# OLASILIK ve NORMAL DAĞILIM

Dr. Gökhan AKSU



## Soru

- Ahmet, Sağlık Bilimleri fakültesi 3. sınıf öğrencisidir.
  - Biyoistatistik sınavından 70 puan
  - İletiŞim sınavından 85 puan almıŞtır.

Ahmet bu iki dersten hangisinde daha başarılıdır?



## Olasılık

- Bir deneme sonrasında ilgilenilen olayın tüm olaylar içinde ortaya çıkma veya gözlenme oranıdır
- Olasılık, denemelerin olası sonuçları ile ilgilenir.



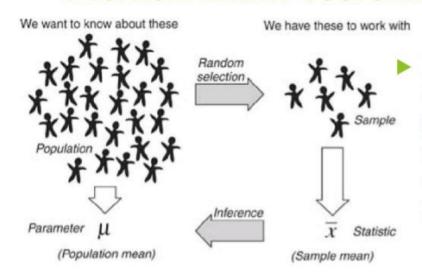
### Olasılık

- Olasılığı 3 Şekilde hesaplarız:
  - **Subjektif**: olayın gerçekle**ş**me durumuyla ilgili ki**ş**isel inancımızdır. 2050 yılında kıyametin kopaca**ğ**ına inanmak gibi.
  - Frekans hesabı: deneyimizi tekrarlamamız halinde olayın meydana gelme olasılığıdır.
  - Önsel: olasılık dağılımının önceden bilinen bir modeline göre hesaplanır. Kalıtım teorisiyle ilgili oluşturacağımız modellere dayanarak mavi gözlü bir anne ve kahverengi gözlü bir babadan olacak çocuğun göz rengini tahmin edebiliriz.

## Olasılık

- Araştırmacılar genellikle evrenin tümünde çalışmazlar,
  - Evreni temsil edecek örneklemlerle çalıŞırlar
  - Bu durumda olaylar ya da olgular arasında gözlenen ili**Ş**kilerin evrende olup olmadı**ğ**ını veya
  - Örneklemden elde edilen istatistiklerin evrendeki de**ğ**erlerini tahmin etmeye çalı**Ş**ırlar.
- Bir öğrencinin başarısını yorumlamada olasılık kavramıyla ilişkili olan normal dağılım ve bu dağılımdaki alan ilişkilerinden yararlanılabilir
  - Çan e**ğ**risi

#### Merkezi Limit Teoremi



Merkezi limit teoremine göre, örneklem boyutu büyüdükçe, ortalamaya ait örneklem dağılımı, popülasyon (kitle) hangi dağılıma sahip olursa olsun, normal dağılıma yaklaşacaktır.

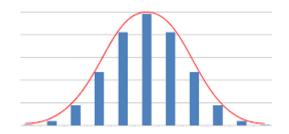
Ortalamaların dağılımı, örneklem boyutu arttıkça **standart normal dağılıma** yaklaşır.

Sample 2 Sample 3 Merkezi Limit Teoremi'ne göre; ortalaması μ ve standart sapması σ olan herhangi bir anakütleden X<sub>1</sub>, X<sub>2</sub>, ....X<sub>n</sub> olmak üzere n hacimli rassal olarak çekilen örneklemlerin ortalaması  $\mu_{\bar{x}} = \mu$  ve standart sapması  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$  olan standart normal dağılıma yaklaşır ve standart değişken  $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\bar{x}}}$  olur.

Sample 1

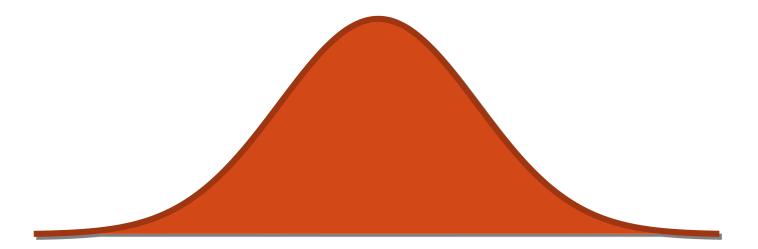
# NORMAL DAĞILIM

- İstatistiksel dağılımlarda en çok kullanılan dağılımlardan birisidir.
- Bu nedenle günlük yaşamda karşılaşılan pek çok sürekli rassal değişken normal dağılır.
  - İnsanların boy uzunlukları
  - Ağırlıkları
  - Sınav sonuçları
  - Paketlerin ağırlıkları
  - Elektronik cihazların ömrü örnek verilebilir.



