Some Exercises

Q.1) The total lifetime (in years) of five-year-old dogs of a terrier breed is a r.v. whose dist. function is given by

$$F(x) = \begin{cases} 0 & \text{for } x \le 5 \\ 1 - \frac{25}{x^2} & \text{for } x > 5 \end{cases}$$

Find the following probabilities.

a) $P(X > 10)$

b) $P(X < 8)$

c) $P(12 < X < 15)$

Solutions a) $1 - P(X \le 10) = 1 - F(10) = 1 - \left(1 - \frac{25}{100}\right) = 0.25$

b) $F(8) = 1 - \frac{25}{64} = \frac{39}{64} = 0.6094$

c) $F(15) - F(12) = 1 - \frac{25}{15^2} - 1 + \frac{25}{12^2} = \frac{25}{144} - \frac{25}{225} = 0.0625$

Q.2) Find the dist. function of the r.v. X whose prob. density is given by

$$f(x) = \begin{cases} \frac{x}{2}, & \text{for } 0 < x \le 1 \\ \frac{1}{2}, & \text{for } 1 < x \le 2 \\ \frac{3-x}{2}, & \text{for } 2 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Solution :

$$F(x) = \int_0^x f(u)\,du$$

$$\int_0^x \frac{u}{2}\,du = \frac{1}{2}\frac{u^2}{2}\Big|_0^x = \frac{x^2}{4}$$

$$\int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,du = \frac{1}{2}\frac{x^2}{2}\Big|_0^1 + \frac{1}{2}u\Big|_1^x = \frac{1}{4} + \frac{1}{2}(x-1)$$

$$= \frac{x}{2} + \frac{1}{4} - \frac{1}{2} = \frac{2x-1}{4}$$

$$\int_0^1 \frac{x}{2}\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x \frac{3-u}{2}\,du = \frac{1}{4} + \frac{1}{2}x\Big|_1^2 + \frac{1}{2}\left(3u - \frac{u^2}{2}\right)\Big|_2^x$$

$$= \frac{1}{4} + \frac{1}{2}(2-1) + \frac{1}{2}\left(3x - \frac{x^2}{2} - 6 + \frac{4}{2}\right) = \frac{1}{4}(6x - x^2 - 5)$$

So;

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^2}{4}, & 0 < x \leq 1 \\ \frac{1}{4}(2x-1), & 1 < x \leq 2 \\ \frac{1}{4}(6x - x^2 - 5), & 2 < x < 3 \\ 1, & x \geq 3 \end{cases}$$

Q.3) A company employing 10000 workers offers deluxe medical coverage, standard medical coverage, and economy medical coverage. Of the employees 30% have deluxe coverage, 60% have standard coverage, and 10% have economy coverage. From past experience, the prob. that an employee with deluxe coverage will submit no claims during the next year is 0.1, the prob. of an employee with standard coverage submitting no claim is 0.4, and the prob. of an employee with economy coverage submitting no claim is 0.7. If an employee is selected at random,

a) What is the prob. that the selected employee has standard coverage and will submit no claim?

b) What is the prob. that the selected employee will submit no claim?

Solutions

a) $A_1 \to$ deluxe coverage $\quad P(A_1) = 0.30 \quad$ $B \to$ submit no claim

$A_2 \to$ standard " $\quad P(A_2) = 0.60$

$A_3 \to$ economy " $\quad P(A_3) = 0.10$

$$P(A_2 \cap B) = P(B/A_2) \cdot P(A_2)$$
$$= 0.4 \times 0.6 = 0.24$$

b) $P(B) = P(B|A_1).P(A_1) + P(B|A_2).P(A_2) + P(B|A_3).P(A_3)$

Total probability

$= 0.30 \times 0.1 + 0.4 \times 0.6 + 0.7 \times 0.10$

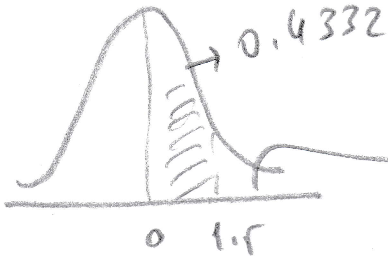
$= 0.03 + 0.24 + 0.07 = 0.34$ //

Q.4) If Z denotes a standard normal r.v., then find;

