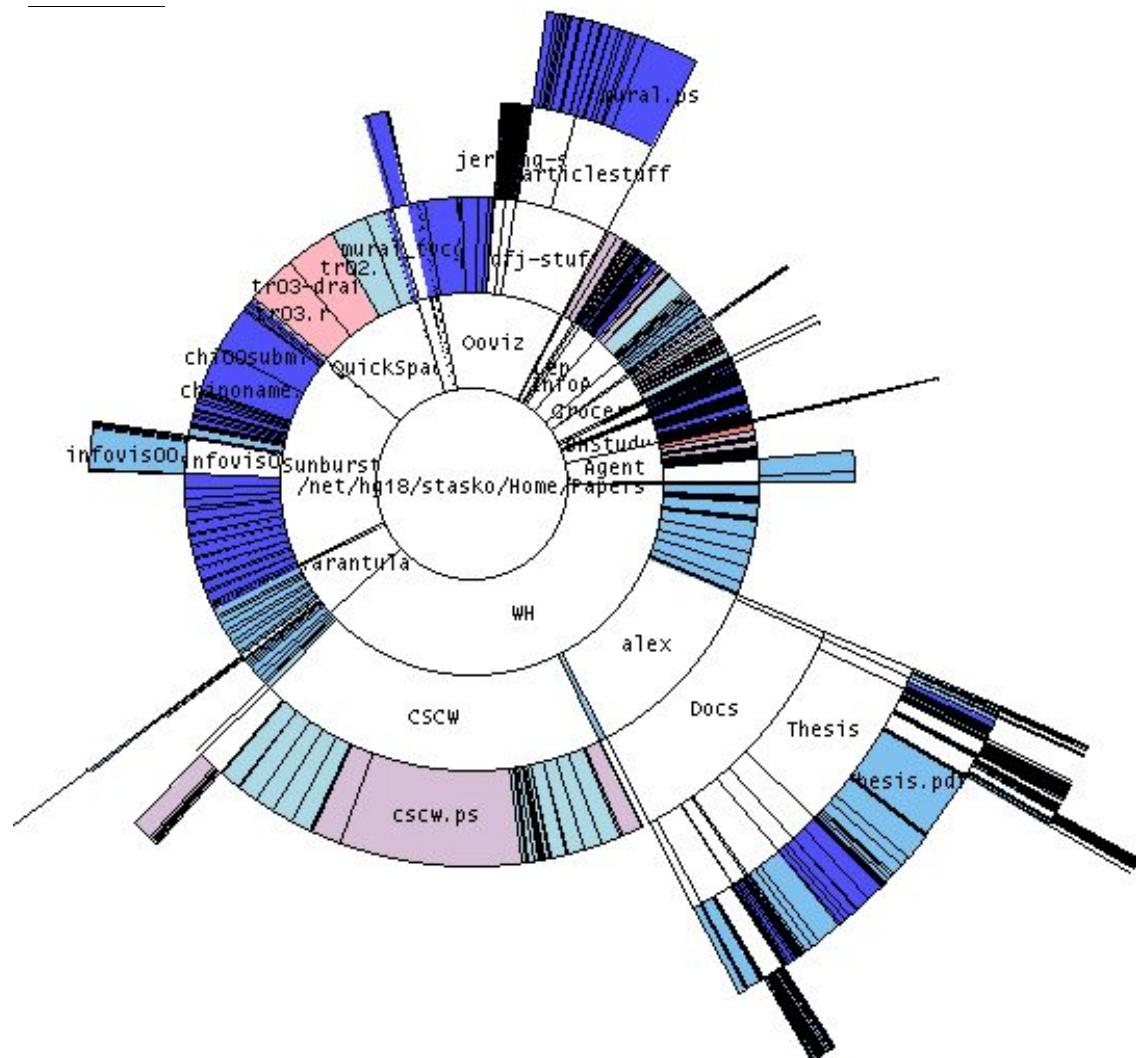
Adjacency

Inclusion

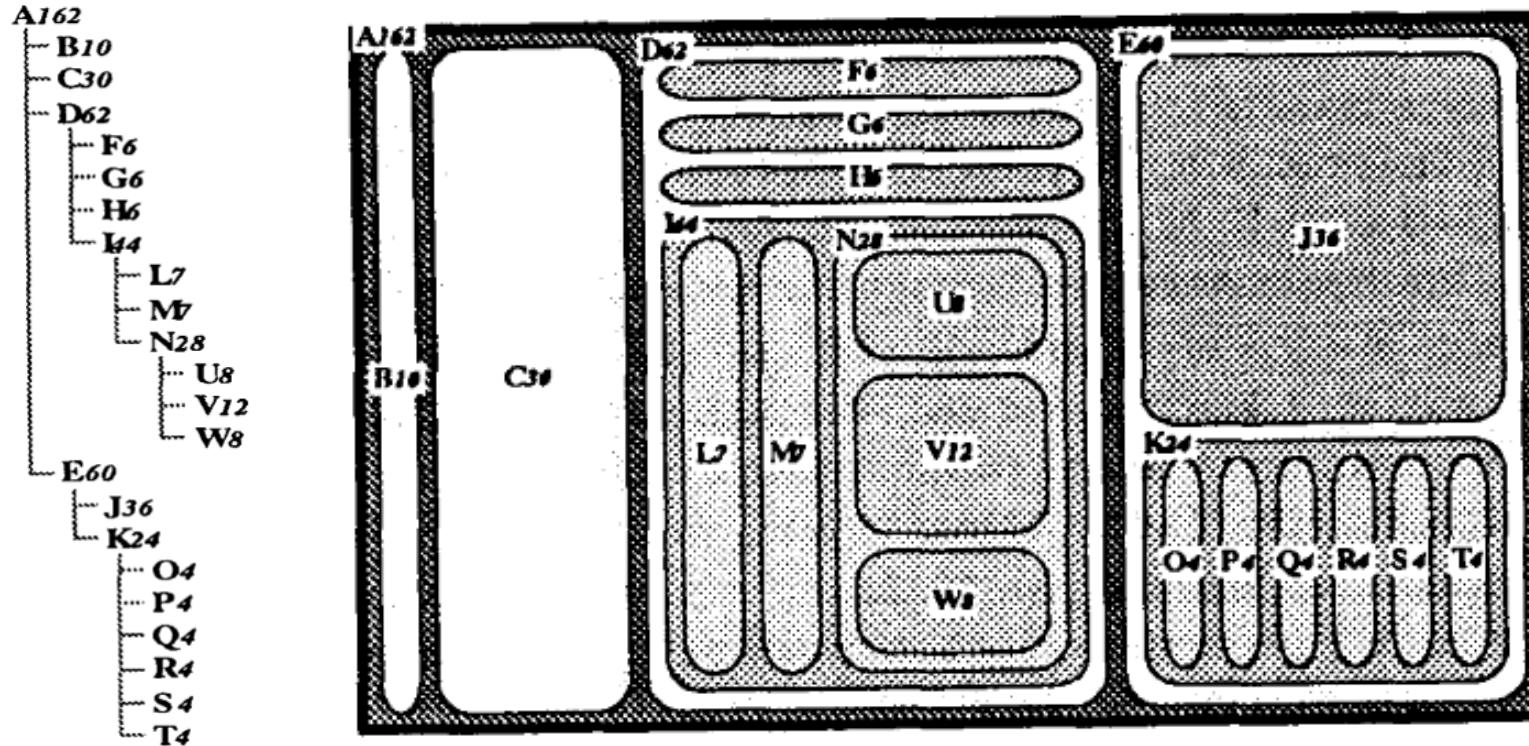
ADJACENCY: ICICLE PLOT (1983)



