STATISTICS

Normal Probability Distributions

Chapter Outline

- 5.1 Introduction to Normal Distributions and the Standard Normal Distribution
- 5.2 Normal Distributions: Finding Probabilities
- 5.3 Normal Distributions: Finding Values
- 5.4 Sampling Distributions and the Central Limit Theorem
- 5.5 Normal Approximations to Binomial Distributions

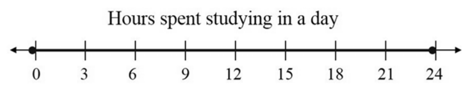
Section 5.1

Introduction to Normal Distributions and the Standard Normal Distributions

Properties of a Normal Distribution (1 of 4)

Continuous random variable

- Has an infinite number of possible values that can be represented by an interval on the number line.



The time spent studying can be any number between 0 and 24.

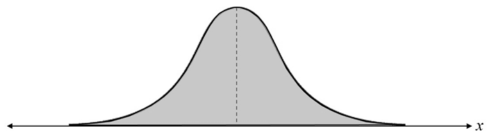
Continuous probability distribution

- The probability distribution of a continuous random variable.

Properties of a Normal Distribution (2 of 4)

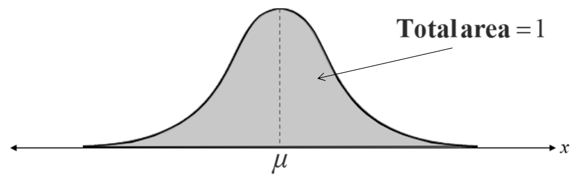
Normal distribution

- A continuous probability distribution for a random variable, *x*.
- The most important continuous probability distribution in statistics.
- The graph of a normal distribution is called the **normal curve**.



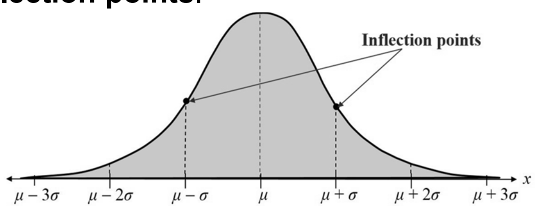
Properties of a Normal Distribution (3 of 4)

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and is symmetric about the mean.
3. The total area under the normal curve is equal to one.
4. The normal curve approaches, but never touches the *x*-axis as it extends farther and farther away from the mean.



Properties of a Normal Distribution (4 of 4)

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.

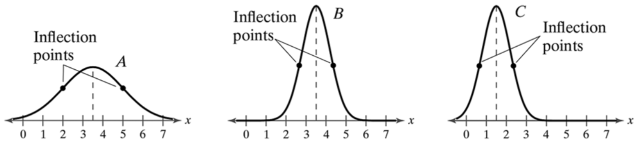


Probability Density Function (PDF)

- A discrete probability distribution can be graphed with a histogram.
- For a continuous probability distribution, you can use a **probability density function (pdf).**
- A probability density function has two requirements:
  1. the total area under the curve is equal to 1
  2. the function can never be negative.

Means and Standard Deviations

- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.



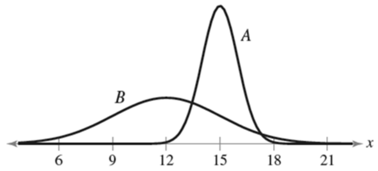
Mean: $\mu = 3.5$
Standard deviation: $\sigma = 1.5$

Mean: $\mu = 3.5$
Standard deviation: $\sigma = 0.7$

Mean: $\mu = 1.5$
Standard deviation: $\sigma = 0.7$

Example: Understanding Mean and Standard Deviation (1 of 2)

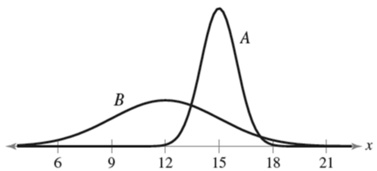
1. Which curve has the greater mean?



Solution:
Curve *A* has the greater mean (The line of symmetry of curve *A* occurs at $x = 15$. The line of symmetry of curve *B* occurs at $x = 12$.)

Example: Understanding Mean and Standard Deviation (2 of 2)

2. Which curve has the greater standard deviation?

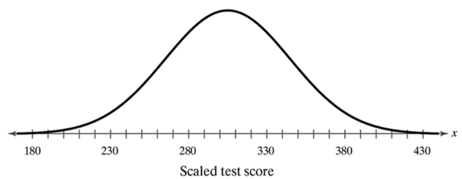


Solution:
Curve *B* has the greater standard deviation (Curve *B* is more spread out than curve *A*.)

