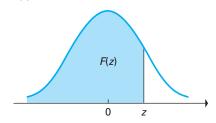
APPENDIX TABLES

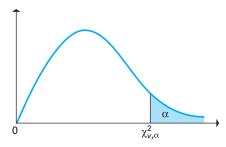
 Table 1
 Cumulative Distribution Function, F(z), of the Standard Normal Distribution Table



| Z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |

Dr. William L. Carlson, prepared using Minitab 16.

 Table 7a
 Upper Critical Values of Chi-Square Distribution with ν Degrees of Freedom

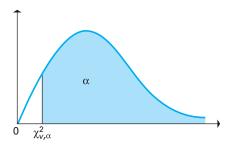


For selected probabilities α , the table shows the values $\chi^2_{v,\alpha}$ such that $P(\chi^2_v > \chi^2_{v,\alpha}) = \alpha$, where χ^2_v is a chi-square random variable with v degrees of freedom. For example, the probability is .100 that a chi-square random variable with 10 degrees of freedom is greater than 15.987.

| | I | PROBABILITY OF EXCEE | DING THE CRITICAL V | ALUE | |
|-----|---------|----------------------|---------------------|---------|---------|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.001 |
| 1 | 2.706 | 3.841 | 5.024 | 6.635 | 10.828 |
| 2 | 4.605 | 5.991 | 7.378 | 9.210 | 13.816 |
| 3 | 6.251 | 7.815 | 9.348 | 11.345 | 16.266 |
| 4 | 7.779 | 9.488 | 11.143 | 13.277 | 18.467 |
| 5 | 9.236 | 11.070 | 12.833 | 15.086 | 20.515 |
| 6 | 10.645 | 12.592 | 14.449 | 16.812 | 22.458 |
| 7 | 12.017 | 14.067 | 16.013 | 18.475 | 24.322 |
| 8 | 13.362 | 15.507 | 17.535 | 20.090 | 26.125 |
| 9 | 14.684 | 16.919 | 19.023 | 21.666 | 27.877 |
| 10 | 15.987 | 18.307 | 20.483 | 23.209 | 29.588 |
| 11 | 17.275 | 19.675 | 21.920 | 24.725 | 31.264 |
| 12 | 18.549 | 21.026 | 23.337 | 26.217 | 32.910 |
| 13 | 19.812 | 22.362 | 24.736 | 27.688 | 34.528 |
| 14 | 21.064 | 23.685 | 26.119 | 29.141 | 36.123 |
| 15 | 22.307 | 24.996 | 27.488 | 30.578 | 37.697 |
| 16 | 23.542 | 26.296 | 28.845 | 32.000 | 39.25 |
| 17 | 24.769 | 27.587 | 30.191 | 33.409 | 40.790 |
| 18 | 25.989 | 28.869 | 31.526 | 34.805 | 42.312 |
| 19 | 27.204 | 30.144 | 32.852 | 36.191 | 43.820 |
| 20 | 28.412 | 31.410 | 34.170 | 37.566 | 45.315 |
| 21 | 29.615 | 32.671 | 35.479 | 38.932 | 46.797 |
| 22 | 30.813 | 33.924 | 36.781 | 40.289 | 48.268 |
| 23 | 32.007 | 35.172 | 38.076 | 41.638 | 49.728 |
| 24 | 33.196 | 36.415 | 39.364 | 42.980 | 51.179 |
| 25 | 34.382 | 37.652 | 40.646 | 44.314 | 52.620 |
| 26 | 35.563 | 38.885 | 41.923 | 45.642 | 54.052 |
| 27 | 36.741 | 40.113 | 43.195 | 46.963 | 55.476 |
| 28 | 37.916 | 41.337 | 44.461 | 48.278 | 56.892 |
| 29 | 39.087 | 42.557 | 45.722 | 49.588 | 58.30 |
| 30 | 40.256 | 43.773 | 46.979 | 50.892 | 59.703 |
| 40 | 51.805 | 55.758 | 59.342 | 63.691 | 73.40 |
| 50 | 63.167 | 67.505 | 71.420 | 76.154 | 86.66 |
| 60 | 74.397 | 79.082 | 83.298 | 88.379 | 99.60 |
| 70 | 85.527 | 90.531 | 95.023 | 100.425 | 112.31 |
| 80 | 96.578 | 101.879 | 106.629 | 112.329 | 124.839 |
| 90 | 107.565 | 113.145 | 118.136 | 124.116 | 137.20 |
| 100 | 118.498 | 124.342 | 129.561 | 135.807 | 149.44 |

