STATISTICS

Hypothesis Testing with One Sample

Chapter Outline

- Introduction to Hypothesis Testing
- Steps of Hypothesis testing
- Hypothesis Testing for the Mean (Large sample)
- Hypothesis Testing for the Mean (Small sample)
- Hypothesis Testing for Proportions

INTRODUCTION

- A **statistical hypothesis** is a statement about a population parameter.
- A parameter is a value that defines the characteristic of a whole population which means that population parameter consists of mean, variance, standard deviation, mode, median or proportion of the population or objects under consideration.
- Researchers and scientists in various areas for instance business, marketing, medicine, agriculture and education utilize hypothesis testing to make decisions about the population parameters based on sample information.

STATISTICAL HYPOTHESIS TESTING

Some examples of statistical hypotheses are given as follows:

- The mean age of students in a classroom is 26.5 years.
- The proportion of people wearing eyeglasses is almost four times higher than that the proportion of the contact lens users.
- The variance of the fuel consumption of a certain model car under various road and traffic condition is 0.76 litres in 100 kilometres.
- The battery manufacturer claims that the average number of charge cycle is about 900 charge cycles for the specific type of rechargeable batteries.
- The average number of hours per week students spent in the university library is 1818 hours during the final's week.

STEPS IN HYPOTHESIS TESTING

- To test the statistical expressions the following steps can be followed:
- Step 1: State null and alternate hypotheses
- Step 2: Select a level of significance
- Step 3: Identify the test statistic
- Step 4: Formulate a decision rule
- Step 5: Take a sample and make the decision

Step 1: State the Null and Alternate Hypotheses

- H_0 : null hypothesis
- H_1 : alternative hypothesis
- To test a population parameter, you should carefully state a pair of hypotheses—one that represents the claim and the other, its complement.
- When one of these hypotheses is false, the other must be true.

DEFINITION

1. A **null hypothesis H_0** is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis H_a** is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

H_0 is read as "H sub-zero" or "H naught" and H_a is read as "H sub-a."

Step 1: State the Null and Alternate Hypotheses

- To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement. Then, write its complement.

$$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases} \quad \begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases} \quad \begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$$

- Similar statements can be made to test other population parameters, such as *p, or σ, σ²*.

Step 1: State the Null and Alternate Hypotheses

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.*
. . . equal to k. *. . . k.* *. . . exactly k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.*

Step 1: State the Null and Alternate Hypotheses

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

H_0: $p = 0.61$ (Claim)

H_a: $p \neq 0.61$

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

H_0: $\mu \geq 15$ minutes

H_a: $\mu < 15$ minutes (Claim)

3. A company advertises that the mean life of its furnaces is more than 18 years.

H_0: $\mu \leq 18$ years

H_a: $\mu > 18$ years (Claim)

Step 1: State the Null and Alternate Hypotheses

- Suppose that the quality control engineer is interested in the diameter of the certain cylindrical machine part through the manufacturing process.
- At this time the diameter of the cylindrical machine part can be described by a probability distribution and suppose that the quality control engineer is interested in the mean diameter of the cylindrical part.
- Specifically, the quality control engineer is interested in deciding whether or not the mean diameter of the cylindrical machine part is 25 millimeters.

H_0: $\mu = 25$ milimeters

H_1: $\mu \neq 25$ milimeters

- the quality control engineer may specify the diameter of the cylindrical part which could be greater than 25 millimeters or less than 25 millimeters

H_0: $\mu = 25$ milimeters $\quad$ H_0: $\mu = 25$ milimeters

H_1: $\mu > 25$ milimeters $\quad$ H_1: $\mu < 25$ milimeters

Step 2: Select the Level of Significance

- A hypothesis test is started by assuming that the equality condition in the null hypothesis is true.
- When you perform a hypothesis test, you make one of two decisions:

  1. reject the null hypothesis or
  2. fail to reject the null hypothesis.

Step 2: Select the Level of Significance

- Because your decision is based on a sample rather than the entire population, there is always the possibility you will make the wrong decision.
- For instance, suppose you claim that a certain coin is not fair.

- To test your claim, you flip the coin 100 times and get 49 heads and 51 tails.
  – You would probably agree that you do not have enough evidence to support your claim
- But what if you flip the coin 100 times and get 21 heads and 79 tails?
  – So, you probably have enough evidence to support your claim that the coin is not fair.

