9

Testing the Difference Between Two Means, Two Proportions, and Two Variances

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OUTLINE

OBJECTIVES

After completing this chapter, you should be able to:

1. Test the difference between two means, using the *z* test.
2. Test the difference between two means for independent samples, using the *t* test.
3. Test the difference between two means for dependent samples.
4. Test the difference between two proportions.
5. Test the difference between two variances or standard deviations.

STATISTICS TODAY

To Vaccinate or Not to Vaccinate? Small versus Large Nursing Homes

Influenza is a serious disease among the elderly, especially those living in nursing homes. Those residents are more susceptible to influenza than elderly persons living in the community because the former are usually older and more debilitated, and they live in a closed environment where they are exposed more so than community residents to the virus if it is introduced into the home. Three researchers decided to investigate the use of vaccine and its value in determining outbreaks of influenza in small nursing homes.

These researchers surveyed 83 randomly selected licensed homes in seven counties in Michigan. Part of the study consisted of comparing the number of people being vaccinated in small nursing homes (100 or fewer beds) with the number in larger nursing homes (more than 100 beds). Unlike the statistical methods presented in Chapter 8, these researchers used the techniques explained in this chapter to compare two sample proportions to see if there was a significant difference in the vaccination rates of patients in small nursing homes compared to those in large nursing homes. See Statistics Today—Revisited at the end of the chapter.

Source: Nancy Arden, Arnold S. Monto, and Suzanne E. Ohmit, “Vaccine Use and the Risk of Outbreaks in a Sample of Nursing Homes During an Influenza Epidemic,” *American Journal of Public Health* 85, no. 3, pp. 399–401. Copyright by the American Public Health Association.

Introduction

The basic concepts of hypothesis testing were explained in Chapter 8. With the z, t, and χ^2 tests, a sample mean, variance, or proportion can be compared to a specific population mean, variance, or proportion to determine whether the null hypothesis should be rejected.

There are, however, many instances when researchers wish to compare two sample means, using experimental and control groups. For example, the average lifetimes of two different brands of bus tires might be compared to see whether there is any difference in tread wear. Two different brands of fertilizer might be tested to see whether one is better than the other for growing plants. Or two brands of cough syrup might be tested to see whether one brand is more effective than the other.

In the comparison of two means, the same basic steps for hypothesis testing shown in Chapter 8 are used, and the z and t tests are also used. When comparing two means by using the t test, the researcher must decide if the two samples are *independent* or *dependent*. The concepts of independent and dependent samples will be explained in Sections 9–2 and 9–3.

The z test can be used to compare two proportions, as shown in Section 9–4. Finally, two variances can be compared by using an F test as shown in Section 9–5.

9–1 Testing the Difference Between Two Means: Using the *z* Test

OBJECTIVE 1

Test the difference between two means, using the z test.

Suppose a researcher wishes to determine whether there is a difference in the average age of nursing students who enroll in a nursing program at a community college and those who enroll in a nursing program at a university. In this case, the researcher is not interested in the average age of all beginning nursing students; instead, he is interested in *comparing* the means of the two groups. His research question is, Does the mean age of nursing students who enroll at a community college differ from the mean age of nursing students who enroll at a university? Here, the hypotheses are

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$

where

μ_1 = mean age of all beginning nursing students at a community college
μ_2 = mean age of all beginning nursing students at a university

Another way of stating the hypotheses for this situation is

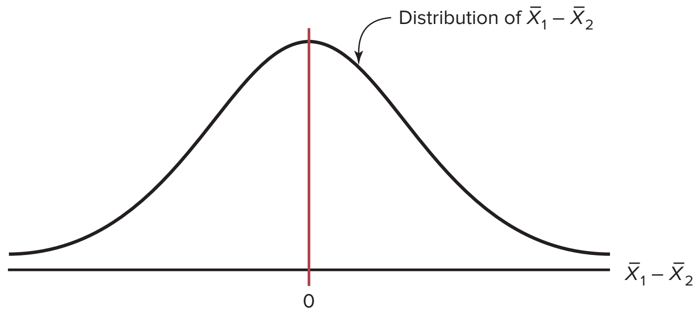
$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 \neq 0$$

If there is no difference in population means, subtracting them will give a difference of zero. If they are different, subtracting will give a number other than zero. Both methods of stating hypotheses are correct; however, the first method will be used in this text.

If two samples are *independent* of each other, the subjects selected for the first sample in no way influence the way the subjects are selected in the second sample. For example, if a group of 50 people were randomly divided into two groups of 25 people each in order to test the effectiveness of a new drug, where one group gets the drug and the other group gets a placebo, the samples would be independent of each other.

On the other hand, two samples would be *dependent* if the selection of subjects for the first group in some way influenced the selection of subjects for the other group. For example, suppose you wanted to determine if a person's right foot was slightly larger than his or her left foot. In this case, the samples are dependent because once you selected a

FIGURE 9–1
Differences of Means of Pairs of Samples



person's right foot for sample 1, you must select his or her left foot for sample 2 because you are using the same person for both feet.

Before you can use the z test to test the difference between two independent sample means, you must make sure that the following assumptions are met.

Assumptions for the z Test to Determine the Difference Between Two Means

1. Both samples are random samples.
2. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
3. The standard deviations of both populations must be known; and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

In this book, the assumptions will be stated in the exercises; however, when encountering statistics in other situations, you must check to see that these assumptions have been met before proceeding.

The theory behind testing the difference between two means is based on selecting pairs of samples and comparing the means of the pairs. The population means need not be known.

All possible pairs of samples are taken from populations. The means for each pair of samples are computed and then subtracted, and the differences are plotted. If both populations have the same mean, then most of the differences will be zero or close to zero. Occasionally, there will be a few large differences due to chance alone, some positive and others negative. If the differences are plotted, the curve will be shaped like a normal distribution and have a mean of zero, as shown in Figure 9–1.

Unusual Stats

Adult children who live with their parents spend more than 2 hours a day doing household chores. According to a study, daughters contribute about 17 hours a week and sons about 14.4 hours.

The variance of the difference $\overline{X}_1 - \overline{X}_2$ is equal to the sum of the individual variances of $\overline{X}_1$ and $\overline{X}_2$. That is,

$$\sigma^2_{\overline{X}_1 - \overline{X}_2} = \sigma^2_{\overline{X}_1} + \sigma^2_{\overline{X}_2}$$

where $\sigma^2_{\overline{X}_1} = \dfrac{\sigma_1^2}{n_1}$ and $\sigma^2_{\overline{X}_2} = \dfrac{\sigma_2^2}{n_2}$

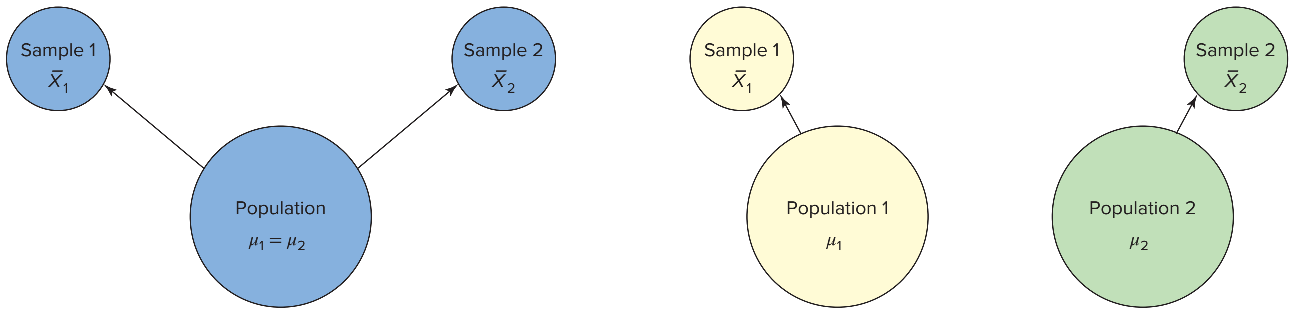
So the standard deviation of $\overline{X}_1 - \overline{X}_2$ is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for the z Test for Comparing Two Means from Independent Populations

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

FIGURE 9–2 Hypothesis-Testing Situations in the Comparison of Means



(a) Difference is not significant. The means of the populations are the same. Do not reject H_0: $\mu_1 = \mu_2$ since $\overline{X}_1 - \overline{X}_2$ is not significant.

(b) Difference is significant. The means of the populations are different. Reject H_0: $\mu_1 = \mu_2$ since $\overline{X}_1 - \overline{X}_2$ is significant.

This formula is based on the general format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where $\overline{X}_1 - \overline{X}_2$ is the observed difference, and the expected difference $\mu_1 - \mu_2$ is zero when the null hypothesis is $\mu_1 = \mu_2$, since that is equivalent to $\mu_1 - \mu_2 = 0$. Finally, the standard error of the difference is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

In the comparison of two sample means, the difference may be due to chance, in which case the null hypothesis will not be rejected and the researcher can assume that the means of the populations are basically the same. The difference in this case is not significant. See Figure 9–2(a). On the other hand, if the difference is significant, the null hypothesis is rejected and the researcher can conclude that the population means are different. See Figure 9–2(b).

These tests can also be one-tailed, using the following hypotheses:

Right-tailed			Left-tailed		
H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 > \mu_2$	or	H_0: $\mu_1 - \mu_2 = 0$ H_1: $\mu_1 - \mu_2 > 0$	H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 < \mu_2$	or	H_0: $\mu_1 - \mu_2 = 0$ H_1: $\mu_1 - \mu_2 < 0$

The same critical values used in Section 8–2 are used here. They can be obtained from Table E in Appendix A.

The basic format for hypothesis testing using the traditional method is reviewed here.

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s).

Step 3 Compute the test value.

Step 4 Make the decision.

Step 5 Summarize the results.

EXAMPLE 9–1 Leisure Time

A study using two random samples of 35 people each found that the average amount of time those in the age group of 26–35 years spent per week on leisure activities was 39.6 hours, and those in the age group of 46–55 years spent 35.4 hours. Assume that the population standard deviation for those in the first age group found by previous studies is 6.3 hours, and the population standard deviation of those in the second group found by previous studies was 5.8 hours. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the average times each group spends on leisure activities?

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0\text{: } \mu_1 = \mu_2 \qquad \text{and} \qquad H_1\text{: } \mu_1 \neq \mu_2 \text{ (claim)}$$

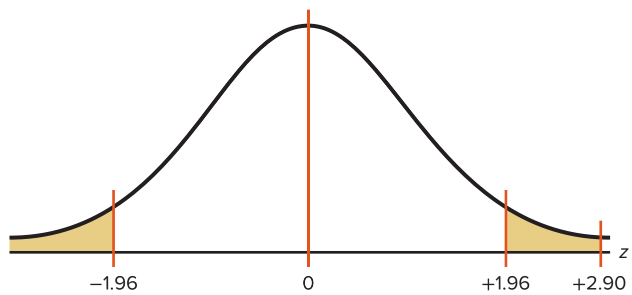
Step 2 Find the critical values. Since $\alpha = 0.05$, the critical values are +1.96 and −1.96.

Step 3 Compute the test value.

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{(39.6 - 35.4) - 0}{\sqrt{\dfrac{6.3^2}{35} + \dfrac{5.8^2}{35}}} = \frac{4.2}{1.447} = 2.90$$

Step 4 Make the decision. Reject the null hypothesis at $\alpha = 0.05$ since $2.90 > 1.96$. See Figure 9–3.

FIGURE 9–3 Critical and Test Values for Example 9–1



Step 5 Summarize the results. There is enough evidence to support the claim that the means are not equal. That is, the average of the times spent on leisure activities is different for the groups.

The P-values for this test can be determined by using the same procedure shown in Section 8–2. For example, if the test value for a two-tailed test is 2.90, then the P-value obtained from Table E is 0.0038. This value is obtained by looking up the area for $z = 2.90$, which is 0.9981. Then 0.9981 is subtracted from 1.0000 to get 0.0019. Finally, this value is doubled to get 0.0038 since the test is two-tailed. If $\alpha = 0.05$, the decision would be to reject the null hypothesis, since P-value $< \alpha$ (that is, $0.0038 < 0.05$). *Note:* The P-value obtained on the TI-84 is 0.0037.

The P-value method for hypothesis testing for this chapter also follows the same format as stated in Chapter 8. The steps are reviewed here.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the P-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 9–2 illustrates these steps.

EXAMPLE 9–2 College Sports Offerings

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A random sample of the number of sports offered by colleges for males and females is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$.

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Source: USA TODAY.

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0: \mu_1 = \mu_2 \qquad \text{and} \qquad H_1: \mu_1 > \mu_2 \text{ (claim)}$$

Step 2 Compute the test value. Using a calculator or the formula in Chapter 3, find the mean for each data set.

For the males $\quad \overline{X}_1 = 8.6 \quad$ and $\quad \sigma_1 = 3.3$

For the females $\quad \overline{X}_2 = 7.9 \quad$ and $\quad \sigma_2 = 3.3$

Substitute in the formula.

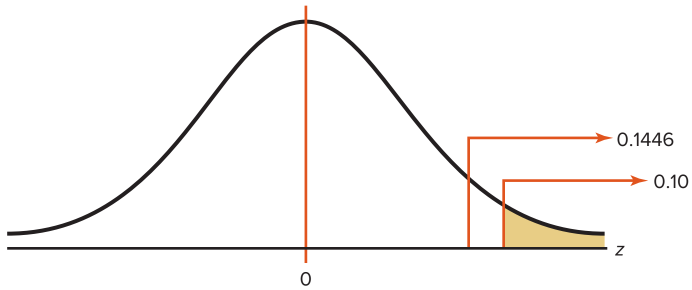
$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{(8.6 - 7.9) - 0}{\sqrt{\dfrac{3.3^2}{50} + \dfrac{3.3^2}{50}}} = 1.06^*$$

Step 3 Find the P-value from Table E. For $z = 1.06$, the area is 0.8554, and $1.0000 - 0.8554 = 0.1446$, or a P-value of 0.1446.

Step 4 Make the decision. Since the P-value is larger than α (that is, $0.1446 > 0.10$), the decision is to not reject the null hypothesis. See Figure 9–4.

Step 5 Summarize the results. There is not enough evidence to support the claim that colleges offer more sports for males than they do for females at the 0.10 level of significance.

FIGURE 9–4 *P*-Value and α Value for Example 9–2



**Note:* Calculator results may differ due to rounding.

Sometimes, the researcher is interested in testing a specific difference in means other than zero. For example, he or she might hypothesize that the nursing students at a community college are, on average, 3.2 years older than those at a university. In this case, the hypotheses are

$$H_0\colon \mu_1 - \mu_2 = 3.2 \qquad \text{and} \qquad H_1\colon \mu_1 - \mu_2 > 3.2$$

The formula for the z test is still

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

where $\mu_1 - \mu_2$ is the hypothesized difference or expected value. In this case, $\mu_1 - \mu_2 = 3.2$.

Confidence intervals for the difference between two means can also be found. When you are hypothesizing a difference of zero, if the confidence interval contains zero, the null hypothesis is not rejected. If the confidence interval does not contain zero, the null hypothesis is rejected.

Confidence intervals for the difference between two means can be found by using this formula:

Formula for the z Confidence Interval for Difference Between Two Means

$$(\overline{X}_1 - \overline{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

EXAMPLE 9–3 Leisure Time

Find the 95% confidence interval for the difference between the means in Example 9–1.

SOLUTION

Substitute in the formula, using $z_{\alpha/2} = 1.96$.

$$(\overline{X}_1 - \overline{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(39.6 - 35.4) - 1.96\sqrt{\frac{6.3^2}{35} + \frac{5.8^2}{35}} < \mu_1 - \mu_2 < (39.6 - 35.4) + 1.96\sqrt{\frac{6.3^2}{35} + \frac{5.8^2}{35}}$$

$$4.2 - 2.8 < \mu_1 - \mu_2 < 4.2 + 2.8$$

$$1.4 < \mu_1 - \mu_2 < 7.0$$

(The confidence interval obtained from the TI-84 is $1.363 < \mu_1 - \mu_2 < 7.037$.)

Since the confidence interval does not contain zero, the decision is to reject the null hypothesis, which agrees with the previous result.

Applying the Concepts 9–1

Home Runs

For a sports radio talk show, you are asked to research the question whether more home runs are hit by players in the National League or by players in the American League. You decide to use the home run leaders from each league for a 40-year period as your data. The numbers are shown.

National League									
47	49	73	50	65	70	49	47	40	43
46	35	38	40	47	39	49	37	37	36
40	37	31	48	48	45	52	38	38	36
44	40	48	45	45	36	39	44	52	47
American League									
47	57	52	47	48	56	56	52	50	40
46	43	44	51	36	42	49	49	40	43
39	39	22	41	45	46	39	32	36	32
32	32	37	33	44	49	44	44	49	32

Using the data given, answer the following questions.

1. Define a population.
2. What kind of sample was used?
3. Do you feel that the samples are representative?
4. What are your hypotheses?
5. What significance level will you use?
6. What statistical test will you use?
7. What are the test results? (Assume $\sigma_1 = 8.8$ and $\sigma_2 = 7.8$.)
8. What is your decision?
9. What can you conclude?
10. Do you feel that using the data given really answers the original question asked?
11. What other data might be used to answer the question?

See page 544 for the answers.

Exercises 9–1

1. Explain the difference between testing a single mean and testing the difference between two means.

2. When a researcher selects all possible pairs of samples from a population in order to find the difference between the means of each pair, what will be the shape of the distribution of the differences when the original distributions are normally distributed? What will be the mean of the distribution? What will be the standard deviation of the distribution?

3. What three assumptions must be met when you are using the z test to test differences between two means when σ_1 and σ_2 are known?

4. Show two different ways to state that the means of two populations are equal.

For Exercises 5 through 16, perform each of the following steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

5. Recreational Time A researcher wishes to see if there is a difference between the mean number of hours per week that a family with no children participates in recreational activities and a family with children participates in recreational activities. She selects two random samples and the data are shown. At $\alpha = 0.10$, is there a difference between the means?

	$\bar{X}$	σ	n
No children	8.6	2.1	36
Children	10.6	2.7	36

6. Teachers' Salaries Teachers' Salaries New York and Massachusetts lead the list of average teacher's salaries.

The New York average is \$76,409 while teachers in Massachusetts make an average annual salary of \$73,195. Random samples of 45 teachers from each state yielded the following.

	Massachusetts	New York
Sample means	\$73,195	\$76,409
Population standard deviation	8,200	7,800

At $\alpha = 0.10$, is there a difference in means of the salaries?

Source: World Almanac.

7. **Commuting Times** The U.S. Census Bureau reports that the average commuting time for citizens of both Baltimore, Maryland, and Miami, Florida, is approximately 29 minutes. To see if their commuting times appear to be any different in the winter, random samples of 40 drivers were surveyed in each city and the average commuting time for the month of January was calculated for both cities. The results are shown. At the 0.05 level of significance, can it be concluded that the commuting times are different in the winter?

	Miami	Baltimore
Sample size	40	40
Sample mean	28.5 min	35.2 min
Population standard deviation	7.2 min	9.1 min

Source: www.census.gov

8. **Heights of 9-Year-Olds** At age 9 the average weight (21.3 kg) and the average height (124.5 cm) for both boys and girls are exactly the same. A random sample of 9-year-olds yielded these results. At $\alpha = 0.05$, do the data support the given claim that there is a difference in heights?

	Boys	Girls
Sample size	60	50
Mean height, cm	123.5	126.2
Population variance	98	120

Source: www.healthepic.com

9. **Length of Hospital Stays** The average length of "short hospital stays" for men is slightly longer than that for women, 5.2 days versus 4.5 days. A random sample of recent hospital stays for both men and women revealed the following. At $\alpha = 0.01$, is there sufficient evidence to conclude that the average hospital stay for men is longer than the average hospital stay for women?