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Table 7b Lower Critical Values of Chi-Square Distribution with ν Degrees of Freedom

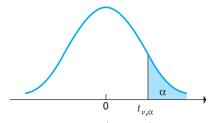


For selected probabilities α , the table shows the values $\chi^2_{v,\alpha}$ such that $P(\chi^2_v > \chi^2_{v,\alpha}) = \alpha$, where χ^2_v is a chi-square random variable with v degrees of freedom. For example, the probability is 0.90 that a chi-square variable with 10 degrees of freedom is greater than 4.865.

| | | Probability of Exceeding the Critical Value | | | | | | | |
|-------|--------|---|--------|--------|--------|--|--|--|--|
| ν | 0.90 | 0.95 | 0.975 | 0.99 | 0.999 | | | | |
| 1 | .016 | .004 | .001 | .000 | .000 | | | | |
| 2 | .211 | .103 | .051 | .020 | .002 | | | | |
| 3 | .584 | .352 | .216 | .115 | .024 | | | | |
| 4 | 1.064 | .711 | .484 | .297 | .091 | | | | |
| 5 | 1.610 | 1.145 | .831 | .554 | .210 | | | | |
| 6 | 2.204 | 1.635 | 1.237 | .872 | .381 | | | | |
| 7 | 2.833 | 2.167 | 1.690 | 1.239 | .598 | | | | |
| 8 | 3.490 | 2.733 | 2.180 | 1.646 | .857 | | | | |
| 9 | 4.168 | 3.325 | 2.700 | 2.088 | 1.152 | | | | |
| 10 | 4.865 | 3.940 | 3.247 | 2.558 | 1.479 | | | | |
| 11 | 5.578 | 4.575 | 3.816 | 3.053 | 1.834 | | | | |
| 12 | 6.304 | 5.226 | 4.404 | 3.571 | 2.214 | | | | |
| 13 | 7.042 | 5.892 | 5.009 | 4.107 | 2.617 | | | | |
| 14 | 7.790 | 6.571 | 5.629 | 4.660 | 3.041 | | | | |
| 15 | 8.547 | 7.261 | 6.262 | 5.229 | 3.483 | | | | |
| 16 | 9.312 | 7.962 | 6.908 | 5.812 | 3.942 | | | | |
| 17 | 10.085 | 8.672 | 7.564 | 6.408 | 4.416 | | | | |
| 18 | 10.865 | 9.390 | 8.231 | 7.015 | 4.905 | | | | |
| 19 | 11.651 | 10.117 | 8.907 | 7.633 | 5.407 | | | | |
| 20 | 12.443 | 10.851 | 9.591 | 8.260 | 5.921 | | | | |
| 21 | 13.240 | 11.591 | 10.283 | 8.897 | 6.447 | | | | |
| 22 | 14.041 | 12.338 | 10.982 | 9.542 | 6.983 | | | | |
| 23 | 14.848 | 13.091 | 11.689 | 10.196 | 7.529 | | | | |
| 24 | 15.659 | 13.848 | 12.401 | 10.856 | 8.085 | | | | |
| 25 | 16.473 | 14.611 | 13.120 | 11.524 | 8.649 | | | | |
| 26 | 17.292 | 15.379 | 13.844 | 12.198 | 9.222 | | | | |
| 27 | 18.114 | 16.151 | 14.573 | 12.879 | 9.803 | | | | |
| 28 | 18.939 | 16.928 | 15.308 | 13.565 | 10.391 | | | | |
| 29 | 19.768 | 17.708 | 16.047 | 14.256 | 10.986 | | | | |
| 30 | 20.599 | 18.493 | 16.791 | 14.953 | 11.588 | | | | |
| 40 | 29.051 | 26.509 | 24.433 | 22.164 | 17.916 | | | | |
| 50 | 37.689 | 34.764 | 32.357 | 29.707 | 24.674 | | | | |
| 60 | 46.459 | 43.188 | 40.482 | 37.485 | 31.738 | | | | |
| 70 | 55.329 | 51.739 | 48.758 | 45.442 | 39.036 | | | | |
| 80 | 64.278 | 60.391 | 57.153 | 53.540 | 46.520 | | | | |
| 90 | 73.291 | 69.126 | 65.647 | 61.754 | 54.155 | | | | |
| 100 | 82.358 | 77.929 | 74.222 | 70.065 | 61.918 | | | | |