Step 2: Select the Level of Significance

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

	Truth of H_0	
Decision	H_0 is true.	H_0 is false.
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Step 2: Select the Level of Significance

Example: Identifying Type I and Type II Errors

- The *U.S. Department of Agriculture* (USDA) limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit.
- You perform a hypothesis test to determine whether the meat inspector's claim is true.
- When will a type I or type II error occur? Which is more serious?

Solution H_0: $p \leq 0.2$ The proportion is less than or equal to 20%.

H_a: $p > 0.2$ (Claim) The proportion is greater than 20%.

Step 2: Select the Level of Significance

- A type I error will occur if the actual proportion of contaminated chicken is less than or equal to 0.2, but you reject H_0.
- A type II error will occur if the actual proportion of contaminated chicken is greater than 0.2, but you do not reject H_0
- With a type I error, you might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits.
- With a type II error, you could be allowing chicken that exceeded the USDA contamination limit to be sold to consumers.
- A type II error is more serious because it could result in sickness or even death.

Step 2: Select the Level of Significance

	H_0 is true	H_0 is false
Reject the null hypothesis (H_0)	Type I error (α)	Correct decision
Fail to reject the null hypothesis (H_0)	Correct decision	Type II error (β)

DEFINITION

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

The probability of a type II error is denoted by β, the lowercase Greek letter beta.

Step 3: Identify the Test Statistic

- The statistic that is compared with the parameter in the null hypothesis is called the **test statistic.**
- The type of test used and the sampling distribution are based on the *test statistic*

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, $n \geq 30$), t (Section 7.3, $n < 30$)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

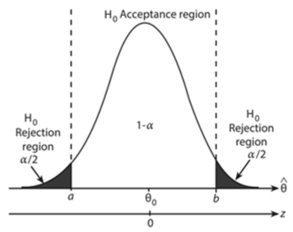
- In hypothesis testing for the mean (μ) of a single normal population and the standard deviation of the population σ is known, the test statistic z can be obtained as follows;

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Step 4: Formulate a Decision Rule

- Hypothesis thesis or statistical decision rules amount to rejection or acceptance of the null hypothesis (H0) with *α* significance level.
- Critical value of the test statistics specifies or separates the rejection and acceptance region of the hypothesis thesis.
- Critical values depend on the significance level *α*,

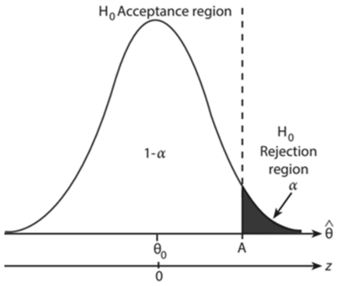
$$H_0: \mu = k$$
$$H_a: \mu \neq k$$



Critical Values and Acceptance and Rejection Region for **The Two-Sided Tests**

Step 4: Formulate a Decision Rule

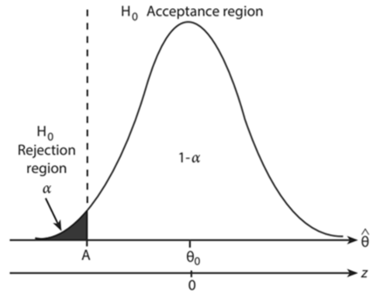
H_0: $\mu \leq k$
H_a: $\mu > k$



Critical Values and Acceptance and
Rejection Region for <u>The One-Sided Right Tale</u>

Step 4: Formulate a Decision Rule

H_0: $\mu \geq k$
H_a: $\mu < k$



Critical Values and Acceptance and
Rejection Region for <u>The One-Sided Left Tale</u>

Step 4: Formulate a Decision Rule

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

H_0: $p = 0.61$ (Claim)
H_a: $p \neq 0.61$

a two-tailed hypothesis test.

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

H_0: $\mu \geq 15$ minutes
H_a: $\mu < 15$ minutes (Claim)

a left-tailed hypothesis test

3. A company advertises that the mean life of its furnaces is more than 18 years.

H_0: $\mu \leq 18$ years
H_a: $\mu > 18$ years (Claim)

a right-tailed hypothesis test

Step 5: Use Sample Information and Make a Decision

- There are only two possible outcomes to a hypothesis test:
  - (1) reject the null hypothesis and
  - (2) fail to reject the null hypothesis.