	Men	Women
Sample size	32	30
Sample mean	5.5 days	4.2 days
Population standard deviation	1.2 days	1.5 days

Source: www.cdc.gov/nchs

10. **Home Prices** A real estate agent compares the selling prices of randomly selected homes in two municipalities in southwestern Pennsylvania to see if there is a difference. The results of the study are shown. Is there enough evidence to reject the claim that the average cost of a home in both locations is the same? Use $\alpha = 0.01$.

Scott	Ligonier
$\bar{X}_1 = \$93,430^*$	$\bar{X}_2 = \$98,043^*$
$\sigma_1 = \$5602$	$\sigma_2 = \$4731$
$n_1 = 35$	$n_2 = 40$

*Based on information from RealSTATs.

11. **Manual Dexterity Differences** A researcher wishes to see if there is a difference in the manual dexterity of athletes and that of band members. Two random samples of 30 are selected from each group and are given a manual dexterity test. The mean of the athletes' test was 87, and the mean of the band members' test was 92. The population standard deviation for the test is 7.2. At $\alpha = 0.01$, is there a significant difference in the mean scores?

12. **ACT Scores** A random survey of 1000 students nationwide showed a mean ACT score of 21.4. Ohio was not used. A survey of 500 randomly selected Ohio scores showed a mean of 20.8. If the population standard deviation is 3, can we conclude that Ohio is below the national average? Use $\alpha = 0.05$.

Source: Report of WFIN radio.

13. **Per Capita Income** The average per capita income for Wisconsin is reported to be \$37,314, and for South Dakota it is \$37,375—almost the same thing. A random sample of 50 workers from each state indicated the following sample statistics.

	Wisconsin	South Dakota
Size	50	50
Mean	\$40,275	\$38,750
Population standard deviation	\$10,500	\$12,500

At $\alpha = 0.05$, can we conclude a difference in means of the personal incomes?

Source: New York Times Almanac.

14. **Monthly Social Security Benefits** The average monthly Social Security benefit for a specific year for retired workers was \$954.90 and for disabled workers was \$894.10. Researchers used data from the Social Security records to test the claim that the difference in monthly benefits between the two groups was greater than \$30. Based on the following information, can the researchers' claim be supported at the 0.05 level of significance?

	Retired	Disabled
Sample size	60	60
Mean benefit	\$960.50	\$902.89
Population standard deviation	\$98	\$101

Source: New York Times Almanac.

15. Self-Esteem Scores In a study of a group of women science majors who remained in their profession and a group who left their profession within a few months of graduation, the researchers collected the data shown here on a self-esteem questionnaire. At $\alpha = 0.05$, can it be concluded that there is a difference in the self-esteem scores of the two groups? Use the P-value method.

Leavers	**Stayers**
$\overline{X}_1 = 3.05$	$\overline{X}_2 = 2.96$
$\sigma_1 = 0.75$	$\sigma_2 = 0.75$
$n_1 = 103$	$n_2 = 225$

Source: Paula Rayman and Belle Brett, "Women Science Majors: What Makes a Difference in Persistence after Graduation?" *The Journal of Higher Education.*

16. Ages of College Students The dean of students wants to see whether there is a significant difference in ages of resident students and commuting students. She selects a random sample of 50 students from each group. The ages are shown here. At $\alpha = 0.05$, decide if there is enough evidence to reject the claim of no difference in the ages of the two groups. Use the P-value method. Assume $\sigma_1 = 3.68$ and $\sigma_2 = 4.7$.

Resident students

22	25	27	23	26	28	26	24
25	20	26	24	27	26	18	19
18	30	26	18	18	19	32	23
19	19	18	29	19	22	18	22
26	19	19	21	23	18	20	18
22	21	19	21	21	22	18	20
19	23						

Commuter students

18	20	19	18	22	25	24	35
23	18	23	22	28	25	20	24
26	30	22	22	22	21	18	20
19	26	35	19	19	18	19	32
29	23	21	19	36	27	27	20
20	21	18	19	23	20	19	19
20	25						

17. Working Breath Rate Two random samples of 32 individuals were selected. One sample participated in an activity which simulates hard work. The average breath rate of these individuals was 21 breaths per minute. The other sample did some normal walking. The mean breath rate of these individuals was 14. Find the 90% confidence interval of the difference in the breath rates if the population standard deviation was 4.2 for breath rate per minute.

18. Traveling Distances Find the 95% confidence interval of the difference in the distance that day students travel to school and the distance evening students travel to school. Two random samples of 40 students are taken, and the data are shown. Find the 95% confidence interval of the difference in the means.

	$\overline{X}$	σ	n
Day students	4.7	1.5	40
Evening Students	6.2	1.7	40

19. Literacy Scores Adults aged 16 or older were assessed in three types of literacy: prose, document, and quantitative. The scores in document literacy were the same for 19- to 24-year-olds and for 40- to 49-year-olds. A random sample of scores from a later year showed the following statistics.

Age group	**Mean score**	**Population standard deviation**	**Sample size**
19–24	280	56.2	40
40–49	315	52.1	35

Construct a 95% confidence interval for the true difference in mean scores for these two groups. What does your interval say about the claim that there is no difference in mean scores?

Source: www.nces.ed.gov

20. Age Differences In a large hospital, a nursing director selected a random sample of 30 registered nurses and found that the mean of their ages was 30.2. The population standard deviation for the ages is 5.6. She selected a random sample of 40 nursing assistants and found the mean of their ages was 31.7. The population standard deviation of the ages for the assistants is 4.3. Find the 99% confidence interval of the differences in the ages.

21. Television Watching The average number of hours of television watched per week by women over age 55 is 48 hours. Men over age 55 watch an average of 43 hours of television per week. Random samples of 40 men and 40 women from a large retirement community yielded the following results. At the 0.01 level of significance, can it be concluded that women watch more television per week than men?

	Sample size	**Mean**	**Population standard deviation**
Women	40	48.2	5.6
Men	40	44.3	4.5

Source: World Almanac 2012.

22. Commuting Times for College Students The mean travel time to work for Americans is 25.3 minutes. An employment agency wanted to test the mean commuting times for college graduates and those with only some college. Thirty-five college graduates spent a mean time of 40.5 minutes commuting to work with a population variance of 67.24. Thirty workers who had completed some college had a mean commuting time of 34.8 minutes with a population variance of 39.69. At the 0.05 level of significance, can a difference in means be concluded?

Source: World Almanac 2012.

23. Store Sales A company owned two small Bath and Body Goods stores in different cities. It was desired to see if there was a difference in their mean daily sales. The following results were obtained from a random sample of daily sales over a six-week period. At $\alpha = 0.01$, can a difference in sales be concluded? Use the P-value method.

Store	Mean	Population standard deviation	Sample size
A	\$995	\$120	30
B	1120	250	30

24. Home Prices According to the almanac, the average sales price of a single-family home in the metropolitan Dallas/Ft. Worth/Irving, Texas, area is \$215,200. The average home price in Orlando, Florida, is \$198,000. The mean of a random sample of 45 homes in the Texas metroplex was \$216,000 with a population standard deviation of \$30,000. In the Orlando, Florida, area a sample of 40 homes had a mean price of \$203,000 with a population standard deviation of \$32,500. At the 0.05 level of significance, can it be concluded that the mean price in Dallas exceeds the mean price in Orlando? Use the P-value method.

Source: World Almanac.

Extending the Concepts

25. Exam Scores at Private and Public Schools A researcher claims that students in a private school have exam scores that are at most 8 points higher than those of students in public schools. Random samples of 60 students from each type of school are selected and given an exam. The results are shown. At $\alpha = 0.05$, test the claim.

Private school	Public school
$\bar{X}_1 = 110$	$\bar{X}_2 = 104$
$\sigma_1 = 15$	$\sigma_2 = 15$
$n_1 = 60$	$n_2 = 60$

26. Sale Prices for Houses The average sales price of new one-family houses in the Midwest is \$250,000 and in the South is \$253,400. A random sample of 40 houses in each region was examined with the following results. At the 0.05 level of significance, can it be concluded that the difference in mean sales price for the two regions is greater than \$3400?

	South	Midwest
Sample size	40	40
Sample mean	\$261,500	\$248,200
Population standard deviation	\$10,500	\$12,000

Source: New York Times Almanac.

27. Average Earnings for College Graduates The average earnings of year-round full-time workers with bachelor's degrees or more is \$88,641 for men and \$58,000 for women—a difference of slightly over \$30,000 a year. One hundred of each were randomly sampled, resulting in a sample mean of \$90,200 for men, and the population standard deviation is \$15,000; and a mean of \$57,800 for women, and the population standard deviation is \$12,800. At the 0.01 level of significance, can it be concluded that the difference in means is not \$30,000?

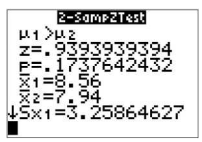
Source: New York Times Almanac.

Technology Step by Step

TI-84 Plus Step by Step

Hypothesis Test for the Difference Between Two Means and z Distribution (Data)

This refers to Example 9–2 in the text.

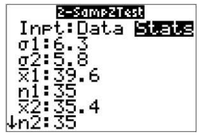


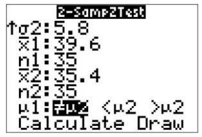
Example TI9–1

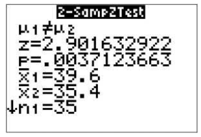
1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **3** for 2-SampZTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Means and *z* Distribution (Statistics)

This refers to Example 9–1 in the text.







Example TI9–2

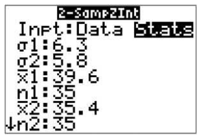
1. Press **STAT** and move the cursor to TESTS.
2. Press **3** for 2-SampZTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

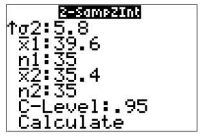
Confidence Interval for the Difference Between Two Means and *z* Distribution (Data)

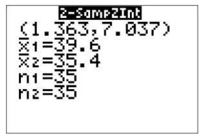
1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **9** for 2-SampZInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *z* Distribution (Statistics)

This refers to Example 9–3 in the text.







Example TI9–3

1. Press **STAT** and move the cursor to TESTS.
2. Press **9** for 2-SampZInt.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to Calculate and press **ENTER.**

EXCEL
Step by Step

z Test for the Difference Between Two Means

Excel has a two-sample *z* test included in the Data Analysis Add-in. To perform a *z* test for the difference between the means of two populations, given two independent samples, do this:

1. Enter the first sample data set into column A.
2. Enter the second sample data set into column B.
3. Select the Data tab from the toolbar. Then select Data Analysis.
4. In the Analysis Tools box, select *z* test: Two Sample for Means.
5. Type the ranges for the data in columns A and B and type a value (usually 0) for the Hypothesized Mean Difference.
6. Type the known population variances in for Variable 1 Variance (known) and Variable 2 Variance (known).
7. Specify the confidence level Alpha.
8. Specify a location for the output, and click [OK].

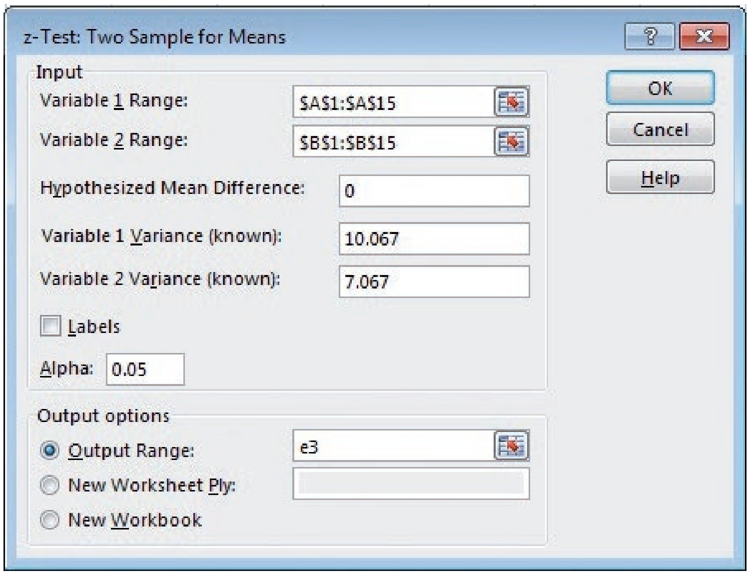
Example XL9–1

Test the claim that the two population means are equal, using the sample data provided here, at $\alpha = 0.05$. Assume the population variances are $\sigma_A^2 = 10.067$ and $\sigma_B^2 = 7.067$.

Set A	10	2	15	18	13	15	16	14	18	12	15	15	14	18	16
Set B	5	8	10	9	9	11	12	16	8	8	9	10	11	7	6

The two-sample *z* test dialog box is shown (before the variances are entered); the results appear in the table that Excel generates. Note that the *P*-value and critical *z* value are provided for both the one-tailed test and the two-tailed test. The *P*-values here are expressed in scientific notation: 7.09045E-06 = 7.09045×10^{-6} = 0.00000709045. Because this value is less than 0.05, we reject the null hypothesis and conclude that the population means are not equal.

Two-Sample *z* Test Dialog Box



z-Test: Two Sample for Means		
	Variable 1	*Variable 2*
Mean	14.06666667	9.266666667
Known Variance	10.067	7.067
Observations	15	15
Hypothesized Mean Difference	0	
z	4.491149228	
P(Z<=z) one-tail	3.54522E-06	
z Critical one-tail	1.644853	
P(Z<=z) two-tail	7.09045E-06	
z Critical two-tail	1.959961082	

9–2 Testing the Difference Between Two Means of Independent Samples: Using the *t* Test

OBJECTIVE 2

Test the difference between two means for independent samples, using the *t* test.

In Section 9–1, the *z* test was used to test the difference between two means when the population standard deviations were known and the variables were normally or approximately normally distributed, or when both sample sizes were greater than or equal to 30. In many situations, however, these conditions cannot be met—that is, the population standard deviations are not known. In these cases, a *t* test is used to test the difference between means when the two samples are independent and when the samples are taken from two normally or approximately normally distributed populations. Samples are **independent samples** when they are not related. Also it will be assumed that the variances are not equal.

Formula for the *t* Test for Testing the Difference Between Two Means, Independent Samples

Variances are assumed to be unequal:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where the degrees of freedom are equal to the smaller of $n_1 - 1$ or $n_2 - 1$.

The formula

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where $\overline{X}_1 - \overline{X}_2$ is the observed difference between sample means and where the expected value $\mu_1 - \mu_2$ is equal to zero when no difference between population means is hypothesized. The denominator $\sqrt{s_1^2/n_1 + s_2^2/n_2}$ is the standard error of the difference between two means. This formula is similar to the one used when σ_1 and σ_2 are known; but when we use this t test, σ_1 and σ_2 are unknown, so s_1 and s_2 are used in the formula in place of σ_1 and σ_2. Since mathematical derivation of the standard error is somewhat complicated, it will be omitted here.

Before you can use the testing methods to determine whether two independent sample means differ when σ_1 and σ_2 are unknown, the following assumptions must be met.

Assumptions for the t Test for Two Independent Means When σ_1 and σ_2 Are Unknown

1. The samples are random samples.
2. The sample data are independent of one another.
3. When the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

In this book, the assumptions will be stated in the exercises; however, when encountering statistics in other situations, you must check to see that these assumptions have been met before proceeding.

Again the hypothesis test here follows the same steps as those in Section 9–1; however, the formula uses s_1 and s_2 and Table F to get the critical values.

EXAMPLE 9–4 Work Absences

A study was done to see if there is a difference between the number of sick days men take and the number of sick days women take. A random sample of 9 men found that the mean of the number of sick days taken was 5.5. The standard deviation of the sample was 1.23. A random sample of 7 women found that the mean was 4.3 days and a standard deviation of 1.19 days. At $\alpha = 0.05$, can it be concluded that there is a difference in the means?

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0\colon \mu_1 = \mu_2 \qquad \text{and} \qquad H_1\colon \mu_1 \neq \mu_2 \text{ (claim)}$$

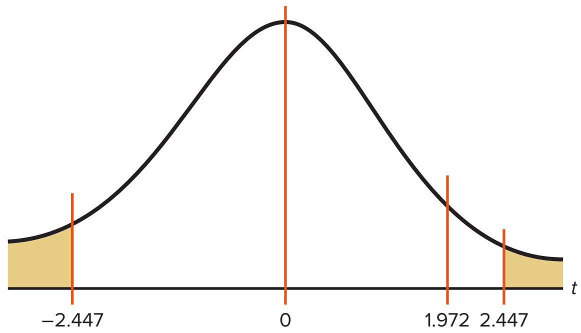
Step 2 Find the critical values. Since the test is two-tailed and $\alpha = 0.05$, the degrees of freedom are the smaller of $n_1 - 1$ and $n_2 - 1$. In this case, $n_1 - 1 = 9 - 1 = 8$ and $n_2 - 1 = 7 - 1 = 6$. So d.f. = 6. From Table F, the critical values are +2.447 and −2.447.

Step 3 Compute the test value.

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(5.5 - 4.3) - 0}{\sqrt{\frac{1.23^2}{9} + \frac{1.19^2}{7}}} = 1.972$$

Step 4 Make the decision. Do not reject the null hypothesis since 1.972 < 2.447. See Figure 9–5.

FIGURE 9–5 Critical and Test Values for Example 9–4



Step 5 Summarize the results. There is not enough evidence to support the claim that the means are different.

When raw data are given in the exercises, use your calculator or the formulas in Chapter 3 to find the means and variances for the data sets. Then follow the procedures shown in this section to test the hypotheses.

Confidence intervals can also be found for the difference of two means with this formula:

Confidence Intervals for the Difference of Two Means: Independent Samples

Variances assumed to be unequal:

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\text{d.f.} = \text{smaller value of } n_1 - 1 \text{ or } n_2 - 1$$

EXAMPLE 9–5

Find the 95% confidence interval for the data in Example 9–4.

SOLUTION

Substitute in the formula.

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(5.5 - 4.3) - 2.447\sqrt{\frac{1.23^2}{9} + \frac{1.19^2}{7}} < \mu_1 - \mu_2 < (5.5 - 4.3) + 2.447\sqrt{\frac{1.23^2}{9} + \frac{1.19^2}{7}}$$

$$1.2 - 1.489 < \mu_1 - \mu_2 < 1.2 + 1.489$$

$$-0.289 < \mu_1 - \mu_2 < 2.689$$

Since 0 is contained in the interval, there is not enough evidence to support the claim that the means are different.

In many statistical software packages, a different method is used to compute the degrees of freedom for this t test. They are determined by the formula

$$\text{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

This formula will not be used in this textbook.

There are actually two different options for the use of t tests. *One option is used when the variances of the populations are not equal, and the other option is used when the variances are equal.* To determine whether two sample variances are equal, the researcher can use an F test, as shown in Section 9–5.

When the variances are assumed to be equal, this formula is used and

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

For the numerator, the terms are the same as in the previously given formula. However, a note of explanation is needed for the denominator of the second test statistic. Since both populations are assumed to have the same variance, the standard error is computed with what is called a pooled estimate of the variance. A **pooled estimate of the variance** is a weighted average of the variance using the two sample variances and the *degrees of freedom* of each variance as the weights. Again, since the algebraic derivation of the standard error is somewhat complicated, it is omitted.

Note, however, that not all statisticians are in agreement about using the F test before using the t test. Some believe that conducting the F and t tests at the same level of significance will change the overall level of significance of the t test. Their reasons are beyond the scope of this text. Because of this, we will assume that $\sigma_1 \neq \sigma_2$ in this text.

Applying the Concepts 9–2

Too Long on the Telephone

A company collects data on the lengths of telephone calls made by employees in two different divisions. The sample mean and the sample standard deviation for the sales division are 10.26 and 8.56, respectively. The sample mean and sample standard deviation for the shipping and receiving division are 6.93 and 4.93, respectively. A hypothesis test was run, and the computer output follows.

Degrees of freedom = 56
Confidence interval limits = −0.18979, 6.84979
Test statistic t = 1.89566
Critical value t = −2.0037, 2.0037
P-value = 0.06317
Significance level = 0.05

1. Are the samples independent or dependent?
2. Which number from the output is compared to the significance level to check if the null hypothesis should be rejected?
3. Which number from the output gives the probability of a type I error that is calculated from the sample data?
4. Was a right-, left-, or two-tailed test done? Why?
5. What are your conclusions?
6. What would your conclusions be if the level of significance were initially set at 0.10?