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 Table 8
 Upper Critical Values of Student's t Distribution with ν Degrees of Freedom

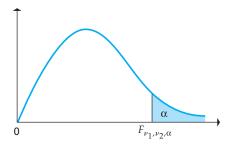


For selected probabilities, α , the table shows the values $t_{\nu,\alpha}$ such that $P(t_{\nu} > t_{\nu,\alpha}) = \alpha$, where t_{ν} is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

| | | Probabili | ty of Exceeding th | e Critical Value | | |
|-----|-------|-----------|--------------------|------------------|--------|---------|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.313 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.782 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.499 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.296 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.143 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.024 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.929 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

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 Table 9a
 Upper Critical Values of the F Distribution



For probabilities $\alpha=0.5$ and $\alpha=.01$, the tables show the values $F_{\nu_1,\nu_2,\alpha}$ such that $P(F_{\nu_1,\nu_2}>F_{\nu_1,\nu_2,\alpha})=\alpha$, where F_{ν_1,ν_2} is an F random variable, with numerator degrees of freedom ν_1 and denominator degrees of freedom ν_2 . For example, the probability is .05 that an $F_{3,7}$ random variables exceeds 4.347.

| For ν_1 | For ν_1 Numerator Degrees of Freedom and ν_2 Denominator Degrees of Freedom 5% Significance Level $F_{.05}(\nu_1,\nu_2)$ | | | | | | | | | |
|---------------|--|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| ν_2/ν_1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 161.448 | 199.500 | 215.707 | 224.583 | 230.162 | 233.986 | 236.768 | 238.882 | 240.543 | 241.882 |
| 2 | 18.513 | 19.000 | 19.164 | 19.247 | 19.296 | 19.330 | 19.353 | 19.371 | 19.385 | 19.396 |
| 3 | 10.128 | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.887 | 8.845 | 8.812 | 8.786 |
| 4 | 7.709 | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 6.094 | 6.041 | 5.999 | 5.964 |
| 5 | 6.608 | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.876 | 4.818 | 4.772 | 4.735 |
| 6 | 5.987 | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.207 | 4.147 | 4.099 | 4.060 |
| 7 | 5.591 | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.787 | 3.726 | 3.677 | 3.637 |