	Claim	
Decision	Claim is H_0.	Claim is H_a.
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Review

- EXAMPLE - Writing the Hypotheses
- A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours. How would you write the null and alternative hypotheses if
- (1) you are on the research team and want to support the claim?

H_0: $\mu \geq 96$
H_a: $\mu < 96$ (Claim)

- (2) you are on an opposing team and want to reject the claim?

H_0: $\mu \leq 96$ (Claim)
H_a: $\mu > 96$

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Hypothesis Testing for the Mean (Large sample)

Hypothesis Testing for the Mean (Large sample)

- Normal population OR random sample is large (n≥30)
- In hypothesis testing for the mean (μ) of a single normal population and the standard deviation of the population σ is known, the test statistic z can be obtained as follows;

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Using P-values to Make a Decision

DECISION RULE BASED ON *P*-VALUE

To use a P-value to make a conclusion in a hypothesis test, compare the P-value with α.

1. If $P \leq \alpha$, then reject H_0.

2. If $P > \alpha$, then fail to reject H_0.

Example: Interpreting a P-value

- The P-value for a hypothesis test is P = 0.0237. What is your decision if the level of significance is

α = 0.05?

- Solution
- Because 0.0237 < 0.05, you should reject the null hypothesis.

α = 0.01?

- Solution
- Because 0.0237 > 0.01, you fail to reject the null hypothesis.

Finding the P-value for a Hypothesis Test

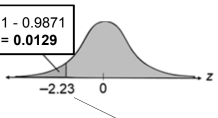
- After determining the hypothesis test's standardized test statistic and the test statistic's corresponding area, do one of the following to find the P-value.
- For a left-tailed test, P = (Area in left tail).
- For a right-tailed test, P = (Area in right tail).
- For a two-tailed test, P = 2*(Area in tail of standardized test statistic).

Example: Finding the P-value for a Left-Tailed Test

- Find the P-value for a left-tailed hypothesis test with a test statistic of z = –2.23. Decide whether to reject H_0 if the level of significance is α = 0.01.
- Solution
- For a left-tailed test, P = (Area in left tail)

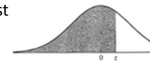
Example: Finding the P-value for a Left-Tailed Test

P= 1 - 0.9871
P = **0.0129**



Because 0.0129 > 0.01, you should **fail to reject H_0.**

TABLE A.2 Cumulative normal distribution (continued)



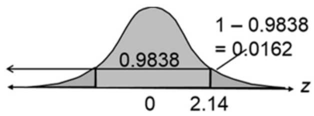
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Example: Finding the P-value for a Two-Tailed Test

- Find the P-value for a two-tailed hypothesis test with a test statistic of z = 2.14. Decide whether to reject H_0 if the level of significance is α = 0.05.

Solution

- For a two-tailed test, P = 2(Area in tail of standardized test statistic)



- P = 2(0.0162) = 0.0324

Because 0.0324 < 0.05, you should **reject H_0.**

z-Test for a Mean μ

- Can be used when
- Sample is random
- The population is normally distributed, or for any population when the sample size n is at least 30.

The **test statistic** is the sample mean $\bar{x}$

The **standardized test statistic** is z

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \frac{\sigma}{\sqrt{n}} = \text{standard error} = \sigma_{\bar{x}}$$

σ is known.

Using P-values for a z-Test for Mean μ (1 of 3)

In Words	In Symbols
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.

Using P-values for a z-Test for Mean μ (2 of 3)

In Words	In Symbols
4. Find the standardized test statistic. (skip)	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
5. Find the area that corresponds to z.	Use Table 4 in Appendix B.
6. Find the P-value. (Use z-interval) a. left-tailed test, P = (Area in left tail). b. right-tailed test, P = (Area in right tail). c. two-tailed test, P = 2(Area in tail of standardized test statistic).	

Using P-values for a z-Test for Mean μ (3 of 3)

In Words	In Symbols
7. Make a decision to reject or fail to reject the null hypothesis.	If $P \leq \alpha$, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Example 1: Hypothesis Testing Using P-values (1 of 2)

- In auto racing, a pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random selection of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at α = 0.01? Use a P-value.