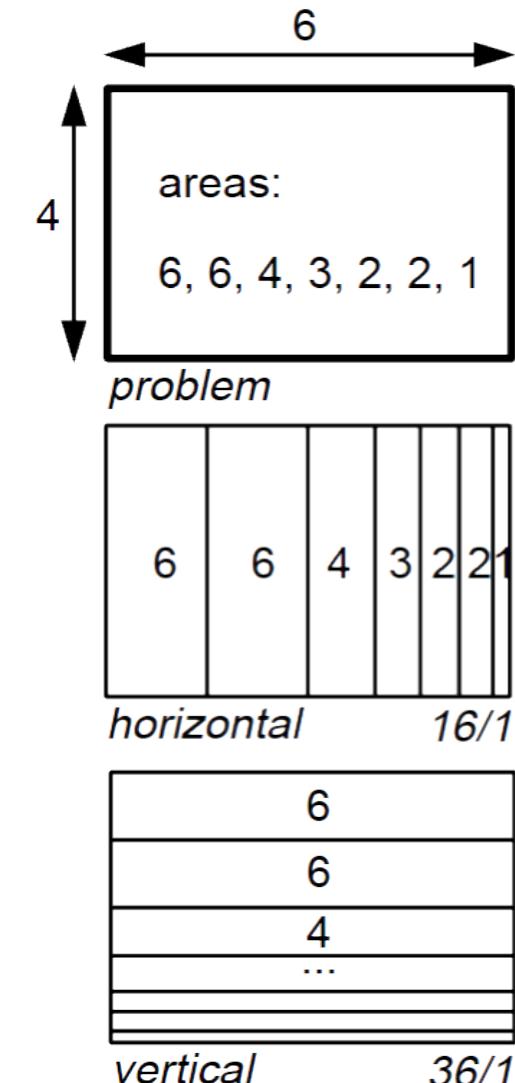
ADJACENCY: SUNBURST (2000)



INCLUSION: TREEMAP (SLICE & DICE)



picture source: [Johnson & Shneiderman 1991]