a) $P(Z > 1.5)$

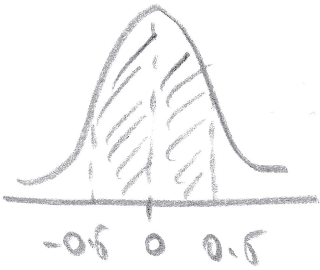
b) $P(-0.5 < Z < 0.5)$

<u>Solutions</u> a)



$$\begin{array}{r} 0.5 \\ - 0.4332 \\ \hline 0.0668 \end{array}$$ //

b)



$0.1915 \times 2 = 0.383$ //

Q.5) Let the prob. density function of X is given by;

$$f(x) = \begin{cases} k.e^{-3x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find k

b) Find $P(0.5 \leq X \leq 1)$

<u>Solutions</u>

$$\int_0^\infty k.e^{-3x}dx = 1$$

$$k.\frac{e^{-3x}}{-3}\Big|_0^\infty = 1 \Rightarrow -\frac{k}{3}(\cancel{e^{-\infty}}^{0} - e^{-0}) = 1$$

$$\frac{k}{3} = 1 \quad k = 3$$

b) $3.\int_{0.5}^{1} e^{-3x}dx = \cancel{3}.\frac{e^{-3x}}{-\cancel{3}}\Big|_{0.5}^{1} = -e^{-3x}\Big|_{0.5}^{1} = -e^{-3x} + e^{-1.5}$

$= e^{-1.5} - e^{-3}$ //

Q.6) Ten percent of computer parts produced by a certain supplier are defective. What is the prob. that a sample of 10 parts contains more than 3 defective ones?

Solution: $P(X>3) = 1 - P(X\leq 3)$

$= 1 - \left\{ \binom{10}{0}(0.10)^0(0.90)^{10} + \binom{10}{1}(0.10)^1(0.90)^9 + \binom{10}{2}(0.10)^2(0.90)^8 + \binom{10}{3}(0.10)^3(0.90)^7 \right\}$

(Binomial dist.)

Find the rest.

Q.7) In a shopping center, one company selling refrigerators, they sell in an average of 4 refrigerators during a week. Thus in the following week find the prob. of this company selling at most one refrigerator.

Solution: $\lambda = 4$

(Poisson dist.)

$P(X\leq 1) = P(X=0) + P(X=1)$

$= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!}$

Find the rest.

Q.8) A lab network consisting of 10 computers was attacked by a computer virus. This virus enters each computer with prob. 0.3, independently of other computers. A computer manager checks the lab computers to see if they were infected by the virus. What is the prob. that she has to test at least 9 computers to find the first infected one?

Solution

$$p = 0.3$$

$$P(X \geq 9) = P(X=9) + P(X=10) \quad \text{(Geometric dist.)}$$

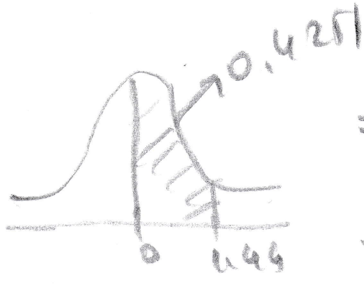
$$= 0.7^8 \cdot 0.3 + 0.7^9 \cdot 0.3$$

Q.9) The amount of space radiation that any person receiving while flying from North to South of USA by an airplane is normally distributed with a mean of 4.35 mrem (one thousand of a rem), and a standard deviation of 0.59 mrem. In such a flight find the prob. of a person receiving a space radiation more than 5.20 mrem?

Solution $\mu = 4.35$, $\sigma = 0.59$

$P(X > 5.20) = ?$ → by using standardization

$$P\left(\frac{X-\mu}{\sigma} > \frac{5.20 - 4.35}{0.59}\right)$$



$$= P\left(Z > \frac{0.85}{0.59}\right)$$

$$= P(Z > 1.44) \Rightarrow 0.5 - 0.4251 = 0.0749 \,//$$