H_0: $\mu \geq 13$ seconds and H_a: $\mu < 13$ seconds. (Claim)

The level of significance is $\alpha = 0.01$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{Because } n \geq 30, \text{ use the } z\text{-test.}$$

$$\approx \frac{12.9 - 13}{0.19/\sqrt{32}} \qquad \text{Because } n \geq 30, \text{ use } \sigma \approx s = 0.19. \text{ Assume } \mu = 13.$$

$$\approx -2.98.$$

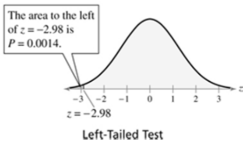
Example 1: Hypothesis Testing Using P-values (2 of 2)

Solution

- H_0: $\mu \geq 13$ sec
- H_a: $\mu < 13$ sec
- $\alpha = 0.01$
- **Test Statistic:**

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

- ***P*-value**



- **Decision:** 0.0014 < 0.01

Reject H_0

At the 1% level of significance, you have sufficient evidence to conclude the mean pit stop time is less than 13 seconds.

Example 2: Hypothesis Testing Using P-values (1 of 2)

- According to a study, the mean cost of bariatric (weight loss) surgery is \$22,500. You think this information is incorrect. You randomly select 30 bariatric surgery patients and find that the average cost for their surgeries is \$21,545. The population standard deviation is known to be \$3,015 and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P-value.

Solution

H_0: $\mu = \$22{,}500$ $\qquad$ H_a: $\mu \neq \$22{,}500$. (Claim)

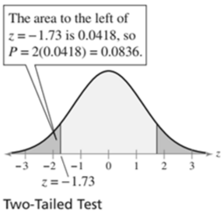
The level of significance is $\alpha = 0.05$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because $n \geq 30$, use the z-test.

$$\approx \frac{21{,}545 - 22{,}500}{3015/\sqrt{30}}$$ Because $n \geq 30$, use $\sigma \approx s = 3015$. Assume $\mu = 22{,}500$.

$$\approx -1.73.$$

Example 2: Hypothesis Testing Using P-values (2 of 2)



- **Decision:** 0.0836 > 0.05

Fail to reject H_0

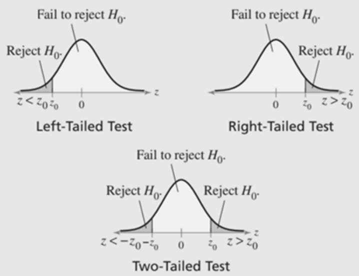
At the 5% level of significance, there is not sufficient evidence to support the claim that the mean cost of bariatric surgery is different from \$22,500.

Using Rejection Regions For a Z-test

DECISION RULE BASED ON REJECTION REGION

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic z. If the standardized test statistic

1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Using Rejection Regions For a Z-test

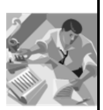
GUIDELINES

Using Rejection Regions for a z-Test for a Mean μ

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the critical value(s).	Use Table 4 in Appendix B.
4. Determine the rejection region(s).	
5. Find the standardized test statistic and sketch the sampling distribution.	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$, or, if $n \geq 30$, use $\sigma \approx s$.
6. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.	

Example 1: Testing μ Using a Rejection Region (1 of 2)

- Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of the one of its competitors, which is \$68,000. A random sample of 30 of the company's mechanical engineers has a mean salary of \$66,900. Assume the population standard deviation is \$5500 and the population is normally distributed. At $\alpha = 0.05$, test the employees' claim.



H_0: $\mu \geq \$68{,}000$ and H_a: $\mu < \$68{,}000$. (Claim)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because $n \geq 30$, use the z-test.

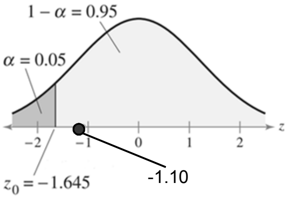
$$\approx \frac{66{,}900 - 68{,}000}{5500/\sqrt{30}}$$ Because $n \geq 30$, use $\sigma \approx s = 5500$. Assume $\mu = 68{,}000$.

$$\approx -1.10.$$

Example 1: Testing μ Using a Rejection Region (2 of 2)

Solution

- ***H_0*: *μ* ≥ $68,000**
- ***H_a*: *μ* < $68,000**
- ***α* = 0.05**
- **Rejection Region:**



$$\approx \frac{66{,}900 - 68{,}000}{5500/\sqrt{30}}$$
$$\approx -1.10.$$

Because *z* is not in the rejection region, you fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 5% level of significance to support the employees' claim that the mean salary is less than $68,000

