APPLIED STATISTICS

Week 2: Introduction to Sampling Distributions

Outline

Sampling Distributions

Sampling Distributions

Inference about the Population from Sample Information

parameter (μ)
(truth about the population)

Population

Sample #1
(random sample of size n)

Sample #1

Calculate the Statistic

$\bar{X}_1$

Sample #2

$\bar{X}_2$

Sample #n

$\bar{X}_n$

What's the distribution of these values?

reasonable estimate of μ

Sampling Distributions

– A sampling distribution is a distribution of all possible values of a statistic for a given sample size (such as n) selected from a population!

Sampling Distributions

Please see the video demo:

https://www.youtube.com/watch?v=JyfyDXNFLiU

Case 1 :

If the Population is Normal :

- If a population is normal with mean μ and standard deviation σ, the sampling distribution of $\bar{X}$ is also normally distributed with

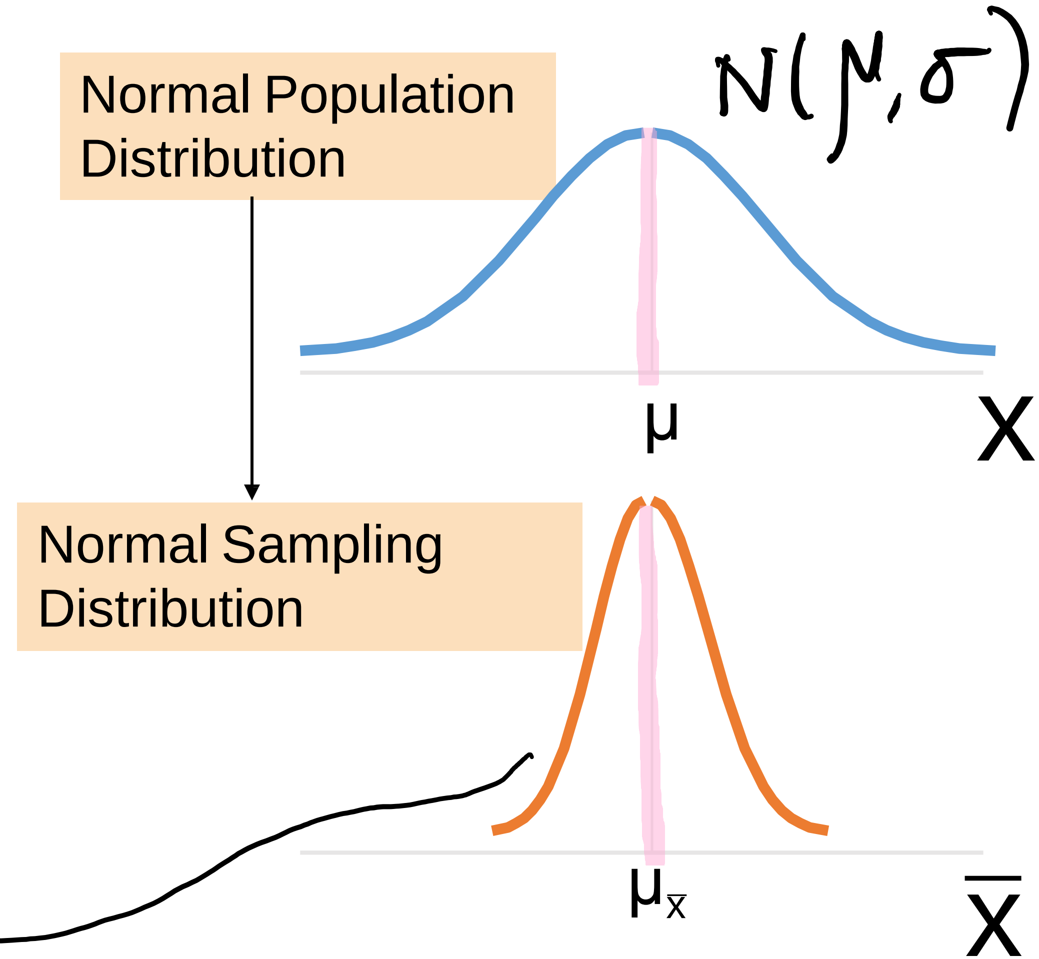
$$\boxed{\mu_{\bar{X}} = \mu} \quad \text{and} \quad \boxed{\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}}$$

sample size !!

Sampling Distribution Properties

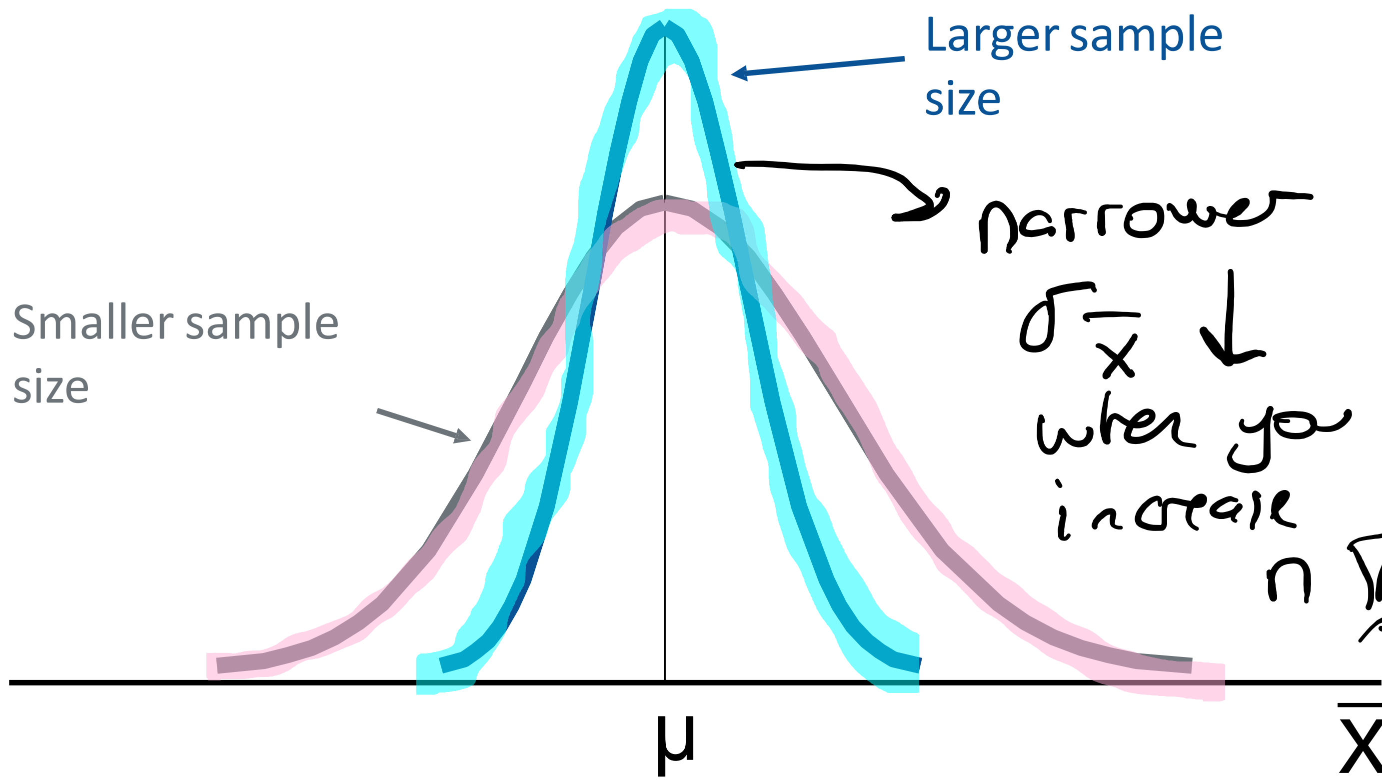
$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



Sampling Distribution Properties

As n increases ($\uparrow$), $\sigma_{\bar{x}}$ decreases !



narrower

$\sigma_{\bar{x}} \downarrow$ when you increase n !