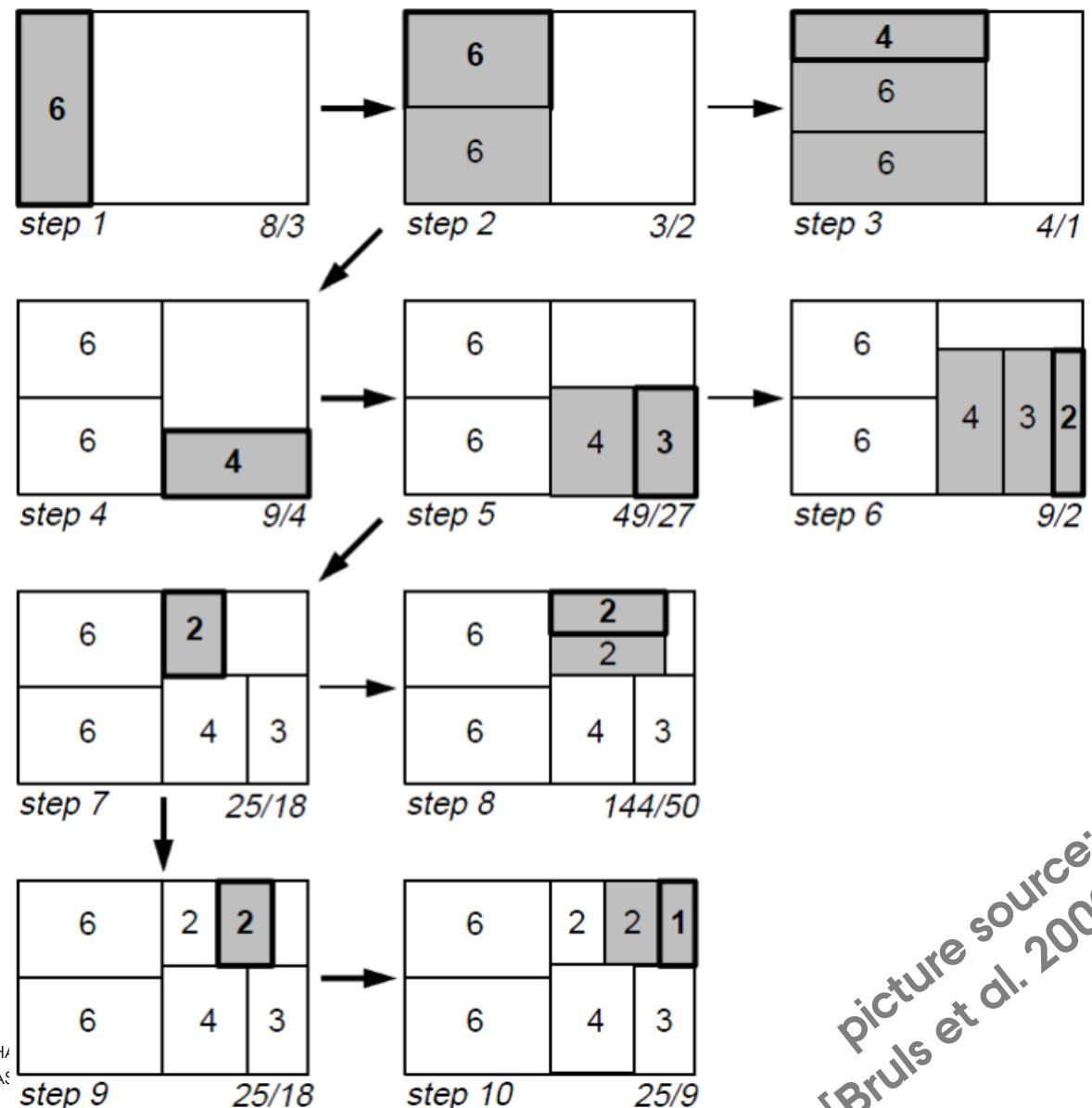
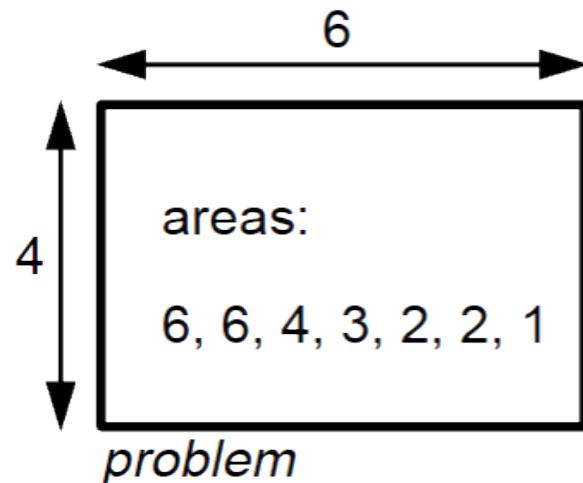


picture source: [Bruls et al. 2000]