| 8 | 5.318 | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.500 | 3.438 | 3.388 | 3.347 |
| 9 | 5.117 | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.293 | 3.230 | 3.179 | 3.137 |
| 10 | 4.965 | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 3.135 | 3.072 | 3.020 | 2.978 |
| 11 | 4.844 | 3.982 | 3.587 | 3.357 | 3.204 | 3.095 | 3.012 | 2.948 | 2.896 | 2.854 |
| 12 | 4.747 | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.913 | 2.849 | 2.796 | 2.753 |
| 13 | 4.667 | 3.806 | 3.411 | 3.179 | 3.025 | 2.915 | 2.832 | 2.767 | 2.714 | 2.671 |
| 14 | 4.600 | 3.739 | 3.344 | 3.112 | 2.958 | 2.848 | 2.764 | 2.699 | 2.646 | 2.602 |
| 15 | 4.543 | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.707 | 2.641 | 2.588 | 2.544 |
| 16 | 4.494 | 3.634 | 3.239 | 3.007 | 2.852 | 2.741 | 2.657 | 2.591 | 2.538 | 2.494 |
| 17 | 4.451 | 3.592 | 3.197 | 2.965 | 2.810 | 2.699 | 2.614 | 2.548 | 2.494 | 2.450 |
| 18 | 4.414 | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.577 | 2.510 | 2.456 | 2.412 |
| 19 | 4.381 | 3.522 | 3.127 | 2.895 | 2.740 | 2.628 | 2.544 | 2.477 | 2.423 | 2.378 |
| 20 | 4.351 | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.514 | 2.447 | 2.393 | 2.348 |
| 21 | 4.325 | 3.467 | 3.072 | 2.840 | 2.685 | 2.573 | 2.488 | 2.420 | 2.366 | 2.321 |
| 22 | 4.301 | 3.443 | 3.049 | 2.817 | 2.661 | 2.549 | 2.464 | 2.397 | 2.342 | 2.297 |
| 23 | 4.279 | 3.422 | 3.028 | 2.796 | 2.640 | 2.528 | 2.442 | 2.375 | 2.320 | 2.275 |
| 24 | 4.260 | 3.403 | 3.009 | 2.776 | 2.621 | 2.508 | 2.423 | 2.355 | 2.300 | 2.255 |
| 25 | 4.242 | 3.385 | 2.991 | 2.759 | 2.603 | 2.490 | 2.405 | 2.337 | 2.282 | 2.236 |
| 26 | 4.225 | 3.369 | 2.975 | 2.743 | 2.587 | 2.474 | 2.388 | 2.321 | 2.265 | 2.220 |
| 27 | 4.210 | 3.354 | 2.960 | 2.728 | 2.572 | 2.459 | 2.373 | 2.305 | 2.250 | 2.204 |
| 28 | 4.196 | 3.340 | 2.947 | 2.714 | 2.558 | 2.445 | 2.359 | 2.291 | 2.236 | 2.190 |
| 29 | 4.183 | 3.328 | 2.934 | 2.701 | 2.545 | 2.432 | 2.346 | 2.278 | 2.223 | 2.177 |
| 30 | 4.171 | 3.316 | 2.922 | 2.690 | 2.534 | 2.421 | 2.334 | 2.266 | 2.211 | 2.165 |
| 40 | 4.085 | 3.232 | 2.839 | 2.606 | 2.449 | 2.336 | 2.249 | 2.180 | 2.124 | 2.077 |
| 60 | 4.001 | 3.150 | 2.758 | 2.525 | 2.368 | 2.254 | 2.167 | 2.097 | 2.040 | 1.993 |
| 100 | 3.936 | 3.087 | 2.696 | 2.463 | 2.305 | 2.191 | 2.103 | 2.032 | 1.975 | 1.927 |

(continued)

 Table 9a
 Upper Critical Values of the F Distribution (Continued)