Case 2:

If the Population is not Normal

We have the Central Limit Theorem

- Even if the population is not normal, sample means from the population will be approximately normal as long as the sample size is large enough

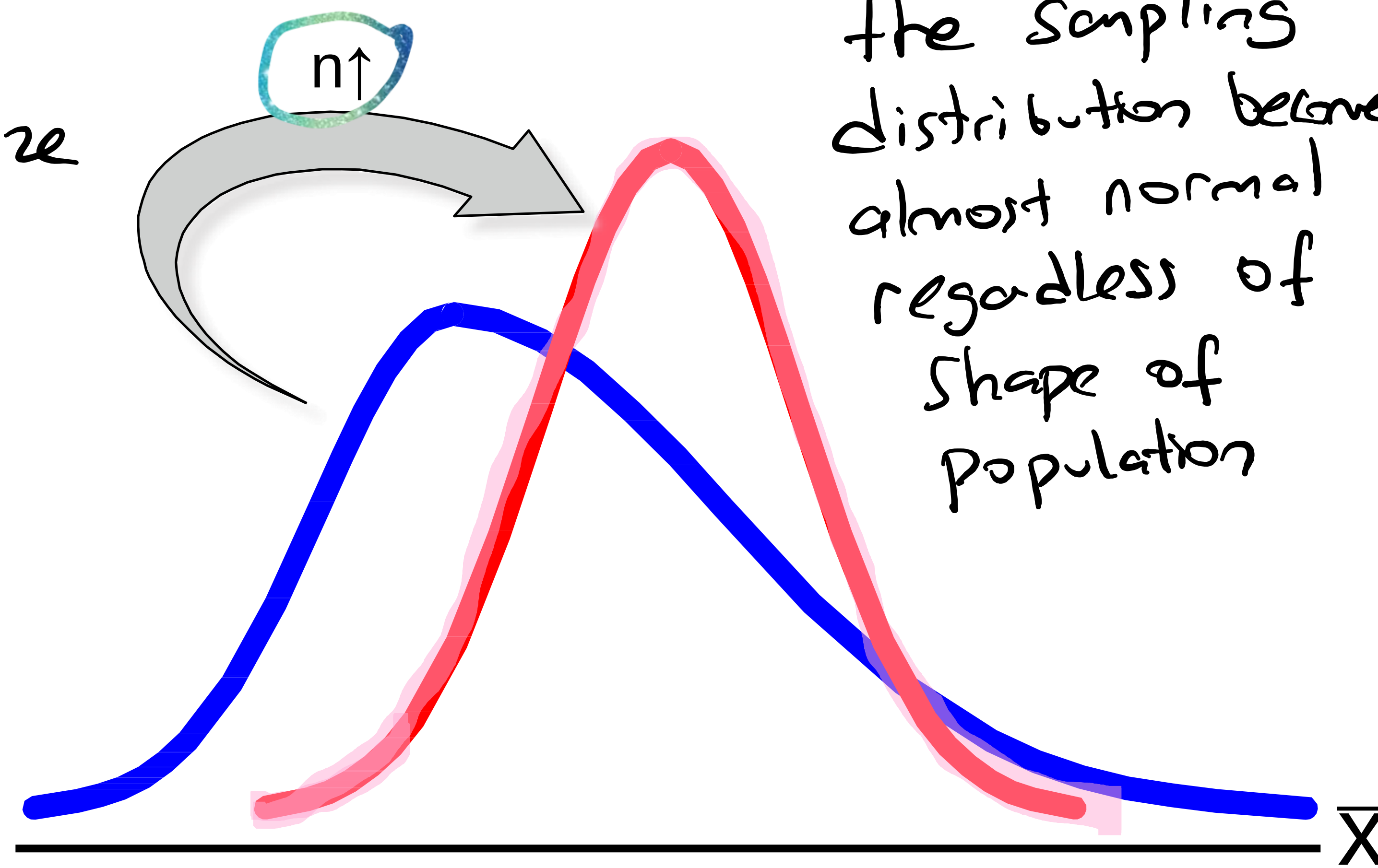
Properties of the sampling distribution:

$$\boxed{\mu_{\bar{x}} = \mu} \quad \text{and} \quad \boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Central Limit Theorem

(Most fundamental and profound concept in statistics)

As the sample size gets large enough



the sampling distribution becomes almost normal regardless of shape of population

If the Population is **not** Normal (if n is large enough)