Example: Interpreting Graphs of Normal Distributions

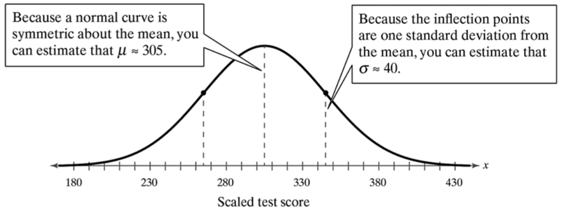
The scaled test scores for New York State Grade 4 Common Core Mathematics Test are normally distributed. The normal curve shown below represents this distribution. What is the mean test score? Estimate the standard deviation of this normal distribution.
(Adapted from New York State Education Department)

Mathematics Test



Solution: Interpreting Graphs of Normal Distributions (1 of 2)

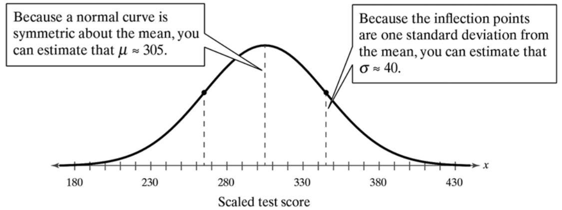
Solution:



The scaled test scores for the New York State Grade 4 Common Core Mathematics Test are normally distributed with a mean of about 305 and a standard deviation of about 40.

Solution: Interpreting Graphs of Normal Distributions (2 of 2)

Solution:

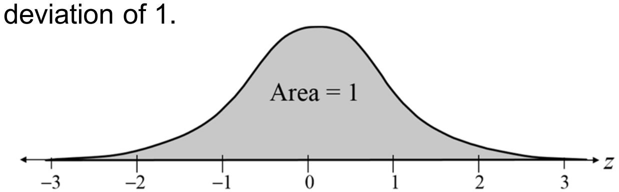


Using the Empirical, you know that about 68% of the scores are between 265 and 345, about 95% of the scores are between 225 and 385, and about 99.7% of the scores are between 185 and 425.

The Standard Normal Distribution (1 of 2)

Standard normal distribution

- A normal distribution with a mean of 0 and a standard deviation of 1.

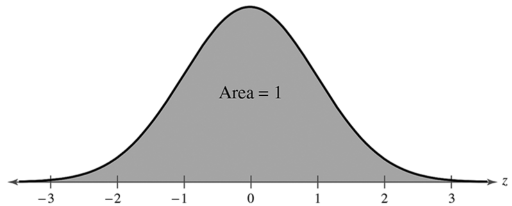


- Any *x*-value can be transformed into a *z*-score by using the formula

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution (2 of 2)

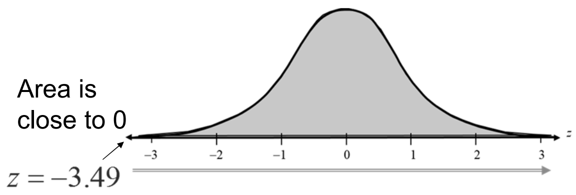
The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1. The total area under its normal curve is 1.



Standard Normal Distribution

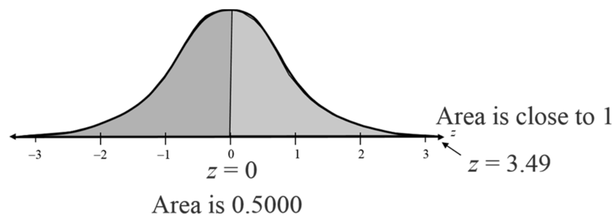
Properties of the Standard Normal Distribution (1 of 2)

1. The cumulative area is close to 0 for *z*-scores close to $z = -3.49$.
2. The cumulative area increases as the *z*-scores increase.



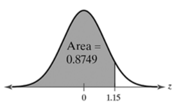
Properties of the Standard Normal Distribution (2 of 2)

3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for *z*-scores close to $z = 3.49$.



Example: Using The Standard Normal Table (1 of 2)

1. Find the cumulative area that corresponds to a *z*-score of 1.15.

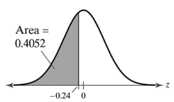


z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

Solution:
Find 1.1 in the left hand column.
Move across the row to the column under 0.05
The area to the left of $z = 1.15$ is 0.8749.

Example: Using The Standard Normal Table (2 of 2)

2. Find the cumulative area that corresponds to a *z*-score of -0.24.

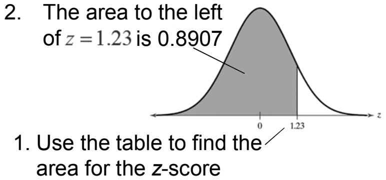


z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

Solution:
Find -0.2 in the left hand column.
Move across the row to the column under 0.04.
The area to the left of $z = -0.24$ is 0.4052.