| For ν_1 | Numerato | r Degrees (| of Freedom | and ν_2 Den | NOMINATOR | Degrees of | FREEDOM 5 | % Significa | NCE LEVEL | $F_{.05}(\nu_1, \nu_2)$ |
|---------------|----------|-------------|------------|-----------------|-----------|------------|-----------|-------------|-----------|-------------------------|
| ν_2/ν_1 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 242.983 | 243.906 | 244.690 | 245.364 | 245.950 | 246.464 | 246.918 | 247.323 | 247.686 | 248.013 |
| 2 | 19.405 | 19.413 | 19.419 | 19.424 | 19.429 | 19.433 | 19.437 | 19.440 | 19.443 | 19.446 |
| 3 | 8.763 | 8.745 | 8.729 | 8.715 | 8.703 | 8.692 | 8.683 | 8.675 | 8.667 | 8.660 |
| 4 | 5.936 | 5.912 | 5.891 | 5.873 | 5.858 | 5.844 | 5.832 | 5.821 | 5.811 | 5.803 |
| 5 | 4.704 | 4.678 | 4.655 | 4.636 | 4.619 | 4.604 | 4.590 | 4.579 | 4.568 | 4.558 |
| 6 | 4.027 | 4.000 | 3.976 | 3.956 | 3.938 | 3.922 | 3.908 | 3.896 | 3.884 | 3.874 |
| 7 | 3.603 | 3.575 | 3.550 | 3.529 | 3.511 | 3.494 | 3.480 | 3.467 | 3.455 | 3.445 |
| 8 | 3.313 | 3.284 | 3.259 | 3.237 | 3.218 | 3.202 | 3.187 | 3.173 | 3.161 | 3.150 |
| 9 | 3.102 | 3.073 | 3.048 | 3.025 | 3.006 | 2.989 | 2.974 | 2.960 | 2.948 | 2.936 |
| 10 | 2.943 | 2.913 | 2.887 | 2.865 | 2.845 | 2.828 | 2.812 | 2.798 | 2.785 | 2.774 |
| 11 | 2.818 | 2.788 | 2.761 | 2.739 | 2.719 | 2.701 | 2.685 | 2.671 | 2.658 | 2.646 |
| 12 | 2.717 | 2.687 | 2.660 | 2.637 | 2.617 | 2.599 | 2.583 | 2.568 | 2.555 | 2.544 |
| 13 | 2.635 | 2.604 | 2.577 | 2.554 | 2.533 | 2.515 | 2.499 | 2.484 | 2.471 | 2.459 |
| 14 | 2.565 | 2.534 | 2.507 | 2.484 | 2.463 | 2.445 | 2.428 | 2.413 | 2.400 | 2.388 |
| 15 | 2.507 | 2.475 | 2.448 | 2.424 | 2.403 | 2.385 | 2.368 | 2.353 | 2.340 | 2.328 |
| 16 | 2.456 | 2.425 | 2.397 | 2.373 | 2.352 | 2.333 | 2.317 | 2.302 | 2.288 | 2.276 |
| 17 | 2.413 | 2.381 | 2.353 | 2.329 | 2.308 | 2.289 | 2.272 | 2.257 | 2.243 | 2.230 |
| 18 | 2.374 | 2.342 | 2.314 | 2.290 | 2.269 | 2.250 | 2.233 | 2.217 | 2.203 | 2.191 |
| 19 | 2.340 | 2.308 | 2.280 | 2.256 | 2.234 | 2.215 | 2.198 | 2.182 | 2.168 | 2.155 |
| 20 | 2.310 | 2.278 | 2.250 | 2.225 | 2.203 | 2.184 | 2.167 | 2.151 | 2.137 | 2.124 |
| 21 | 2.283 | 2.250 | 2.222 | 2.197 | 2.176 | 2.156 | 2.139 | 2.123 | 2.109 | 2.096 |
| 22 | 2.259 | 2.226 | 2.198 | 2.173 | 2.151 | 2.131 | 2.114 | 2.098 | 2.084 | 2.071 |
| 23 | 2.236 | 2.204 | 2.175 | 2.150 | 2.128 | 2.109 | 2.091 | 2.075 | 2.061 | 2.048 |
| 24 | 2.216 | 2.183 | 2.155 | 2.130 | 2.108 | 2.088 | 2.070 | 2.054 | 2.040 | 2.027 |
| 25 | 2.198 | 2.165 | 2.136 | 2.111 | 2.089 | 2.069 | 2.051 | 2.035 | 2.021 | 2.007 |
| 26 | 2.181 | 2.148 | 2.119 | 2.094 | 2.072 | 2.052 | 2.034 | 2.018 | 2.003 | 1.990 |
| 27 | 2.166 | 2.132 | 2.103 | 2.078 | 2.056 | 2.036 | 2.018 | 2.002 | 1.987 | 1.974 |
| 28 | 2.151 | 2.118 | 2.089 | 2.064 | 2.041 | 2.021 | 2.003 | 1.987 | 1.972 | 1.959 |
| 29 | 2.138 | 2.104 | 2.075 | 2.050 | 2.027 | 2.007 | 1.989 | 1.973 | 1.958 | 1.945 |
| 30 | 2.126 | 2.092 | 2.063 | 2.037 | 2.015 | 1.995 | 1.976 | 1.960 | 1.945 | 1.932 |
| 40 | 2.038 | 2.003 | 1.974 | 1.948 | 1.924 | 1.904 | 1.885 | 1.868 | 1.853 | 1.839 |
| 60 | 1.952 | 1.917 | 1.887 | 1.860 | 1.836 | 1.815 | 1.796 | 1.778 | 1.763 | 1.748 |
| 100 | 1.886 | 1.850 | 1.819 | 1.792 | 1.768 | 1.746 | 1.726 | 1.708 | 1.691 | 1.676 |

