BME 3005 Biostatistics

Lecture 2: *Discrete Statistics, Probability*

Burcu Tunç Çamlıbel

How to Summarize Data?

First goal: to summarize data collected on a single variable in a way that best describes the larger, unobserved population.

– Mean
– Median
– Standard deviation and variance
– Percentiles
– Normal distribution
– Whole population versus a sample of that population – Standard error of the mean

Chapter 02
Descriptive Statistics

Introduction

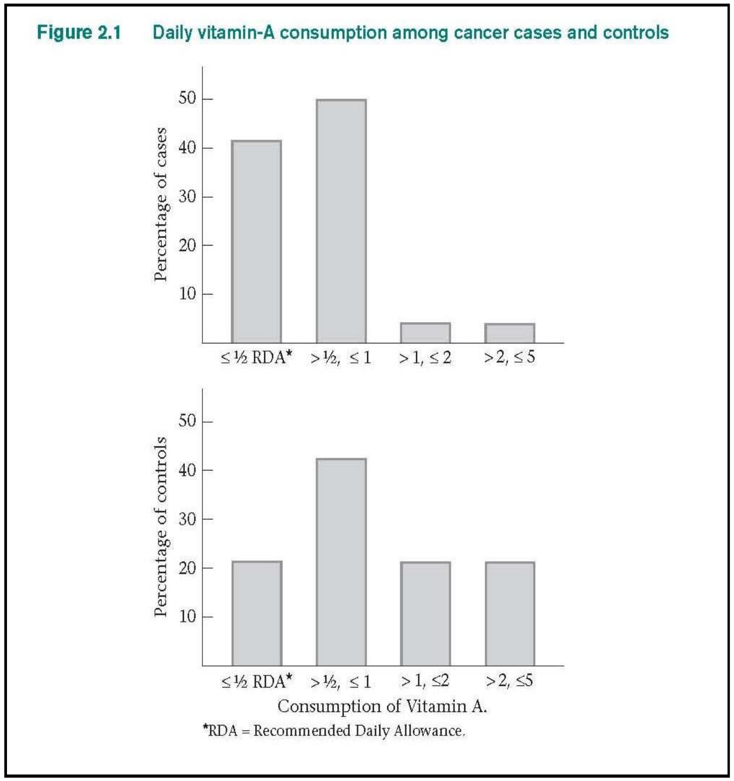
The first step in data analysis is to describe the data in some concise manner.

Descriptive statistics that involve numeric or graphic display are crucial in capturing and conveying the final results of studies in publications.

Features of good numeric or graphic form of data summarization:

- Self-contained
- Understandable without reading the text
- Clearly labeled of attributes with well-defined terms
- Indicate principal trends in data

Example: Bar graphs



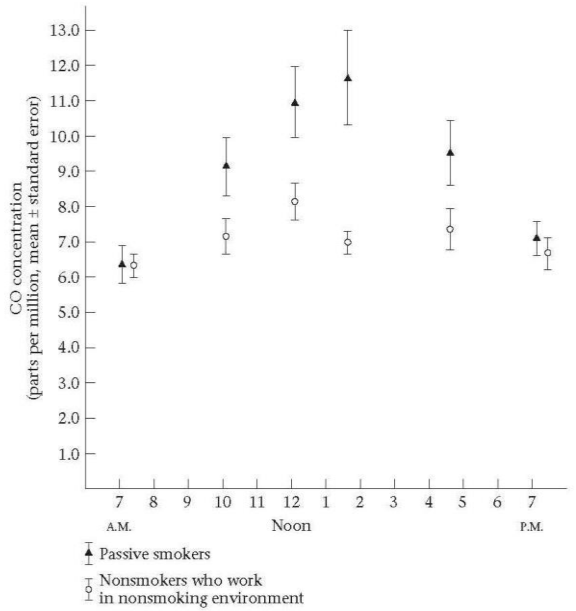
Vitamin A consumption prevents cancer

Total cancer cases: 200
Total matched controls: 200

The bar graphs show that the Vitamin A consumed by controls is more than that consumed by the patients with cancer. In some cases, the levels exceed the recommended daily allowance (RDA).