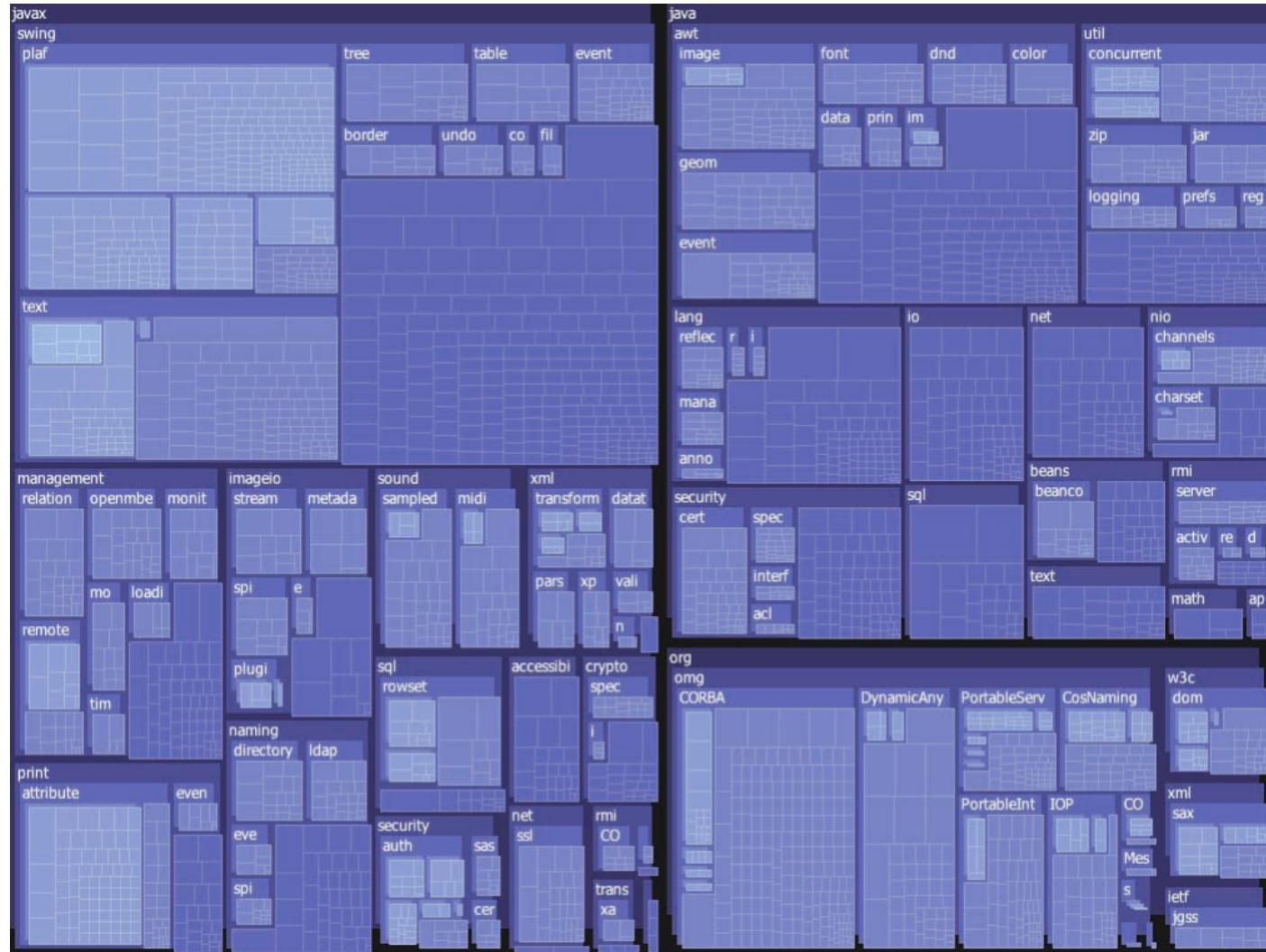


INCLUSION: TREEMAP (SQUARIFIED)

Idea: choose greedily which axis to add a region to -> always the one that yields the better aspect ratio
(start with the largest regions)



OVERLAP: CASCADED TREEMAPS (2008)



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treevis.net - A Visual Bibliography of Tree Visualization 2.0 by Hans-Jörg Schulz