See pages 544–545 for the answers.

Exercises 9–2

For these exercises, perform each of these steps. Assume that all variables are normally or approximately normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified and assume the variances are unequal.

1. **Waterfall Heights** Is there a significant difference at $\alpha = 0.10$ in the mean heights in feet of waterfalls in Europe and the ones in Asia? The data are shown.

Europe			Asia		
487	1246	1385	614	722	964
470	1312	984	1137	320	830
900	345	820	350	722	1904

Source: World Almanac and Book of Facts.

2. **Tax-Exempt Properties** A tax collector wishes to see if the mean values of the tax-exempt properties are different for two cities. The values of the tax-exempt properties for the two random samples are shown. The data are given in millions of dollars. At $\alpha = 0.05$, is there enough evidence to support the tax collector's claim that the means are different?

City A				City B			
113	22	14	8	82	11	5	15
25	23	23	30	295	50	12	9
44	11	19	7	12	68	81	2
31	19	5	2	20	16	4	5

3. **Noise Levels in Hospitals** The mean noise level of 20 randomly selected areas designated as "casualty doors" was 63.1 dBA, and the sample standard deviation is 4.1 dBA. The mean noise level for 24 randomly selected areas designated as operating theaters was 56.3 dBA, and the sample standard deviation was 7.5 dBA. At $\alpha = 0.05$, can it be concluded that there is a difference in the means?

4. **Ages of Gamblers** The mean age of a random sample of 25 people who were playing the slot machines is 48.7 years, and the standard deviation is 6.8 years. The mean age of a random sample of 35 people who were playing roulette is 55.3 with a standard deviation of 3.2 years. Can it be concluded at $\alpha = 0.05$ that the mean age of those playing the slot machines is less than those playing roulette?

5. **Carbohydrates in Candies** The number of grams of carbohydrates contained in 1-ounce servings of randomly selected chocolate and nonchocolate candy is listed here. Is there sufficient evidence to conclude that the difference in the means is statistically significant? Use $\alpha = 0.10$.

Chocolate:	29	25	17	36	41	25	32	29
	38	34	24	27	29			
Nonchocolate:	41	41	37	29	30	38	39	10
	29	55	29					

Source: The Doctor's Pocket Calorie, Fat, and Carbohydrate Counter.

6. **Weights of Vacuum Cleaners** Upright vacuum cleaners have either a hard body type or a soft body type. Shown are the weights in pounds of a random sample of each type. At $\alpha = 0.05$, can it be concluded that the means of the weights are different?

Hard body types				Soft body types			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

7. **Weights of Running Shoes** The weights in ounces of a sample of running shoes for men and women are shown. Test the claim that the means are different. Use the P-value method with $\alpha = 0.05$.

Men		Women		
10.4	12.6	10.6	10.2	8.8
11.1	14.7	9.6	9.5	9.5
10.8	12.9	10.1	11.2	9.3
11.7	13.3	9.4	10.3	9.5
12.8	14.5	9.8	10.3	11.0

8. **Teacher Salaries** A researcher claims that the mean of the salaries of elementary school teachers is greater than the mean of the salaries of secondary school teachers in a large school district. The mean of the salaries of a random sample of 26 elementary school teachers is \$48,256, and the sample standard deviation is \$3,912.40. The mean of the salaries of a random sample of 24 secondary school teachers is \$45,633. The sample standard deviation is \$5533. At $\alpha = 0.05$, can it be concluded that the mean of the salaries of the elementary school teachers is greater than the mean of the salaries of the secondary school teachers? Use the P-value method.

9. Find the 90% confidence for the difference of the means in Exercise 1 of this section.

10. Find the 95% confidence interval for the difference of the means in Exercise 6 of this section.

11. **Hours Spent Watching Television** According to Nielsen Media Research, children (ages 2–11) spend an average of 21 hours 30 minutes watching television per week while teens (ages 12–17) spend an average of 20 hours 40 minutes. Based on the sample statistics shown, is there sufficient evidence to conclude a

difference in average television watching times between the two groups? Use $\alpha = 0.01$.

	Children	Teens
Sample mean	22.45	18.50
Sample variance	16.4	18.2
Sample size	15	15

Source: Time Almanac.

12. Professional Golfers' Earnings Two random samples of earnings of professional golfers were selected. One sample was taken from the Professional Golfers Association, and the other was taken from the Ladies Professional Golfers Association. At $\alpha = 0.05$, is there a difference in the means? The data are in thousands of dollars.

PGA

446	1147	1344	9188	5687
10,508	4910	8553	7573	375

LPGA

48	76	122	466	863
100	1876	2029	4364	2921

13. Cyber School Enrollment The data show the number of students attending cyber charter schools in Allegheny County and the number of students attending cyber schools in counties surrounding Allegheny County. At $\alpha = 0.01$, is there enough evidence to support the claim that the average number of students in school districts in Allegheny County who attend cyber schools is greater than those who attend cyber schools in school districts outside Allegheny County? Give a factor that should be considered in interpreting this answer.

Allegheny County						Outside Allegheny County					
25	75	38	41	27	32	57	25	38	14	10	29

Source: Pittsburgh Tribune-Review.

14. Hockey's Highest Scorers The number of points held by random samples of the NHL's highest scorers for both the Eastern Conference and the Western Conference is shown. At $\alpha = 0.05$, can it be concluded that there is a difference in means based on these data?

Eastern Conference				Western Conference			
83	60	75	58	77	59	72	58
78	59	70	58	37	57	66	55
62	61	59		61			

Source: www.foxsports.com

15. Hospital Stays for Maternity Patients Health Care Knowledge Systems reported that an insured woman spends on average 2.3 days in the hospital for a routine childbirth, while an uninsured woman spends on average 1.9 days. Assume two random samples of 16 women each were used in both samples. The standard deviation of the first sample is equal to 0.6 day, and the standard deviation of the second sample is 0.3 day. At $\alpha = 0.01$, test the claim that the means are equal. Find the 99% confidence interval for the differences of the means. Use the P-value method.

Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

16. Ages of Homes Whiting, Indiana, leads the "Top 100 Cities with the Oldest Houses" list with the average age of houses being 66.4 years. Farther down the list resides Franklin, Pennsylvania, with an average house age of 59.4 years. Researchers selected a random sample of 20 houses in each city and obtained the following statistics. At $\alpha = 0.05$, can it be concluded that the houses in Whiting are older? Use the P-value method.

	Whiting	Franklin
Mean age	62.1 years	55.6 years
Standard deviation	5.4 years	3.9 years

Source: www.city-data.com

17. Medical School Enrollments A random sample of enrollments from medical schools that specialize in research and from those that are noted for primary care is listed. Find the 90% confidence interval for the difference in the means.

Research				Primary care			
474	577	605	663	783	605	427	728
783	467	670	414	546	474	371	107
813	443	565	696	442	587	293	277
692	694	277	419	662	555	527	320
884							

Source: U.S. News & World Report Best Graduate Schools.

18. Out-of-State Tuitions The out-of-state tuitions (in dollars) for random samples of both public and private four-year colleges in a New England state are listed. Find the 95% confidence interval for the difference in the means.

Private		Public	
13,600	13,495	7,050	9,000
16,590	17,300	6,450	9,758
23,400	12,500	7,050	7,871
		16,100	

Source: New York Times Almanac.

19. Gasoline Prices A random sample of monthly gasoline prices was taken from 2011 and from 2015. The samples are shown. Using $\alpha = 0.01$, can it be concluded that gasoline cost more in 2015? Use the P-value method.

2011	2.02	2.47	2.50	2.70	3.13	2.56	
2015	2.36	2.46	2.63	2.76	3.00	2.85	2.77

20. Miniature Golf Scores A large group of friends went miniature golfing together at a par 54 course and decided to play on two teams. A random sample of scores from each of the two teams is shown. At $\alpha = 0.05$, is there a difference in mean scores between the two teams? Use the P-value method.

Team 1	61	44	52	47	56	63	62	55
Team 2	56	40	42	58	48	52	51	

21. Home Runs Two random samples of professional baseball players were selected and the number of home runs hit were recorded. One sample was obtained from the National League, and the other sample was obtained from the American League. At $\alpha = 0.10$, is there a difference in the means?

National League				American League			
18	4	8	2	6	11	18	11
9	2	6	5	3	12	25	4
6	8	29	25	24	9	12	5

22. Batting Averages Random samples of batting averages from the leaders in both leagues prior to the All-Star break are shown. At the 0.05 level of significance, can a difference be concluded?

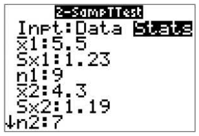
National	.360	.654	.652	.338	.313	.309
American	.340	.332	.317	.316	.314	.306

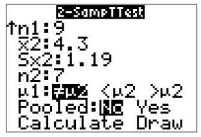
Technology Step by Step

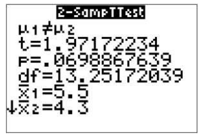
TI-84 Plus Step by Step

Hypothesis Test for the Difference Between Two Means and *t* Distribution (Statistics)

This refers to Example 9–4 in the text.







Example TI9–4

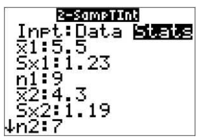
1. Press **STAT** and move the cursor to TESTS.
2. Press **4** for 2-SampTTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

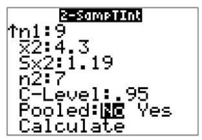
Confidence Interval for the Difference Between Two Means and *t* Distribution (Data)

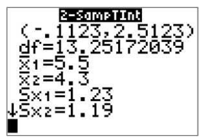
1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **0** for 2-SampTInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *t* Distribution (Statistics)

This refers to Example 9–5 in the text.







Example TI9–5

1. Press **STAT** and move the cursor to TESTS.
2. Press **0** for 2-SampTInt.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

EXCEL Step by Step

Testing the Difference Between Two Means: Independent Samples

Excel has a two-sample *t* test included in the Data Analysis Add-in. The following example shows how to perform a *t* test for the difference between two means.

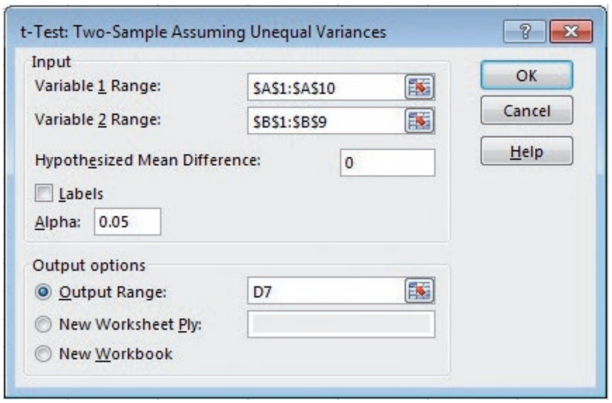
Example XL9–2

Test the claim that there is no difference between population means based on these sample data. Assume the population variances are not equal. Use $\alpha = 0.05$.

Set A	32	38	37	36	36	34	39	36	37	42
Set B	30	36	35	36	31	34	37	33	32	

1. Enter the 10-number data set A into column A.
2. Under the Home tab, select Format > enter the 9-number data set B into column B.
3. Select the Data tab from the toolbar. Then select Data Analysis.
4. In the Data Analysis box, under Analysis Tools select *t*-test: Two-Sample Assuming Unequal Variances, and click [OK].
5. In Input, type in the Variable 1 Range: **A1:A10** and the Variable 2 Range: **B1:B9.**
6. Type **0** for the Hypothesized Mean Difference.
7. Type **0.05** for Alpha.
8. In Output options, type D7 for the Output Range, then click [OK].

Two-Sample *t* Test in Excel



t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	Variable 2
Mean	36.7	33.77777778
Variance	7.344444444	5.944444444
Observations	10	9
Hypothesized Mean Difference	0	
df	17	
t Stat	2.474205364	
P(T<=t) one-tail	0.012095	
t Critical one-tail	1.739606716	
P(T<=t) two-tail	0.024189999	
t Critical two-tail	2.109815559	

Note: You may need to increase the column width to see all the results. To do this:

1. Highlight the columns D, E, and F.
2. Select **Format>AutoFit** Column Width.

The output reports both one- and two-tailed *P*-values.

MINITAB
Step by Step

Test the Difference Between Two Means: Independent Samples*

MINITAB will calculate the test statistic and *P*-value for differences between the means for two populations when the population standard deviations are unknown.

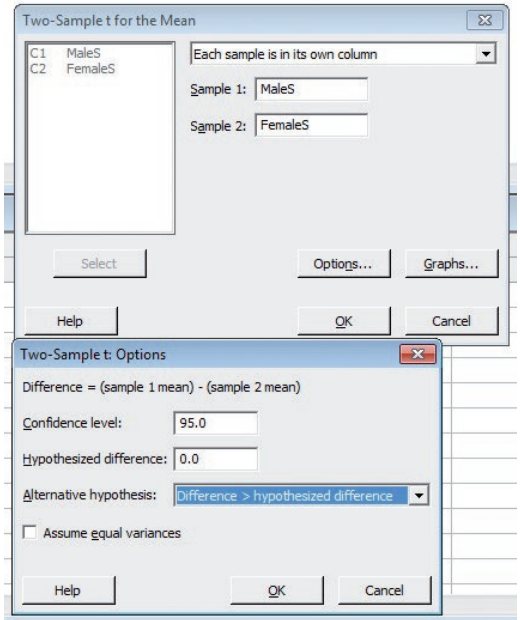
For Example 9–2, is the average number of sports for men higher than the average number for women?

1. Enter the data for Example 9–2 into C1 and C2. Name the columns **MaleS** and **FemaleS.**
2. Select **Stat>Basic Statistics>2-Sample t.**
3. Select Each sample is in its own column from the drop down menu.

*MINITAB does not calculate a *z* test statistic. This statistic can be used instead.

There is one sample in each column.

4. Click in the box for Sample 1. Double-click C1 MaleS in the list.
5. Click in the box for Sample 2, then double-click C2 FemaleS in the list. Do not check the box for Assume equal variances. MINITAB will use the large sample formula. The completed dialog box is shown.
6. Click [Options].
   a) Type in **90** for the Confidence level and **0** for the Hypothesized difference.
   b) Select Difference > hypothesized difference for the Alternative hypothesis. Make sure that Assume equal variances is not checked.
7. Click [OK] twice. Since the P-value is greater than the significance level, 0.172 > 0.1, do not reject the null hypothesis.



Two-Sample T-Test and CI: MaleS, FemaleS

Two–sample T for MaleS vs FemaleS

	N	Mean	StDev	SE Mean
MaleS	50	8.56	3.26	0.46
FemaleS	50	7.94	3.27	0.46

Difference = μ (MaleS) − μ (FemaleS)
Estimate for difference: 0.620
95% lower bound for difference: −0.464
T-Test of difference = 0 (vs >): T–Value = 0.95 P-Value = 0.172 DF = 97

9–3 Testing the Difference Between Two Means: Dependent Samples

OBJECTIVE 3

Test the difference between two means for dependent samples.

In Section 9–1, the z test was used to compare two sample means when the samples were independent and σ_1 and σ_2 were known. In Section 9–2, the t test was used to compare two sample means when the samples were independent. In this section, a different version of the t test is explained. This version is used when the samples are dependent. Samples are considered to be **dependent samples** when the subjects are paired or matched in some way. Dependent samples are sometimes called matched-pair samples.

For example, suppose a medical researcher wants to see whether a drug will affect the reaction time of its users. To test this hypothesis, the researcher must pretest the subjects in the sample. That is, they are given a test to ascertain their normal reaction times. Then after taking the drug, the subjects are tested again, using a posttest. Finally, the means of the two tests are compared to see whether there is a difference. Since the same subjects are used in both cases, the samples are *related;* subjects scoring high on the pretest will generally score high on the posttest, even after consuming the drug. Likewise, those scoring lower on the pretest will tend to score lower on the posttest. To take this effect into account, the researcher employs a t test, using the differences between the pretest values and the posttest values. Thus, only the gain or loss in values is compared.

Here are some other examples of dependent samples. A researcher may want to design an SAT preparation course to help students raise their test scores the second time they take the SAT. Hence, the differences between the two exams are compared. A medical specialist may want to see whether a new counseling program will help subjects lose weight. Therefore, the preweights of the subjects will be compared with the postweights.

Besides samples in which the same subjects are used in a pre-post situation, there are other cases where the samples are considered dependent. For example, students might be matched or paired according to some variable that is pertinent to the study; then one student is assigned to one group, and the other student is assigned to a second group. For instance, in a study involving learning, students can be selected and paired according to their IQs. That is, two students with the same IQ will be paired. Then one will be assigned to one sample group (which might receive instruction by computers), and the other student will be assigned to another sample group (which might receive instruction by the lecture discussion method). These assignments will be done randomly. Since a student's IQ is important to learning, it is a variable that should be controlled. By matching subjects on IQ, the researcher can eliminate the variable's influence, for the most part. Matching, then, helps to reduce type II error by eliminating extraneous variables.

Two notes of caution should be mentioned. First, when subjects are matched according to one variable, the matching process does not eliminate the influence of other variables. Matching students according to IQ does not account for their mathematical ability or their familiarity with computers. Since not all variables influencing a study can be controlled, it is up to the researcher to determine which variables should be used in matching. Second, when the same subjects are used for a pre-post study, sometimes the knowledge that they are participating in a study can influence the results. For example, if people are placed in a special program, they may be more highly motivated to succeed simply because they have been selected to participate; the program itself may have little effect on their success.

When the samples are dependent, a special *t* test for dependent means is used. This test employs the difference in values of the matched pairs. The hypotheses are as follows:

Two-tailed	Left-tailed	Right-tailed
H_0: $\mu_D = 0$ H_1: $\mu_D \neq 0$	H_0: $\mu_D = 0$ H_1: $\mu_D < 0$	H_0: $\mu_D = 0$ H_1: $\mu_D > 0$

Here, μ_D is the symbol for the expected mean of the difference of the matched pairs. The general procedure for finding the test value involves several steps.

First, find the differences of the values of the pairs of data.

$$D = X_1 - X_2$$

Second, find the mean $\overline{D}$ of the differences, using the formula

$$\overline{D} = \frac{\Sigma D}{n}$$

where n is the number of data pairs. Third, find the standard deviation s_D of the differences, using the formula

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Fourth, find the estimated standard error $s_{\overline{D}}$ of the differences, which is

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

Finally, find the test value, using the formula

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

The formula in the final step follows the basic format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where the observed value is the mean of the differences. The expected value μ_D is zero if the hypothesis is $\mu_D = 0$. The standard error of the difference is the standard deviation of the difference, divided by the square root of the sample size. Both populations must be normally or approximately normally distributed.

Before you can use the testing method presented in this section, the following assumptions must be met.

Assumptions for the t Test for Two Means When the Samples Are Dependent

1. The sample or samples are random.
2. The sample data are dependent.
3. When the sample size or sample sizes are less than 30, the population or populations must be normally or approximately normally distributed.

In this book, the assumptions will be stated in the exercises; however, when encountering statistics in other situations, you must check to see that these assumptions have been met before proceeding.

The formulas for this t test are given next.

Formulas for the t Test for Dependent Samples

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

with d.f. $= n - 1$ and where

$$\overline{D} = \frac{\Sigma D}{n} \qquad \text{and} \qquad s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

The steps for this t test are summarized in the Procedure Table.

Procedure Table

Testing the Difference Between Means for Dependent Samples

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s).

Step 3 Compute the test value.

a. Make a table, as shown.

X_1	X_2	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
⋮	⋮	$\Sigma D =$ ______	$\Sigma D^2 =$ ______

b. Find the differences and place the results in column A.

$$D = X_1 - X_2$$

c. Find the mean of the differences.

$$\overline{D} = \frac{\Sigma D}{n}$$

d. Square the differences and place the results in column B. Complete the table.

$$D^2 = (X_1 - X_2)^2$$

e. Find the standard deviation of the differences.