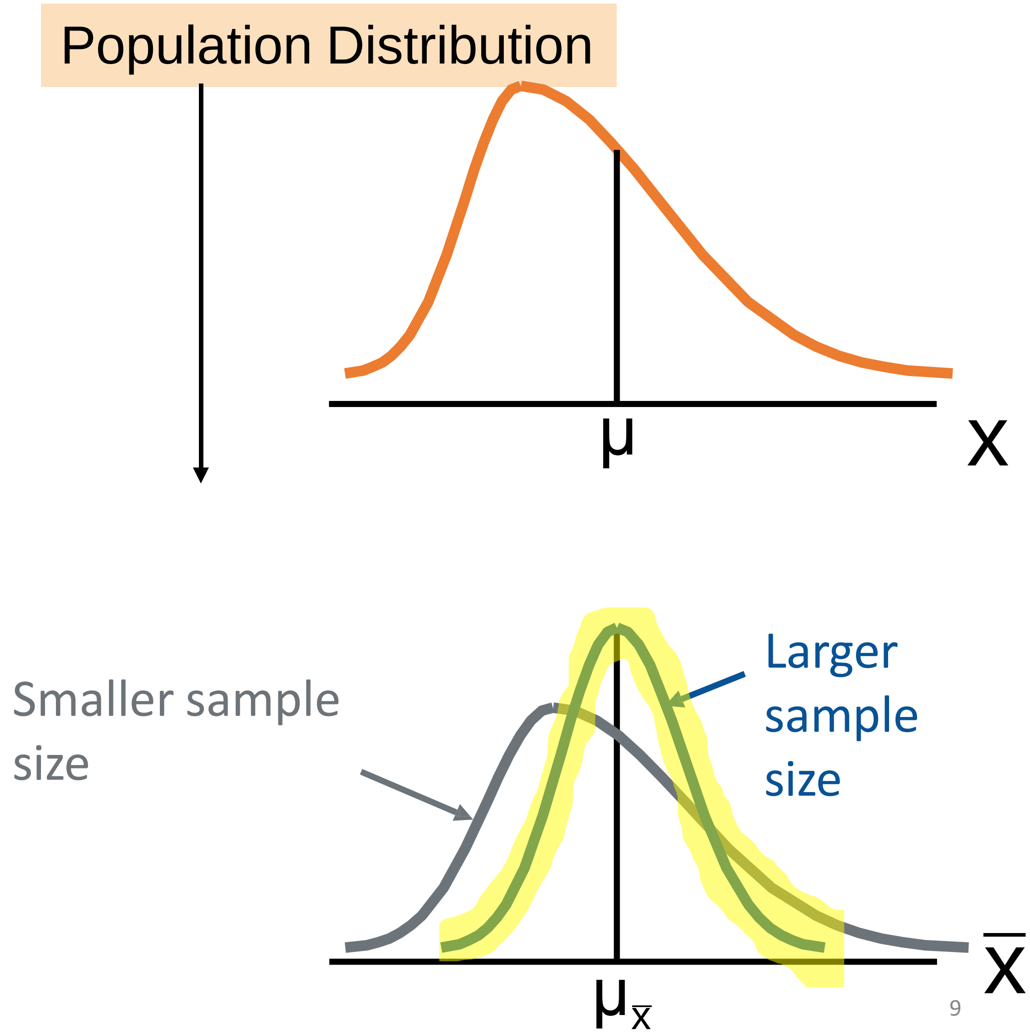
Sampling distribution properties:

Central Tendency

$$\mu_{\bar{x}} = \mu$$

Variation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



How Large is Large Enough? $\boxed{n = 30}$?

- For most distributions, n >=30 will give a sampling distribution that is nearly normal.

Sample size

- If n < 30 , the approximation is good only if the population is not too different from a normal distribution.

- Moreover, if the population is known to be normal, the sampling distribution of the mean will follow a normal distribution exactly, no matter how small the size of the samples.

An animation from CreatureCast on Central Limit Theorem ☺

Please watch this video!!!

https://vimeo.com/75089338

Central Limit Theorem

If $\overline{X}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution

$$\boxed{Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}}$$

(z-score)

as $n \to \infty$, is the standard normal distribution

$n(z : 0, 1)$

Assessing Normality

1. Assessing Normality

Not all continuous random variables are normally distributed.

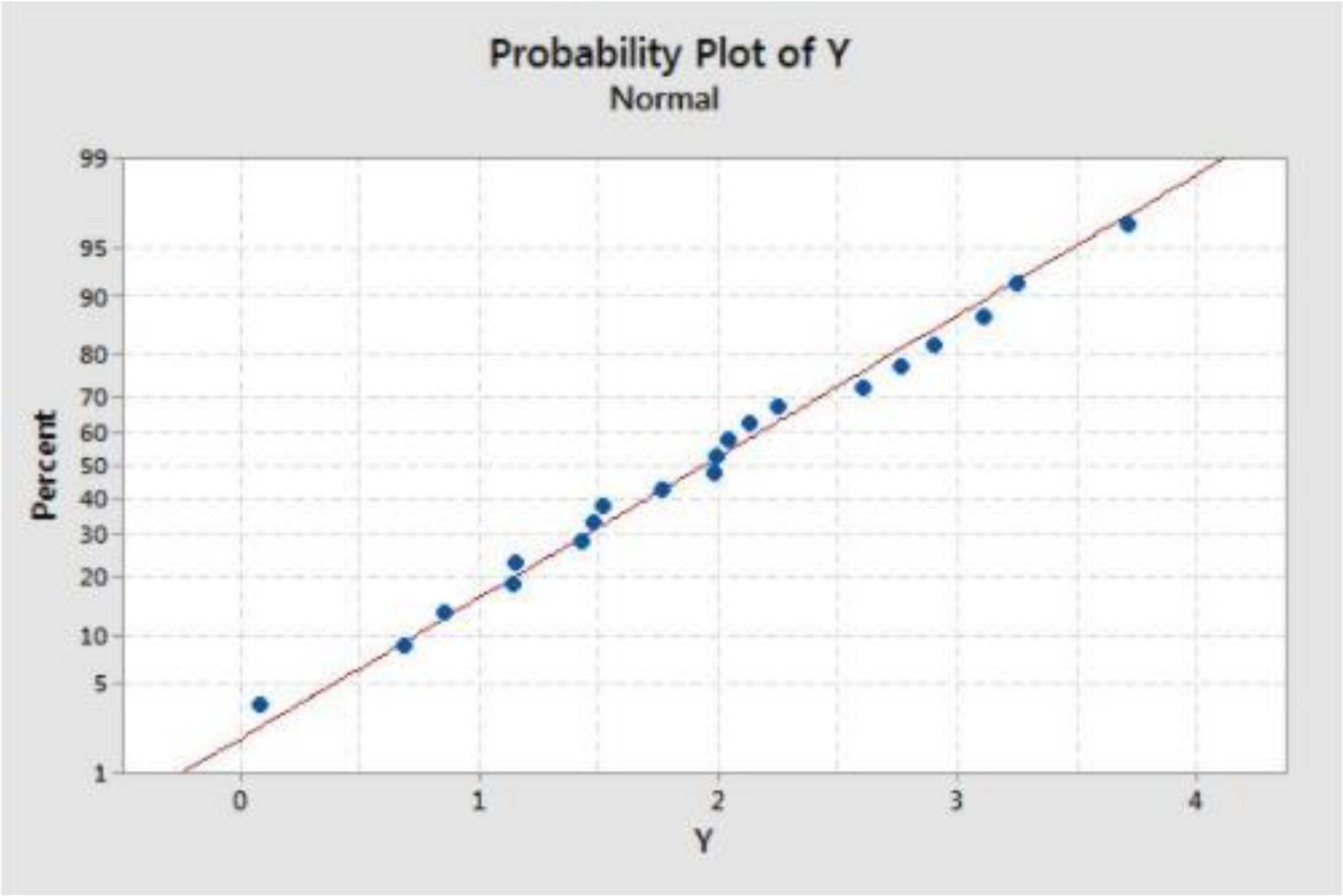
It is important to evaluate how well the data is approximated by a normal distribution.

