Exam Calculus
April 3, 2013, 14:00-17:00

- The exam is a closed-book exam
- Use of calculators is forbidden during the exam
- You are allowed to use a sheet of hand-written A4 paper (only one side of it!) prepared in advance; it has to be written by hand and in a normal size (min. 12pt)
- You have 180 minutes for this exam
- Always explain your answer
- You can get maximally 80 points from this exam

1. Compute the following limits, if they exist, or explain why they do not exist:
   a. (2 points) $\lim\limits_{x\to -3} \frac{x+3}{x-3}$
   b. (3 points) $\lim\limits_{x\to 1} \frac{|x-1|}{x^2-1}$
   c. (3 points) $\lim\limits_{x\to 0} \frac{2\,x^2}{\mathrm{e}^x}$
   d. (4 points) $\lim\limits_{x\to \infty} (\sqrt{9\,x^2+3\,x}-3\,x)$
2. Let $f(x)=\frac{x+\sqrt{2}}{x^2-2}$
   a. (2 points) Determine the domain of f. Is f continuous on its domain? Why?
   b. (3 points) For what values of x is f increasing and decreasing? Does it have local minimum(s), maximum(s)? If so, at what values of x?
   c. (4 points) For what values of x is f convex (=concave up) and concave (=concave down)? Does it have inflection points? If so, at what x?
   d. (4 points) Does f have vertical, horizontal, and/or oblique asymptotes? Why?
   e. (2 point) Is f even or odd?
   f. (3 points) Based on your previous answers (a.–e.) and perhaps with drawing some extra points lying on the graph of f, sketch function f.
3. a. (3 points) Compute the derivative: $\left(a^{\sin(x^2)}\right)'$
   b. (3 points) Compute the derivative: $\left(\frac{2\,x}{x^2-4}\right)''$
   c. (3 points) Compute the derivative: $(|x-3|)'$
   d. (4 points) Find equation of the tangent line of $f(x)=x^2+2\,x-3$ at point $(4, f(4))$.
4. Compute the following integrals:
   a. (3 points) $\int\limits_0^1 \mathrm{e}^x\,\sqrt{1+\mathrm{e}^x}\,\mathrm{d}\,x$
   b. (3 points) $\int \arcsin(x)\,\mathrm{d}\,x$
   c. (4 points) $\int \frac{\mathrm{d}\,x}{\sqrt{3-x^2}}$
   d. (4 points) $\int\limits_2^3 \frac{x^2}{x^2-1}\,\mathrm{d}\,x$

5. Determine whether the series

   a. (3 points) $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ converges or diverges by using any appropriate test

   b. (3 points) $\sum_{n=1}^{\infty} \frac{77^n}{n!}$ converges or diverges by using any appropriate test

   c. (3 points) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\,\pi^n}$ converges absolutely, converges conditionally, or diverges, by using any appropriate test

6. Let $f(x, y) = x\,y + 5$.

   a. (4 points) Find equation of the tangent plane of f at point $(1, 2, f(1, 2))$.

   b. (3 points) Compute $\frac{\partial^2 f(x,y)}{\partial x \partial y}$

   c. (4 points) Evaluate $\int\int_T f(x, y)\,\mathrm{d}x\,\mathrm{d}y$ over the triangle T with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$. What is the interpretation of this integral?

   d. (3 points) Evaluate $\int\int_T 1\,\mathrm{d}x\,\mathrm{d}y$ over the same triangle. What is the interpretation of this integral?