Example 2: Testing μ Using a Rejection Region (1 of 2)

- A researcher claims that the mean cost of raising a child from birth to age 2 by husband-wife families in the U.S. is $13,120. A random sample of 500 children (age 2) has a mean cost of $12,925. Assume the population standard deviation is $1745. At α = 0.10, is there enough evidence to reject the claim?
- (Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion)

H_0: μ = $13,120 (Claim)

H_a: $\mu \neq$ $13,120.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because $n \geq 30$, use the z-test.

$$\approx \frac{12{,}925 - 13{,}120}{1745/\sqrt{500}}$$ Because $n \geq 30$, use $\sigma \approx s = 1745$. Assume $\mu = 13{,}120$.

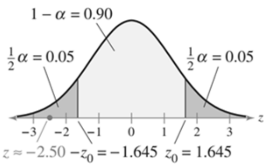
$$\approx -2.50.$$

Alpha	Tail	z
0.10	Left	−1.28
	Right	1.28
	Two	±1.645
0.05	Left	−1.645
	Right	1.645
	Two	±1.96
0.01	Left	−2.33
	Right	2.33
	Two	±2.575

Example 2: Testing μ Using a Rejection Region (2 of 2)

H_0: μ = $13,120 (Claim)

H_a: $\mu \neq$ $13,120.



5% Level of Significance

Test statistic

$$\approx \frac{12{,}925 - 13{,}120}{1745/\sqrt{500}}$$
$$\approx -2.50.$$

- **Decision: Reject *H_0***

Section 7.2 Summary

- Found and interpreted P-values and used them to test a mean μ
- Used P-values for a z-test for a mean μ when σ is known
- Found critical values and rejection regions in the standard normal distribution
- Used rejection regions for a z-test for a mean μ when σ is known

Hypothesis Testing for the Mean (Small sample, σ Unknown)

Study tip

σ known n ≥ 30 or n<30	σ unknown n ≥ 30	σ unknown n<30
• USE Z-test	• You can use z-test or t-test	• USE T-test

Finding Critical Values in a t Distribution

1. Identify the level of significance α.
2. Identify the degrees of freedom d.f. = n − 1.
3. Find the critical value(s) using T-Table in the row with n – 1 degrees of freedom. If the hypothesis test is
   a. **left-tailed**, use “One Tail, α” column with a negative sign,
   b. **right-tailed**, use “One Tail, α” column with a positive sign,
   c. **two-tailed**, use “Two Tails, α” column with a negative and a positive sign.

Table 5—*t*-Distribution

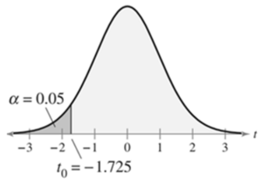
d.f.	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921

Example 1: Finding Critical Values for t

- Find the critical value t_0 for a left-tailed test given α = 0.05 and n = 21

Solution

- The degrees of freedom are
  d.f. = n – 1 = **21 – 1 = 20.**
- Use t-Table .
- Look at α = 0.05 in the “One Tail, α” column.
- Because the test is left-tailed, the critical value is negative.



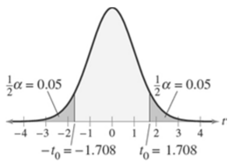
5% Level of Significance

Example 2: Finding Critical Values for t

- Find the critical values $-t_0$ and t_0 for a two-tailed test given α = 0.10 and n = 26.

Solution

- The degrees of freedom are d.f. = n – 1 = **26 – 1 = 25.**
- Look at α = 0.10 in the “Two Tail, α” column.
- Because the test is two-tailed, one critical value is negative and one is positive.



10% Level of Significance

t-Test for a Mean μ (σ Unknown)

***t*-TEST FOR A MEAN μ**

The ***t*-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad \text{Standardized test statistic for } \mu \ (\sigma \text{ unknown})$$

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

The degrees of freedom are

d.f. = n − 1.

Using P-values for a t-Test for Mean μ (σ Unknown) (1 of 3)

	In Words	In Symbols
1.	Verify that σ is not known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2.	State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3.	Specify the level of significance.	Identify α.