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 Table 9b
 Upper Critical Values of the F Distribution

| For | ν_1 Numerat | or Degrees | of Freedo | m and ν_2 De | ENOMINATOI | r Degrees o | f Freedom | 1% Signific | ance Level | $F_{\cdot 01}(\nu_1, \nu_2)$ |
|---------------|-----------------|------------|-----------|------------------|------------|-------------|-----------|-------------|------------|------------------------------|
| ν_2/ν_1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 4052.19 | 4999.52 | 5403.34 | 5624.62 | 5763.65 | 5858.97 | 5928.33 | 5981.10 | 6022.50 | 6055.85 |
| 2 | 98.502 | 99.000 | 99.166 | 99.249 | 99.300 | 99.333 | 99.356 | 99.374 | 99.388 | 99.399 |
| 3 | 34.116 | 30.816 | 29.457 | 28.710 | 28.237 | 27.911 | 27.672 | 27.489 | 27.345 | 27.229 |
| 4 | 21.198 | 18.000 | 16.694 | 15.977 | 15.522 | 15.207 | 14.976 | 14.799 | 14.659 | 14.546 |
| 5 | 16.258 | 13.274 | 12.060 | 11.392 | 10.967 | 10.672 | 10.456 | 10.289 | 10.158 | 10.051 |
| 6 | 13.745 | 10.925 | 9.780 | 9.148 | 8.746 | 8.466 | 8.260 | 8.102 | 7.976 | 7.874 |
| 7 | 12.246 | 9.547 | 8.451 | 7.847 | 7.460 | 7.191 | 6.993 | 6.840 | 6.719 | 6.620 |
| 8 | 11.259 | 8.649 | 7.591 | 7.006 | 6.632 | 6.371 | 6.178 | 6.029 | 5.911 | 5.814 |
| 9 | 10.561 | 8.022 | 6.992 | 6.422 | 6.057 | 5.802 | 5.613 | 5.467 | 5.351 | 5.257 |
| 10 | 10.044 | 7.559 | 6.552 | 5.994 | 5.636 | 5.386 | 5.200 | 5.057 | 4.942 | 4.849 |
| 11 | 9.646 | 7.206 | 6.217 | 5.668 | 5.316 | 5.069 | 4.886 | 4.744 | 4.632 | 4.539 |
| 12 | 9.330 | 6.927 | 5.953 | 5.412 | 5.064 | 4.821 | 4.640 | 4.499 | 4.388 | 4.296 |
| 13 | 9.074 | 6.701 | 5.739 | 5.205 | 4.862 | 4.620 | 4.441 | 4.302 | 4.191 | 4.100 |
| 14 | 8.862 | 6.515 | 5.564 | 5.035 | 4.695 | 4.456 | 4.278 | 4.140 | 4.030 | 3.939 |
| 15 | 8.683 | 6.359 | 5.417 | 4.893 | 4.556 | 4.318 | 4.142 | 4.004 | 3.895 | 3.805 |
| 16 | 8.531 | 6.226 | 5.292 | 4.773 | 4.437 | 4.202 | 4.026 | 3.890 | 3.780 | 3.691 |
| 17 | 8.400 | 6.112 | 5.185 | 4.669 | 4.336 | 4.102 | 3.927 | 3.791 | 3.682 | 3.593 |
| 18 | 8.285 | 6.013 | 5.092 | 4.579 | 4.248 | 4.015 | 3.841 | 3.705 | 3.597 | 3.508 |
| 19 | 8.185 | 5.926 | 5.010 | 4.500 | 4.171 | 3.939 | 3.765 | 3.631 | 3.523 | 3.434 |
| 20 | 8.096 | 5.849 | 4.938 | 4.431 | 4.103 | 3.871 | 3.699 | 3.564 | 3.457 | 3.368 |
| 21 | 8.017 | 5.780 | 4.874 | 4.369 | 4.042 | 3.812 | 3.640 | 3.506 | 3.398 | 3.310 |
| 22 | 7.945 | 5.719 | 4.817 | 4.313 | 3.988 | 3.758 | 3.587 | 3.453 | 3.346 | 3.258 |
| 23 | 7.881 | 5.664 | 4.765 | 4.264 | 3.939 | 3.710 | 3.539 | 3.406 | 3.299 | 3.211 |
| 24 | 7.823 | 5.614 | 4.718 | 4.218 | 3.895 | 3.667 | 3.496 | 3.363 | 3.256 | 3.168 |
| 25 | 7.770 | 5.568 | 4.675 | 4.177 | 3.855 | 3.627 | 3.457 | 3.324 | 3.217 | 3.129 |
| 26 | 7.721 | 5.526 | 4.637 | 4.140 | 3.818 | 3.591 | 3.421 | 3.288 | 3.182 | 3.094 |
| 27 | 7.677 | 5.488 | 4.601 | 4.106 | 3.785 | 3.558 | 3.388 | 3.256 | 3.149 | 3.062 |
| 28 | 7.636 | 5.453 | 4.568 | 4.074 | 3.754 | 3.528 | 3.358 | 3.226 | 3.120 | 3.032 |
| 29 | 7.598 | 5.420 | 4.538 | 4.045 | 3.725 | 3.499 | 3.330 | 3.198 | 3.092 | 3.005 |
| 30 | 7.562 | 5.390 | 4.510 | 4.018 | 3.699 | 3.473 | 3.305 | 3.173 | 3.067 | 2.979 |
| 40 | 7.314 | 5.179 | 4.313 | 3.828 | 3.514 | 3.291 | 3.124 | 2.993 | 2.888 | 2.801 |
| 60 | 7.077 | 4.977 | 4.126 | 3.649 | 3.339 | 3.119 | 2.953 | 2.823 | 2.718 | 2.632 |
| 100 | 6.895 | 4.824 | 3.984 | 3.513 | 3.206 | 2.988 | 2.823 | 2.694 | 2.590 | 2.503 |