# NORMAL DAĞILIM

- Değişkenlere ilişkin verilerin oluşturduğu
  - Çan eğrisine benzer eğriye normal dağılım eğrisi
  - Eğrinin yatay eksene göre gösterdiği dağılıma da normal dağılım denir.



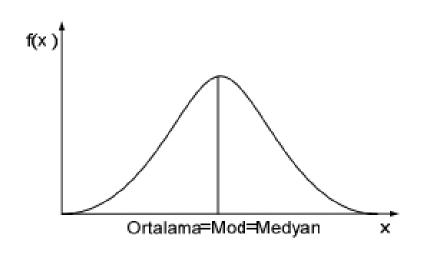
# Normal dağılımın özellikleri

- Çan eğrisi Şeklindedir
- Simetriktir
- Mod, ortalama ve ortanca birbirine e**Ş**ittir
- Puanlar merkez (ortalama) etrafında kümelenme eğilimindedir
- Varysans arttığında yassılaşır, varyans azaldığında ise sivrileşir (ortalama sabit)
- Normal dağılımın parametreleri

evren ortalaması=

evren varyansı=  $\sigma^2$ 

# Normal dağılımın özellikleri



#### Normal Dağılımın Olasılık Yoğunluk fonksiyonu

$$f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} &, -\infty < x < \infty \\ 0 &, diger \quad yerlerde \end{cases}$$

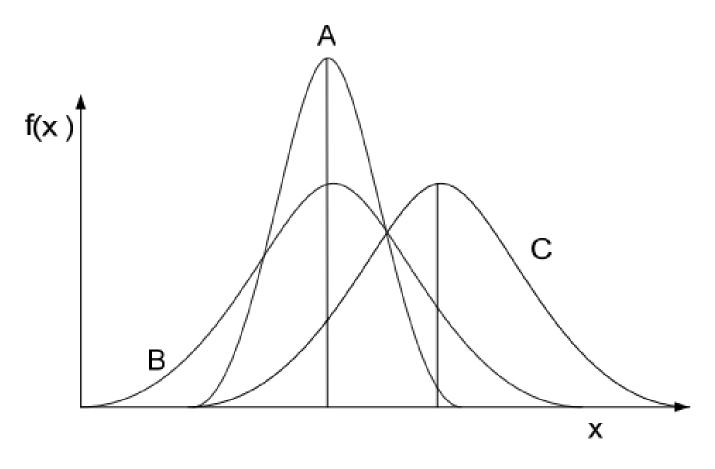
 $\pi = 3,14159...$ 

e = 2,71828

 $\sigma$  = populasyon standart sapması

 $\mu$  = populasyon ortalaması

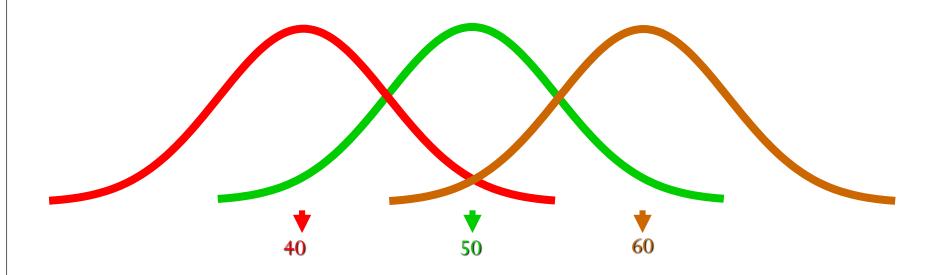
# $\mu$ ve $\sigma^2$ değişikliklerinin dağılımın şekli üzerindeki etkisi



$$\mu_A = \mu_B < \mu_C \qquad \sigma_A^2 < \sigma_B^2 = \sigma_C^2$$

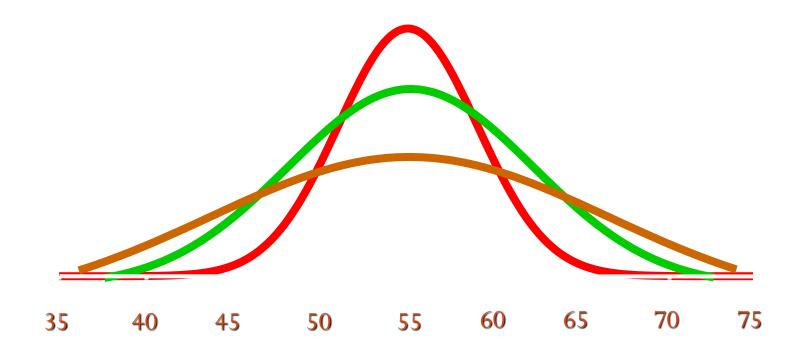
# NORMAL DAĞILIM....