The Normal Probability Plot

Normal probability plot:

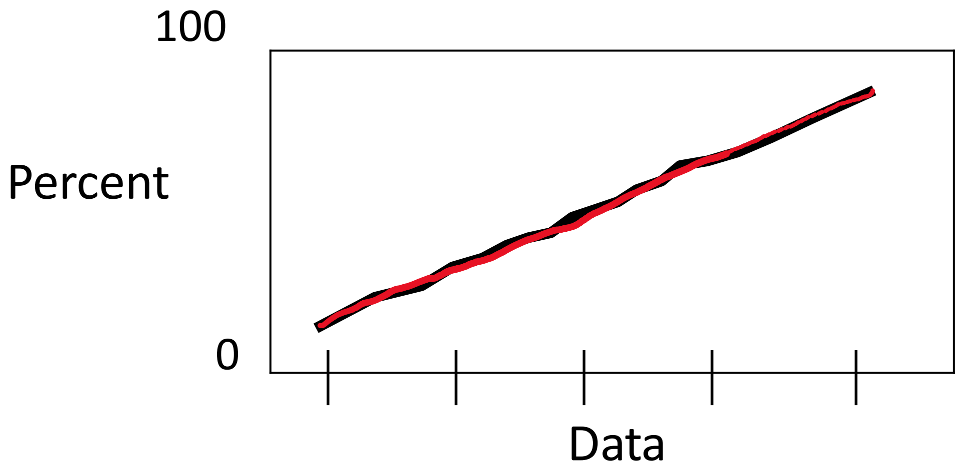
- Arrange data from low to high values
- Find cumulative normal probabilities for all values
- Examine a plot of the observed values vs. cumulative probabilities (with the cumulative normal probability on the vertical axis and the observed data values on the horizontal axis)
- Evaluate the plot for evidence of linearity

The Normal Probability Plot



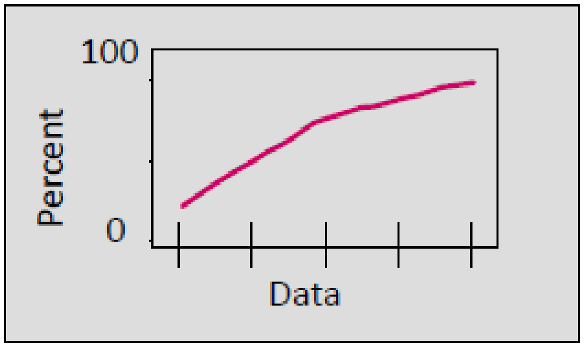
The Normal Probability Plot

A normal probability plot for data from a normal distribution will be approximately linear:

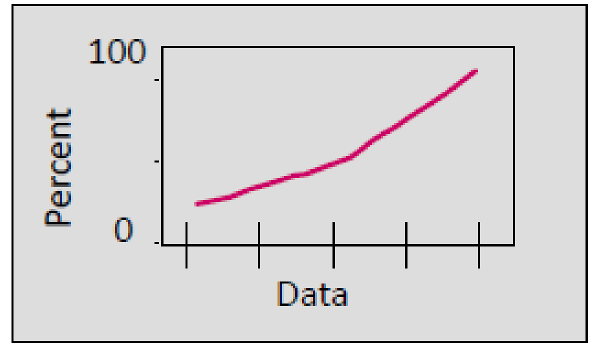


The Normal Probability Plot

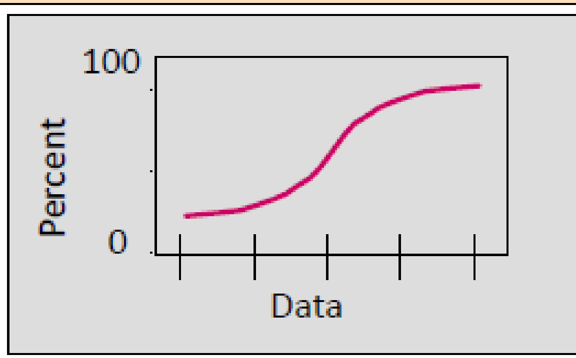
Left-Skewed



Right-Skewed



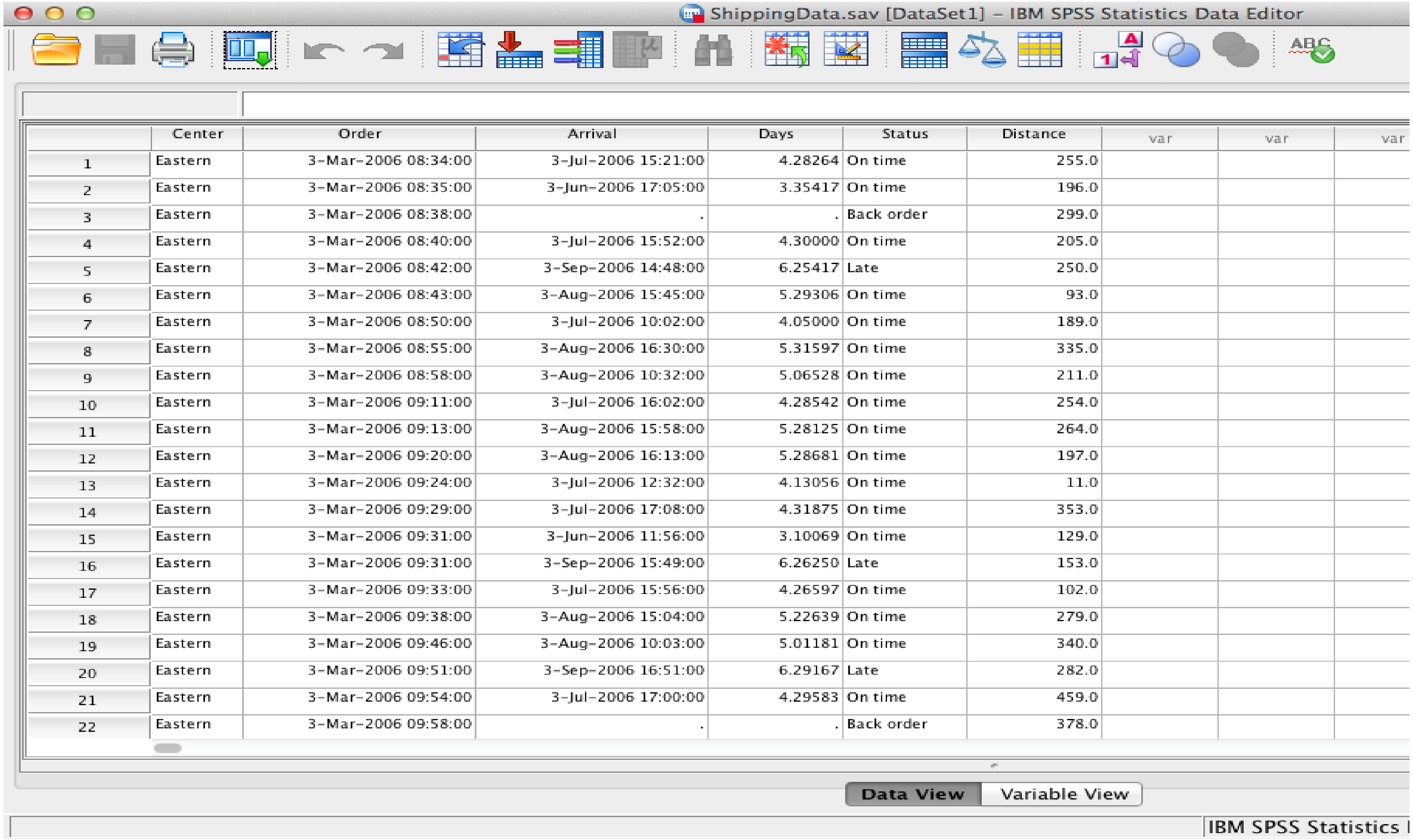
Uniform



Nonlinear plots indicate a deviation from normality

Checking Normality using SPSS

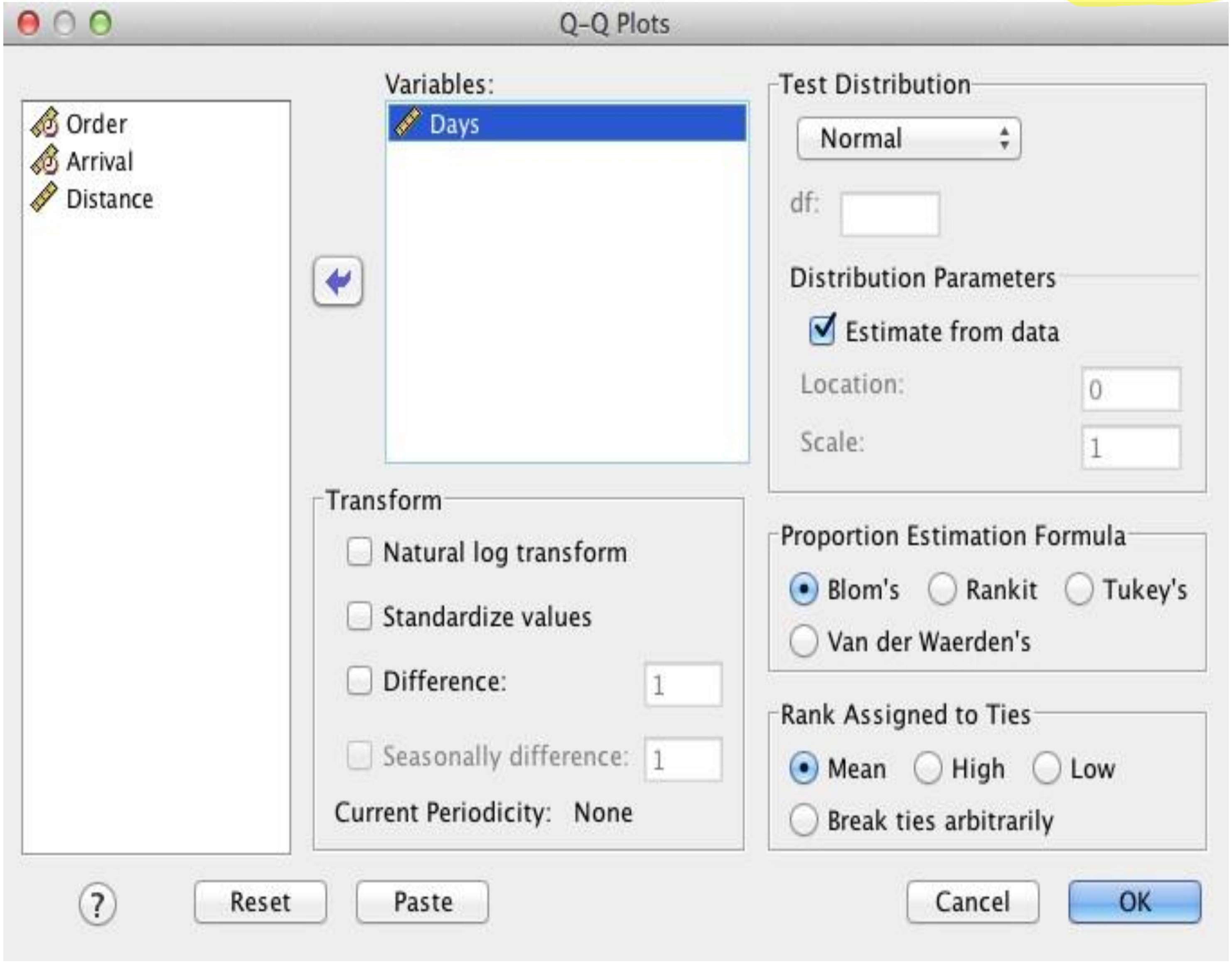
See Shipping Data, Days.