Using P-values for a t-Test for Mean μ (σ Unknown) (2 of 3)

	In Words	In Symbols
4.	Identify the degrees of freedom.	d.f. = n – 1
5.	Find the critical values.	
6.	Determine the rejection region(s).	Use Table 4 in Appendix B.
7.	Find the standardized test statistic and sketch the sampling distribution.	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

Using P-values for a t-Test for Mean μ (σ Unknown) (3 of 3)

In Words	In Symbols
8. Make a decision to reject or fail to reject the null hypothesis.	If t is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

Example 1: Testing μ with a Small Sample (1 of 2)

- A used car dealer says that the mean price of a two-year-old sedan is at least \$20,500. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$19,850 and a standard deviation of \$1084. Is there enough evidence to reject the dealer's claim at α = 0.05? Assume the population is normally distributed. (Adapted from Kelley Blue Book)

H_0: $\mu \geq \$20{,}500$ (Claim)

and

H_a: $\mu < \$20{,}500$.

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Because σ is unknown and the population is normally distributed, use the t-test.

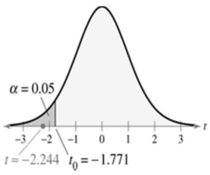
$= \dfrac{19{,}850 - 20{,}500}{1084/\sqrt{14}}$ Assume $\mu = 20{,}500$.

$\approx -2.244.$ Round to three decimal places.

Example 1: Testing μ with a Small Sample (2 of 2)

Solution

- H_0: $\mu \geq \$20{,}500$
- H_a: $\mu < \$20{,}500$
- $\alpha = 0.05$
- df = 14 – 1 = 13
- **Rejection Region:**



- **Test Statistic:**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{19{,}850 - 20{,}500}{1084/\sqrt{14}} \approx -2.244$$

- **Decision: Reject H_0**

At the 0.05 level of significance, there is enough evidence to reject the claim that the mean price of a two-year-old sedan is at least \$20,500.

Example 2: Testing μ with a Small Sample (1 of 2)

- An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company's claim at α = 0.05? Assume the population is normally distributed.

H_0: $\mu = 6.8$ (Claim) and H_a: $\mu \neq 6.8$.

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Because σ is unknown and $n \geq 30$, use the t-test.

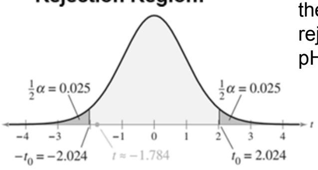
$= \dfrac{6.7 - 6.8}{0.35/\sqrt{39}}$ Assume $\mu = 6.8$.

$\approx -1.784.$ Round to three decimal places.

Example 2: Testing μ with a Small Sample (2 of 2)

Solution

- H_0: $\mu = 6.8$
- H_a: $\mu \neq 6.8$
- $\alpha = 0.05$
- df = 39 – 1 = 38
- **Rejection Region:**



5% Level of Significance

- **Test Statistic:**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{6.7 - 6.8}{0.35/\sqrt{39}} \approx -1.784$$

- **Decision: Fail to reject H_0**

At the 0.05 level of significance, there is not enough evidence to reject the claim that the mean pH is 6.8.

Example 2: Testing μ with a Small Sample (1 of 2)

- The price of a certain electronic product at a chain store is 180 TL. The same electronic product is also available through various online stores and five of the store prices are as follows: 155, 179, 175, 175, 161 TL. Let's assume that online prices are normally distributed; at the significance level α = 0.1, can we conclude that the mean online store price of the product is less than 180 TL?

$H_0: \mu \geq 180$
$H_1: \mu < 180$ (claim)

$$\bar{x} = \frac{\sum x}{n} = 169 \text{ and } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = 10.39$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{169 - 180}{10.39/\sqrt{5}} = \frac{-11}{4.64} = -2.367.$$

Example 2: Testing μ with a Small Sample (2 of 2)

- The degrees of freedom is
  d.f. = 5 – 1 = 4

$$t < -t_{\alpha,4} = -t_{0.0,14} = -1.533.$$

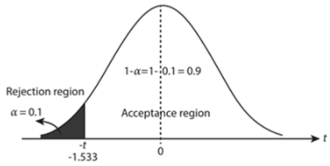


Figure 3.14 Left Tailed *t* Test for α=0.1.

we reject
the null hypothesis H_0

In conclusion,
we conclude that the average price of the product at online stores is less than 180 TL at the significance level *α* = 0.1

Hypothesis Testing for Proportions

z-Test for a Population Proportion

z-Test for a Population Proportion

- A statistical test for a population proportion.
- Can be used when a binomial distribution is given such that *np* ≥ 5 and *nq* ≥ 5.
- The **test statistic** is the sample proportion $\hat{p}$.
- The **standardized test statistic** is *z*.