• Ortalamaları farklı standart sapmaları aynı normal dağılımlar



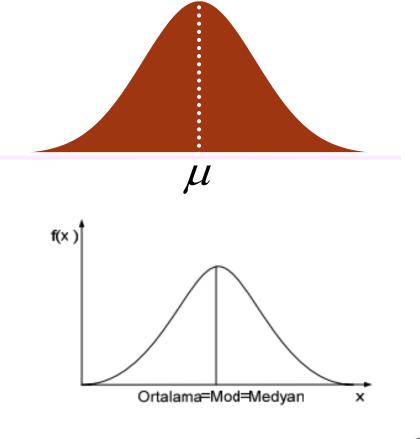
# NORMAL DAĞILIM.....

• Ortalamaları aynı standart sapmaları farklı normal dağılımlar.



# NORMAL DAĞILIM GRAFİĞİ

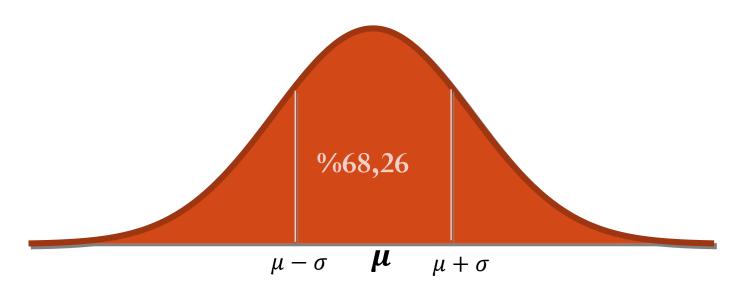
- Dağılım ortalamaya göre simetriktir
- Alanın % 50'si ortalamadan geçen dikey çizginin sağına, % 50'si soluna düşer.
- Eğri altında kalan toplam alan bir birim karedir.
- Aritmetik ortalama, ortanca ve tepe değeri birbirine eşittir.



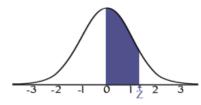
# NORMAL DAĞILIM...

• Ortalaması  $\mu$  ve varyansı  $\sigma^2$  olan normal dağılıma sahip bir x rastgele değişkeni için olasılıklar aşağıdaki gibidir:

$$(\mu - \sigma)$$
 ile  $(\mu + \sigma)$  arasında olma olasılığı 0,68



$$P(\mu - \sigma \le x \le \mu + \sigma) = 0.6826$$



#### STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for z = 1.25 the area under the curve between the mean (0) and z is 0.3944.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0		0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.8749	9.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.1175	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

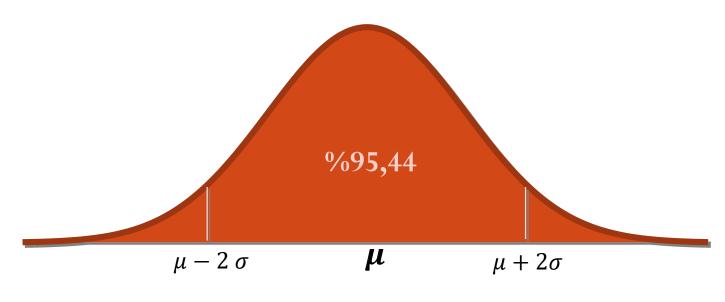


$$\mu - \sigma \mu + \sigma$$

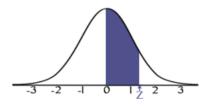
$$P(\mu - \sigma \le x \le \mu + \sigma) = 0.6826$$

# NORMAL DAĞILIM....

• Ortalaması  $\mu$  ve varyansı  $\sigma^2$  olan normal dağılıma sahip bir x rastgele değişkeni için olasılıklar aşağıdaki gibidir:  $(\mu - 2\sigma)$  ile  $(\mu + 2\sigma)$  arasında olma olasılığı 0,95