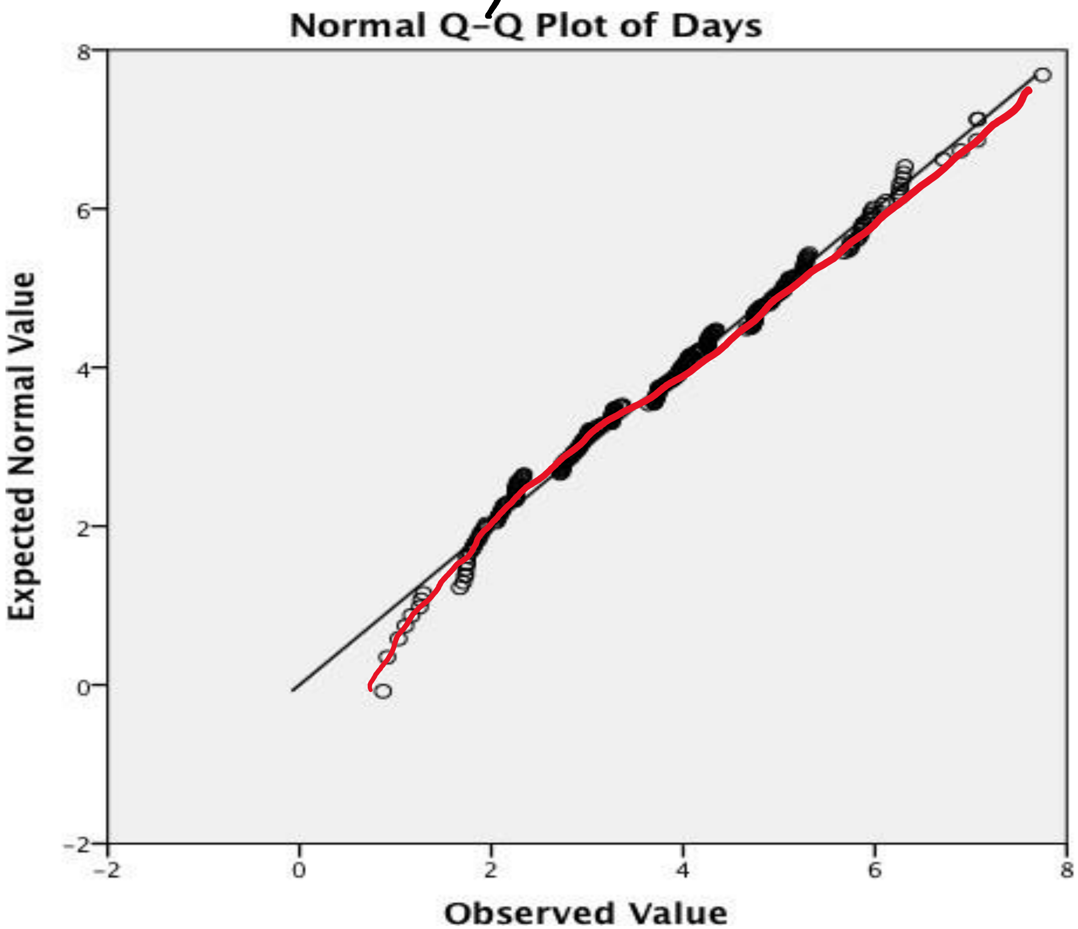
	Center	Order	Arrival	Days	Status	Distance	var	var	var
1	Eastern	3-Mar-2006 08:34:00	3-Jul-2006 15:21:00	4.28264	On time	255.0			
2	Eastern	3-Mar-2006 08:35:00	3-Jun-2006 17:05:00	3.35417	On time	196.0			
3	Eastern	3-Mar-2006 08:38:00	.	.	Back order	299.0			
4	Eastern	3-Mar-2006 08:40:00	3-Jul-2006 15:52:00	4.30000	On time	205.0			
5	Eastern	3-Mar-2006 08:42:00	3-Sep-2006 14:48:00	6.25417	Late	250.0			
6	Eastern	3-Mar-2006 08:43:00	3-Aug-2006 15:45:00	5.29306	On time	93.0			
7	Eastern	3-Mar-2006 08:50:00	3-Jul-2006 10:02:00	4.05000	On time	189.0			
8	Eastern	3-Mar-2006 08:55:00	3-Aug-2006 16:30:00	5.31597	On time	335.0			
9	Eastern	3-Mar-2006 08:58:00	3-Aug-2006 10:32:00	5.06528	On time	211.0			
10	Eastern	3-Mar-2006 09:11:00	3-Jul-2006 16:02:00	4.28542	On time	254.0			
11	Eastern	3-Mar-2006 09:13:00	3-Aug-2006 15:58:00	5.28125	On time	264.0			
12	Eastern	3-Mar-2006 09:20:00	3-Aug-2006 16:13:00	5.28681	On time	197.0			
13	Eastern	3-Mar-2006 09:24:00	3-Jul-2006 12:32:00	4.13056	On time	11.0			
14	Eastern	3-Mar-2006 09:29:00	3-Jul-2006 17:08:00	4.31875	On time	353.0			
15	Eastern	3-Mar-2006 09:31:00	3-Jun-2006 11:56:00	3.10069	On time	129.0			
16	Eastern	3-Mar-2006 09:31:00	3-Sep-2006 15:49:00	6.26250	Late	153.0			
17	Eastern	3-Mar-2006 09:33:00	3-Jul-2006 15:56:00	4.26597	On time	102.0			
18	Eastern	3-Mar-2006 09:38:00	3-Aug-2006 15:04:00	5.22639	On time	279.0			
19	Eastern	3-Mar-2006 09:46:00	3-Aug-2006 10:03:00	5.01181	On time	340.0			
20	Eastern	3-Mar-2006 09:51:00	3-Sep-2006 16:51:00	6.29167	Late	282.0			
21	Eastern	3-Mar-2006 09:54:00	3-Jul-2006 17:00:00	4.29583	On time	459.0			
22	Eastern	3-Mar-2006 09:58:00	.	.	Back order	378.0			

another tool for assessing normality

SPSS->Analyze->Descriptive Statistics->QQ-Plots



Q–Q : Quantile - Quantile Plot
for assessing
normality



It looks like delivery days follow approximately a normal distribution!

PROBLEMS

Question 1:

The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

(a) Find the mean μ and the variance σ^2 of X.

(b) Find the mean $\mu_{\bar{X}}$ and the variance $\sigma^2_{\bar{X}}$ of the mean $\bar{X}$ for random samples of 36 cherry puffs

(c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

Question_1 (Cherry Puff Problem)

a) $\boxed{\mu = \sum x \cdot f(x)}$

$\mu = 4(0.2) + 5(0.4) + 6(0.3) + 7(0.1)$

$\mu = \boxed{5.3}$ //

x	4	5	6	7
$f(x)$	0.2	0.4	0.3	0.1

$\sigma^2 = E[x^2] - E[x]^2$

$\boxed{\sigma^2 = E[x^2] - \mu^2} \rightarrow \sigma^2 = 28.9 - (5.3)^2$

$\sigma^2 = \boxed{0.81}$ //

$E[x^2] = \sum x^2 \cdot f(x) = 4^2(0.2) + 5^2(0.4)$
$+ 6^2(0.3) + 7^2(0.1) = 28.9$

b) $\boxed{n = 36} \rightarrow$ sample size is given !

$\mu_{\bar{x}} = \mu = \boxed{5.3}$ //

$$\boxed{\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}}$$

or

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma^2_{\bar{x}} = \frac{0.81}{36} \Rightarrow \boxed{0.0225}$$ //