v.10-APR-2019

Dimensionality



Representation



Alignment

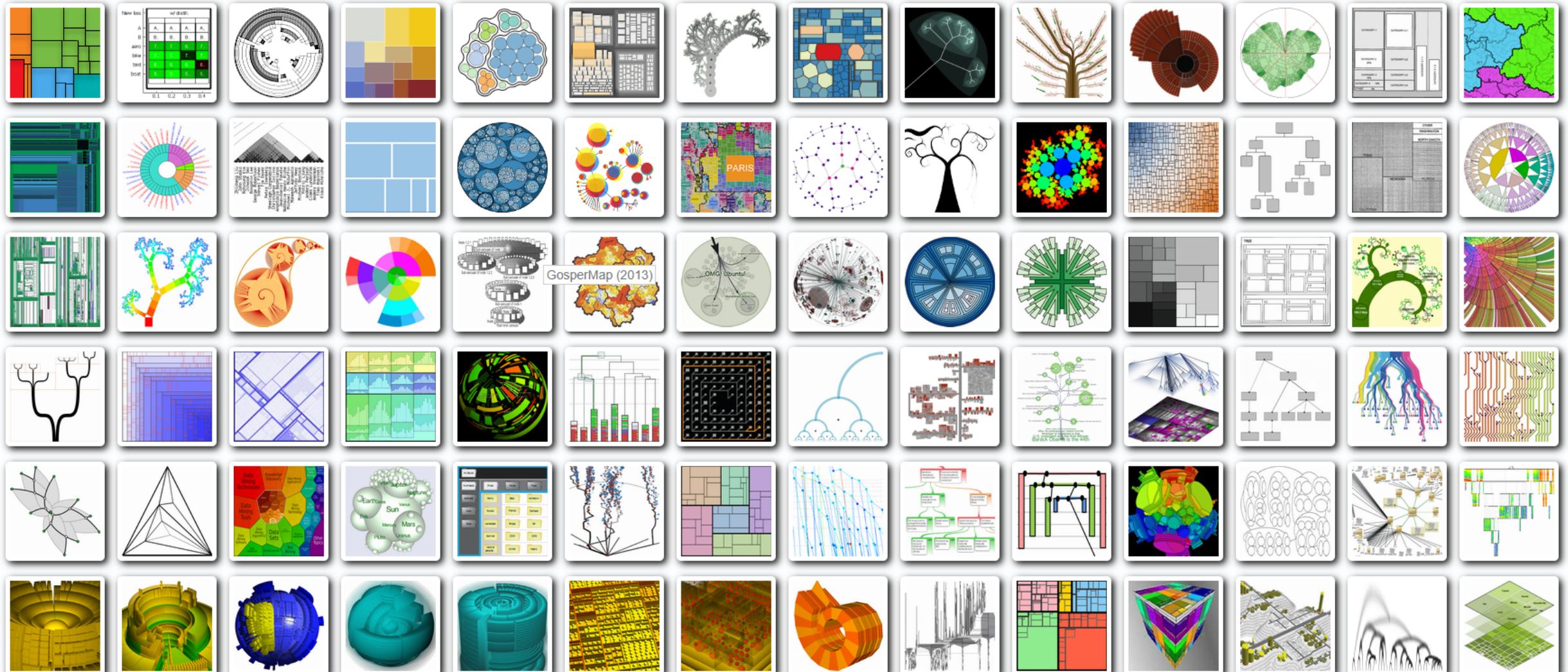


Fulltext Search

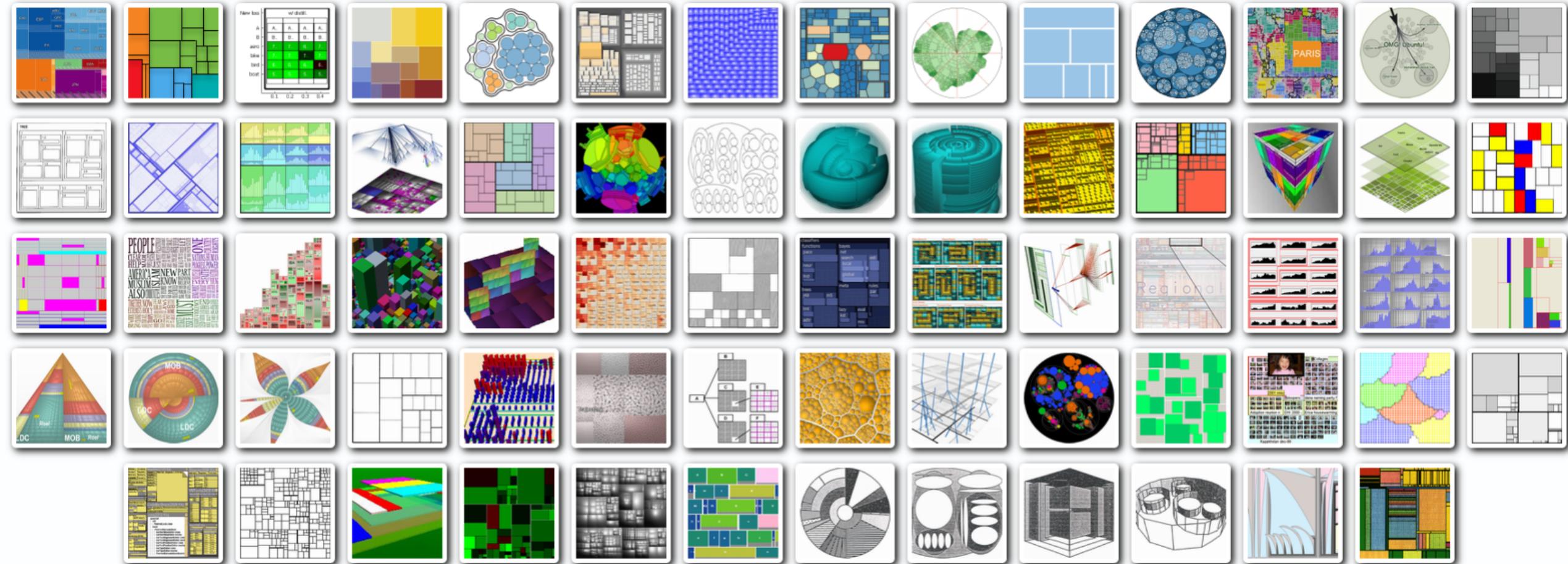
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Techniques Shown