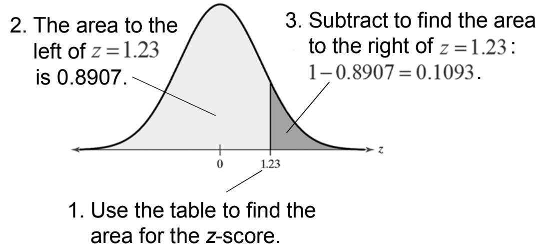
Finding Areas Under the Standard Normal Curve (1 of 3)

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
   a. To find the area to the *left* of *z*, find the area that corresponds to *z* in the Standard Normal Table.



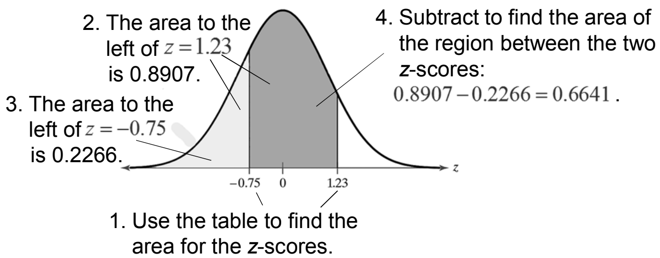
Finding Areas Under the Standard Normal Curve (2 of 3)

b. To find the area to the *right* of *z*, use the Standard Normal Table to find the area that corresponds to *z*. Then subtract the area from 1.



Finding Areas Under the Standard Normal Curve (3 of 3)

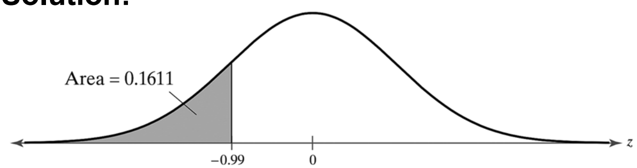
c. To find the area *between* two *z*-scores, find the area corresponding to each *z*-score in the Standard Normal Table. Then subtract the smaller area from the larger area.



Example: Finding Area Under the Standard Normal Curve (1 of 3)

Find the area under the standard normal curve to the left of $z = -0.99$.

Solution:



From the Standard Normal Table, the area is equal to 0.1611.

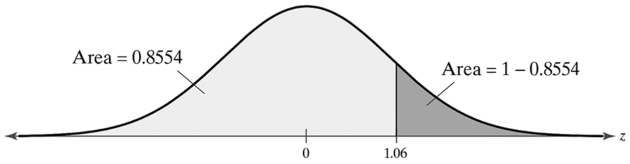
EXCEL

	A	B
1	NORM.S.DIST(-0.99,TRUE)	
2		0.16108706

Example: Finding Area Under the Standard Normal Curve (2 of 3)

Find the area under the standard normal curve to the right of $z = 1.06$.

Solution:



From the Standard Normal Table, the area is equal to 0.1446.

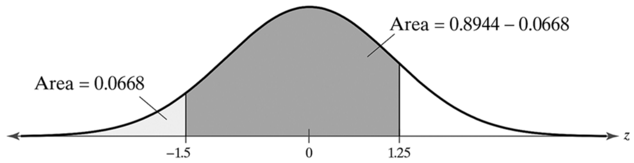
EXCEL

	A	B
1	1-NORM.S.DIST(1.06,TRUE)	
2		0.1445723

Example: Finding Area Under the Standard Normal Curve (3 of 3)

Find the area under the standard normal curve between $z = -1.5$ and $z = 1.25$.

Solution:



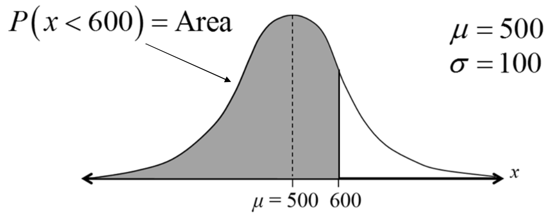
From the Standard Normal Table, the area is equal to 0.8276. So, 82.76% of the area under the curve falls between $z = -1.5$ and $z = 1.25$.

Section 5.2

Normal Distributions: Finding Probabilities

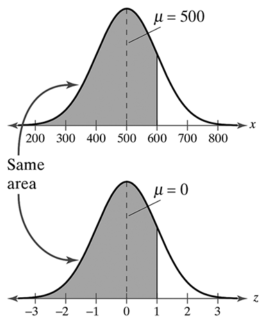
Probability and Normal Distributions (1 of 2)

- If a random variable *x* is normally distributed, you can find the probability that *x* will fall in a given interval by calculating the area under the normal curve for that interval.



Probability and Normal Distributions (2 of 2)

- Consider a normal curve with $\mu = 500$ and $\sigma = 100$ (upper right).
- The value of *x* one standard deviation above the mean is $\mu + \sigma = 500 + 100 = 600$.
- Now consider the standard normal curve (lower right).
- The value of *z* one standard deviation above the mean is $\mu + \sigma = 0 + 1 = 1$.
- The *z*-score of 1 corresponds to an *x*-value of 600, and areas are not changed with a transformation to a standard normal curve, the shaded areas at the right are equal.