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

f. Find the test value.

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

Step 4 Make the decision.

Step 5 Summarize the results.

Unusual Stat

About 4% of Americans spend at least one night in jail each year.

EXAMPLE 9–6 Bank Deposits

A random sample of nine local banks shows their deposits (in billions of dollars) 3 years ago and their deposits (in billions of dollars) today. At $\alpha = 0.05$, can it be concluded that the average in deposits for the banks is greater today than it was 3 years ago? Use $\alpha = 0.05$. Assume the variable is normally distributed.

Source: SNL Financial.

Bank	1	2	3	4	5	6	7	8	9
3 years ago	11.42	8.41	3.98	7.37	2.28	1.10	1.00	0.9	1.35
Today	16.69	9.44	6.53	5.58	2.92	1.88	1.78	1.5	1.22

SOLUTION

Step 1 State the hypothesis and identify the claim. Since we are interested to see if there has been an increase in deposits, the deposits 3 years ago must be less than the deposits today; hence, the deposits must be significantly less 3 years ago than they are today. Hence, the mean of the differences must be less than zero.

$$H_0\colon \mu_D = 0 \qquad \text{and} \qquad H_1\colon \mu_D < 0 \text{ (claim)}$$

Step 2 Find the critical value. The degrees of freedom are $n - 1$, or $9 - 1 = 8$. Using Table F, the critical value for a left-tailed test with $\alpha = 0.05$ is -1.860.

Step 3 Compute the test value.

a. Make a table.

3 years ago (X_1)	Today (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
11.42	16.69		
8.41	9.44		
3.98	6.53		
7.37	5.58		
2.28	2.92		
1.10	1.88		
1.00	1.78		
0.90	1.50		
1.35	1.22		

b. Find the differences and place the results in column A.

$$\begin{aligned}
11.42 - 16.69 &= -5.27 \\
8.41 - 9.44 &= -1.03 \\
3.98 - 6.53 &= -2.55 \\
7.37 - 5.58 &= +1.79 \\
2.28 - 2.92 &= -0.64 \\
1.10 - 1.88 &= -0.78 \\
1.00 - 1.78 &= -0.78 \\
0.9 - 1.50 &= -0.60 \\
1.35 - 1.22 &= \underline{+0.13} \\
\Sigma D &= -9.73
\end{aligned}$$

c. Find the means of the differences.

$$\overline{D} = \frac{\Sigma D}{n} = \frac{-9.73}{9} = -1.081$$

d. Square the differences and place the results in column B.

$$
\begin{aligned}
(-5.27)^2 &= 27.7729 \\
(-1.03)^2 &= 1.0609 \\
(-2.55)^2 &= 6.5025 \\
(+1.79)^2 &= 3.2041 \\
(-0.64)^2 &= 0.4096 \\
(-0.78)^2 &= 0.6084 \\
(-0.78)^2 &= 0.6084 \\
(-0.60)^2 &= 0.3600 \\
(+0.13)^2 &= \underline{0.0169} \\
\Sigma D^2 &= 40.5437
\end{aligned}
$$

The completed table is shown next.

3 years ago (X_1)	Today (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
11.42	16.69	−5.27	27.7729
8.41	9.44	−1.03	1.0609
3.98	6.53	−2.55	6.5025
7.37	5.58	+1.79	3.2041
2.28	2.92	−0.64	0.4096
1.10	1.88	−0.78	0.6084
1.00	1.78	−0.78	0.6084
0.90	1.50	−0.60	0.3600
1.35	1.22	+0.13	0.0169
		$\Sigma D = -9.73$	$\Sigma D^2 = 40.5437$

e. Find the standard deviation of the differences.

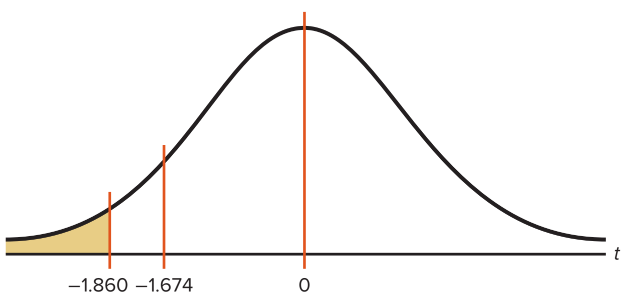
$$
\begin{aligned}
s_D &= \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}} \\
&= \sqrt{\frac{9(40.5437) - (-9.73)^2}{9(9-1)}} \\
&= \sqrt{\frac{270.2204}{72}} \\
&= 1.937
\end{aligned}
$$

f. Find the test value.

$$
t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} = \frac{-1.081 - 0}{1.937/\sqrt{9}} = -1.674
$$

Step 4 Make the decision. Do not reject the null hypothesis since the test value, −1.674, is greater than the critical value, −1.860. See Figure 9–6.

FIGURE 9–6 Critical and Test Values for Example 9–6



Step 5 Summarize the results. There is not enough evidence to show that the deposits have increased over the last 3 years.

EXAMPLE 9–7 Cholesterol Levels

A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six randomly selected subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at $\alpha = 0.10$? Assume the variable is approximately normally distributed.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

SOLUTION

Step 1 State the hypotheses and identify the claim. If the diet is effective, the before cholesterol levels should be different from the after levels.

$$H_0: \mu_D = 0 \quad \text{and} \quad H_1: \mu_D \neq 0 \text{ (claim)}$$

Step 2 Find the critical value. The degrees of freedom are $6 - 1 = 5$. At $\alpha = 0.10$, the critical values are ± 2.015.

Step 3 Compute the test value.

a. Make a table.

Before (X_1)	After (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
210	190		
235	170		
208	210		
190	188		
172	173		
244	228		

b. Find the differences and place the results in column A.

$$\begin{aligned} 210 - 190 &= 20 \\ 235 - 170 &= 65 \\ 208 - 210 &= -2 \\ 190 - 188 &= 2 \\ 172 - 173 &= -1 \\ 244 - 228 &= \underline{16} \\ \Sigma D &= 100 \end{aligned}$$

c. Find the mean of the differences.

$$\overline{D} = \frac{\Sigma D}{n} = \frac{100}{6} = 16.7$$

d. Square the differences and place the results in column B.

$$\begin{aligned} (20)^2 &= 400 \\ (65)^2 &= 4225 \\ (-2)^2 &= 4 \\ (2)^2 &= 4 \\ (-1)^2 &= 1 \\ (16)^2 &= \underline{256} \\ \Sigma D^2 &= 4890 \end{aligned}$$

Then complete the table as shown.

Before (X_1)	After (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
210	190	20	400
235	170	65	4225
208	210	–2	4
190	188	2	4
172	173	–1	1
244	228	16	256
		$\Sigma D = 100$	$\Sigma D^2 = 4890$

e. Find the standard deviation of the differences.

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

$$= \sqrt{\frac{6 \cdot 4890 - 100^2}{6(6-1)}}$$

$$= \sqrt{\frac{29{,}340 - 10{,}000}{30}}$$

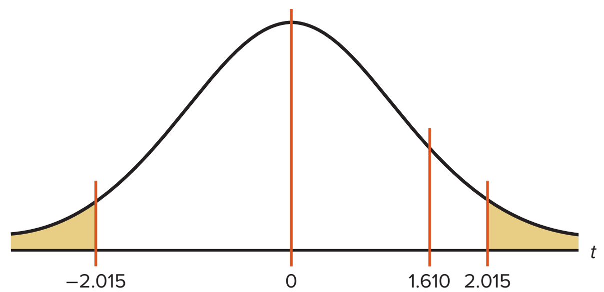
$$= 25.4$$

f. Find the test value.

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} = \frac{16.7 - 0}{25.4/\sqrt{6}} = 1.610$$

Step 4 Make the decision. The decision is to not reject the null hypothesis, since the test value 1.610 is in the noncritical region, as shown in Figure 9–7.

FIGURE 9–7 Critical and Test Values for Example 9–7



Step 5 Summarize the results. There is not enough evidence to support the claim that the mineral changes a person's cholesterol level.

The P-values for the t test are found in Table F. For a two-tailed test with d.f. = 5 and $t = 1.610$, the P-value is found between 1.476 and 2.015; hence, $0.10 < P\text{-value} < 0.20$. Thus, the null hypothesis cannot be rejected at $\alpha = 0.10$.

If a specific difference is hypothesized, this formula should be used

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

where μ_D is the hypothesized difference.

SPEAKING OF STATISTICS Can Video Games Save Lives?

Can playing video games help doctors perform surgery? The answer is yes. A study showed that surgeons who played video games for at least 3 hours each week made about 37% fewer mistakes and finished operations 27% faster than those who did not play video games.

The type of surgery that they performed is called *laparoscopic* surgery, where the surgeon inserts a tiny video camera into the body and uses a joystick to maneuver the surgical instruments while watching the results on a television monitor. This study compares two groups and uses proportions. What statistical test do you think was used to compare the percentages? (See Section 9–4.)

For example, if a dietitian claims that people on a specific diet will lose an average of 3 pounds in a week, the hypotheses are

$$H_0\colon \mu_D = 3 \qquad \text{and} \qquad H_1\colon \mu_D \neq 3$$

The value 3 will be substituted in the test statistic formula for μ_D.

Confidence intervals can be found for the mean differences with this formula.

Confidence Interval for the Mean Difference

$$\overline{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \overline{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$
$$\text{d.f.} = n - 1$$

EXAMPLE 9–8

Find the 90% confidence interval for the data in Example 9–7.

SOLUTION

Substitute in the formula.

$$\overline{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \overline{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

$$16.7 - 2.015 \cdot \frac{25.4}{\sqrt{6}} < \mu_D < 16.7 + 2.015 \cdot \frac{25.4}{\sqrt{6}}$$

$$16.7 - 20.89 < \mu_D < 16.7 + 20.89$$

$$-4.19 < \mu_D < 37.59$$

$$-4.2 < \mu_D < 37.6$$

Since 0 is contained in the interval, the decision is to not reject the null hypothesis $H_0\colon \mu_D = 0$. Hence, there is not enough evidence to support the claim that the mineral changes a person's cholesterol, as previously shown.

Applying the Concepts 9–3

Air Quality

As a researcher for the EPA, you have been asked to determine if the air quality in the United States has changed over the past 2 years. You select a random sample of 10 metropolitan areas and find the number of days each year that the areas failed to meet acceptable air quality standards. The data are shown.

Year 1	18	125	9	22	138	29	1	19	17	31
Year 2	24	152	13	21	152	23	6	31	34	20

Source: The World Almanac and Book of Facts.

Based on the data, answer the following questions.

1. What is the purpose of the study?
2. Are the samples independent or dependent?
3. What hypotheses would you use?
4. What is (are) the critical value(s) that you would use?
5. What statistical test would you use?
6. How many degrees of freedom are there?
7. What is your conclusion?
8. Could an independent means test have been used?
9. Do you think this was a good way to answer the original question?

See page 545 for the answers.

Exercises 9–3

1. Classify each as independent or dependent samples.
   a. Heights of identical twins
   b. Test scores of the same students in English and psychology
   c. The effectiveness of two different brands of aspirin on two different groups of people
   d. Effects of a drug on reaction time of two different groups of people, measured by a before-and-after test
   e. The effectiveness of two different diets on two different groups of individuals

For Exercises 2 through 12, perform each of these steps. Assume that all variables are normally or approximately normally distributed.

   a. State the hypotheses and identify the claim.
   b. Find the critical value(s).
   c. Compute the test value.
   d. Make the decision.
   e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

2. Retention Test Scores A random sample of non-English majors at a selected college was used in a study to see if the student retained more from reading a 19th-century novel or by watching it in DVD form. Each student was assigned one novel to read and a different one to watch, and then they were given a 100-point written quiz on each novel. The test results are shown. At $\alpha = 0.05$, can it be concluded that the book scores are higher than the DVD scores?

Book	90	80	90	75	80	90	84
DVD	85	72	80	80	70	75	80

3. Improving Study Habits As an aid for improving students' study habits, nine students were randomly selected to attend a seminar on the importance of education in life. The table shows the number of hours each student studied per week before and after the seminar. At $\alpha = 0.10$, did attending the seminar increase the number of hours the students studied per week?

Before	9	12	6	15	3	18	10	13	7
After	9	17	9	20	2	21	15	22	6

4. **Obstacle Course Times** An obstacle course was set up on a campus, and 8 randomly selected volunteers were given a chance to complete it while they were being timed. They then sampled a new energy drink and were given the opportunity to run the course again. The "before" and "after" times in seconds are shown. Is there sufficient evidence at $\alpha = 0.05$ to conclude that the students did better the second time? Discuss possible reasons for your results.

Student	1	2	3	4	5	6	7	8
Before	67	72	80	70	78	82	69	75
After	68	70	76	65	75	78	65	68

5. **Cholesterol Levels** A medical researcher wishes to see if he can lower the cholesterol levels through diet in 6 people by showing a film about the effects of high cholesterol levels. The data are shown. At $\alpha = 0.05$, did the cholesterol level decrease on average?

Patient	1	2	3	4	5	6
Before	243	216	214	222	206	219
After	215	202	198	195	204	213

6. **PGA Golf Scores** At a recent PGA tournament (the Honda Classic at Palm Beach Gardens, Florida) the following scores were posted for eight randomly selected golfers for two consecutive days. At $\alpha = 0.05$, is there evidence of a difference in mean scores for the two days?

Golfer	1	2	3	4	5	6	7	8
Thursday	67	65	68	68	68	70	69	70
Friday	68	70	69	71	72	69	70	70

Source: Washington Observer-Reporter.

7. **Reducing Errors in Grammar** A composition teacher wishes to see whether a new smartphone app will reduce the number of grammatical errors her students make when writing a two-page essay. She randomly selects six students, and the data are shown. At $\alpha = 0.025$, can it be concluded that the number of errors has been reduced?

Student	1	2	3	4	5	6
Errors before	12	9	0	5	4	3
Errors after	9	6	1	3	2	3

8. **Overweight Dogs** A veterinary nutritionist developed a diet for overweight dogs. The total volume of food consumed remains the same, but one-half of the dog food is replaced with a low-calorie "filler" such as canned green beans. Six overweight dogs were randomly selected from her practice and were put on this program. Their initial weights were recorded, and they were weighed again after 4 weeks. At the 0.05 level of significance, can it be concluded that the dogs lost weight?

Before	42	53	48	65	40	52
After	39	45	40	58	42	47

9. **Pulse Rates of Identical Twins** A researcher wanted to compare the pulse rates of identical twins to see whether there was any difference. Eight sets of twins were randomly selected. The rates are given in the table as number of beats per minute. At $\alpha = 0.01$, is there a significant difference in the average pulse rates of twins? Use the P-value method. Find the 99% confidence interval for the difference of the two.

Twin A	87	92	78	83	88	90	84	93
Twin B	83	95	79	83	86	93	80	86

10. **Toy Assembly Test** An educational researcher devised a wooden toy assembly project to test learning in 6-year-olds. The time in seconds to assemble the project was noted, and the toy was disassembled out of the child's sight. Then the child was given the task to repeat. The researcher would conclude that learning occurred if the mean of the second assembly times was less than the mean of the first assembly times. At $\alpha = 0.01$, can it be concluded that learning took place? Use the P-value method, and find the 99% confidence interval of the difference in means.

Child	1	2	3	4	5	6	7
Trial 1	100	150	150	110	130	120	118
Trial 2	90	130	150	90	105	110	120

11. **Reducing Errors in Spelling** A ninth-grade teacher wishes to see if a new spelling program will reduce the spelling errors in his students' writing. The number of spelling errors made by the students in a five-page report before the program is shown. Then the number of spelling errors made by students in a five-page report after the program is shown. At $\alpha = 0.05$, did the program work?

Before	8	3	10	5	9	11	12
After	6	4	8	1	4	7	11

12. **Mistakes in a Song** A random sample of six music students played a short song, and the number of mistakes in music each student made was recorded. After they practiced the song 5 times, the number of mistakes each student made was recorded. The data are shown. At $\alpha = 0.05$, can it be concluded that there was a decrease in the mean number of mistakes?

Student	A	B	C	D	E	F
Before	10	6	8	8	13	8
After	4	2	2	7	8	9

Extending the Concepts

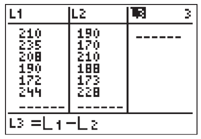
13. Instead of finding the mean of the differences between X_1 and X_2 by subtracting $X_1 - X_2$, you can find it by finding the means of X_1 and X_2 and then subtracting the means. Show that these two procedures will yield the same results.

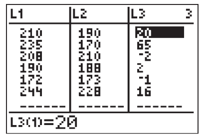
Technology Step by Step

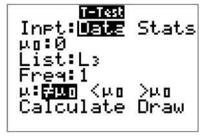
TI-84 Plus Step by Step

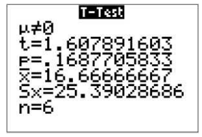
Hypothesis Test for the Difference Between Two Means: Dependent Samples

This refers to Example 9–7 in the text.







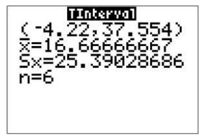


Example TI9–6

1. Enter the data values into L_1 and L_2.
2. Move the cursor to the top of the L_3 column so that L_3 is highlighted.
3. Type $\mathbf{L_1 - L_2}$, then press **ENTER.**
4. Press **STAT** and move the cursor to TESTS.
5. Press **2** for TTest.
6. Move the cursor to Data and press **ENTER.**
7. Type in the appropriate values, using **0** for μ_0 and $\mathbf{L_3}$ for the list.
8. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
9. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means: Dependent Samples

This refers to Example 9–8 in the text.



Example TI9–7

1. Enter the data values into L_1 and L_2.
2. Move the cursor to the top of the L_3 column so that L_3 is highlighted.
3. Type $\mathbf{L_1 - L_2}$, then press **ENTER.**
4. Press **STAT** and move the cursor to TESTS.
5. Press **8** for TInterval.
6. Move the cursor to Data and press **ENTER.**
7. Type in the appropriate values, using $\mathbf{L_3}$ for the list.
8. Move the cursor to Calculate and press **ENTER.**

EXCEL Step by Step

Testing the Difference Between Two Means: Dependent Samples

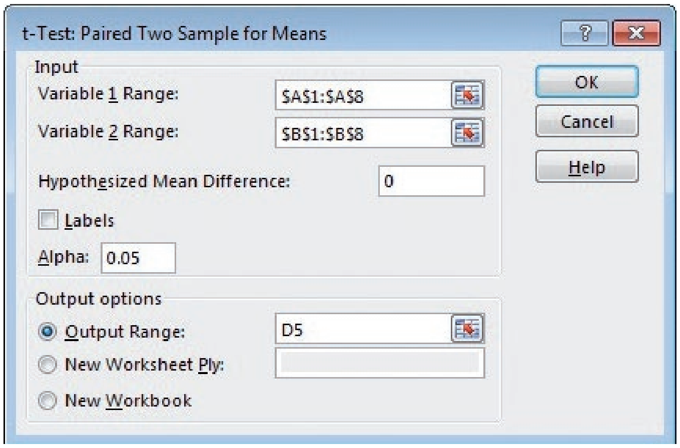
Example XL9–3

Test the claim that there is no difference between population means based on these sample paired data. Use $\alpha = 0.05$.

Set A	33	35	28	29	32	34	30	34
Set B	27	29	36	34	30	29	28	24

1. Enter the 8-number data set A into column A.
2. Enter the 8-number data set B into column B.
3. Select the Data tab from the toolbar. Then select Data Analysis.
4. In the Data Analysis box, under Analysis Tools select *t*-test: Paired Two Sample for Means, and click [OK].

