

PROBLEM SET RELATED TO MIDTERM EXAM

1) MOM & MLE

Let X_1, X_2, \dots, X_n be a simple random sample of Pareto random variables with density:

$$f_X(x|\beta) = \frac{\beta}{x^{\beta+1}} \quad , \quad x > 1$$

- a) Find the Method of Moment (MOM) estimator $\hat{\beta}$ for β .
- b) Find the Maximum Likelihood Estimator (MLE) $\hat{\beta}$ for β .

2) C.I. & Hypothesis Test on μ (Z test)

A doctor claims that a particular hospital contains more than 100 diabetes patients with a sugar level of 234 or more. To verify this claim, a random test was conducted on 90 diabetes patients. The test resulted in a mean blood sugar level of 279 with standard deviation of 18.

- a) Determine 95% lower confidence bound on mean blood sugar level.
- b) Test the hypothesis that the mean blood sugar level is above 234 with $\alpha = 0.05$.
- c) Find Type 1 error probability.
- d) What is the power of test if actual mean blood sugar is 240.

3) C.I. & Hypothesis Test on μ (t test)

Wilson Electronics claims that their new line of Plasma TVs will last for 60.000 hours. A customer research group decides to test this claim. The group randomly selects 50 TVs to test. The data from this sample shows that the mean life of a TV is 57.000 hours with a standard deviation of 1200 hours.

- a) Determine 95% Confidence Interval (C.I.) for the mean life of a TV.
- b) Test the hypothesis that the mean life of a TV is 60.000 hours for $\alpha = 0.05$.
- c) Find P-value and interpret the result.

4) C.I. & Hypothesis Test on 1-variance

A large candy manufacturer produces, packages and sells packs of candy targeted to weigh 52 grams. A quality control manager working for the company was concerned that the variation in the actual weights of the targeted 52-gram packs was larger than acceptable. That is, he was concerned that some packs weighed significantly less than 52-grams and some weighed significantly more than 52 grams. In an attempt to estimate σ , the standard deviation of the weights of all of the 52-gram packs the manufacturer

makes, he took a random sample of $n = 10$ packs off of the factory line. The random sample yielded a sample variance of 4.2 grams.

- a) Use the random sample to derive a 95% confidence interval for σ .
- b) Perform hypothesis test which claims that the variance is 5 grams with $\alpha = 0.05$.

5) C.I. & Hypothesis Test on population proportion

Suppose that a market research firm is hired to estimate the percent of adults living in a large city who have cell phones. Five hundred randomly selected adult residents in this city are surveyed to determine whether they have cell phones. Of the 500 people sampled, 421 responded yes- they own cell phones.

- a) Using a 95% confidence level, compute a confidence interval estimate for the true proportion of adult residents of this city who have cell phones.
- b) Test whether the population proportion is equal to 80% or not with confidence level 95%.

6) Goodness of Fit Test

Assume that we collect a random sample of ten bags. Each bag has 100 pieces of candy and five flavors. Total number of pieces (in 10 bags) of candy of each flavor are given in the table below.

Flavor	Number of Pieces of Candy (10 bags)
Apple	180
Lime	250
Cherry	120
Ginger	225
Grape	225

Test the hypothesis that the proportions of the five flavors in each bag are the same.

7) Contingency Table

In 2000 the Vermont State legislature approved a bill authorizing civil unions. The vote can be broken down by gender to produce the following table:

	Vote	
	Yes	No
Women	35	9
Men	60	41

Test whether the voting behaviour is independent of the gender or not.