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Example: Finding Probabilities for Normal Distributions (1 of 2)

A national study found that college students with jobs worked an average of 22 hours per week. The standard deviation is 9 hours. A college student with a job is selected at random. Find the probability that the student works for less than 4 hours per week. Assume that the lengths of time college students work are normally distributed and are represented by the variable *x*.

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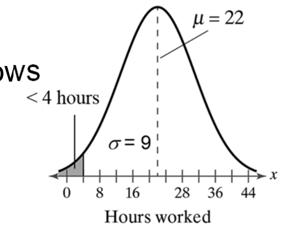
Solution: Finding Probabilities for Normal Distributions (1 of 3)

Solution

- The *z*-score that corresponds to 4 hours is

$$z = \frac{x - \mu}{\sigma} = \frac{4 - 22}{9} = -2.$$

- The Standard Normal Table shows that $P(z < -2) = 0.0228$.
- The probability that the student works for less that 4 hours per week is 0.0228.



So, 2.28% of college students with jobs worked for less than 4 hours per week. Because 2.28% is less than 5%, this is an unusual event.

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Example: Finding Probabilities for Normal Distributions (2 of 2)

A survey indicates that for each trip to a supermarket, a shopper spends an average of 43 minutes with a standard deviation of 12 minutes in the store. The lengths of time spent in the store are normally distributed and are represented by the variable *x*. A shopper enters the store. a) Find the probability that the shopper will be in the store for each interval of time listed below.
b) When 200 shoppers enter the store, how many shoppers would you expect to be in the store for each interval of time listed below?

1. Between 22 and 52 minutes
2. More than 37 minutes

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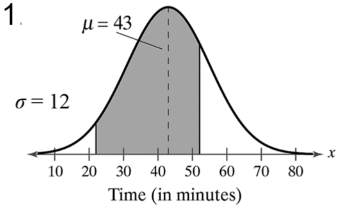
Solution: Finding Probabilities for Normal Distributions (2 of 3)

1.



a) $z_1 = \dfrac{x-\mu}{\sigma} = \dfrac{22-43}{12} = -1.75$

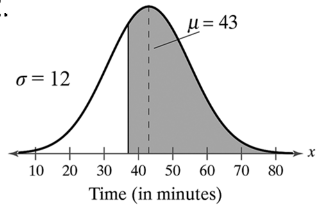
$z_2 = \dfrac{x-\mu}{\sigma} = \dfrac{52-43}{12} = 0.75$

$$P(22 < x < 52) = P(-1.75 < z < 0.75)$$
$$= 0.7734 - 0.0401 = 0.7333$$

b) When 200 shoppers enter the store, you would expect about $200(0.7333) = 146.66 \approx 147$ shoppers to be in the store between 22 and 52 minutes.

Solution: Finding Probabilities for Normal Distributions (3 of 3)

2.



a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{37-43}{12} = -0.5$

$$P(x > 37) = P(z > -0.5) = 1 - 0.3085 = 0.6915$$

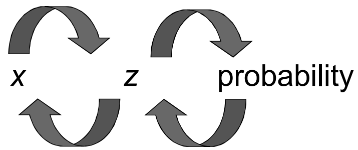
b) When 200 shoppers enter the store, you would expect about $200(0.6915) = 138.3 \approx 138$ shoppers to be in the store more than 37 minutes.

Section 5.3

Normal Distributions: Finding Values

Finding values Given a Probability

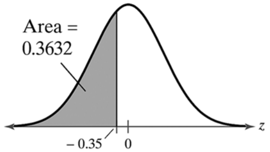
- In a previous section we were given a normally distributed random variable *x* and we were asked to find a probability.
- In this section, we will be given a probability and we will be asked to find the value of the random variable *x*.



Example: Finding a *z*-Score Given an Area (1 of 2)

1. Find the *z*-score that corresponds to a cumulative area of 0.3632.

Solution:



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Solution: Finding a *z*-Score Given an Area (1 of 2)

- Locate 0.3632 in the body of the Standard Normal Table.

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090

- The values at the beginning of the corresponding row and at the top of the column give the *z*-score.

The *z*-score is -0.35**.**

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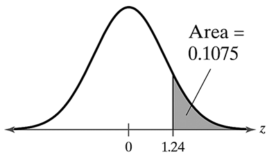
Example: Finding a *z*-Score Given an Area (2 of 2)

2. Find the *z*-score that has 10.75% of the distribution's area to its right.

Solution:



Because the area to the right is 0.1075, the cumulative area is $1-0.1075=0.8925$.