5. In Input, type in the Variable 1 Range: **A1:A8** and the Variable 2 Range: **B1:B8.**
6. Type **0** for the Hypothesized Mean Difference.
7. Type **0.05** for Alpha.
8. In Output options, type **D5** for the Output Range, then click [OK].



t-Test: Paired Two Sample for Means		
	Variable 1	*Variable 2*
Mean	31.875	29.625
Variance	6.696428571	14.55357143
Observations	8	8
Pearson Correlation	-0.757913399	
Hypothesized Mean Difference	0	
df	7	
t Stat	1.057517468	
P(T<=t) one-tail	0.1626994	
t Critical one-tail	1.894578604	
P(T<=t) two-tail	0.3253988	
t Critical two-tail	2.364624251	

Note: You may need to increase the column width to see all the results. To do this:

1. Highlight the columns D, E, and F.
2. Under the Home tab, select **Format>AutoFit** Column Width.

The output shows a *P*-value of 0.3253988 for the two-tailed case. This value is greater than the alpha level of 0.05, so we fail to reject the null hypothesis.

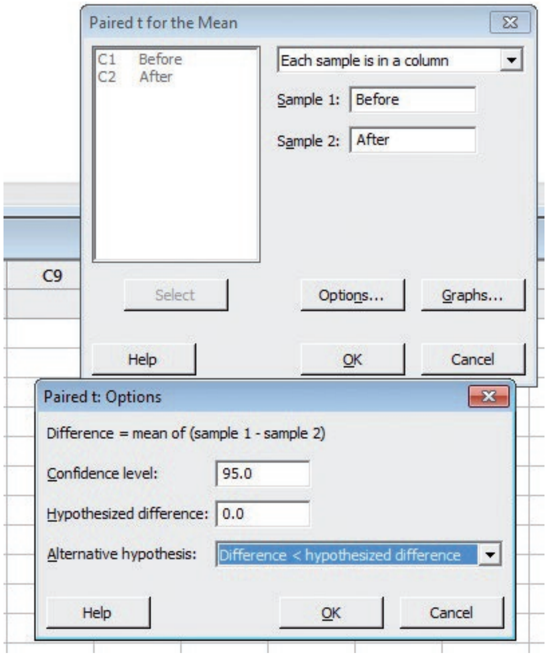
MINITAB
Step by Step

Test the Difference Between Two Means: Dependent Samples

A sports fitness trainer claims by taking a special vitamin, a weight lifter can increase his strength. Eight athletes are selected and given a test of strength, using the standard bench press. After 2 weeks of regular training, supplemented with the vitamin, they are tested again. Test the effectiveness of the vitamin regimen at $\alpha = 0.05$. Each value in these data represents the maximum number of pounds the athlete can bench-press. Assume that the variable is approximately normally distributed.

Athlete	1	2	3	4	5	6	7	8
Before (X_1)	210	230	182	205	262	253	219	216
After (X_2)	219	236	179	204	270	250	222	216

1. Enter the data into C1 and C2. Name the columns **Before** and **After.**
2. Select **Stat>Basic Statistics>Paired t.**
3. Select Each sample is in a column from the drop down menu.
4. Double click C1 Before for Sample 1.
5. Double click C2 After for Sample 2. The second sample will be subtracted from the first. The differences are not stored or displayed.
6. Click [Options].
7. Change Alternative hypothesis to Difference < hypothesized difference.
8. Click [OK] twice.



Paired T-Test and CI: Before, After

Paired T for Before – After

	N	Mean	StDev	SE Mean
Before	8	222.13	25.92	9.16
After	8	224.50	27.91	9.87
Difference	8	−2.38	4.84	1.71

95% upper bound for mean difference: 0.87
T-Test of mean difference = 0 (vs < 0) : T–Value = −1.39 P–Value = 0.104.

Since the P-value is 0.104, do not reject the null hypothesis. The sample difference of −2.38 in the strength measurement is not statistically significant.

9–4 Testing the Difference Between Proportions

OBJECTIVE 4

Test the difference between two proportions.

In Chapter 8, an inference about a single proportion was explained. In this section, testing the difference between two sample proportions will be explained.

The z test with some modifications can be used to test the equality of two proportions. For example, a researcher might ask, Is the proportion of men who exercise regularly less than the proportion of women who exercise regularly? Is there a difference in the percentage of students who own a mobile device and the percentage of nonstudents who own one? Is there a difference in the proportion of college graduates who pay cash for purchases and the proportion of non-college graduates who pay cash?

Recall from Chapter 7 that the symbol $\hat{p}$ ("p hat") is the sample proportion used to estimate the population proportion, denoted by p. For example, if in a sample of 30 college students, 9 are on probation, then the sample proportion is $\hat{p} = \frac{9}{30}$, or 0.3. The population proportion p is the number of all students who are on probation, divided by the number of students who attend the college. The formula for the sample proportion $\hat{p}$ is

$$\hat{p} = \frac{X}{n}$$

where

X = number of units that possess the characteristic of interest
n = sample size

When you are testing the difference between two population proportions p_1 and p_2, the hypotheses can be stated thus, if no specific difference between the proportions is hypothesized.

$$H_0\colon p_1 = p_2 \qquad\qquad H_0\colon p_1 - p_2 = 0$$
$$\text{or}$$
$$H_1\colon p_1 \neq p_2 \qquad\qquad H_1\colon p_1 - p_2 \neq 0$$

Similar statements using < or > in the alternate hypothesis can be formed for one-tailed tests.

For two proportions, $\hat{p}_1 = X_1/n_1$ is used to estimate p_1 and $\hat{p}_2 = X_2/n_2$ is used to estimate p_2. The standard error of the difference is

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\sigma_{p_1}^2 + \sigma_{p_2}^2} = \sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}$$

where $\sigma_{p_1}^2$ and $\sigma_{p_2}^2$ are the variances of the proportions, $q_1 = 1 - p_1$, $q_2 = 1 - p_2$, and n_1 and n_2 are the respective sample sizes.

Since p_1 and p_2 are unknown, a weighted estimate of p can be computed by using the formula

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

and $\bar{q} = 1 - \bar{p}$. This weighted estimate is based on the hypothesis that $p_1 = p_2$. Hence, $\bar{p}$ is a better estimate than either $\hat{p}_1$ or $\hat{p}_2$, since it is a combined average using both $\hat{p}_1$ and $\hat{p}_2$.

Since $\hat{p}_1 = X_1/n_1$ and $\hat{p}_2 = X_2/n_2$, $\bar{p}$ can be simplified to

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

Finally, the standard error of the difference in terms of the weighted estimate is

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

The formula for the test value is shown next.

Formula for the *z* Test Value for Comparing Two Proportions

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

This formula follows the format

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

Before you can test the difference between two sample proportions, the following assumptions must be met.

Assumptions for the *z* Test for Two Proportions

1. The samples must be random samples.
2. The sample data are independent of one another.
3. For both samples $np \geq 5$ and $nq \geq 5$.

In this book, the assumptions will be stated in the exercises; however, when encountering statistics in other situations, you must check to see that these assumptions have been met before proceeding.

The hypothesis-testing procedure used here follows the five-step procedure presented previously except that $\hat{p}_1$, $\hat{p}_2$, $\bar{p}$, and $\bar{q}$ must be computed.

EXAMPLE 9–9 Vaccination Rates in Nursing Homes

In the nursing home study mentioned in the chapter-opening Statistics Today, the researchers found that 12 out of 34 randomly selected small nursing homes had a resident vaccination rate of less than 80%, while 17 out of 24 randomly selected large nursing homes had a vaccination rate of less than 80%. At $\alpha = 0.05$, test the claim that there is no difference in the proportions of the small and large nursing homes with a resident vaccination rate of less than 80%.

Source: Nancy Arden, Arnold S. Monto, and Suzanne E. Ohmit, "Vaccine Use and the Risk of Outbreaks in a Sample of Nursing Homes During an Influenza Epidemic," *American Journal of Public Health.*

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0: p_1 = p_2 \text{ (claim)} \qquad \text{and} \qquad H_1: p_1 \neq p_2$$

Step 2 Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and -1.96.

Step 3 Compute the test value. First compute $\hat{p}_1$, $\hat{p}_2$, $\bar{p}$, and $\bar{q}$. Then substitute in the formula.

Let $\hat{p}_1$ be the proportion of the small nursing homes with a vaccination rate of less than 80% and $\hat{p}_2$ be the proportion of the large nursing homes with a vaccination rate of less than 80%. Then

$$\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35 \qquad \text{and} \qquad \hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{12 + 17}{34 + 24} = \frac{29}{58} = 0.5$$

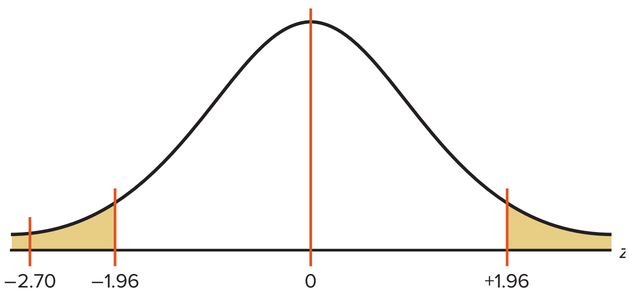
$$\bar{q} = 1 - \bar{p} = 1 - 0.5 = 0.5$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

$$= \frac{(0.35 - 0.71) - 0}{\sqrt{(0.5)(0.5)\left(\dfrac{1}{34} + \dfrac{1}{24}\right)}} = \frac{-0.36}{0.1333} = -2.70$$

Step 4 Make the decision. Reject the null hypothesis, since $-2.70 < -1.96$. See Figure 9–8.

FIGURE 9–8 Critical and Test Values for Example 9–9



Step 5 Summarize the results. There is enough evidence to reject the claim that there is no difference in the proportions of small and large nursing homes with a resident vaccination rate of less than 80%.

EXAMPLE 9–10 Criminal Arrests

A study found that in a random sample of 50 burglaries, 16% of the criminals were arrested. In a random sample of 50 car thefts, 12% of the criminals were arrested. At $\alpha = 0.10$, can it be concluded that the percentages of people who committed burglaries and were arrested was greater than the percentages of people who committed car thefts and were arrested.

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0: p_1 = p_2 \qquad \text{and} \qquad H_1: p_1 > p_2 \text{ (claim)}$$

Step 2 Find the critical value, using Table E. At $\alpha = 0.10$ the critical value is 1.28.

Step 3 Compute the test value. Since percentages are given, you need to compare $\bar{p}$ and $\bar{q}$.

$$X_1 = \hat{p}_1 n_1 = 0.16(50) = 8$$

$$X_2 = \hat{p}_2 n_2 = 0.12(50) = 6$$

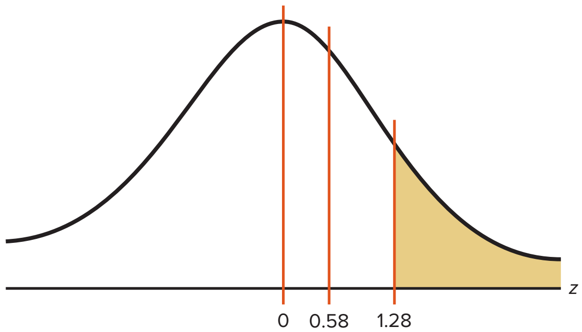
$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{8 + 6}{50 + 50} = \frac{14}{100} = 0.14$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.14 = 0.86$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.16 - 0.12) - 0}{\sqrt{(0.14)(0.86)\left(\frac{1}{50} + \frac{1}{50}\right)}} = \frac{0.04}{0.069} = 0.58$$

Step 4 Make the decision. Do not reject the null hypothesis since $0.58 < 1.28$. That is, 0.58 falls in the noncritical region. See Figure 9–9.

FIGURE 9–9 Critical and Test Value for Example 9–10



Step 5 Summarize the results. There is not enough evidence to support the claim that the percentage of people who are arrested for burglaries is greater than the percentage of people who are arrested who committed car thefts.

SPEAKING OF STATISTICS Is More Expensive Better?

An article in the *Journal of the American Medical Association* explained a study done on placebo pain pills. Researchers randomly assigned 82 healthy people to two groups. The individuals in the first group were given sugar pills, but they were told that the pills were a new, fast-acting opioid pain reliever similar to codeine and that they were listed at \$2.50 each. The individuals in the other group received the same sugar pills but were told that the pills had been marked down to 10¢ each.

Each group received electrical shocks before and after taking the pills. They were then asked if the pills reduced the pain. Eighty-five percent of the group who were told that the pain pills cost \$2.50 said that they were effective, while 61% of the group who received the supposedly discounted pills said that they were effective.

© Comstock/PictureQuest RF

State possible null and alternative hypotheses for this study. What statistical test could be used in this study? What might be the conclusion of the study?

The P-value for the difference of proportions can be found from Table E as shown in Section 9–1. In Example 9–10, the table value for 0.58 is 0.7190, and $1 - 0.7190 = 0.2810$. Hence, $0.2810 > 0.01$; thus the decision is to not reject the null hypothesis.

The sampling distribution of the difference of two proportions can be used to construct a confidence interval for the difference of two proportions. The formula for the confidence interval for the difference between two proportions is shown next.

Confidence Interval for the Difference Between Two Proportions

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Here, the confidence interval uses a standard deviation based on estimated values of the population proportions, but the hypothesis test uses a standard deviation based on the assumption that the two population proportions are equal. As a result, you may obtain different conclusions when using a confidence interval or a hypothesis test. So when testing for a difference of two proportions, you use the z test rather than the confidence interval.

EXAMPLE 9–11

Find the 95% confidence interval for the difference of proportions for the data in Example 9–9.

SOLUTION

$$\hat{p}_1 = \frac{12}{34} = 0.35 \qquad \hat{q}_1 = 0.65$$

$$\hat{p}_2 = \frac{17}{24} = 0.71 \qquad \hat{q}_2 = 0.29$$

Substitute in the formula.

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$(0.35 - 0.71) - 1.96\sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}} < p_1 - p_2 < (0.35 - 0.71) + 1.96\sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}}$$

$$-0.36 - 0.242 < p_1 - p_2 < -0.36 + 0.242$$

$$-0.602 < p_1 - p_2 < -0.118$$

Applying the Concepts 9–4

Smoking and Education

You are researching the hypothesis that there is no difference in the percent of public school students who smoke and the percent of private school students who smoke. You find these results from a recent survey.

School	Percent who smoke
Public	32.3
Private	14.5

Based on these figures, answer the following questions.

1. What hypotheses would you use if you wanted to compare percentages of the public school students who smoke with the private school students who smoke?
2. What critical value(s) would you use?
3. What statistical test would you use to compare the two percentages?
4. What information would you need to complete the statistical test?
5. Suppose you found that 1000 randomly selected individuals in each group were surveyed. Could you perform the statistical test?
6. If so, complete the test and summarize the results.

See page 545 for the answers.

Exercises 9–4

1. Find the proportions $\hat{p}$ and $\hat{q}$ for each.

a. $n = 52, X = 32$

b. $n = 80, X = 66$

c. $n = 36, X = 12$

d. $n = 42, X = 7$

e. $n = 160, X = 50$

2. Find $\hat{p}$ and $\hat{q}$ for each.

a. $n = 36, X = 20$

b. $n = 50, X = 35$

c. $n = 64, X = 16$

d. $n = 200, X = 175$

e. $n = 148, X = 16$

3. Find each X, given $\hat{p}$.

a. $\hat{p} = 0.60, n = 240$
b. $\hat{p} = 0.20, n = 320$
c. $\hat{p} = 0.60, n = 520$
d. $\hat{p} = 0.80, n = 50$
e. $\hat{p} = 0.35, n = 200$

4. Find each X, given $\hat{p}$.

a. $\hat{p} = 0.24, n = 300$
b. $\hat{p} = 0.09, n = 200$
c. $\hat{p} = 88\%, n = 500$
d. $\hat{p} = 40\%, n = 480$
e. $\hat{p} = 32\%, n = 700$

5. Find $\hat{p}$ and $\hat{q}$ for each.

a. $X_1 = 25, n_1 = 75, X_2 = 40, n_2 = 90$
b. $X_1 = 9, n_1 = 15, X_2 = 7, n_2 = 20$
c. $X_1 = 3, n_1 = 20, X_2 = 5, n_2 = 40$
d. $X_1 = 21, n_1 = 50, X_2 = 32, n_2 = 50$
e. $X_1 = 20, n_1 = 150, X_2 = 30, n_2 = 50$

6. Find $\bar{p}$ and $\bar{q}$.

a. $X_1 = 6, n_1 = 15, X_2 = 9, n_2 = 15$
b. $X_1 = 21, n_1 = 100, X_2 = 43, n_2 = 150$
c. $X_1 = 20, n_1 = 80, X_2 = 65, n_2 = 120$
d. $X_1 = 15, n_1 = 50, X_2 = 3, n_2 = 12$
e. $X_1 = 24, n_1 = 40, X_2 = 18, n_2 = 36$

For Exercises 7 through 27, perform these steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

7. Lecture versus Computer-Assisted Instruction A survey found that 83% of the men questioned preferred computer-assisted instruction to lecture and 75% of the women preferred computer-assisted instruction to lecture. There were 100 randomly selected individuals in each sample. At $\alpha = 0.05$, test the claim that there is no difference in the proportion of men and the proportion of women who favor computer-assisted instruction over lecture. Find the 95% confidence interval for the difference of the two proportions.

8. Leisure Time In a sample of 150 men, 132 said that they had less leisure time today than they had 10 years ago. In a random sample of 250 women, 240 women said that they had less leisure time than they had 10 years ago. At $\alpha = 0.10$, is there a difference in the proportions? Find the 90% confidence interval for the difference of the two proportions. Does the confidence interval contain 0? Give a reason why this information would be of interest to a researcher.

Source: Based on statistics from Market Directory.

9. Desire to Be Rich In a random sample of 80 Americans, 44 wished that they were rich. In a random sample of 90 Europeans, 41 wished that they were rich. At $\alpha = 0.01$, is there a difference in the proportions? Find the 99% confidence interval for the difference of the two proportions.

10. Animal Bites of Postal Workers In Cleveland, a random sample of 73 mail carriers showed that 10 had been bitten by an animal during one week. In Philadelphia, in a random sample of 80 mail carriers, 16 had received animal bites. Is there a significant difference in the proportions? Use $\alpha = 0.05$. Find the 95% confidence interval for the difference of the two proportions.

11. Pet Ownership A recent random survey of households found that 14 out of 50 householders had a cat and 21 out of 60 householders had a dog. At $\alpha = 0.05$, test the claim that fewer household owners have cats than household owners who have dogs as pets.

12. Seat Belt Use In a random sample of 200 men, 130 said they used seat belts. In a random sample of 300 women, 63 said they used seat belts. Test the claim that men are more safety-conscious than women, at $\alpha = 0.01$. Use the P-value method.

13. Victims of Violence A random survey of 80 women who were victims of violence found that 24 were attacked by relatives. A random survey of 50 men found that 6 were attacked by relatives. At $\alpha = 0.10$, can it be shown that the percentage of women who were attacked by relatives is greater than the percentage of men who were attacked by relatives?

14. Hypertension It has been found that 26% of men 20 years and older suffer from hypertension (high blood pressure) and 31.5% of women are hypertensive. A random sample of 150 of each gender was selected from recent hospital records, and the following results were obtained. Can you conclude that a higher percentage of women have high blood pressure? Use $\alpha = 0.05$.

Men	43 patients had high blood pressure
Women	52 patients had high blood pressure

Source: www.nchs.gov

15. Commuters A recent random survey of 100 individuals in Michigan found that 80 drove to work alone. A similar survey of 120 commuters in

New York found that 62 drivers drove alone to work. Find the 95% confidence interval for the difference in proportions.

16. **Smoking Survey** National statistics show that 23% of men smoke and 18.5% of women smoke. A random sample of 180 men indicated that 50 were smokers, and a random sample of 150 women surveyed indicated that 39 smoked. Construct a 98% confidence interval for the true difference in proportions of male and female smokers. Comment on your interval—does it support the claim that there is a difference?

Source: www.nchs.gov

17. **Senior Workers** It seems that people are choosing or finding it necessary to work later in life. Random samples of 200 men and 200 women age 65 or older were selected, and 80 men and 59 women were found to be working. At $\alpha = 0.01$, can it be concluded that the proportions are different?