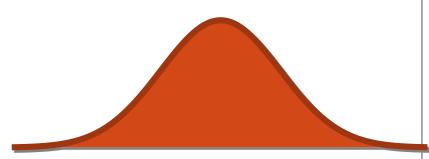
$$P(\mu - 2\sigma \le x \le \mu + 2\sigma) = 0.9544$$



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0.	<b>4</b> 0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.	<b>5</b> 0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.	<b>6</b> 0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.	<b>7</b> 0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.	<b>9</b> 0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.	<b>2</b> 0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
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1.	_	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.		0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.			0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.	_	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
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2.		0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.		0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.		0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.		0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.		0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.	_	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.		0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.	_	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.		0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.		0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.	<b>4</b> 0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

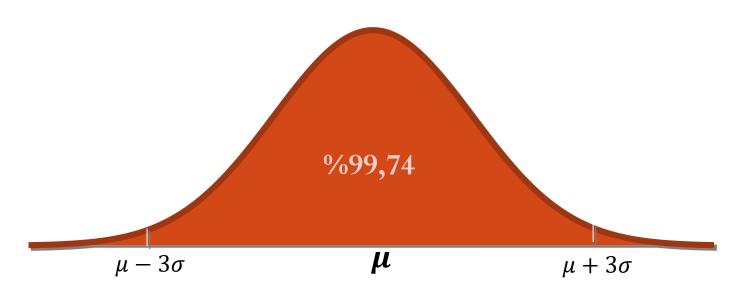


$$P(\mu - 2\sigma \le x \le \mu + 2\sigma) = 0.9544$$

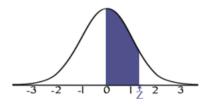
$$2x0,4772=0,95$$

# NORMAL DAĞILIM

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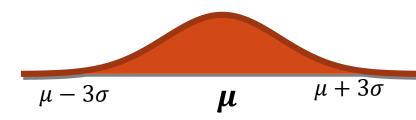
$$P(\mu - 3\sigma \le x \le \mu + 3\sigma) = 0.9974$$



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0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0		0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4940	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998



$$P(\mu - 3\sigma \le x \le \mu + 3\sigma) = 0.9974$$

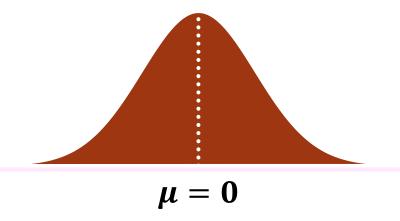
2x0,4987=0,99

- Olasılık hesaplamasındaki zorluktan dolayı normal dağılım gösteren hesaplamalar için standart normal dağılım yaklaşımından yararlanılır.
- O Böylece tek bir olasılık tablosu kullanılarak normal dağılımla ilgili olasılık hesaplamaları yapılmış olur.

- Normal Dağılımın özel bir biçimidir. Normal dağılıma dayalı hesaplamalarda kullanıcılara kolaylık sağlar.
- Ortalama ( $\mu$ ) = 0 ve Varyans ( $\sigma^2$ ) = 1 dir.
- Standart normal de**ğ**i**ş**ken z ile gösterilir.

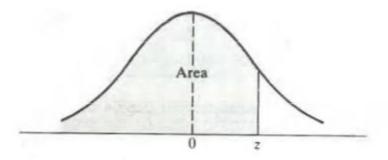
- Eğer bir x değişkeninin normal dağıldığı biliniyorsa
- Yandaki eŞitlik ile elde edilen z değerleri ortalaması 0 ve varyansı 1 olan standart normal dağılıma uyar
- Dağılımın grafiği yandaki gibidir.

$$z = \frac{x - \mu}{\sigma}$$



- Bu özellik, ortalama ve standart sapmanın değerine bağlı değildir.
- Ortalama ve standart sapma ne olursa olsun x değişkeninin normal dağılması bu özelliğin geçerliği için yeterlidir.
- Çeşitli z değerleri için 0 ile z arasında kalan alanı gösteren **Z** tablosu geliştirilmiştir.
- Bu tablodan yararlanarak normal dağılıma dayalı hesaplamalar yapılabilir.
- Z de**ğ**eri ile merkez (ortalama) arasında kalan alanı tablo bize verir.
- Z değerlerinin her birine standart skorlar da denir.