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Solution: Finding a *z*-Score Given an Area (2 of 2)

- Locate 0.8925 in the body of the Standard Normal Table.

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131

- The values at the beginning of the corresponding row and at the top of the column give the *z*-score.

The *z*-score is 1.24.

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Transforming a *z*-Score to an *x*-Value

To transform a standard *z*-score to a data value *x* in a given population, use the formula

$$x = \mu + z\sigma$$

Example: Finding an x-Value Corresponding to a z-score (1 of 2)

A veterinarian records the weights of cats treated at a clinic. The weights are normally distributed, with a mean of 9 pounds and a standard deviation of 2 pounds. Find the weights *x* corresponding to each *z*-score. Interpret the results.

1. $z = 1.96$ 2. $z = -0.44$ 3. $z = 0$.

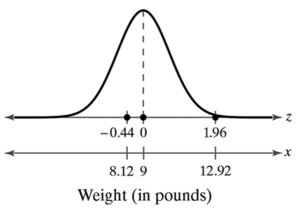
Solution: Use the formula $x = \mu + z\sigma$

1. $z = 1.96$: $x = 9 + 1.96(2) = 12.92$ pounds
2. $z = -0.44$: $x = 9 + (-0.44)(2) = 8.12$ pounds
3. $z = 0$: $x = 9 + 1.96(0) = 9$ pounds

Example: Finding an x-Value Corresponding to a z-score (2 of 2)

Solution:

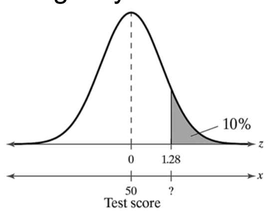
From the figure, you can see that 12.92 pounds is to the right of the mean, 8.12 pounds is to the left of the mean, and 9 pounds is equal to the mean.



Example: Finding a Specific Data Value for a Given Probability

Scores for the California Peace Officer Standards and Training test are normally distributed, with a mean of 50 and a standard deviation of 10. An agency will only hire applicants with scores in the top 10%. What is the lowest score you can earn and still be eligible to be hired by the agency?

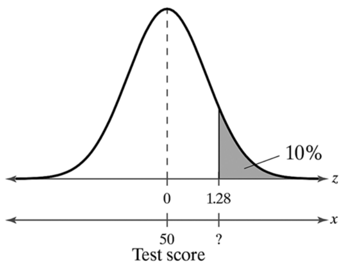
Solution:



An exam score in the top 10% is any score above the 90th percentile. Find the *z*-score that corresponds to a cumulative area of 0.9.

Solution: Finding a Specific Data Value for a Given Probability (1 of 3)

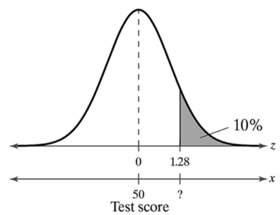
From the Standard Normal Table, the area closest to 0.9 is 0.8997. So the *z*-score that corresponds to an area of 0.9 is $z = 1.28$.



Solution: Finding a Specific Data Value for a Given Probability (2 of 3)

Using the equation $x = \mu + z\sigma$

$$x = 50 + 1.28(10) \approx 62.8$$



The lowest score you can earn and still be eligible to be hired by the agency is about 63.

Section 5.4

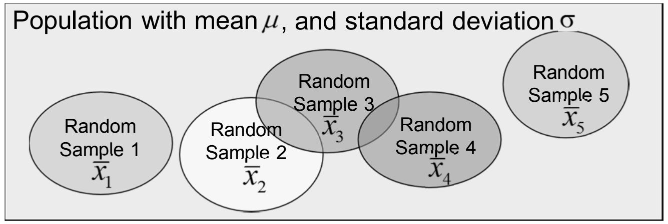
Sampling Distributions and the Central Limit Theorem

Sampling Distributions

Sampling distribution

- The probability distribution of a sample statistic that is formed when random samples of size *n* are repeatedly taken from a population.
- If the sample statistic is the sample mean, then the distribution is the **Sampling distribution of sample means**

Sampling Distribution of Sample Means



The sampling distribution consists of the values of the sample means, $\overline{x}_1, \overline{x}_2, \overline{x}_3, \overline{x}_4, \overline{x}_5, \ldots$

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Properties of Sampling Distributions of Sample Means

1. The mean of the sample means, $\mu_{\overline{x}}$, is equal to the population mean μ.

$$\mu_{\overline{x}} = \mu$$

2. The standard deviation of the sample means, $\sigma_{\overline{x}}$, is equal to the population standard deviation, σ divided by the square root of the sample size, *n*.

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

   • Called the **standard error of the mean**.