Source: Based on www.census.gov

18. **Airlines On-Time Arrivals** The percentages of on-time arrivals for major U.S. airlines range from 68.6 to 91.1. Two regional airlines were surveyed with the following results. At $\alpha = 0.01$, is there a difference in proportions?

	Airline A	**Airline B**
No. of flights	300	250
No. of on-time flights	213	185

Source: New York Times Almanac.

19. **College Education** The percentages of adults 25 years of age and older who have completed 4 or more years of college are 23.6% for females and 27.8% for males. A random sample of women and men who were 25 years old or older was surveyed with these results. Estimate the true difference in proportions with 95% confidence, and compare your interval with the *Almanac* statistics.

	Women	**Men**
Sample size	350	400
No. who completed 4 or more years	100	115

Source: New York Times Almanac.

20. **Married People** In a specific year 53.7% of men in the United States were married and 50.3% of women were married. Two independent random samples of 300 men and 300 women found that 178 men and 139 women were married (not to each other). At the 0.05 level of significance, can it be concluded that the proportion of men who were married is greater than the proportion of women who were married?

Source: New York Times Almanac.

21. **Undergraduate Financial Aid** A study is conducted to determine if the percent of women who receive financial aid in undergraduate school is different from the percent of men who receive financial aid in undergraduate school. A random sample of undergraduates revealed these results. At $\alpha = 0.01$, is there significant evidence to reject the null hypothesis?

	Women	**Men**
Sample size	250	300
Number receiving aid	200	180

Source: U.S. Department of Education, National Center for Education Statistics.

22. **High School Graduation Rates** The overall U.S. public high school graduation rate is 73.4%. For Pennsylvania it is 83.5% and for Idaho 80.5%—a difference of 3%. Random samples of 1200 students from each state indicated that 980 graduated in Pennsylvania and 940 graduated in Idaho. At the 0.05 level of significance, can it be concluded that there is a difference in the proportions of graduating students between the states?

Source: World Almanac.

23. **Interview Errors** It has been found that many first-time interviewees commit errors that could very well affect the outcome of the interview. An astounding 77% are guilty of using their cell phones or texting during the interview! A researcher wanted to see if the proportion of male offenders differed from the proportion of female ones. Out of 120 males, 72 used their cell phone and 80 of 150 females did so. At the 0.01 level of significance is there a difference?

Source: Careerbuilder.com

24. **Medical Supply Sales** According to the U.S. Bureau of Labor Statistics, approximately equal numbers of men and women are engaged in sales and related occupations. Although that may be true for total numbers, perhaps the proportions differ by industry. A random sample of 200 salespersons from the industrial sector indicated that 114 were men, and in the medical supply sector, 80 of 200 were men. At the 0.05 level of significance, can we conclude that the proportion of men in industrial sales differs from the proportion of men in medical supply sales?

25. **Coupon Use** In today's economy, everyone has become savings savvy. It is still believed, though, that a higher percentage of women than men clip coupons. A random survey of 180 female shoppers indicated that 132 clipped coupons while 56 out of 100 men did so. At $\alpha = 0.01$, is there sufficient evidence that the proportion of couponing women is higher than the proportion of couponing men? Use the *P*-value method.

26. **Never Married People** The percentage of males 18 years and older who have never married is 30.4. For females the percentage is 23.6. Looking at the records in a particular populous county, a random sample of 250 men showed that 78 had never married and 58 of 200 women had never married. At the 0.05 level of significance, is the proportion of men greater than the proportion of women? Use the *P*-value method.

27. Bullying Bullying is a problem at any age but especially for students aged 12 to 18. A study showed that 7.2% of all students in this age bracket reported being bullied at school during the past six months with 6th grade having the highest incidence at 13.9% and 12th grade the lowest at 2.2%. To see if there is a difference between public and private schools, 200 students were randomly selected from each. At the 0.05 level of significance, can a difference be concluded?

	Private	**Public**
Sample size	200	200
No. bullied	13	16

Source: www.nces.ed.gov

Extending the Concepts

28. If there is a significant difference between p_1 and p_2 and between p_2 and p_3, can you conclude that there is a significant difference between p_1 and p_3?

Technology Step by Step

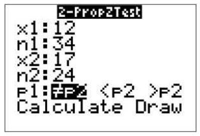
TI-84 Plus Step by Step

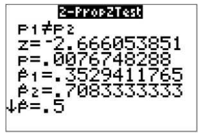
Hypothesis Test for the Difference Between Two Proportions

Example TI9–8

This refers to Example 9–9 in the text.

1. Press **STAT** and move the cursor to TESTS.
2. Press **6** for 2-PropZTEST.
3. Type in the appropriate values.
4. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
5. Move the cursor to Calculate and press **ENTER.**



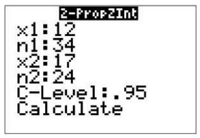


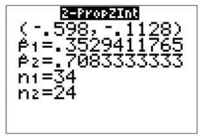
Confidence Interval for the Difference Between Two Proportions

Example TI9–9

This refers to Example 9–11 in the text.

1. Press **STAT** and move the cursor to TESTS.
2. Press **B (ALPHA APPS)** for 2-PropZInt.
3. Type in the appropriate values.
4. Move the cursor to Calculate and press **ENTER.**





EXCEL Step by Step

Testing the Difference Between Two Proportions

Excel does not have a procedure to test the difference between two population proportions. However, you may conduct this test using the MegaStat Add-in available in your online resources. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

We will use the summary information from Example 9–9.

1. From the toolbar, select Add-Ins, **MegaStat>Hypothesis Tests>Compare Two Independent Proportions.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.
2. Under Group 1, type **12** for p and **34** for n. Under Group 2, type **17** for p and **24** for n. MegaStat automatically changes p to X unless a decimal value less than 1 is typed in for these.
3. Type **0** for the Hypothesized difference, select the not equal Alternative, and click [OK].

Hypothesis Test for Two Independent Proportions

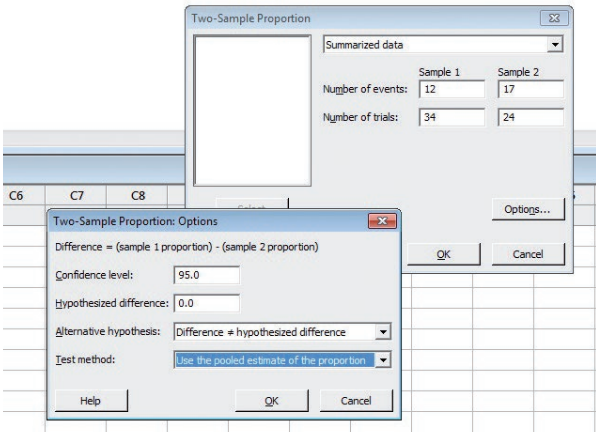
p_1	p_2		p_c	
0.3529	0.7083		0.5	p (as decimal)
12/34	17/24		29/58	p (as fraction)
12.	17.		29.	X
34	24		58	n
	−0.3554	Difference		
	0.	Hypothesized difference		
	0.1333	Standard error		
	−2.67	z		
	0.0077	P-value (two-tailed)		

MINITAB
Step by Step

Test the Difference Between Two Proportions

For Example 9–9, test for a difference in the resident vaccination rates between small and large nursing homes.

1. This test does not require data. It doesn't matter what is in the worksheet.
2. Select Summarized data from the drop down menu.
3. Click the button for Summarized data.
4. Press **TAB** to move cursor to the Sample 1 box for Number of events.
   a) Enter **12, TAB,** then enter **34.**
   b) Press **TAB** or click in the Sample 2 text box for Number of events.
   c) Enter **17, TAB,** then enter **24.**
5. Click on [Options]. The Confidence level should be 95%, and the Hypothesized difference should be 0.
   a) For the Alternative hypothesis, select Difference ≠ hypothesized difference.
   b) For the Test method, select Use the pooled estimate of the proportion.
6. Click [OK] twice. The results are shown in the session window.



Test and CI for Two Proportions

Sample	X	N	Sample p
1	12	34	0.352941
2	17	24	0.708333

Difference = p (1) – p (2)
Estimate for difference: −0.355392
95% CI for difference: (−0.598025, −0.112759)
Test for difference = 0 (vs ≠ 0): Z = −2.67 P–Value = 0.008

The *P*-value of the test is 0.008. Reject the null hypothesis. The difference is statistically significant. Of all small nursing homes 35%, compared to 71% of all large nursing homes, have an immunization rate of less than 80%. We can't tell why, only that there is a difference.

9–5 Testing the Difference Between Two Variances

OBJECTIVE 5

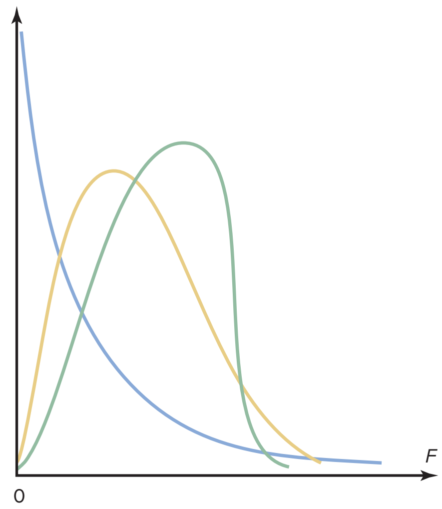
Test the difference between two variances or standard deviations.

In addition to comparing two means, statisticians are interested in comparing two variances or standard deviations. For example, is the variation in the temperatures for a certain month for two cities different?

In another situation, a researcher may be interested in comparing the variance of the cholesterol of men with the variance of the cholesterol of women. For the comparison of two variances or standard deviations, an ***F* test** is used. The *F* test should not be confused with the chi-square test, which compares a single sample variance to a specific population variance, as shown in Chapter 8.

Figure 9–10 shows the shapes of several curves for the F distribution.

FIGURE 9–10
The F Family of Curves



If two independent samples are selected from two normally distributed populations in which the population variances are equal ($\sigma_1^2 = \sigma_2^2$) and if the sample variances s_1^2 and s_2^2 are compared as $\frac{s_1^2}{s_2^2}$, the sampling distribution of the variances is called the ***F* distribution.**

Characteristics of the *F* Distribution

1. The values of F cannot be negative, because variances are always positive or zero.
2. The distribution is positively skewed.
3. The mean value of F is approximately equal to 1.
4. The F distribution is a family of curves based on the degrees of freedom of the variance of the numerator and the degrees of freedom of the variance of the denominator.

Formula for the *F* Test

$$F = \frac{s_1^2}{s_2^2}$$

where the larger of the two variances is placed in the numerator regardless of the subscripts. (See note on page 534.)

The F test has two values for the degrees of freedom: that of the numerator, $n_1 - 1$, and that of the denominator, $n_2 - 1$, where n_1 is the sample size from which the larger variance was obtained.

When you are finding the F test value, *the larger of the variances is placed in the numerator of the F formula;* this is not necessarily the variance of the larger of the two sample sizes.

Table H in Appendix A gives the F critical values for $\alpha = 0.005, 0.01, 0.025, 0.05$, and 0.10 (each α value involves a separate table in Table H). These are one-tailed values; if a two-tailed test is being conducted, then the $\alpha/2$ value must be used. For example, if a two-tailed test with $\alpha = 0.05$ is being conducted, then the $0.05/2 = 0.025$ table of Table H should be used.

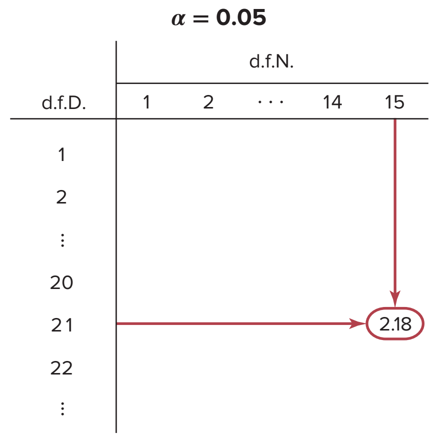
EXAMPLE 9–12

Find the critical value for a right-tailed F test when $\alpha = 0.05$, the degrees of freedom for the numerator (abbreviated d.f.N.) are 15, and the degrees of freedom for the denominator (d.f.D.) are 21.

SOLUTION

Since this test is right-tailed with $\alpha = 0.05$, use the 0.05 table. The d.f.N. is listed across the top, and the d.f.D. is listed in the left column. The critical value is found where the row and column intersect in the table. In this case, it is 2.18. See Figure 9–11.

FIGURE 9–11 Finding the Critical Value in Table H for Example 9–12



As noted previously, when the F test is used, the larger variance is always placed in the numerator of the formula. When you are conducting a two-tailed test, α is split; and even though there are two values, only the right tail is used. The reason is that the F test value is always greater than or equal to 1.

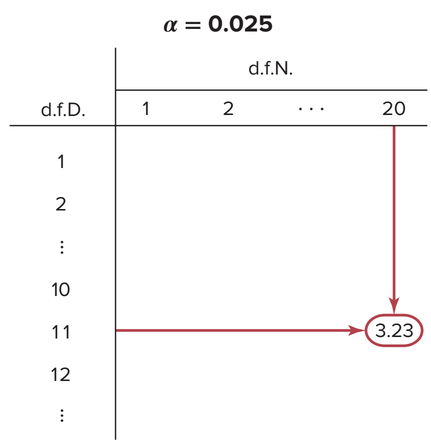
EXAMPLE 9–13

Find the critical value for a two-tailed F test with $\alpha = 0.05$ when the sample size from which the variance for the numerator was obtained was 21 and the sample size from which the variance for the denominator was obtained was 12.

SOLUTION

Since this is a two-tailed test with $\alpha = 0.05$, the $0.05/2 = 0.025$ table must be used. Here, d.f.N. = 21 − 1 = 20, and d.f.D. = 12 − 1 = 11; hence, the critical value is 3.23. See Figure 9–12.

FIGURE 9–12 Finding the Critical Value in Table H for Example 9–13



If the exact degrees of freedom are not specified in Table H, the closest smaller value should be used. For example, if $\alpha = 0.05$ (right-tailed test), d.f.N. = 18, and d.f.D. = 20, use the column d.f.N. = 15 and the row d.f.D. = 20 to get $F = 2.20$. Using the smaller value is the more conservative approach.

When you are testing the equality of two variances, these hypotheses are used:

Right-tailed	Left-tailed	Two-tailed
H_0: $\sigma_1^2 = \sigma_2^2$ H_1: $\sigma_1^2 > \sigma_2^2$	H_0: $\sigma_1^2 = \sigma_2^2$ H_1: $\sigma_1^2 < \sigma_2^2$	H_0: $\sigma_1^2 = \sigma_2^2$ H_1: $\sigma_1^2 \neq \sigma_2^2$

There are four key points to keep in mind when you are using the *F* test.

Unusual Stat

Of all U.S. births, 2% are twins.

Notes for the Use of the *F* Test

1. The larger variance should always be placed in the numerator of the formula regardless of the subscripts. (See note on page 534.)

$$F = \frac{s_1^2}{s_2^2}$$

2. For a two-tailed test, the α value must be divided by 2 and the critical value placed on the right side of the *F* curve.
3. If the standard deviations instead of the variances are given in the problem, they must be squared for the formula for the *F* test.
4. When the degrees of freedom cannot be found in Table H, the closest value on the smaller side should be used.

Before you can use the testing method to determine the difference between two variances, the following assumptions must be met.

Assumptions for Testing the Difference Between Two Variances

1. The samples must be random samples.
2. The populations from which the samples were obtained must be normally distributed. (*Note:* The test should not be used when the distributions depart from normality.)
3. The samples must be independent of one another.

In this book, the assumptions will be stated in the exercises; however, when encountering statistics in other situations, you must check to see that these assumptions have been met before proceeding.

Remember also that in tests of hypotheses using the traditional method, these five steps should be taken:

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value.

Step 3 Compute the test value.

Step 4 Make the decision.

Step 5 Summarize the results.

This procedure is not robust, so minor departures from normality will affect the results of the test. So this test should not be used when the distributions depart from normality because standard deviations are not a good measure of the spread in nonsymmetrical distributions. The reason is that the standard deviation is not resistant to outliers or extreme values. These values increase the value of the standard deviation when the distribution is skewed.

EXAMPLE 9–14 Heart Rates of Smokers

A medical researcher wishes to see whether the variance of the heart rates (in beats per minute) of smokers is different from the variance of heart rates of people who do not smoke. Two samples are selected, and the data are shown. Using $\alpha = 0.05$, is there enough evidence to support the claim? Assume the variable is normally distributed.

Smokers	Nonsmokers
$n_1 = 26$	$n_2 = 18$
$s_1^2 = 36$	$s_2^2 = 10$

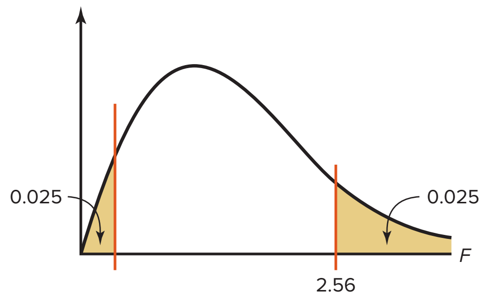
SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma_1^2 = \sigma_2^2 \qquad \text{and} \qquad H_1\colon \sigma_1^2 \neq \sigma_2^2 \text{ (claim)}$$

Step 2 Find the critical value. Use the 0.025 table in Table H since $\alpha = 0.05$ and this is a two-tailed test. Here, d.f.N. = 26 − 1 = 25, and d.f.D. = 18 − 1 = 17. The critical value is 2.56 (d.f.N. = 24 was used). See Figure 9–13.

FIGURE 9–13 Critical Value for Example 9–14



Step 3 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{36}{10} = 3.6$$

Step 4 Make the decision. Reject the null hypothesis, since 3.6 > 2.56.

Step 5 Summarize the results. There is enough evidence to support the claim that the variance of the heart rates of smokers and nonsmokers is different.

EXAMPLE 9–15 Grade Point Averages

A researcher selected a random sample of 10 psychology graduates and found the standard deviation of their grade point average was 0.72. Then she selected a random sample of 14 engineering students and found that the standard deviation of their grade point average was 0.51. At $\alpha = 0.01$, can we conclude that the variance of the grade point averages of the psychology graduates is greater than the variance of the grade point averages of the engineering graduates?

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma_1^2 = \sigma_2^2 \qquad \text{and} \qquad H_1\colon \sigma_1^2 > \sigma_2^2 \text{ (claim)}$$

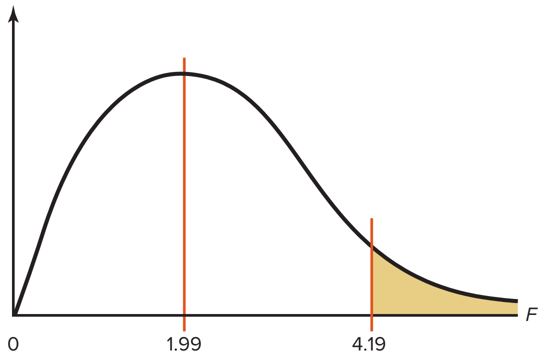
Step 2 Find the critical value. Hence, d.f.N. = 10 − 1 = 9 and d.f.D = 14 − 1 = 13. From Table H at $\alpha = 0.01$ the critical value is 4.19.

Step 3 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{0.72^2}{0.51^2} = 1.99$$

Step 4 Make the decision. Do not reject the null hypothesis since 1.99 < 4.19. That is, 1.99 does not fall in the critical region. See Figure 9–14.

FIGURE 9–14 Critical and Test Value for Example 9–15



Step 5 Summarize the results. There is not enough evidence to support the claim that the variance in the grade point average of psychology graduates is greater than the variance in the grade point average of the engineering graduates.

Finding P-values for the F test statistic is somewhat more complicated since it requires looking through all the F tables (Table H in Appendix A) using the specific d.f.N. and d.f.D. values. For example, suppose that a certain test has $F = 3.58$, d.f.N. = 5, and d.f.D. = 10. To find the P-value interval for $F = 3.58$, you must first find the corresponding F values for d.f.N. = 5 and d.f.D. = 10 for α equal to 0.005, 0.01, 0.025, 0.05, and 0.10 in Table H. Then make a table as shown.