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Using a z-Test for a Proportion p

GUIDELINES

Using a *z*-Test for a Proportion *p*

IN WORDS	IN SYMBOLS
1. Verify that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution.	$np \geq 5, nq \geq 5$
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$.
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Example 1: Hypothesis Test for a Proportion (1 of 2)

- A research center claims that less than 40% of U.S. adults have accessed the Internet over a wireless network with a laptop computer. In a random sample of 100 adults, 31% say they have accessed the Internet over a wireless network with a laptop computer. At α = 0.01, is there enough evidence to support the researcher's claim? (Adopted from Pew Research Center)

Solution

- Verify that np ≥ 5 and nq ≥ 5.
- np = 100(0.40) = 40 and nq = 100(0.60) = 60

H_0: $p \geq 0.4$ and H_a: $p < 0.4$. (Claim)

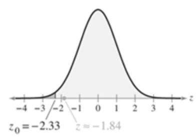
Example 1: Hypothesis Test for a Proportion (2 of 2)

H_0: $p \geq 0.4$ and H_a: $p < 0.4$. (Claim)

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$
Because $np \geq 5$ and $n \geq 5$, you can use the z-test.

$$= \frac{0.31 - 0.4}{\sqrt{(0.4)(0.6)/100}}$$
Assume $p = 0.4$.

$$\approx -1.84.$$
Round to two decimal places.



1% Level of Significance

- **Decision: Fail to reject H_0**
  At the 1% level of significance, there is not enough evidence to support the claim that less than 50% of U.S. adults have accessed the Internet over a wireless network with a laptop computer.

Example 2: Hypothesis Test for a Proportion (1 of 2)

- A researcher claims that 86% of college graduates say their college degree has been a good investment. In a random sample of 1000 graduates, 845 say their college degree has been a good investment. At a = 0.10, is there enough evidence to reject the researcher's claim?

$np = 1000(0.86) = 860$ and $nq = 1000(0.14) = 140$

H_0: $p = 0.86$ (Claim) and H_a: $p \neq 0.86$.

$\hat{p} = \frac{x}{n} = \frac{845}{1000} = 0.845.$

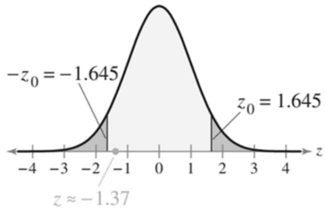
$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$ Because $np \geq 5$ and $nq \geq 5$, you can use the z-test.

$= \frac{0.845 - 0.86}{\sqrt{(0.86)(0.14)/1000}}$ Assume $p = 0.86$.

$\approx -1.37.$ Round to two decimal places.

Example 2: Hypothesis Test for a Proportion (2 of 2)

- Fail to reject the null hypothesis
- Interpretation There is not enough evidence at the 10% level of significance to reject the claim that 86% of college graduates say their college degree has been a good investment.



Example 3

- Department of information technologies of a university claims that more than 60% of students have access to internet via campus wireless network with a laptop computer. In a random sample of 100 students, 70% say that they have accessed the Internet over a ireless network with a laptop computer. At 5% significance level is there enough evidence to support the information Technologies department's claim?

H_0: p = 0.60
H_1: p > 0.60

$n \times p = 100\ (0.6) = 60$ and $n \times q = 100\ (0.4) = 40$, $(q = 1 - p)$

$$z = \frac{\hat{p} - p}{\sqrt{(p \times q)/n}} = \frac{0.70 - 0.60}{\sqrt{(0.6)(0.4)/100}} = \frac{0.10}{0.049} = 2.04$$

Example 3

$z = 2.04 > 1.65$

- Therefore, we reject the null hypothesis
- In conclusion, we conclude that at 5% level of significance to support the claim that more than 60% of students have an access internet on campus wireless network with a laptop computer.