Example: Scatter plot

Figure 2.2 Mean carbon-monoxide concentration (± standard error) by time of day as measured in the working environment of passive smokers and in nonsmokers who work in a nonsmoking environment



Source: Reproduced with permission of *The New England Journal of Medicine*, 302, 720–723, 1980.

CO concentrations are about the same in the working environments of passive smokers and non
smokers early in the day.

This supports the observation that passive smokers have lower pulmonary function than comparable nonsmokers.

Measures of Location

It is easy to lose track of the overall picture when there are too many sample points.

Data summarization is important before any inferences can be made about the population from which the sample points have been obtained.

Measure of location is a type of measure useful for data summarization that defines the center or middle of the sample.

Defining the Middle: The Arithmetic Mean

Table 2.1 Sample of birthweights (g) of live-born infants born at a private hospital in San Diego, California, during a 1-week period

i	x_i	i	x_i	i	x_i	i	x_i
1	3265	6	3323	11	2581	16	2759
2	3260	7	3649	12	2841	17	3248
3	3245	8	3200	13	3609	18	3314
4	3484	9	3031	14	2838	19	3101
5	4146	10	2069	15	3541	20	2834

Arithmetic mean: the sum of all the observations divided by the number of observations.

Statistically expressed as $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Limitation: Oversensitive to extreme values; in which case, it may not be representative of the location of the majority of sample points.

Arithmetic Mean Explained

Sigma ∑ is a summation sign.

$\sum_{i=1}^{n} x_i$ implies $(x_1 + x_2 + \ldots + x_n)$

- If a and b are integers where $a \leq b$, then $\sum_{i=a}^{b} x_i$ meaning $(x_a + x_{a+1} + \ldots + x_b)$

- If $a = b$, then $\sum_{i=a}^{b} x_i = x_a$

- If c is some constant, then $\sum_{i=1}^{n} cx_i = c\left(\sum_{i=1}^{n} x_i\right)$

Median

Sample median is

- $\left(\frac{n+1}{2}\right)$ th the largest observation if n is odd
- Average of the $\left(\frac{n}{2}\right)$ th and the $\left(\frac{n}{2}+1\right)$ h observation if n is even

Example: Calculating the median

Table 2.2 Sample of admission white-blood counts ($\times$ 1000) for all patients entering a hospital in Allentown, PA, on a given day

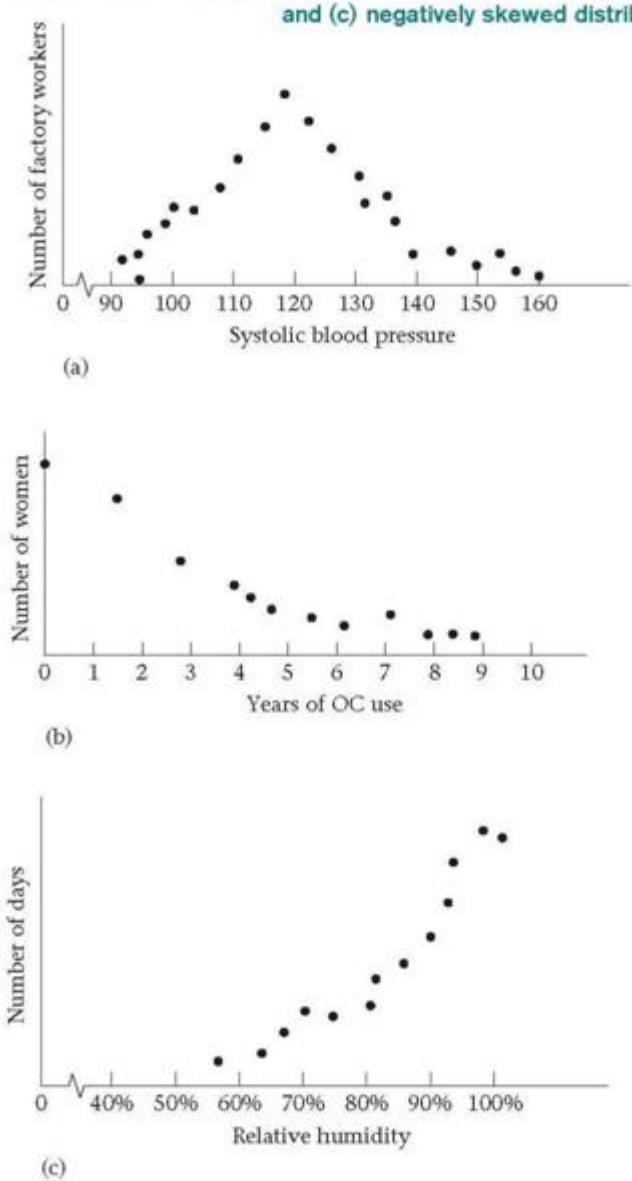
i	x_i	i	x_i
1	7	6	3
2	35	7	10
3	5	8	12
4	9	9	8
5	8		