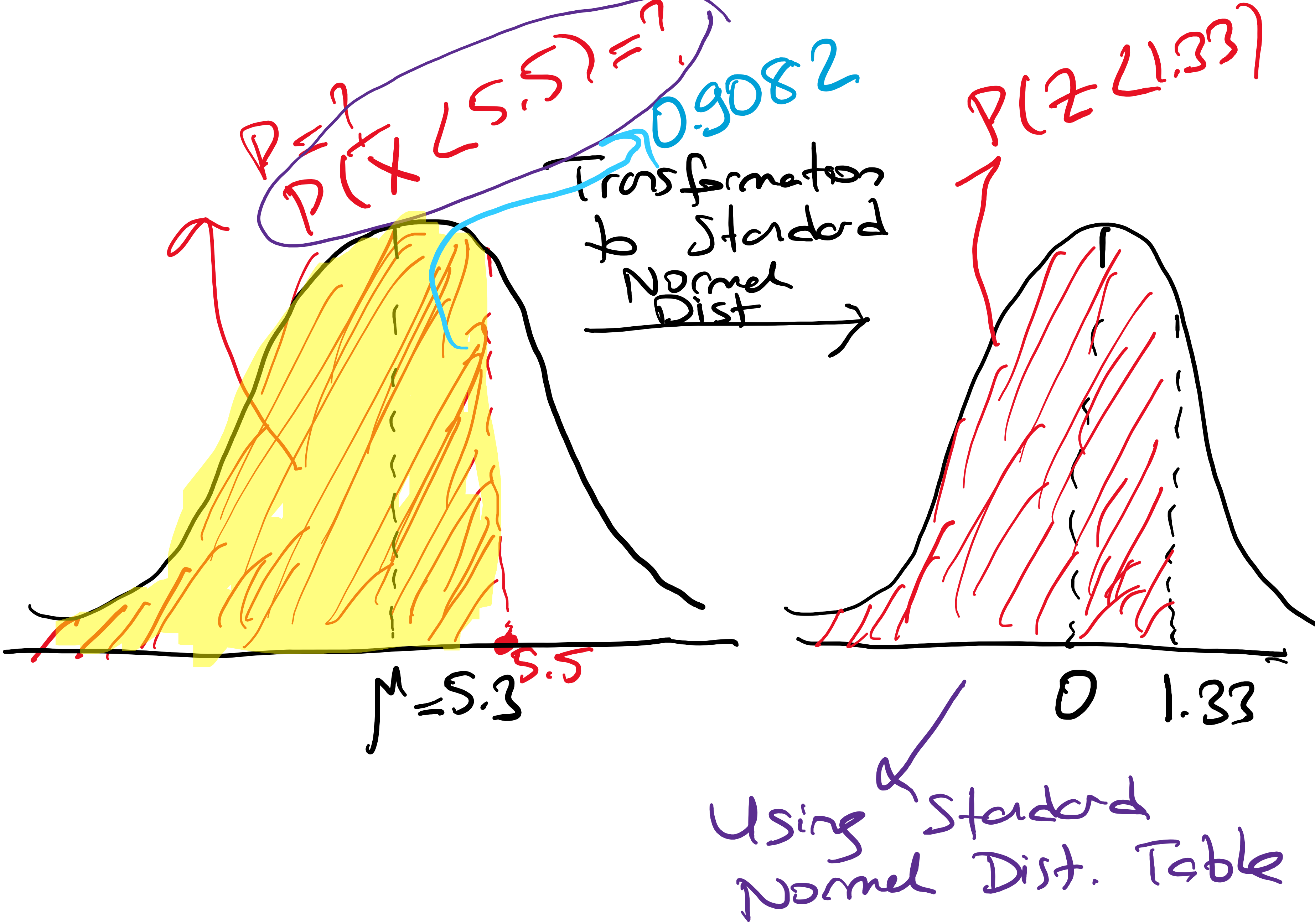
c) $n = 36$ $\mu_{\bar{x}} = 5.3$, $\sigma_{\bar{x}} = 0.15$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow \frac{5.5 - 5.3}{0.15} = 1.33$$

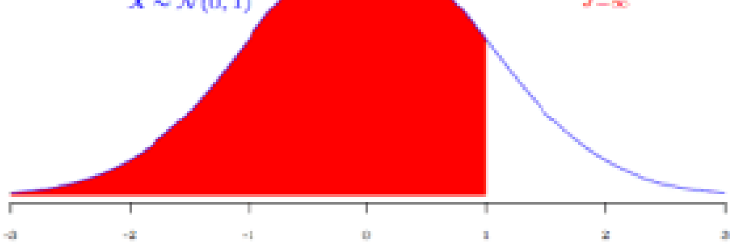
z-score

You should utilize Standard Normal Dist Table

$$P(\bar{x} < 5.5) = P(z < 1.33) = \boxed{0.9082}$$



P = ?
P(X < 5.5) = ?
0.9082
Transformation to Standard Normal Dist
μ = 5.3
5.5
P(Z < 1.33)
0
1.33
Using Standard Normal Dist. Table



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Standard Normal Distribution

$N(0, 1)$

$z = 1.33$

$\Downarrow$

$P = 0.9082$

Question 2:

$n = 64$

The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\mu = 3.2$ minutes and a standard deviation $\sigma = 1.6$ minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's window is

(a) at most 2.7 minutes;

(b) more than 3.5 minutes;

(c) at least 3.2 minutes but less than 3.4 minutes.

a) at most 2.7 minutes

$n = 64 \quad \mu_{\bar{x}} = 3.2 \quad \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.6}{\sqrt{64}}$

$\mu_{\bar{x}} = 3.2 \quad \sigma_{\bar{x}} = 0.2$

$$\underset{\text{z-score}}{z} = \frac{2.7 - 3.2}{0.2} = -2.5$$

So $P(\bar{x} < 2.7 \text{ min}) = P(z < -2.5)$

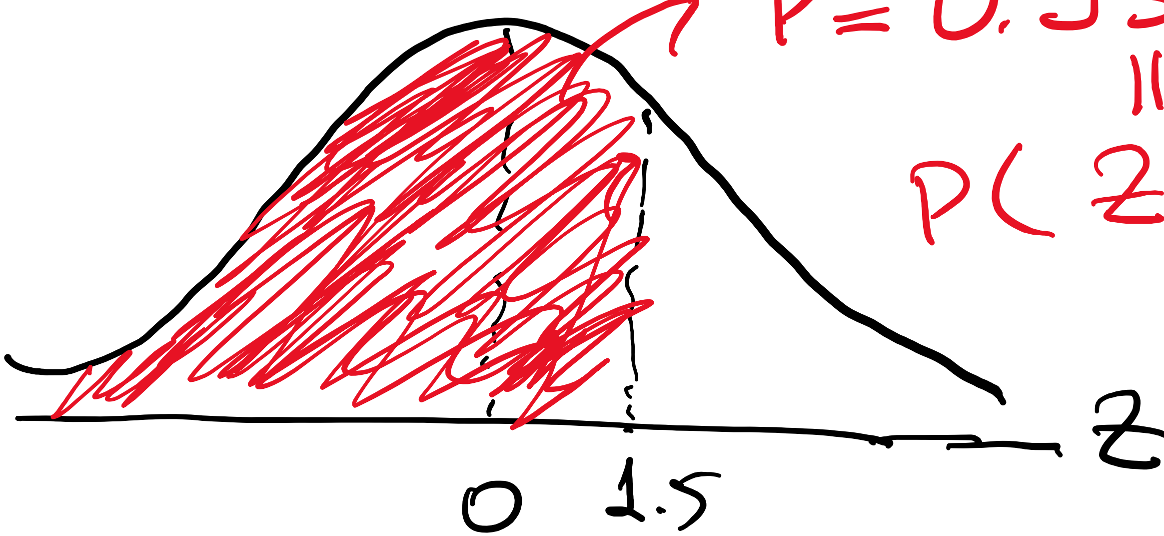
Use Standard Normal Table

$= \boxed{0.0062}$ //

b) more than 3.5 min

$$z = \frac{3.5 - 3.2}{0.2} = 1.5$$

So $P(\bar{X} > 3.5) = \boxed{P(z > 1.5)}$?