# Kümülatif Standart Normal Dağılım Tablosu



 $P[Z \leq z]$ 

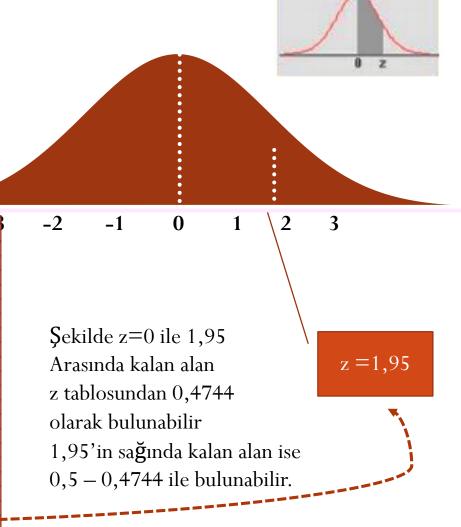
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2			0.0006							
-3.1			0.0009							
-3.0			0.0013							
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6			0.0044							
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3			0.0102							
-2.2	0.0139									
	0.0179									
-2.0			0.0217							

# Z TABLOSU

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036		
0.1	0.040	0.044	0.048	o.c This is	s the formula	for		In ever	ry normal dis 3 of its total o	stribution		
0.2	0.079	0.083	0.087	of x i	erting a give nto its corres	n value ponding		betw	een the med	an and		
0.3	0.118	0.122	0.126	0.1 Z SCO	re:				z = 1.0			
0.4	0.155	0.159	0.163	0.1 Z	= X - P	<u>x</u>	$\longrightarrow$	).3413				
0.5	0.191	0.195	0.198	σ <sub>X</sub> σ <sub>X</sub>								
0.6	0.226	0.229	0.232	0.2								
0.7	0.258	0.261	0.264	0.2			Values of	z×				
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313		
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339		
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358				
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	Z=	1		
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398				
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418		
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432		
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444		
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454		
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463		
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471		
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477		
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482		

• Ortalamanın solunda kalan alan değerleri negatiftir, ancak alan kavramı nedeniyle solda kalan alan pozitif değerlendirilmelidir.

						_				
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09-3
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	<del>- 0.475</del> -	<b>→.+7<del>6</del></b>	0.476	₩77
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482

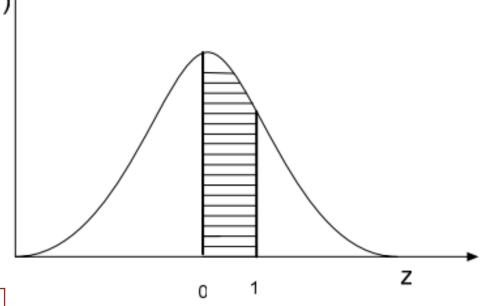


0 ve Z Arasında

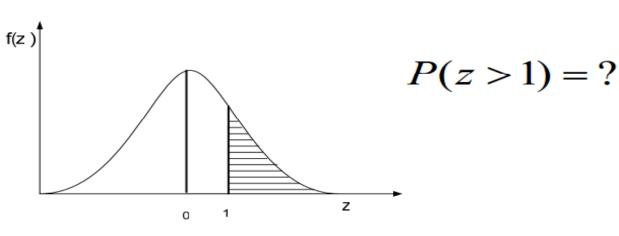
Kalan Alan

$$P(0 < z < 1) = ?$$

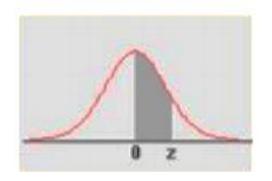
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482