(continued)

 Table 9b
 Upper Critical Values of the F Distribution (Continued)

| For | ν_1 Numerat | TOR DEGREES | of Freedon | m and ν_2 De | NOMINATOR | Degrees of | FREEDOM | 1% Signific | ance Level | $F_{.01}(\nu_1, \nu_2)$ |
|------------|-----------------|-------------|------------|------------------|-----------|------------|---------|-------------|------------|-------------------------|
| ν_2/ u | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 6083.35 | 6106.35 | 6125.86 | 6142.70 | 6157.28 | 6170.12 | 6181.42 | 6191.52 | 6200.58 | 6208.74 |
| 2 | 99.408 | 99.416 | 99.422 | 99.428 | 99.432 | 99.437 | 99.440 | 99.444 | 99.447 | 99.449 |
| 3 | 27.133 | 27.052 | 26.983 | 26.924 | 26.872 | 26.827 | 26.787 | 26.751 | 26.719 | 26.690 |
| 4 | 14.452 | 14.374 | 14.307 | 14.249 | 14.198 | 14.154 | 14.115 | 14.080 | 14.048 | 14.020 |
| 5 | 9.963 | 9.888 | 9.825 | 9.770 | 9.722 | 9.680 | 9.643 | 9.610 | 9.580 | 9.553 |
| 6 | 7.790 | 7.718 | 7.657 | 7.605 | 7.559 | 7.519 | 7.483 | 7.451 | 7.422 | 7.396 |
| 7 | 6.538 | 6.469 | 6.410 | 6.359 | 6.314 | 6.275 | 6.240 | 6.209 | 6.181 | 6.155 |
| 8 | 5.734 | 5.667 | 5.609 | 5.559 | 5.515 | 5.477 | 5.442 | 5.412 | 5.384 | 5.359 |
| 9 | 5.178 | 5.111 | 5.055 | 5.005 | 4.962 | 4.924 | 4.890 | 4.860 | 4.833 | 4.808 |
| 10 | 4.772 | 4.706 | 4.650 | 4.601 | 4.558 | 4.520 | 4.487 | 4.457 | 4.430 | 4.405 |
| 11 | 4.462 | 4.397 | 4.342 | 4.293 | 4.251 | 4.213 | 4.180 | 4.150 | 4.123 | 4.099 |
| 12 | 4.220 | 4.155 | 4.100 | 4.052 | 4.010 | 3.972 | 3.939 | 3.909 | 3.883 | 3.858 |
| 13 | 4.025 | 3.960 | 3.905 | 3.857 | 3.815 | 3.778 | 3.745 | 3.716 | 3.689 | 3.665 |
| 14 | 3.864 | 3.800 | 3.745 | 3.698 | 3.656 | 3.619 | 3.586 | 3.556 | 3.529 | 3.505 |