307



TREEMAPS ON TREEVIS.NET



VISUALIZATION OF PO-SETS



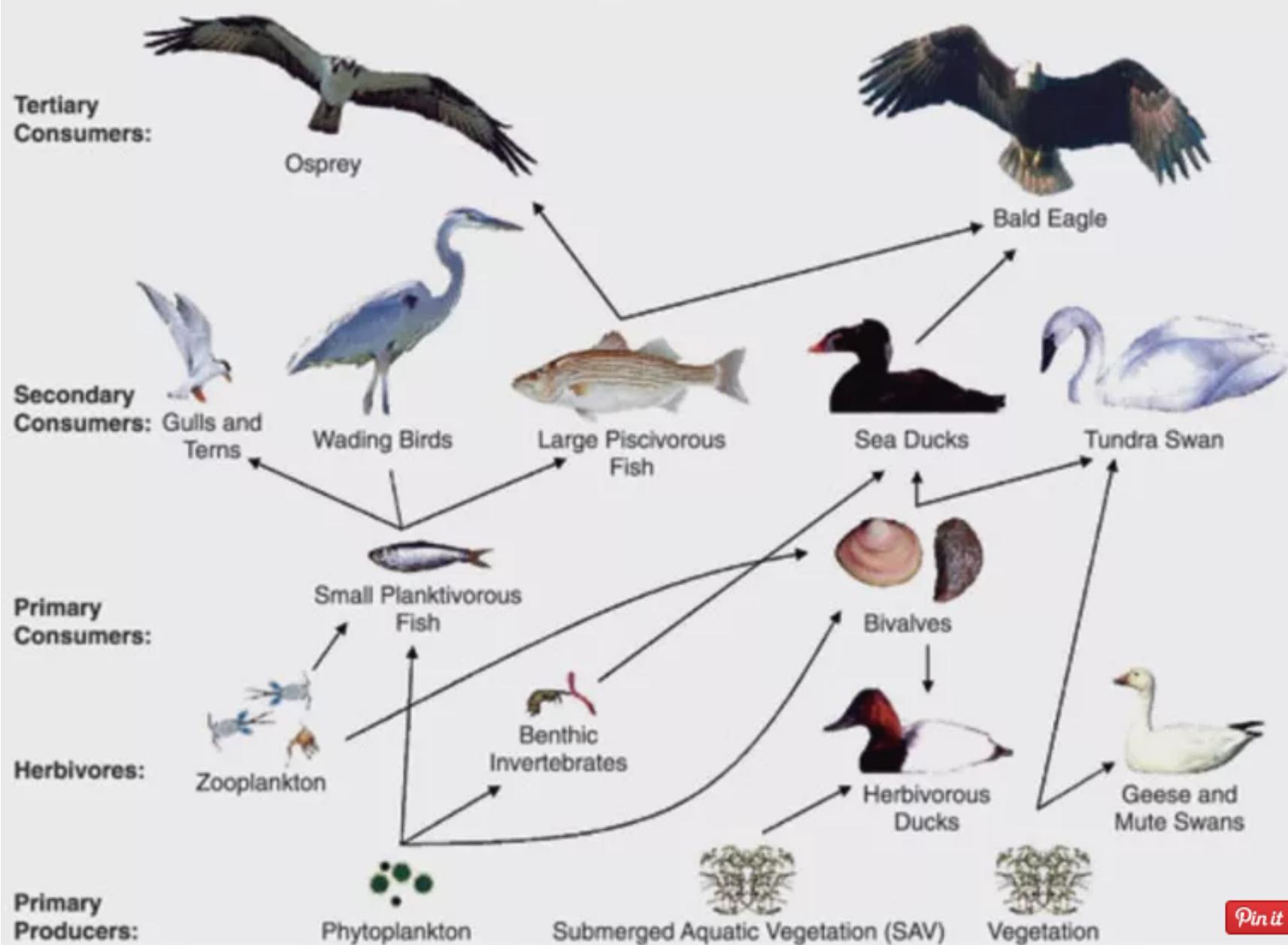
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Chesapeake Bay Waterbird Food Web