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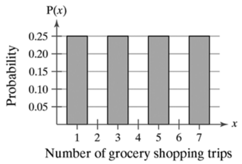
Example: A Sampling Distribution of Sample Means

The number of times four people go grocery shopping in a month is given by the population values {1, 3, 5, 7}. A probability histogram for the data is shown. You randomly choose two of the four people, with replacement. List all possible samples of size $n = 2$ and calculate the mean of each. These means form the sampling distribution of the sample means. Find the mean, variance, and standard deviation of the sample means. Compare your results with the mean $\mu = 4$, variance $\sigma^2 = 5$, and standard deviation $\sigma = \sqrt{5} \approx 2.2$ of the population.



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Solution: A Sampling Distribution of Sample Means (1 of 4)

Solution:

List all 16 samples of size 2 from the population and the mean of each sample.

Sample	Sample mean, $\overline{x}$
1, 1	1
1, 3	2
1, 5	3
1, 7	4
3, 1	2
3, 3	3
3, 5	4
3, 7	5

Sample	Sample mean, $\overline{x}$
5, 1	3
5, 3	4
5, 5	5
5, 7	6
7, 1	4
7, 3	5
7, 5	6
7, 7	7

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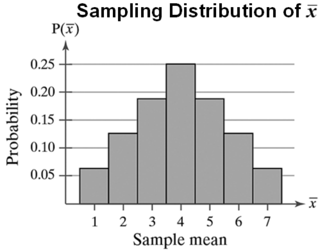
Solution: A Sampling Distribution of Sample Means (2 of 4)

Construct a probability distribution of the sample means. Then, you can graph the sampling distribution using a probability histogram.

Probability Distribution of Sample Means

$\bar{x}$	f	Probability
1	1	1/16
2	2	2/16
3	3	3/16
4	4	4/16
5	3	3/16
6	2	2/16
7	1	1/16

Probability Histogram of Sampling Distribution of $\bar{x}$



Solution: A Sampling Distribution of Sample Means (3 of 4)

The mean, variance, and standard deviation of the 16 sample means are

$$\mu_{\bar{x}} = 4$$ Mean of the sample means

$$\left(\sigma_{\bar{x}}\right)^2 = \frac{5}{2} = 2.5$$ Variance of the sample means

and

$$\sigma_{\bar{x}} = \sqrt{\frac{5}{2}} = \sqrt{2.5} \approx 1.6$$ Standard deviation of the sample means

Solution: A Sampling Distribution of Sample Means (4 of 4)

These results satisfy the properties of sampling distributions because

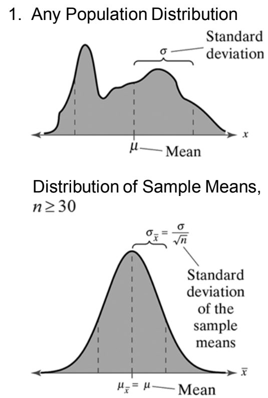
$$\mu_{\bar{x}} = \mu = 4$$

and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{5}}{\sqrt{2}} \approx 1.6.$$

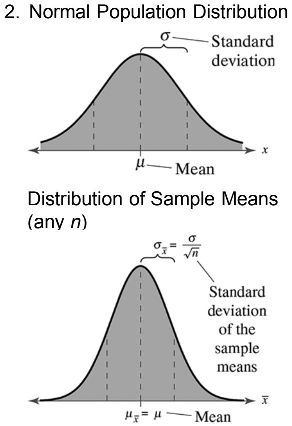
The Central Limit Theorem (1 of 3)

1. If samples of size $n \geq 30$, are drawn from any population with $\text{mean} = \mu$ and $\text{standard deviation} = \sigma$, then the sampling distribution of the sample means approximates a normal distribution. The greater the sample size, the better the approximation.



The Central Limit Theorem (2 of 3)

2. If the population itself is normally distributed, the sampling distribution of the sample means is normally distribution for ***any*** sample size *n*.



The Central Limit Theorem (3 of 3)

- In either case, the sampling distribution of sample means has a mean equal to the population mean.

$$\mu_{\bar{x}} = \mu \quad \text{Mean of the sample means}$$

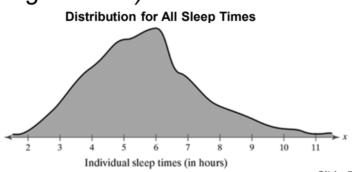
- The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of *n.*

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \quad \text{Variance of the sample means}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{Standard deviation of the sample means (Standard error of the mean)}$$

Example: Interpreting the Central Limit Theorem (1 of 2)

A study analyzed the sleep habits of college students. The study found that the mean sleep time was 6.8 hours, with a standard deviation of 1.4 hours. Random samples of 100 sleep times are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means. Then sketch a graph of the sampling distribution. *(Adapted from The Journal of American College Health)*



Solution: Interpreting the Central Limit Theorem (1 of 4)