| 15 | 3.730 | 3.666 | 3.612 | 3.564 | 3.522 | 3.485 | 3.452 | 3.423 | 3.396 | 3.372 |
| 16 | 3.616 | 3.553 | 3.498 | 3.451 | 3.409 | 3.372 | 3.339 | 3.310 | 3.283 | 3.259 |
| 17 | 3.519 | 3.455 | 3.401 | 3.353 | 3.312 | 3.275 | 3.242 | 3.212 | 3.186 | 3.162 |
| 18 | 3.434 | 3.371 | 3.316 | 3.269 | 3.227 | 3.190 | 3.158 | 3.128 | 3.101 | 3.077 |
| 19 | 3.360 | 3.297 | 3.242 | 3.195 | 3.153 | 3.116 | 3.084 | 3.054 | 3.027 | 3.003 |
| 20 | 3.294 | 3.231 | 3.177 | 3.130 | 3.088 | 3.051 | 3.018 | 2.989 | 2.962 | 2.938 |
| 21 | 3.236 | 3.173 | 3.119 | 3.072 | 3.030 | 2.993 | 2.960 | 2.931 | 2.904 | 2.880 |
| 22 | 3.184 | 3.121 | 3.067 | 3.019 | 2.978 | 2.941 | 2.908 | 2.879 | 2.852 | 2.827 |
| 23 | 3.137 | 3.074 | 3.020 | 2.973 | 2.931 | 2.894 | 2.861 | 2.832 | 2.805 | 2.781 |
| 24 | 3.094 | 3.032 | 2.977 | 2.930 | 2.889 | 2.852 | 2.819 | 2.789 | 2.762 | 2.738 |
| 25 | 3.056 | 2.993 | 2.939 | 2.892 | 2.850 | 2.813 | 2.780 | 2.751 | 2.724 | 2.699 |
| 26 | 3.021 | 2.958 | 2.904 | 2.857 | 2.815 | 2.778 | 2.745 | 2.715 | 2.688 | 2.664 |
| 27 | 2.988 | 2.926 | 2.871 | 2.824 | 2.783 | 2.746 | 2.713 | 2.683 | 2.656 | 2.632 |
| 28 | 2.959 | 2.896 | 2.842 | 2.795 | 2.753 | 2.716 | 2.683 | 2.653 | 2.626 | 2.602 |
| 29 | 2.931 | 2.868 | 2.814 | 2.767 | 2.726 | 2.689 | 2.656 | 2.626 | 2.599 | 2.574 |
| 30 | 2.906 | 2.843 | 2.789 | 2.742 | 2.700 | 2.663 | 2.630 | 2.600 | 2.573 | 2.549 |
| 40 | 2.727 | 2.665 | 2.611 | 2.563 | 2.522 | 2.484 | 2.451 | 2.421 | 2.394 | 2.369 |
| 60 | 2.559 | 2.496 | 2.442 | 2.394 | 2.352 | 2.315 | 2.281 | 2.251 | 2.223 | 2.198 |
| 100 | 2.430 | 2.368 | 2.313 | 2.265 | 2.223 | 2.185 | 2.151 | 2.120 | 2.092 | 2.067 |

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