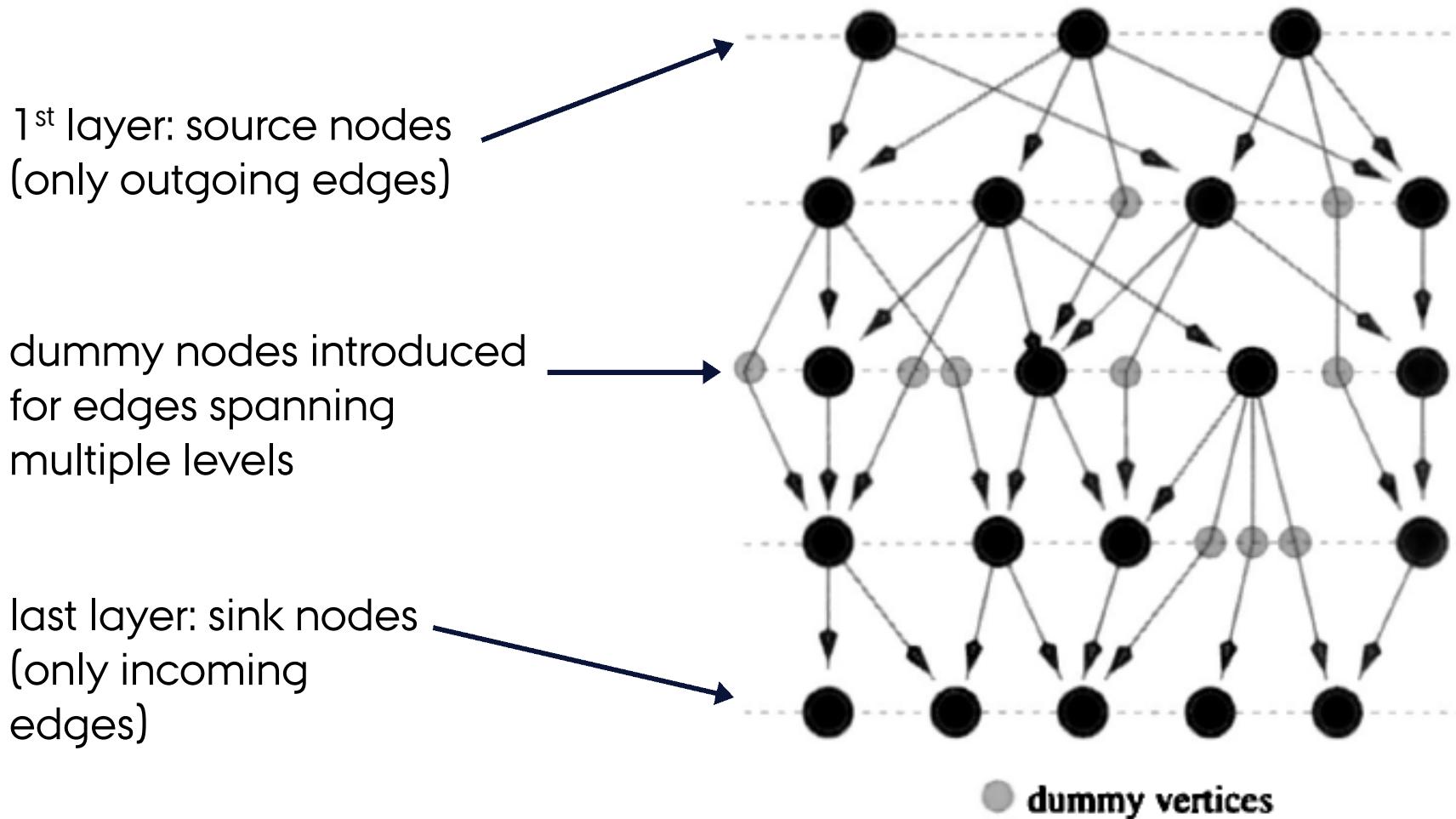


LAYERED DRAWING (SUGIYAMA ET AL. 1981)

3 Steps:

1. **Layer Assignment:** produces a hanging of the graph from its source node(s) -> assigns y-coordinates to each node
2. **Crossing Reduction:** determines an order of the nodes on each layer that minimizes edge crossings
3. **Horizontal Coordinate Assignment:** produces an x-coordinate for each node from the ordering

1. LAYER ASSIGNMENT



picture source: [di Battista et al. 1999]



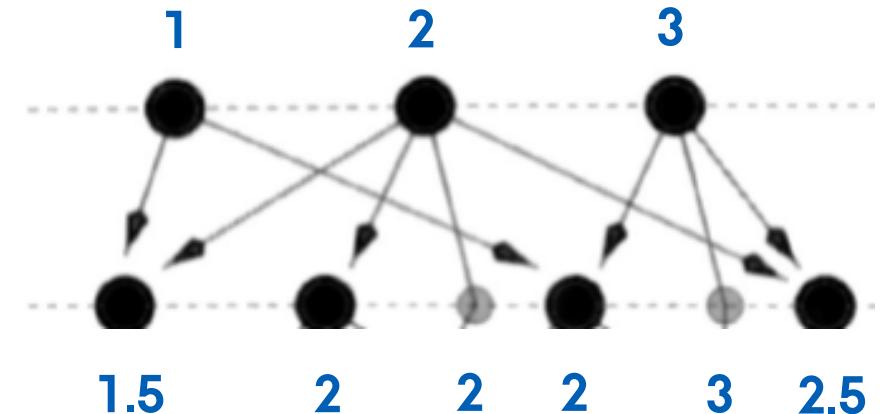
2. CROSSING MINIMIZATION

Local Level-by-Level Optimization

$L_i \Rightarrow i^{\text{th}}$ level, $n \Rightarrow \#$ of levels
for ($i=1 \dots n-1$) **optimize** L_i w.r.t. L_{i-1}



e.g., using the **Barycenter Method** where a node's position on level L_i is the mean of the positions of all connected nodes on level L_{i-1}



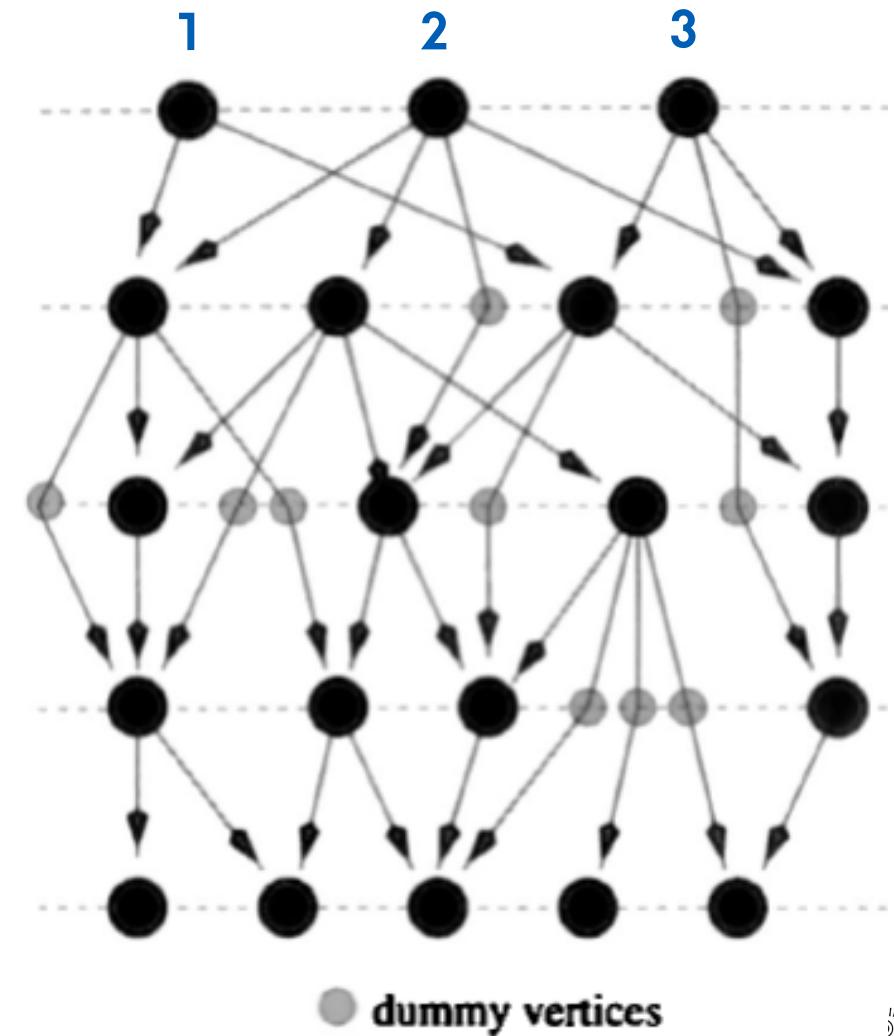
2. CROSSING MINIMIZATION

Local Level-by-Level Optimization

$L_i \Rightarrow i^{\text{th}}$ level, $n \Rightarrow \#$ of levels
for $(i=1 \dots n-1)$ **optimize** L_i w.r.t. L_{i-1}