- The mean of the sampling distribution is equal to the population mean

$$\mu_{\bar{x}} = \mu = 6.8$$

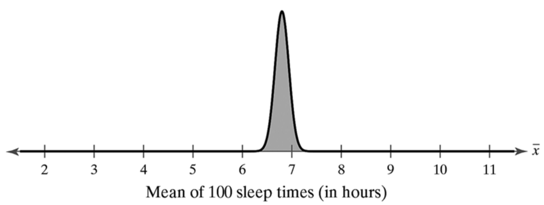
- The standard error of the mean is equal to the population standard deviation divided by $\sqrt{n}$.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.4}{\sqrt{100}} = 0.14$$

Solution: Interpreting the Central Limit Theorem (2 of 4)

- Since the sample size is greater than 30, the sampling distribution can be approximated by a normal distribution with a mean of 6.8 hours and a standard deviation of 0.14 hour.

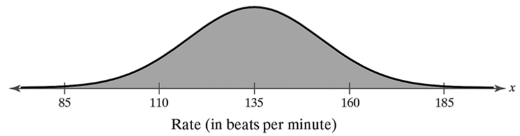
Distribution of Sample Means with $n = 100$



Example: Interpreting the Central Limit Theorem (2 of 2)

The training heart rates of all 20-years old athletes are normally distributed, with a mean of 135 beats per minute and standard deviation of 18 beats per minute. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means.

Distribution of Population Training Heart Rates



Solution: Interpreting the Central Limit Theorem (3 of 4)

- The mean of the sample means

$$\mu_{\bar{x}} = \mu = 135 \text{ beats per minute}$$

- The standard deviation of the sample means

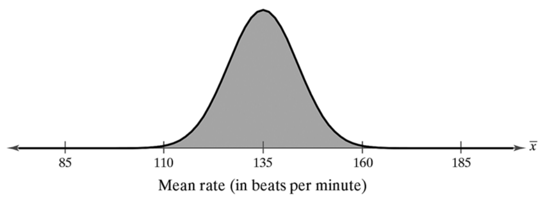
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{4}} = 9 \text{ beats per minute}$$

Solution: Interpreting the Central Limit Theorem (4 of 4)

- Since the population is normally distributed, the sampling distribution of the sample means is also normally distributed.

$$\mu_{\bar{x}} = 135 \qquad \sigma_{\bar{x}} = 9$$

Distribution of Sample Means with $n = 4$



Probability and the Central Limit Theorem

- To transform $\bar{x}$ to a *z*-score

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard Error}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

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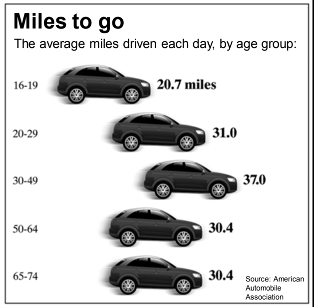
Example: Finding Probabilities for Sampling Distributions (1 of 2)

The figure shows the mean distances traveled by drivers each day. You randomly select 50 drivers ages 16 to 19. What is the probability that the mean distance traveled each day is between 19.4 and 22.5 miles? Assume $\sigma = 6.5 \text{ miles}$.



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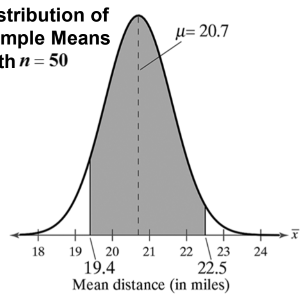
Solution: Finding Probabilities for Sampling Distributions (1 of 7)

From the Central Limit Theorem (sample size is greater than 30), the sampling distribution of sample means is approximately normal with

$$\mu_{\bar{x}} = \mu = 20.7 \text{ miles} \quad \text{and}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.5}{\sqrt{50}} \approx 0.9 \text{ miles}$$



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Solution: Finding Probabilities for Sampling Distributions (2 of 7)

- The *z*-scores that correspond to sample means of 19.4 and 22.5 miles are found as shown.

$$z_1 = \frac{19.4 - 20.7}{6.5/\sqrt{50}} \approx -1.41$$

$$z_2 = \frac{22.5 - 20.7}{6.5/\sqrt{50}} \approx 1.96$$

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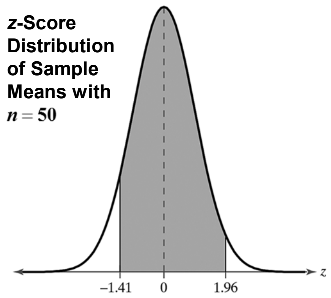
Solution: Finding Probabilities for Sampling Distributions (3 of 7)

- The probability that the mean distance driven each day by the sample of 50 people is between 19.4 and 22.5 miles is

$$\begin{aligned} P(19.4 < \bar{x} < 22.5) &= P(-1.41 < z < 1.96) \\ &= P(z < 1.96) - P(z < -1.41) \\ &= 0.9750 - 0.0793 \\ &= 0.8957 \end{aligned}$$

Solution: Finding Probabilities for Sampling Distributions (4 of 7)

Of all samples of 50 drivers ages 16 to 19, about 90% will drive a mean distance each day between 19.4 and 22.5 miles, as shown in the graph. This implies that, assuming the value of $\mu = 20.7$ is correct, about 10% of such sample means will lie outside the given interval.