$P = 0.9332$

$\parallel$

$P(z < 1.5)$

$1 - 0.9332$

$= \boxed{0.0668}$ //

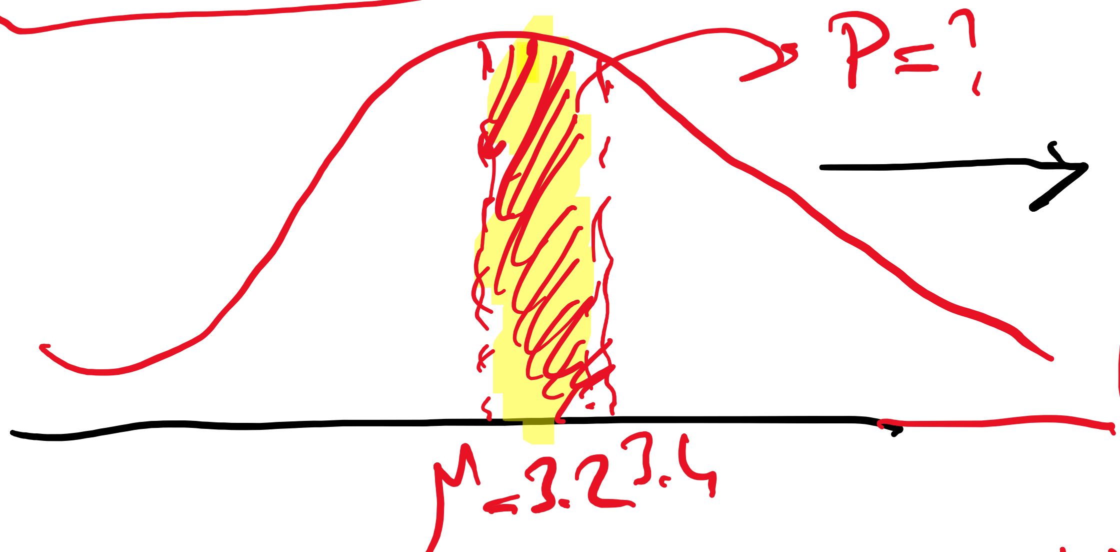
c) at least 3.2 minutes but less than 3.4 minutes

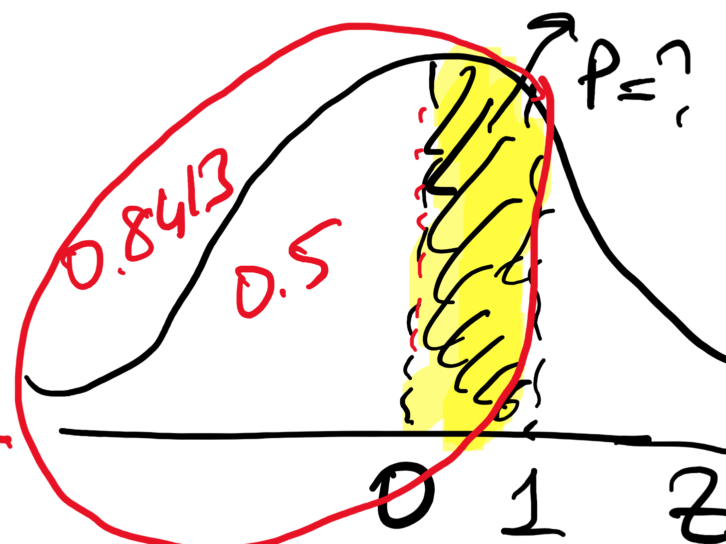
$$\mu_{\bar{x}} = \boxed{3.2}$$

$$z_1 = \frac{3.2 - 3.2}{0.2} = 0, \quad z_2 = \frac{3.4 - 3.2}{0.2}$$

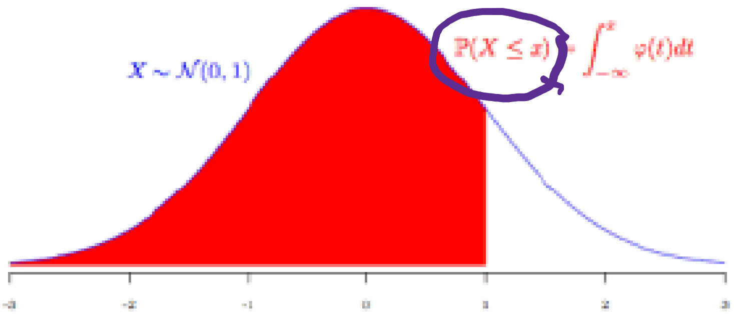
$$z_2 = 1.0$$

$$P(3.2 < \bar{x} < 3.4) = P(0 < z < 1.0)$$





$$= 0.8413 - 0.5 = \boxed{0.3413}$$



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1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
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2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

$P(Z < -2.5) = ?$

$P(Z < 2.5) = \boxed{0.9938}$

$P(Z < -2.5) = 1 - 0.9938 = \boxed{0.0062}$

Question 3:

In a chemical process, the amount of a certain type of impurity in the output is difficult to control and is thus a random variable. Speculation is that the population mean amount of the impurity is 0.20 gram per gram of output. It is known that the standard deviation is 0.1 gram per gram. An experiment is conducted to gain more insight regarding the speculation that $\mu = 0.2$. The process is run on a lab scale 50 times and the sample average $\bar{x}$ turns out to be 0.23 gram per gram. Comment on the speculation that the mean amount of impurity is 0.20 gram per gram. Make use of the Central Limit Theorem in your work.

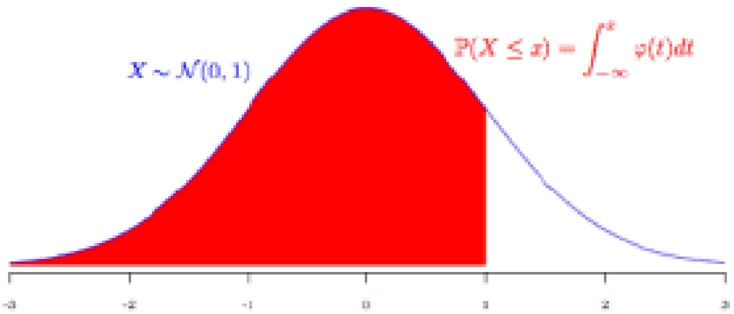
$n = 50$, $\bar{x} = 0.23$ and $\sigma = 0.1$

$$z = \frac{0.23 - 0.20}{0.1/\sqrt{50}} = 2.12$$

$$P(\bar{X} \geq 0.23) = P(Z \geq 2.12)$$
$$= 1 - 0.9830$$
$$= \boxed{0.0170}$$

Comment:

Hence the prob. of having such observations given the mean $\mu = 0.20$ is small. Therefore, the mean amount of to be 0.20 is not likely to be true. ($\approx 2\%$)



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
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2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Question 4:

$n=36$

If the standard deviation of the mean for the sampling distribution of random samples of size 36 from a large or infinite population is 2, how large must the sample size become if the standard deviation is to be reduced to 1.2?

$n_{new}=?$

$n=36, \quad \sigma_{\bar{x}}=2$

Remember that $\Rightarrow \quad \sigma_{\bar{x}}=\dfrac{\sigma}{\sqrt{n}}$

$$\boxed{\sigma=\sigma_{\bar{x}}\cdot\sqrt{n}}$$

$\sigma=2\cdot\sqrt{36}$

$\boxed{\sigma=12}$

$1.2=\dfrac{12}{\sqrt{n_{new}}}$

$1.2\cdot\sqrt{n}=12$

$\sqrt{n}=10$

$\boxed{n=100}$