$$P(0 < z < 1) = 0.3413$$



#### 0 ve Z Arasında Kalan Alan

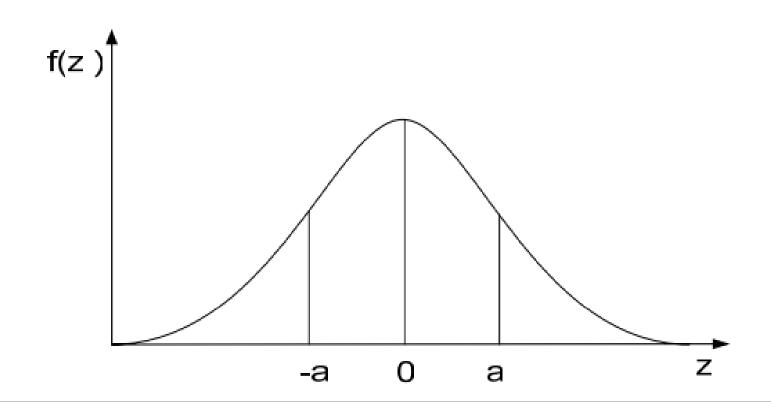


$$0.5 - P(0 < z < 1) = 0.5 - 0.3413 = 0.1587$$

							_			
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482

Simetrik özelliğinden dolayı 0'da eşit uzaklıktaki Z değerlerinin 0 ile arasında kalan alanların değerleri birbirine eşittir.

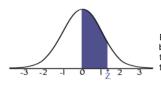
$$P(0 < z < a) = P(-a < z < 0)$$



$$P(-1 < z < 1) = ?$$

$$P(-1 < z < 1) = P(-1 < z < 0) + P(0 < z < 1)$$
$$= 2 * P(0 < z < 1) = 2(0,3413) = 0,6826$$

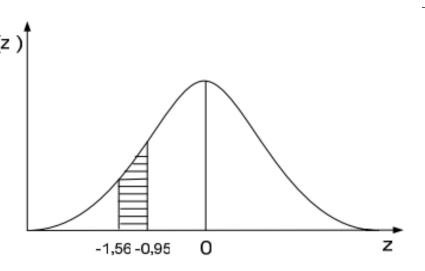
$$P(-1,56 < z < -0.95) = ?$$



#### STANDARD NORMAL TABLE (Z)

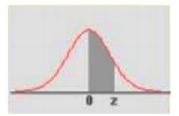
Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for z = 1.25 the area under the curve between the mean (0) and z is 0.3944.

_	0.00	0.04	0.00	0.00	0.04	0.05	0.00	0.07	0.00	0.00
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478 0.0871	0.0517 0.0910	0.0557 0.0948	0.0596 0.0987	0.0636 0.1026	0.0675 0.1064	0.0714	0.0753
0.2										
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879 0.2224
0.6	0.1915 0.2257	0.1950 0.2291	0.1985 0.2324	0.2019 0.2357	0.2054 0.2389	0.2088 0.2422	0.2123 0.2454	0.2157 0.2486	0.2190 0.2517	0.2549
0.7	0.2580	0.2291	0.2642	0.2673	0.2369	0.2422	0.2454	0.2486	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2704	0.2734	0.3051	0.3078	0.2023	0.2032
0.9	0.2001	0.2910	0.2939	0.3238	0.3264		3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3209	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3334	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	_	0.4406	4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

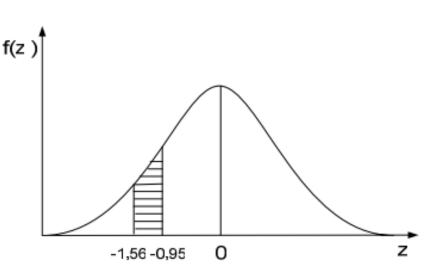


#### 0,4406 - 0,3289 = 0,1117

0 ve Z Arasında Kalan Alan



$$P(-1,56 < z < -0.95) = ?$$

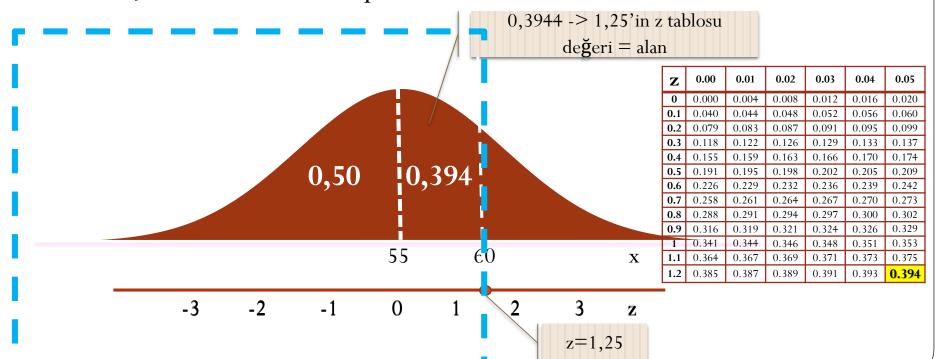


$$P(-1,56 < z < -0.95) = P(-1,56 < z < 0) - P(-0.95 < z < 0)$$

- Oyuncak yarış otomobili montaj süresi  $\mu=55~dakika$ ,  $\sigma=4~dakikadır$ . Montajın en çok 60 dakikada bitmesi olasılığı nedir?
- $p(x \le 60) = P(z \le 1,25)$  ? Z=x-ort/ss = 60-55/4 = 5/4=1,25

60'ın z değeri (60-55)/4 = 1,25 olarak bulunur.