α	0.10	0.05	0.025	0.01	0.005
F	2.52	3.33	4.24	5.64	6.87

Now locate the two F values that the test value 3.58 falls between. In this case, 3.58 falls between 3.33 and 4.24, corresponding to 0.05 and 0.025. Hence, the P-value for a right-tailed test for $F = 3.58$ falls between 0.025 and 0.05 (that is, $0.025 < P\text{-value} < 0.05$). For a right-tailed test, then, you would reject the null hypothesis at $\alpha = 0.05$, but not at $\alpha = 0.01$. The P-value obtained from a calculator is 0.0408. Remember that for a two-tailed test the values found in Table H for α must be doubled. In this case, $0.05 < P\text{-value} < 0.10$ for $F = 3.58$. Once again, if the P-value is less than α, we reject the null hypothesis.

Once you understand the concept, you can dispense with making a table as shown and find the P-value directly from Table H.

EXAMPLE 9–16 Airport Passengers

The CEO of an airport hypothesizes that the variance in the number of passengers for American airports is greater than the variance in the number of passengers for foreign airports. At $\alpha = 0.10$, is there enough evidence to support the hypothesis? The data in millions of passengers per year are shown for selected airports. Use the P-value method. Assume the variable is normally distributed and the samples are random and independent.

American airports		Foreign airports	
36.8	73.5	60.7	51.2
72.4	61.2	42.7	38.6
60.5	40.1		

Source: Airports Council International.

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \text{and} \quad H_1: \sigma_1^2 > \sigma_2^2 \text{ (claim)}$$

Step 2 Compute the test value. Using the formula in Chapter 3 or a calculator, find the variance for each group.

$$s_1^2 = 246.38 \quad \text{and} \quad s_2^2 = 95.87$$

Substitute in the formula and solve.

$$F = \frac{s_1^2}{s_2^2} = \frac{246.38}{95.87} = 2.57$$

Step 3 Find the *P*-value in Table H, using d.f.N. = 6 − 1 = 5 and d.f.D. = 4 − 1 = 3.

α	0.10	0.05	0.025	0.01	0.005
F	5.31	9.01	14.88	28.24	45.39

Since 2.57 is less than 5.31, the *P*-value is greater than 0.10. (The *P*-value obtained from a calculator is 0.234.)

Step 4 Make the decision. The decision is to not reject the null hypothesis since *P*-value > 0.10.

Step 5 Summarize the results. There is not enough evidence to support the claim that the variance in the number of passengers for American airports is greater than the variance in the number of passengers for foreign airports.

Note: It is not absolutely necessary to place the larger variance in the numerator when you are performing the F test. Critical values for left-tailed hypotheses tests can be found by interchanging the degrees of freedom and taking the reciprocal of the value found in Table H.

Also, you should use caution when performing the F test since the data can run contrary to the hypotheses on rare occasions. For example, if the hypotheses are $H_0: \sigma_1^2 \le \sigma_2^2$ (written $H_0: \sigma_1^2 = \sigma_2^2$) and $H_1: \sigma_1^2 > \sigma_2^2$, but if $s_1^2 < s_2^2$, then the F test should not be performed and you would not reject the null hypothesis.

Applying the Concepts 9–5

Automatic Transmissions

Assume the following data values are from a 2016 Auto Guide. The guide compared various parameters of U.S.- and foreign-made cars. This report centers on the price of an optional automatic transmission. Which country has the greater variability in the price of automatic transmissions? Answer the following questions.

Germany (2016)		U.S. Cars (2016)	
BMW 6 Series	\$77,300	Ford Mustang	\$47,795
Audi TT	\$46,400	Chevrolet Corvette	\$55,400
Porsche Boxster	\$82,100	Dodge Challenger	\$62,495
BMW 2 Series	\$50,750	Dodge Viper	\$87,895

1. What is the null hypothesis?
2. What test statistic is used to test for any significant differences in the variances?

3. Is there a significant difference in the variability in the prices between the German cars and the U.S. cars?
4. What effect does a small sample size have on the standard deviations?
5. What degrees of freedom are used for the statistical test?
6. Could two sets of data have significantly different variances without having significantly different means?

See page 545 for the answers.

Exercises 9–5

1. When one is computing the F test value, what condition is placed on the variance that is in the numerator?

2. Why is the critical region always on the right side in the use of the F test?

3. What are the two different degrees of freedom associated with the F distribution?

4. What are the characteristics of the F distribution?

5. Using Table H, find the critical value for each.

a. Sample 1: $s_1^2 = 140$, $n_1 = 25$
Sample 2: $s_2^2 = 125$, $n_2 = 14$
Two-tailed $\alpha = 0.05$

b. Sample 1: $s_1^2 = 43$, $n_1 = 12$
Sample 2: $s_2^2 = 56$, $n_2 = 16$
Right-tailed $\alpha = 0.10$

c. Sample 1: $s_1^2 = 516$, $n_1 = 21$
Sample 2: $s_2^2 = 472$, $n_2 = 18$
Right-tailed $\alpha = 0.01$

6. Using Table H, find the critical value for each.

a. Sample 1: $s_1^2 = 27.3$, $n_1 = 5$
Sample 2: $s_2^2 = 38.6$, $n_2 = 9$
Right-tailed, $\alpha = 0.01$

b. Sample 1: $s_1^2 = 164$, $n_1 = 21$
Sample 2: $s_2^2 = 53$, $n_2 = 17$
Two-tailed, $\alpha = 0.10$

c. Sample 1: $s_1^2 = 92.8$, $n_1 = 11$
Sample 2: $s_2^2 = 43.6$, $n_2 = 11$
Right-tailed, $\alpha = 0.05$

7. Using Table H, find the P-value interval for each F test value.

a. $F = 2.97$, d.f.N. = 9, d.f.D. = 14, right-tailed
b. $F = 3.32$, d.f.N. = 6, d.f.D. = 12, two-tailed
c. $F = 2.28$, d.f.N. = 12, d.f.D. = 20, right-tailed
d. $F = 3.51$, d.f.N. = 12, d.f.D. = 21, right-tailed

8. Using Table H, find the P-value interval for each F test value.

a. $F = 4.07$, d.f.N. = 6, d.f.D. = 10, two-tailed
b. $F = 1.65$, d.f.N. = 19, d.f.D. = 28, right-tailed
c. $F = 1.77$, d.f.N. = 28, d.f.D. = 28, right-tailed
d. $F = 7.29$, d.f.N. = 5, d.f.D. = 8, two-tailed

For Exercises 9 through 24, perform the following steps. Assume that all variables are normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value.
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

9. Wolf Pack Pups Does the variance in the number of pups per pack differ between Montana and Idaho wolf packs? Random samples of packs were selected for each area, and the numbers of pups per pack were recorded. At the 0.05 level of significance, can a difference in variances be concluded?

Montana wolf packs	4	3	5	6	1	2	8	2
	3	1	7	6	5			
Idaho wolf packs	2	4	5	4	2	4	6	3
	1	4	2	1				

Source: www.fws.gov

10. Noise Levels in Hospitals In a hospital study, it was found that the standard deviation of the sound levels from 20 randomly selected areas designated as "casualty doors" was 4.1 dBA and the standard deviation of 24 randomly selected areas designated as operating theaters was 7.5 dBA. At $\alpha = 0.05$, can you substantiate the claim that there is a difference in the standard deviations?

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health.*

11. **Calories in Ice Cream** The numbers of calories contained in $\frac{1}{2}$-cup servings of randomly selected flavors of ice cream from two national brands are listed. At the 0.05 level of significance, is there sufficient evidence to conclude that the variance in the number of calories differs between the two brands?

Brand A		Brand B	
330	300	280	310
310	350	300	370
270	380	250	300
310	300	290	310

Source: The Doctor's Pocket Calorie, Fat and Carbohydrate Counter.

12. **Winter Temperatures** A random sample of daily high temperatures in January and February is listed. At $\alpha = 0.05$, can it be concluded that there is a difference in variances in high temperature between the two months?

Jan.	31	31	38	24	24	42	22	43	35	42
Feb.	31	29	24	30	28	24	27	34	27	

13. **Population and Area** Cities were randomly selected from the list of the 50 largest cities in the United States (based on population). The areas of each in square miles are shown. Is there sufficient evidence to conclude that the variance in area is greater for eastern cities than for western cities at $\alpha = 0.05$? At $\alpha = 0.01$?

Eastern		Western	
Atlanta, GA	132	Albuquerque, NM	181
Columbus, OH	210	Denver, CO	155
Louisville, KY	385	Fresno, CA	104
New York, NY	303	Las Vegas, NV	113
Philadelphia, PA	135	Portland, OR	134
Washington, DC	61	Seattle, WA	84
Charlotte, NC	242		

Source: New York Times Almanac.

14. **Carbohydrates in Candy** The number of grams of carbohydrates contained in 1-ounce servings of randomly selected chocolate and nonchocolate candy is shown. Is there sufficient evidence to conclude that there is a difference between the variation in carbohydrate content for chocolate and nonchocolate candy? Use $\alpha = 0.10$.

Chocolate	29	25	17	36	41	25	32	29
	38	34	24	27	29			
Nonchocolate	41	41	37	29	30	38	39	10
	29	55	29					

Source: The Doctor's Pocket Calorie, Fat and Carbohydrate Counter.

15. **Tuition Costs for Medical School** The yearly tuition costs in dollars for random samples of medical schools that specialize in research and in primary care are listed. At $\alpha = 0.05$, can it be concluded that a difference between the variances of the two groups exists?

Research			Primary care		
30,897	34,280	31,943	26,068	21,044	30,897
34,294	31,275	29,590	34,208	20,877	29,691
20,618	20,500	29,310	33,783	33,065	35,000
21,274			27,297		

Source: U.S. News & World Report Best Graduate Schools.

16. **County Size in Indiana and Iowa** A researcher wishes to see if the variance of the areas in square miles for counties in Indiana is less than the variance of the areas for counties in Iowa. A random sample of counties is selected, and the data are shown. At $\alpha = 0.01$, can it be concluded that the variance of the areas for counties in Indiana is less than the variance of the areas for counties in Iowa?

Indiana				Iowa			
406	393	396	485	640	580	431	416
431	430	369	408	443	569	779	381
305	215	489	293	717	568	714	731
373	148	306	509	571	577	503	501
560	384	320	407	568	434	615	402

Source: The World Almanac and Book of Facts.

17. **Heights of Tall Buildings** Test the claim that the variance of heights of randomly selected tall buildings in Denver is equal to the variance in heights of randomly selected tall buildings in Nashville at $\alpha = 0.10$. The data are given in feet.

Denver			Nashville		
714	698	544	617	524	489
504	438	408	459	453	417
404			410	404	

Source: SkyscraperCenter.com

18. **Reading Program** Summer reading programs are very popular with children. At the Citizens Library, Team Ramona read an average of 23.2 books with a standard deviation of 6.1. There were 21 members on this team. Team Beezus read an average of 26.1 books with a standard deviation of 2.3. There were 23 members on this team. Did the variances of the two teams differ? Use $\alpha = 0.05$.

19. **Weights of Running Shoes** The weights in ounces of a random sample of running shoes for men and women are shown. Calculate the variances for each sample, and test the claim that the variances are equal at $\alpha = 0.05$. Use the P-value method.

Men			Women		
11.9	10.4	12.6	10.6	10.2	8.8
12.3	11.1	14.7	9.6	9.5	9.5
9.2	10.8	12.9	10.1	11.2	9.3
11.2	11.7	13.3	9.4	10.3	9.5
13.8	12.8	14.5	9.8	10.3	11.0

20. **School Teachers' Salaries** A researcher claims that the variation in the salaries of elementary school teachers

is greater than the variation in the salaries of secondary school teachers. A random sample of the salaries of 30 elementary school teachers has a variance of 8324, and a random sample of the salaries of 30 secondary school teachers has a variance of 2862. At $\alpha = 0.05$, can the researcher conclude that the variation in the elementary school teachers' salaries is greater than the variation in the secondary school teachers' salaries? Use the *P*-value method.

21. Ages of Dogs The average age of pet dogs is 4.3 years. Two random samples of pet owners who own dogs are selected. Sample 1 of 13 dog owners was selected from owners who live in Miami. The standard deviation of the ages of the dogs in this sample is 1.3 years. Sample 2 of 8 dog owners was selected from dog owners who live in Boston. The standard deviation of these dogs was 0.7 year. At $\alpha = 0.05$, can it be concluded that there is a difference in the variances?

22. Daily Stock Prices Two portfolios were randomly assembled from the New York Stock Exchange, and the daily stock prices are shown. At the 0.05, level of significance, can it be concluded that a difference in variance in price exists between the two portfolios?

23. Test Scores An instructor who taught an online statistics course and a classroom course feels that the variance of the final exam scores for the students who took the online course is greater than the variance of the final exam scores of the students who took the classroom final exam. The following data were obtained. At $\alpha = 0.05$ is there enough evidence to support the claim?

Online Course	Classroom Course
$s_1 = 3.2$	$s_2 = 2.8$
$n_1 = 11$	$n_2 = 16$

24. Museum Attendance A metropolitan children's museum open year-round wants to see if the variance in daily attendance differs between the summer and winter months. Random samples of 30 days each were selected and showed that in the winter months, the sample mean daily attendance was 300 with a standard deviation of 52, and the sample mean daily attendance for the summer months was 280 with a standard deviation of 65. At $\alpha = 0.05$, can we conclude a difference in variances?

Portfolio A	36.44	44.21	12.21	59.60	55.44	39.42	51.29	48.68	41.59	19.49
Portfolio B	32.69	47.25	49.35	36.17	63.04	17.74	4.23	34.98	37.02	31.48

Source: Washington Observer-Reporter.

Technology Step by Step

TI-84 Plus Step by Step

Hypothesis Test for the Difference Between Two Variances (Data)

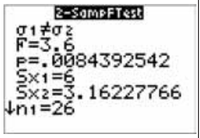
1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **E (ALPHA SIN)** for 2-SampFTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate Alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Variances (Statistics)

Example TI9–10

This refers to Example 9–14 in the text.

1. Press **STAT** and move the cursor to TESTS.
2. Press **E (ALPHA SIN)** for 2-SampFTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate Alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**



EXCEL
Step by Step

F Test for the Difference Between Two Variances

Excel has a two-sample *F* test included in the Data Analysis Add-in. To perform an *F* test for the difference between the variances of two populations, given two independent samples, do this:

1. Enter the first sample data set into column A.
2. Enter the second sample data set into column B.
3. Select the Data tab from the toolbar. Then select Data Analysis.
4. In the Analysis Tools box, select F-test Two-sample for Variances.
5. Type the ranges for the data in columns A and B.
6. Specify the confidence level Alpha.
7. Specify a location for the output, and click [OK].

Example XL9–4

At $\alpha = 0.05$, test the hypothesis that the two population variances are equal, using the sample data provided here.

Set A	63	73	80	60	86	83	70	72	82
Set B	86	93	64	82	81	75	88	63	63

The results appear in the table that Excel generates, shown here. For this example, the output shows that the null hypothesis cannot be rejected at an α level of 0.05.

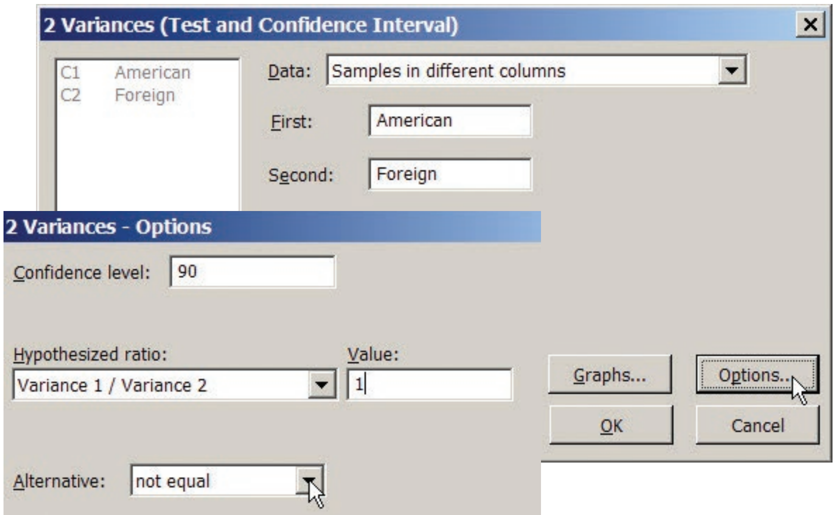
F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	74.33333333	77.22222222
Variance	82.75	132.9444444
Observations	9	9
df	8	8
F	0.622440451	
P(F<=f) one-tail	0.258814151	
F Critical one-tail	0.290858219	

MINITAB
Step by Step

Test for the Difference Between Two Variances

For Example 9–16, test the hypothesis that the variance in the number of passengers for American and foreign airports is different. Use the *P*-value approach.

American airports	Foreign airports
36.8	60.7
72.4	42.7
60.5	51.2
73.5	38.6
61.2	
40.1	



1. Enter the data into two columns of MINITAB.
2. Name the columns American and Foreign.
   a) Select **Stat>Basic Statistics>2-Variances.**
   b) Select Each sample is in its own column from the drop down menu.
   c) Click in the text box for Sample 1, then double-click C1 American.
   d) Double-click C2 Foreign, then click on [Options]. The dialog box is shown. For ratio, select (sample 1 variance) / (sample 2 variance) and change the confidence level to **90**. The hypothesized ratio should be 1. For the Alternative hypothesis, select Ratio > hypothesized ratio. Check the box for Use test and confidence intervals based on normal distribution.
3. Click [OK] twice. A graph window will open that includes a small window that says the *P*-value is 0.234. In the session window, the F-test statistic is shown as the Ratio of variances = 2.570. You can view the session window by closing the graph or clicking and dragging it to the right hand part of your screen.

There is not enough evidence in the sample to conclude there is greater variance in the number of passengers in American airports compared to foreign airports.

Summary

Many times researchers are interested in comparing two parameters such as two means, two proportions, or two variances. These measures are obtained from two samples, then compared using a z test, t test, or an F test.

- If two sample means are compared, when the samples are independent and the population standard deviations are known, a z test is used. If the sample sizes are less than 30, the populations should be normally distributed. (9–1)
- If two means are compared when the samples are independent and the sample standard deviations are used, then a t test is used. The two variances are assumed to be unequal. (9–2)
- When the two samples are dependent or related, such as using the same subjects and comparing the means of before-and-after tests, then the t test for dependent samples is used. (9–3)
- Two proportions can be compared by using the z test for proportions. In this case, each of n_1p_1, n_1q_1, n_2p_2, and n_2q_2 must all be 5 or more. (9–4)
- Two variances can be compared by using an F test. The critical values for the F test are obtained from the F distribution. (9–5)
- Confidence intervals for differences between two parameters can also be found.

Important Terms

dependent samples 507

F distribution 529

F test 528

independent samples 499

pooled estimate of the variance 502

Important Formulas

Formula for the z test for comparing two means from independent populations; σ_1 and σ_2 are known:

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Formula for the confidence interval for difference of two means when σ_1 and σ_2 are known:

$$(\overline{X}_1 - \overline{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for the t test for comparing two means (independent samples, variances not equal), σ_1 and σ_2 unknown:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

and d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

Formula for the confidence interval for the difference of two means (independent samples, variances unequal), σ_1 and σ_2 unknown:

$$(\overline{X}_1 - \overline{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

and d.f. = smaller of $n_1 - 1$ and $n_2 - 1$.

Formula for the t test for comparing two means from dependent samples:

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

d.f. $= n - 1$

where $\overline{D}$ is the mean of the differences

$$\overline{D} = \frac{\Sigma D}{n}$$

and s_D is the standard deviation of the differences

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Formula for confidence interval for the mean of the difference for dependent samples:

$$\overline{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \overline{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

and d.f. $= n - 1$.