Example: Finding Probabilities for Sampling Distributions (2 of 2)

The mean room and board expense per year at four-year colleges is $\$10,453$. You randomly select 9 four-year colleges. What is the probability that the mean room and board is less than $\$10,750$? Assume that the room and board expenses are normally distributed with a standard deviation of $\$1650$. *(Adapted from National Center for Education Statistics)*

Solution: Finding Probabilities for Sampling Distributions (5 of 7)

- Because the population is normally distributed, you can use the Central Limit Theorem to conclude that the distribution of sample means is normally distributed, with a mean and a standard deviation of

$\mu_{\bar{x}} = \mu = \$10{,}453$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{\$1650}{\sqrt{9}} == \$550$.

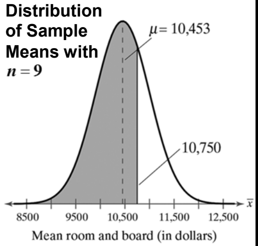
Solution: Finding Probabilities for Sampling Distributions (6 of 7)

- The area to the left of $\$10{,}750$ is shaded. The *z*-score that corresponds to $\$10{,}750$ is

$$z = \frac{10{,}750 - 10{,}453}{1650/\sqrt{9}} = \frac{297}{550} = 0.54.$$

- So, the probability that the mean room and board expense is less than $\$10{,}750$ is

$$P(x < 10{,}750) = P(z < 0.54) = 0.7054.$$



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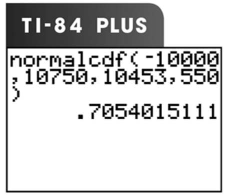
Solution: Finding Probabilities for Sampling Distributions (7 of 7)

You can check this answer using technology. For instance, you can use a TI-84 Plus to find the *x*-value,



So, about 71% of such samples with $n = 9$ will have a mean less than $\$10{,}750$ and about 29% of these sample means will be greater than $\$10{,}750$.

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Example: Finding Probabilities for *x* and *x bar* (1 of 4)

Some college students use credit cards to pay for school-related expenses. For this population, the amount paid is normally distributed, with a mean of $\$1615$ and a standard deviation of $\$550$. *(Adapted from Sallie Mae/Ipsos Public Affairs)*

1. What is the probability that a randomly selected college student, who uses a credit card to pay for school-related expenses, paid less than $\$1400$?
2. You randomly select 25 college students who use credit cards to pay for school-related expenses. What is the probability that their mean amount paid is less than $\$1400$?
3. Compare the probabilities from parts 1 and 2.

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Solution: Finding Probabilities for *x* and *x bar*

1. In this case, you are asked to find the probability associated with a certain value of the random variable *x*. The *z*-score that corresponds to $x = \$1400$ is

$$z = \frac{x - \mu}{\sigma} = \frac{1400 - 1615}{550} = \frac{-215}{550} \approx -0.39.$$

So, the probability that the student paid less than $\$1400$ is

$$P(x < 1400) = P(z < -0.39) = 0.3483.$$

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Example: Finding Probabilities for *x* and *x bar* (2 of 4)

2. Here, you are asked to find the probability associated with a sample mean *x*. The *z*-score that corresponds to $x = \$1400$ is

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{1400 - 1615}{550/\sqrt{25}} = \frac{-215}{110} = -1.95$$

So, the probability that the mean credit card balance of the 25 card holders is less than $\$1400$ is

$$P(x < 1400) = P(z < -1.95) = 0.0256.$$

Example: Finding Probabilities for *x* and *x bar* (3 of 4)

You can check the answers for part 1 and 2 using technology. For instance, you can use Excel to find the probabilities.

EXCEL

	A	B	C
1	NORM.DIST(1400,1615,550,TRUE)		
2			0.347932217

EXCEL

	A	B	C
1	NORM.DIST(1400,1615,110,TRUE)		
2			0.025318372

Example: Finding Probabilities for *x* and *x bar* (4 of 4)

3. Although there is about a 35% chance that a college student who uses a credit card to pay for school-related expenses will pay less than $\$1400$, there is only about a 3% chance that the mean amount a sample of 25 college students will pay is less than $\$1400$. Because there is only a 3% chance that the mean amount a sample of 25 college students will pay is less than $\$1400$, this is an unusual event.