60 dan küçük olan alanların toplamı 0,5+0,3944=0,8944

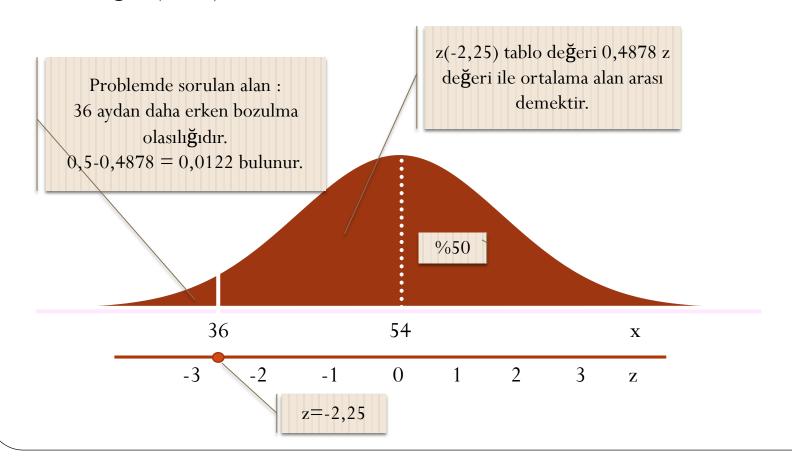


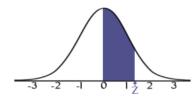
• Bilgisayar üreten bir firma bilgisayarların ortalama ömrünü  $\mu = 54~ay$ ,  $\sigma = 8~ay~dir$ . 36 ay içinde bozulanları değiştirme kampanyası düzenlemiştir. Rasgele satılan bir bilgisayarın değiştirilme olasılığı nedir?

$$Z=x-ort/ss = 36-54/8 = 18/8 = -2,25$$

$$p(x \le 36) = P(z \le -2.25)$$
?

36'nın z değeri (36-54)/8 = -2,25 olarak bulunur.





#### STANDARD NORMAL TABLE (Z)

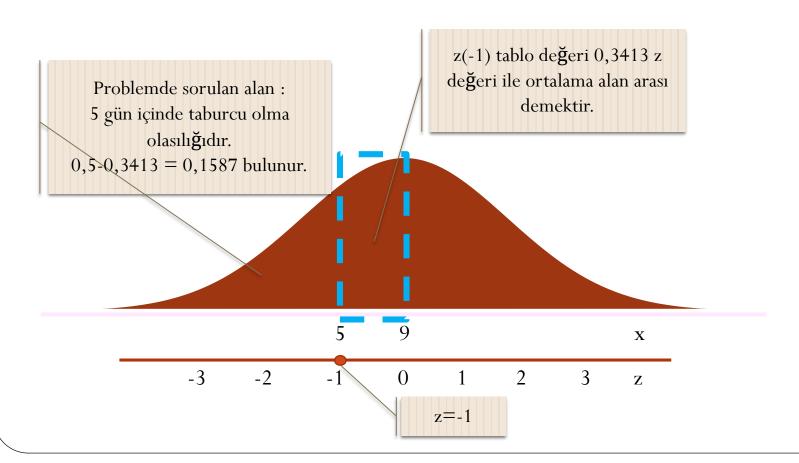
Entries in the table give the area under the curve between the mean and z standard deviation the mean. For example, for z = 1.25 the curve between the man

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	ი.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	6.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	C.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	C.4639	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	C.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	C.4837	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	6.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	C.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	C.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4936	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