e.g., using the **Barycenter Method** where a node's position on level L_i is the mean of the positions of all connected nodes on level L_{i-1}



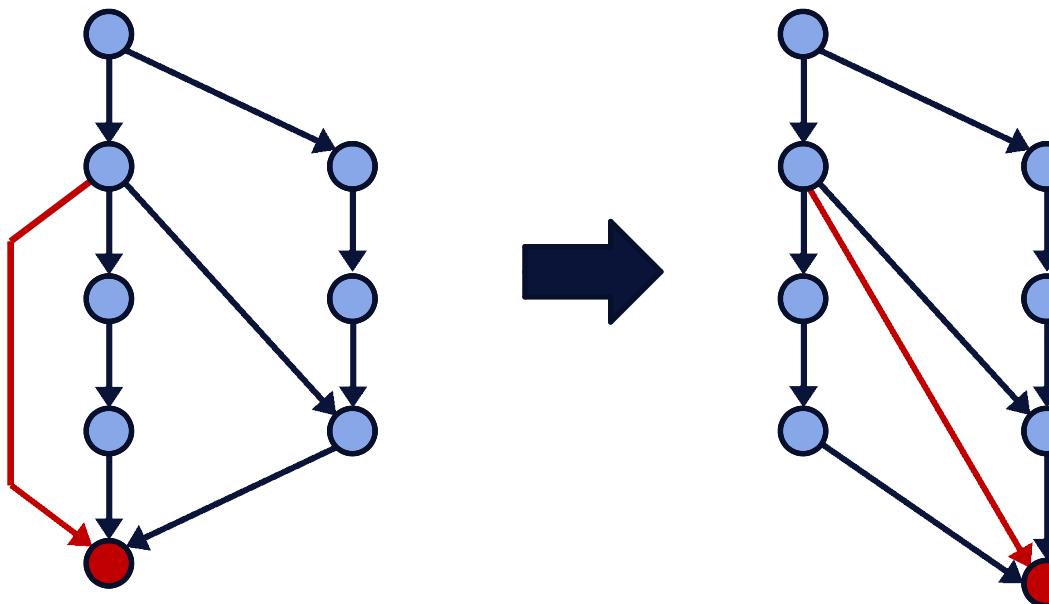
dummy vertices



3. HORIZONTAL COORDINATE ASSIGNMENT

Bend Minimization

- > vertical lines are preferable to straight but diagonal lines
- > diagonal lines are preferable to bent lines
- > bent lines with fewer bends are preferable to those with more bends



LIST OF LITERATURE SOURCES (PART 1)

- Brandes 2001: https://doi.org/10.1007/3-540-44969-8_4
- Barnes & Hut 1986: <https://doi.org/10.1038/324446a0>
- Quigley & Eades 2000: https://doi.org/10.1007/3-540-44541-2_19
- Erten et al. 2004: <https://doi.org/10.1117/12.539245>
- Frishman & Tal 2007: <https://doi.org/10.2312/VisSym/EuroVis07/075-082>
- Gorochowski et al. 2012: <https://doi.org/10.1109/TVCG.2011.142>
- McDonnel & Elmqvist 2009: <https://doi.org/10.1109/TVCG.2009.191>
- Henry et al. 2007: <https://doi.org/10.1109/TVCG.2007.70582>
- Elmqvist et al. 2008: <https://doi.org/10.1109/PACIFICVIS.2008.4475479>
- Shen & Ma 2007: <https://doi.org/10.2312/VisSym/EuroVis07/083-090>
- v.Ham et al. 2009: https://doi.org/10.1007/978-3-642-03658-3_47
- Wu et al. 2008: https://doi.org/10.1007/978-3-540-33037-0_26
- Beinhauer & Ziegler 2003:
https://www.researchgate.net/publication/267236592_Intuitive_Interaction_in_Complex_Information_Spaces_Results_and_Exploration_of_INVITE
- Johnson & Shneiderman 1991: <https://doi.org/10.1109/VISUAL.1991.175815>

LIST OF LITERATURE SOURCES (PART 2)

- Bruls et al. 2000: <https://doi.org/10.2312/VisSym/VisSym00/033-042>
- Bederson & Shneiderman 2002: <https://doi.org/10.1145/571647.571649>
- Icicle Plot (1983): <https://doi.org/10.2307/2685881>
- Sunburst (2000): <https://doi.org/10.1109/INFVIS.2000.885091>
- Cascaded Treemaps (2008): <http://portal.acm.org/citation.cfm?id=1375758>
- Treevis.net: <https://doi.org/10.1109/MCG.2011.103>
- Sugiyama et al. 1981: <https://doi.org/10.1109/TSMC.1981.4308636>

