Exam Statistics

Bachelor Econometrics and Operations Research
Bachelor Econometrics and Data Science
Faculty of Economics and Business Administration
Wednesday, March 30, 2022

Exam:	Statistics
Code:	E_EOR1_STAT
Coordinator:	M.H.C. Nientker
Date:	March 30, 2022
Time:	12:15
Duration:	2 hours
Calculator:	Not allowed
Graphical calculator:	Not allowed
Number of questions:	4
Type of questions:	Open
Answer in:	English
Credit score:	88 credits counts for a 10
Grades:	Made public within 10 working days
Number of pages:	2, including front page

- Read the entire exam carefully before you start answering the questions.
- Be clear and concise in your statements, but justify every step in your derivations.
- The questions should be handed back at the end of the exam. Do not take it home.

Good luck!

Question 1. Let $X_1, \ldots, X_n$ be an independent and identically distributed sequence of random variables from a population in $\{g_\mu \mid \mu \in \mathbb{R}\}$, where

$$g_\mu(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \qquad x > 0.$$

Note that σ^2 is assumed to be known.

(8 points) a. Show that the moment estimator $\hat{\mu}_{MOM}$ of μ_0 is equal to the sample average $\overline{X}$.

(8 points) b. Calculate the mean squared error of $\hat{\mu}_{MOM}$.

(8 points) c. Find a sufficient and complete statistic for μ_0.

(8 points) d. Find an UMVU estimator of μ_0^2. Hint: start from $\overline{X}^2$.

Question 2. Let $X_1, \ldots, X_n$ be an independent and identically distributed sequence of random variables from a population in $\{g_\theta \mid \theta \in \Theta\}$, let W be an unbiased estimator for $\tau(\theta_0)$ and let T be a sufficient statistic for θ_0.

(8 points) a. Give the formal definition of sufficiency. What is the intuitive interpretation based on summarizing the data?

(8 points) b. State the Rao-Blackwell theorem. Why do we need sufficiency for this result?

Question 3. Let $X_1, \ldots, X_n$ be an independent and identically distributed sequence of random variables from a population in $\{g_\lambda \mid \lambda > 0\}$, where

$$g_\lambda(x) = \lambda e^{-\lambda x}, \qquad x > 0.$$

In this question you are allowed to use that $\mathbb{E}_\lambda X_1 = 1/\lambda$ and $\mathbb{V}\mathrm{ar}_\lambda\, X_1 = 1/\lambda^2$.

(8 points) a. Show that $\hat{\lambda}_{ML} = 1/\overline{X}$ is the maximum likelihood estimator of λ_0.

(8 points) b. Show that $\overline{X}$ is an UMVU estimator for $\tau(\lambda_0) = 1/\lambda_0$ using the Cramér-Rao lower bound.

(8 points) c. Find an asymptotic distribution for $\hat{\lambda}_{ML}$ given the general result on the asymptotic distribution of maximum likelihood estimators. Make sure the asymptotic variance does not depend on λ_0.

Question 4. Let X be Uniform$(0, 1)$ distributed, that is, it has pdf

$$g(x) = 1, \qquad 0 \leq x \leq 1.$$

(8 points) a. Show that the location scale family of g is $\{g_{(\mu,\sigma^2)} \mid \mu \in \mathbb{R}, \sigma^2 > 0\}$, where

$$g_{(\mu,\sigma^2)} = \frac{1}{\sigma}, \qquad \mu \leq x \leq \mu + \sigma.$$

(8 points) b. Use the factorization theorem to find a sufficient statistic T for (μ, σ^2). Note that the domain of $g_{(\mu,\sigma^2)}$ depends on the parameters μ and σ^2.