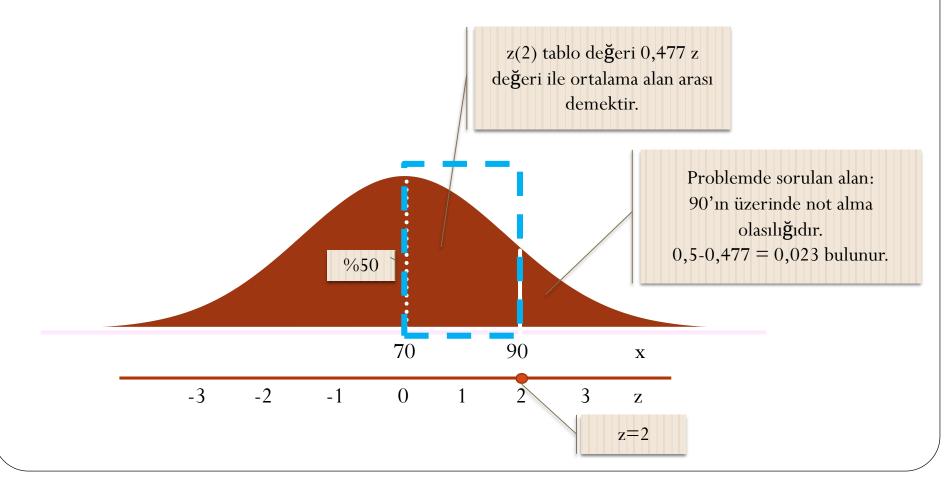
Problemde sorulan alan :
36 aydan daha erken bozulma
olasılı**ğ**ıdır.
0,5-0,4878 = 0,0122 bulunur.

Rasgele satılan bir bilgisayarın de**ğ**i**Ş**tirilme olasılı**ğ**ı %1,22 olur.

- Farabi Hastanesinde yatan hastaların ortalama yatış süresi  $\mu=9$  gün,  $\sigma=4$  gün dür. Rasgele seçilen bir yatan hastanın 5 gün içinde taburcu olma olasılığı nedir?
- $p(x \le 5) = P(z \le -1)$  ?
- 5'in z de $\S$ eri (5-9)/4 = -1 olarak bulunur.



- Üçüncü sınıf biyoistatistik not ortalaması  $\mu = 70~\sigma = 10~dur$ . Rastgele seçilen bir öğrencinin 90 puanın üzerinde not almış olma olasılığı nedir?
- 90'ın z değeri (90-70)/10 = 2 olarak bulunur.
- $p(x \ge 90) = P(z \ge 2)$  ?



• Bir hastanede 20 depresyon hastası, 20 psikotik hasta ve 60 da hipertansiyon hasta vardır. Bu hastaneden rastgele bir hasta seçsek depresyonlu veya psikozlu olma olasılığı nedir?

• Bir serviste 6 kızamık ve 4 astım hastası yatmaktadır. Bu servisten sırayla (rastgele) iki hasta seçsek ikisinin de astımlı olma olasılığı nedir?

• 1000 kişinin katıldığı sınavda öğrencilerin aldıkları puanların aritmetik ortalaması 60, standart sapması 5''tir. Histogram grafiğinde verilerin normal dağıldığı görülmektedir.

Geçme notu 50 olduğuna göre bu öğrencilerin kaçı geçmiştir?

#### Soru

- Ahmet, Sağlık Bilimleri fakültesi 3. sınıf öğrencisidir.
  - İstatistik sınavından 70 puan
    - Sınıf ortalaması: 60, standart sapma: 5

$$Z = \frac{(70-60)}{5} = 2$$

- Ölçme sınavından 85 puan almı**Ş**tır.
  - Sınıf ortalaması: 70, standart sapma: 15

$$Z = \frac{(85-70)}{15} = 1$$

Ahmet bu iki dersten hangisinde daha başarılıdır?

Sonuç: Ahmet, biyoistatistik dersinde daha başarılıdır.

# Kaynak

- 1. Aktürk Z, Acemoğlu H. Sağlık Çalışanları İçin Araştırma ve Pratik İstatistik. Anadolu Ofset: İstanbul, 2011.
- 2. Prof. Dr. Kemal Turhan. Biyoistatistik ppt. Sunumu.
- 3. <a href="http://kisi.deu.edu.tr/hamdi.emec/ist1-7.pdf">http://kisi.deu.edu.tr/hamdi.emec/ist1-7.pdf</a> EriŞim tarihi: 18.11.2014
- 4. Büyüköztürk Ş, Çokluk O, Köklü N. Sosyal Bilimler İçin İstatistik. 15. Baskı, Pegem Akademi, Ankara, 2015.