Formula for the z test for comparing two proportions:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p}\,\overline{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

where

$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\overline{q} = 1 - \overline{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

Formula for confidence interval for the difference of two proportions:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Formula for the F test for comparing two variances:

$$F = \frac{s_1^2}{s_2^2} \qquad \begin{array}{l}\text{d.f.N.} = n_1 - 1\\ \text{d.f.D.} = n_2 - 1\end{array}$$

The larger variance is placed in the numerator.

Review Exercises

For each exercise, perform these steps. Assume that all variables are normally or approximately normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

Section 9–1

1. **Driving for Pleasure** Two groups of randomly selected drivers are surveyed to see how many miles per week they drive for pleasure trips. The data are shown. At $\alpha = 0.01$, can it be concluded that single drivers do more driving for pleasure trips on average than married drivers? Assume $\sigma_1 = 16.7$ and $\sigma_2 = 16.1$.

Single drivers					Married drivers				
106	110	115	121	132	97	104	138	102	115
119	97	118	122	135	133	120	119	136	96
110	117	116	138	142	139	108	117	145	114
115	114	103	98	99	140	136	113	113	150
108	117	152	147	117	101	114	116	113	135
154	86	115	116	104	115	109	147	106	88
107	133	138	142	140	113	119	99	108	105

2. **Average Earnings of College Graduates** The average yearly earnings of male college graduates (with at least a bachelor's degree) are \$58,500 for men aged 25 to 34. The average yearly earnings of female college graduates with the same qualifications are \$49,339. Based on the results below, can it be concluded that there is a difference in mean earnings between male and female college graduates? Use the 0.01 level of significance.

	Male	Female
Sample mean	\$59,235	\$52,487
Population standard deviation	\$8,945	\$10,125
Sample size	40	35

Source: New York Times Almanac.

Section 9–2

3. **Physical Therapy** A recent study of 20 individuals found that the average number of therapy sessions a person takes for a shoulder problem is 9.6. The standard deviation of the sample was 2.8. A study of 25 individuals with a hip problem found that they had a mean of 10.3 sessions. The standard deviation for this sample was 2.3. At $\alpha = 0.01$, is there a significant difference in the means?

4. **Average Temperatures** The average temperatures for a 25-day period for Birmingham, Alabama, and Chicago, Illinois, are shown. Based on the samples, at $\alpha = 0.10$, can it be concluded that it is warmer in Birmingham?

Birmingham					Chicago				
78	82	68	67	68	70	74	73	60	77
75	73	75	64	68	71	72	71	74	76
62	73	77	78	79	71	80	65	70	83
74	72	73	78	68	67	76	75	62	65
73	79	82	71	66	66	65	77	66	64

5. **Teachers' Salaries** A random sample of 15 teachers from Rhode Island has an average salary of \$35,270, with a standard deviation of \$3256. A random sample of 30 teachers from New York has an average salary of \$29,512, with a standard deviation of \$1432. Is there a significant difference in teachers' salaries between the two states? Use $\alpha = 0.02$. Find the 98% confidence interval for the difference of the two means.

6. **Soft Drinks in School** The data show the amounts (in thousands of dollars) of the contracts for soft drinks in randomly selected local school districts. At $\alpha = 0.10$, can it be concluded that there is a difference in the averages? Use the *P*-value method. Give a reason why the result would be of concern to a cafeteria manager.

Pepsi						Coca-Cola		
46	120	80	500	100	59	420	285	57

Source: Local school districts.

Section 9–3

7. **High and Low Temperatures** March is a month of variable weather in the Northeast. The chart shows records of the actual high and low temperatures for a selection of days in March from the weather report for Pittsburgh, Pennsylvania. At the 0.01 level of significance, is there sufficient evidence to conclude that there is more than a 10° difference between average highs and lows?

Maximum	44	46	46	36	34	36	57	62	73	53
Minimum	27	34	24	19	19	26	33	57	46	26

Source: www.wunderground.com

8. **Testing After Review** A statistics class was given a pretest on probability (since many had previous experience in some other class). Then the class was given a six-page review handout to study for two days. At the next class they were given another test. Is there sufficient evidence that the scores improved? Use $\alpha = 0.05$.

Student	1	2	3	4	5	6
Pretest	52	50	40	58	60	52
Posttest	62	65	50	65	68	63

Section 9–4

9. **Lay Teachers in Religious Schools** A study found a slightly lower percentage of lay teachers in religious secondary schools than in elementary schools. A random sample of 200 elementary school and 200 secondary school teachers from religious schools in a large diocese found the following. At the 0.05 level of significance, is there sufficient evidence to conclude a difference in proportions?

	Elementary	Secondary
Sample size	200	200
Lay teachers	49	62

Source: New York Times Almanac.

10. **Gambling** A survey of 60 men found that 36 gamble. Another survey of 50 women found that 28 gamble. At $\alpha = 0.01$, is there a difference in the proportions?

Section 9–5

11. **Noise Levels in Hospitals** In the hospital study cited previously, the standard deviation of the noise levels of the 11 intensive care units was 4.1 dBA, and the standard deviation of the noise levels of 24 nonmedical care areas, such as kitchens and machine rooms, was 13.2 dBA. At $\alpha = 0.10$, is there a significant difference between the standard deviations of these two areas?

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health.*

12. **Heights of World Famous Cathedrals** The heights (in feet) for a random sample of world famous cathedrals are listed. In addition, the heights for a random sample of the tallest buildings in the world are listed. Is there sufficient

evidence at $\alpha = 0.05$ to conclude that there is a difference in the variances in height between the two groups?

Cathedrals	72	114	157	56	83	108	90	151	
Tallest buildings	452	442	415	391	355	344	310	302	209

Source: www.infoplease.com

13. Sodium Content of Cereals The sodium content of brands of cereal produced by two major manufacturers is shown. At $\alpha = 0.01$, is there a significant difference in the variances?

Manufacturer 1			Manufacturer 2		
87	92	96	87	92	93
100	94	94	91	100	94
101	103	98	103	96	98
91	92	96	87	92	91

STATISTICS TODAY

To Vaccinate or Not to Vaccinate? Small or Large? —Revisited

Using a z test to compare two proportions, the researchers found that the proportion of residents in smaller nursing homes who were vaccinated (80.8%) was statistically greater than that of residents in large nursing homes who were vaccinated (68.7%). Using statistical methods presented in later chapters, they also found that the larger size of the nursing home and the lower frequency of vaccination were significant predictions of influenza outbreaks in nursing homes.

Data Analysis

The Data Bank is found in Appendix B, or on the World Wide Web by following links from www.mhhe.com/math/stat/bluman/

1. From the Data Bank, select a variable and compare the mean of the variable for a random sample of at least 30 men with the mean of the variable for the random sample of at least 30 women. Use a z test.
2. Repeat the experiment in Exercise 1, using a different variable and two samples of size 15. Compare the means by using a t test.
3. Compare the proportion of men who are smokers with the proportion of women who are smokers. Use the data in the Data Bank. Choose random samples of size 30 or more. Use the z test for proportions.
4. Select two samples of 20 values from the data in Data Set IV in Appendix B. Test the hypothesis that the mean heights of the buildings are equal.
5. Using the same data obtained in Exercise 4, test the hypothesis that the variances are equal.

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. When you are testing the difference between two means, it is not important to distinguish whether the samples are independent of each other.
2. If the same diet is given to two groups of randomly selected individuals, the samples are considered to be dependent.
3. When computing the F test value, you should place the larger variance in the numerator of the fraction.
4. Tests for variances are always two-tailed.

Select the best answer.

5. To test the equality of two variances, you would use a(n) _______ test.
   a. z *c.* Chi-square
   b. t *d.* F
6. To test the equality of two proportions, you would use a(n) _______ test.
   a. z *c.* Chi-square
   b. t *d.* F
7. The mean value of F is approximately equal to
   a. 0 *c.* 1
   b. 0.5 *d.* It cannot be determined.
8. What test can be used to test the difference between two sample means when the population variances are known?
   a. z *c.* Chi-square
   b. t *d.* F

Complete these statements with the best answer.

9. If you hypothesize that there is no difference between means, this is represented as H_0: ________.

10. When you are testing the difference between two means, the _______ test is used when the population variances are not known.

11. When the t test is used for testing the equality of two means, the populations must be _______.

12. The values of F cannot be _______.

13. The formula for the F test for variances is _______.

For each of these problems, perform the following steps.

a. State the hypotheses and identify the claim.

b. Find the critical value(s).

c. Compute the test value.

d. Make the decision.

e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

14. Cholesterol Levels A researcher wishes to see if there is a difference in the cholesterol levels of two groups of men. A random sample of 30 men between the ages of 25 and 40 is selected and tested. The average level is 223. A second random sample of 25 men between the ages of 41 and 56 is selected and tested. The average of this group is 229. The population standard deviation for both groups is 6. At $\alpha = 0.01$, is there a difference in the cholesterol levels between the two groups? Find the 99% confidence interval for the difference of the two means.

15. Apartment Rental Fees The data shown are the rental fees (in dollars) for two random samples of apartments in a large city. At $\alpha = 0.10$, can it be concluded that the average rental fee for apartments in the east is greater than the average rental fee in the west? Assume $\sigma_1 = 119$ and $\sigma_2 = 103$.

East					West				
495	390	540	445	420	525	400	310	375	750
410	550	499	500	550	390	795	554	450	370
389	350	450	530	350	385	395	425	500	550
375	690	325	350	799	380	400	450	365	425
475	295	350	485	625	375	360	425	400	475
275	450	440	425	675	400	475	430	410	450
625	390	485	550	650	425	450	620	500	400
685	385	450	550	425	295	350	300	360	400

Source: Pittsburgh Post-Gazette.

16. Prices of Low-Calorie Foods The average price of a random sample of 12 bottles of diet salad dressing taken from different stores is \$1.43. The standard deviation is \$0.09. The average price of a random sample of 16 low-calorie frozen desserts is \$1.03. The standard deviation is \$0.10. At $\alpha = 0.01$, is there a significant difference in price? Find the 99% confidence interval of the difference in the means.

17. Jet Ski Accidents The data shown represent the number of accidents people had when using jet skis and other types of wet bikes. At $\alpha = 0.05$, can it be concluded that the average number of accidents per year has increased from one period to the next?

Earlier period			Later period		
376	650	844	1650	2236	3002
1162	1513		4028	4010	

Source: USA TODAY.

18. Salaries of Chemists A random sample of 12 chemists from Washington state shows an average salary of \$39,420 with a standard deviation of \$1659, while a random sample of 26 chemists from New Mexico has an average salary of \$30,215 with a standard deviation of \$4116. Is there a significant difference between the two states in chemists' salaries at $\alpha = 0.02$? Find the 98% confidence interval of the difference in the means.

19. Family Incomes The average income of 15 randomly selected families who reside in a large metropolitan East Coast city is \$62,456. The standard deviation is \$9652. The average income of 11 randomly selected families who reside in a rural area of the Midwest is \$60,213, with a standard deviation of \$2009. At $\alpha = 0.05$, can it be concluded that the families who live in the cities have a higher income than those who live in the rural areas? Use the P-value method.

20. Mathematical Skills In an effort to improve the mathematical skills of 10 students, a teacher provides a weekly 1-hour tutoring session for the students. A pretest is given before the sessions, and a posttest is given after. The results are shown here. At $\alpha = 0.01$, can it be concluded that the sessions help to improve the students' mathematical skills?

Student	1	2	3	4	5	6	7	8	9	10
Pretest	82	76	91	62	81	67	71	69	80	85
Posttest	88	80	98	80	80	73	74	78	85	93

21. Egg Production To increase egg production, a farmer decided to increase the amount of time the lights in his hen house were on. Ten hens were randomly selected, and the number of eggs each produced was recorded. After one week of lengthened light time, the same hens were monitored again. The data are given here. At $\alpha = 0.05$, can it be concluded that the increased light time increased egg production?

Hen	1	2	3	4	5	6	7	8	9	10
Before	4	3	8	7	6	4	9	7	6	5
After	6	5	9	7	4	5	10	6	9	6

22. Factory Worker Literacy Rates In a random sample of 80 workers from a factory in city A, it was found that 5% were unable to read, while in a random sample of 50 workers in city B, 8% were unable to read. Can it be concluded that there is a difference in the proportions of nonreaders in the two cities? Use $\alpha = 0.10$. Find the 90% confidence interval for the difference of the two proportions.

23. **Male Head of Household** A recent survey of 200 randomly selected households showed that 8 had a single male as the head of household. Forty years ago, a survey of 200 randomly selected households showed that 6 had a single male as the head of household. At $\alpha = 0.05$, can it be concluded that the proportion has changed? Find the 95% confidence interval of the difference of the two proportions. Does the confidence interval contain 0? Why is this important to know?

Source: Based on data from the U.S. Census Bureau.

24. **Money Spent on Road Repair** A politician wishes to compare the variances of the amount of money spent for road repair in two different counties. The data are given here. At $\alpha = 0.05$, is there a significant difference in the variances of the amounts spent in the two counties? Use the *P*-value method.

County A	County B
$s_1 = \$11{,}596$	$s_2 = \$14{,}837$
$n_1 = 15$	$n_2 = 18$

25. **Heights of Basketball Players** A researcher wants to compare the variances of the heights (in inches) of four-year college basketball players with those of players in junior colleges. A random sample of 30 players from each type of school is selected, and the variances of the heights for each type are 2.43 and 3.15, respectively. At $\alpha = 0.10$, is there a significant difference between the variances of the heights in the two types of schools?

Data Projects

Use a significance level of 0.05 for all tests below.

1. **Business and Finance** Use the data collected in data project 1 of Chapter 2 to complete this problem. Test the claim that the mean earnings per share for Dow Jones stocks are greater than for NASDAQ stocks.

2. **Sports and Leisure** Use the data collected in data project 2 of Chapter 7 regarding home runs for this problem. Test the claim that the mean number of home runs hit by the American League sluggers is the same as the mean for the National League.

3. **Technology** Use the cell phone data collected for data project 2 in Chapter 8 to complete this problem. Test the claim that the mean length for outgoing calls is the same as that for incoming calls. Test the claim that the standard deviation for outgoing calls is more than that for incoming calls.

4. **Health and Wellness** Use the data regarding BMI that were collected in data project 6 of Chapter 7 to complete this problem. Test the claim that the mean BMI for males is the same as that for females. Test the claim that the standard deviation for males is the same as that for females.

5. **Politics and Economics** Using data from the Internet for the last Presidential election to categorize the 50 states as "red" or "blue" based on who was supported for President in that state, the Democratic or Republican candidate, test the claim that the mean incomes for red states and blue states are equal.

6. **Your Class** Use the data collected in data project 6 of Chapter 2 regarding heart rates. Test the claim that the heart rates after exercise are more variable than the heart rates before exercise.

Answers to Applying the Concepts

Section 9–1 Home Runs

1. The population of all home runs hit by major league baseball players.

2. A cluster sample was used.

3. Answers will vary. While this sample is not representative of all major league baseball players per se, it does allow us to compare the leaders in each league.

4. H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$

5. Answers will vary. Possible answers include the 0.05 and 0.01 significance levels.

6. We will use the z test for the difference in means.

7. Our test statistic is $z = \dfrac{44.75 - 42.88}{\sqrt{\dfrac{8.8^2}{40} + \dfrac{7.8^2}{40}}} = 1.01$, and our *P*-value is 0.3124.

8. We fail to reject the null hypothesis.

9. There is not enough evidence to conclude that there is a difference in the number of home runs hit by National League versus American League baseball players.

10. Answers will vary. One possible answer is that since we do not have a random sample of data from each league, we cannot answer the original question asked.

11. Answers will vary. One possible answer is that we could get a random sample of data from each league from a recent season.

Section 9–2 Too Long on the Telephone

1. These samples are independent.

2. We compare the *P*-value of 0.06317 to the significance level to check if the null hypothesis should be rejected.

3. The P-value of 0.06317 also gives the probability of a type I error.
4. Since two critical values are shown, we know that a two-tailed test was done.
5. Since the P-value of 0.06317 is greater than the significance value of 0.05, we fail to reject the null hypothesis and find that we do not have enough evidence to conclude that there is a difference in the lengths of telephone calls made by employees in the two divisions of the company.
6. If the significance level had been 0.10, we would have rejected the null hypothesis, since the P-value would have been less than the significance level.

Section 9–3 Air Quality

1. The purpose of the study is to determine if the air quality in the United States has changed over the past 2 years.
2. These are dependent samples, since we have two readings from each of 10 metropolitan areas.
3. The hypotheses we will test are H_0: $\mu_D = 0$ and H_1: $\mu_D \neq 0$.
4. We will use the 0.05 significance level and critical values of $t = \pm 2.262$.
5. We will use the t test for dependent samples.
6. There are $10 - 1 = 9$ degrees of freedom.
7. Our test statistic is $t = \dfrac{-6.7 - 0}{11.27/\sqrt{10}} = -1.879$. We fail to reject the null hypothesis and find that there is not enough evidence to conclude that the air quality in the United States has changed over the past 2 years.
8. No, we could not use an independent means test since we have two readings from each metropolitan area.
9. Answers will vary. One possible answer is that there are other measures of air quality that we could have examined to answer the question.

Section 9–4 Smoking and Education

1. Our hypotheses are H_0: $p_1 = p_2$ and H_1: $p_1 \neq p_2$.
2. At the 0.05 significance level, our critical values are $z = \pm 1.96$.
3. We will use the z test for the difference between proportions.
4. To complete the statistical test, we would need the sample sizes.
5. Knowing the sample sizes were 1000, we can now complete the test.
6. Our test statistic is
$$z = \frac{0.323 - 0.145}{\sqrt{(0.234)(0.766)\left(\frac{1}{1000} + \frac{1}{1000}\right)}} = 9.40,$$
and our P-value is very close to zero. We reject the null hypothesis and find that there is enough evidence to conclude that there is a difference in the proportions of public school students and private school students who smoke.

Section 9–5 Variability and Automatic Transmissions

1. The null hypothesis is that the variances are the same: H_0: $\sigma_1^2 = \sigma_2^2$ (H_1: $\sigma_1^2 \neq \sigma_2^2$).
2. We will use an F test.
3. The value of the test statistic is $F = \dfrac{s_1^2}{s_2^2} = \dfrac{18{,}163.58^2}{17{,}400.57^2} =$ 1.090 and the P-value > 0.05. There is not a significant difference in the variability of the prices between the two countries.
4. Small sample sizes are highly impacted by outliers.
5. The degrees of freedom for the numerator and denominator are both 3.
6. Yes, two sets of data can center on the same mean but have very different standard deviations.

Hypothesis-Testing Summary 1

1. Comparison of a sample mean with a specific population mean.

   Example: H_0: $\mu = 100$

   a. Use the z test when σ is known:
$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$
   b. Use the t test when σ is unknown:
$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} \quad \text{with d.f.} = n - 1$$
2. Comparison of a sample variance or standard deviation with a specific population variance or standard deviation.

   Example: H_0: $\sigma^2 = 225$

   Use the chi-square test:
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{with d.f.} = n - 1$$
3. Comparison of two sample means.

   Example: H_0: $\mu_1 = \mu_2$

   a. Use the z test when the population variances are known:
$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

b. Use the t test for independent samples when the population variances are unknown and assume the sample variances are unequal:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

with d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

c. Use the t test for means for dependent samples:

Example: H_0: $\mu_D = 0$

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

where n = number of pairs.

4. Comparison of a sample proportion with a specific population proportion.

Example: H_0: $p = 0.32$

Use the z test:

$$z = \frac{X - \mu}{\sigma} \qquad \text{or} \qquad z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

5. Comparison of two sample proportions.

Example: H_0: $p_1 = p_2$

Use the z test:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\overline{q} = 1 - \overline{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

6. Comparison of two sample variances or standard deviations.

Example: H_0: $\sigma_1^2 = \sigma_2^2$

Use the F test:

$$F = \frac{s_1^2}{s_2^2}$$

where

s_1^2 = larger variance d.f.N. = $n_1 - 1$

s_2^2 = smaller variance d.f.D. = $n_2 - 1$