Order the sample as follows:
3, 5, 6, 7, 7, 8, 8 ,8, 9, 9, 10, 12, 35

Because n is odd, the sample median is the fifth largest point, that is, 8

Comparing Mean and Median

Figure 2.3 Graphic displays of (a) symmetric, (b) positively skewed, and (c) negatively skewed distributions



For symmetric distributions, arithmetic mean is approximately the same as the median

For positively skewed distributions, arithmetic mean tends to be larger than the median

For negatively skewed distributions, the arithmetic mean tends to be smaller than the median

Mode

Mode: the most frequently occurring value among all the observations in a sample.

Data distributions may have one or more modes.

One mode = unimodal

Two modes = bimodal

Three modes = trimodal and so on.

Example

Table 2.3 Sample of time intervals between successive menstrual periods (days) in college-age women

Value	Frequency	Value	Frequency	Value	Frequency
24	5	29	96	34	7
25	10	30	63	35	3
26	28	31	24	36	2
27	64	32	9	37	1
28	185	33	2	38	1

Mode is 28

Geometric Mean

Many types of laboratory data can be expressed as multiples of 2 or a constant multiplied by a power of 2, that is,

$2^k c$ $\qquad$ $k = 0, 1, \ldots$ for some constant c

Example:

Table 2.4 Distribution of minimum inhibitory concentration (MIC) of penicillin G for *N. gonorrhoeae*

Concentration (μg/mL)	Frequency	Concentration (μg/mL)	Frequency
$0.03125 = 2^0(0.03125)$	21	$0.250 = 2^3(0.03125)$	19
$0.0625 = 2^1(0.03125)$	6	$0.50 = 2^4(0.03125)$	17
$0.125 = 2^2(0.03125)$	8	$1.0 = 2^5(0.03125)$	3

Source: Reproduced with permission from *JAMA*, *220*, 205–208, 1972. Copyright 1972, American Medical Association.

$2^k(0.03125)$ for $k = 0, 1, 2, \ldots$

For distributions that are skewed, the log concentrations may be considered.

- Arithmetic mean can be computed in the log scale as

$$\overline{\log x} = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

- The geometric mean is the antilogarithm of $\overline{\log x}$ and can be computed as

$$\overline{\log x} = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

- Log_{10} or $\log_e$ may be used for the calculations

Some Properties of Arithmetic Mean

Original sample : $x_1, \ldots, x_n$

Translated sample : $x_1 + c, \ldots, x_n + c$ (where c is some constant)

Let $y_i = x_i + c$ $\quad\quad\quad\quad$ $i = 1, \ldots, n$ then $\overline{y} = \overline{x} + c$

Table 2.5 **Translated sample for the duration between successive menstrual periods in college-age women**

Value	Frequency	Value	Frequency	Value	Frequency
−4	5	1	96	6	7
−3	10	2	63	7	3
−2	28	3	24	8	2
−1	64	4	9	9	1
0	185	5	2	10	1

Note: $\overline{y} = [(-4)(5) + (-3)(10) + \ldots + (10)(1)] / 500 = 0.54$

$\overline{x} = \overline{y} + 28 = 0.54 + 28 = 28.54$ days the unit or scale changes, then using the **rescaled sample**

$$y_i = cx_i \qquad i = 1, \ldots, n$$

Arithmetic mean is then $\bar{y} = c\bar{x}$

Let $x_1, \ldots, x_n$ be the original sample of data.

Let $y_i = c_1 x_i + c_2$ $\quad i = 1, \ldots, n$ represent a transformed

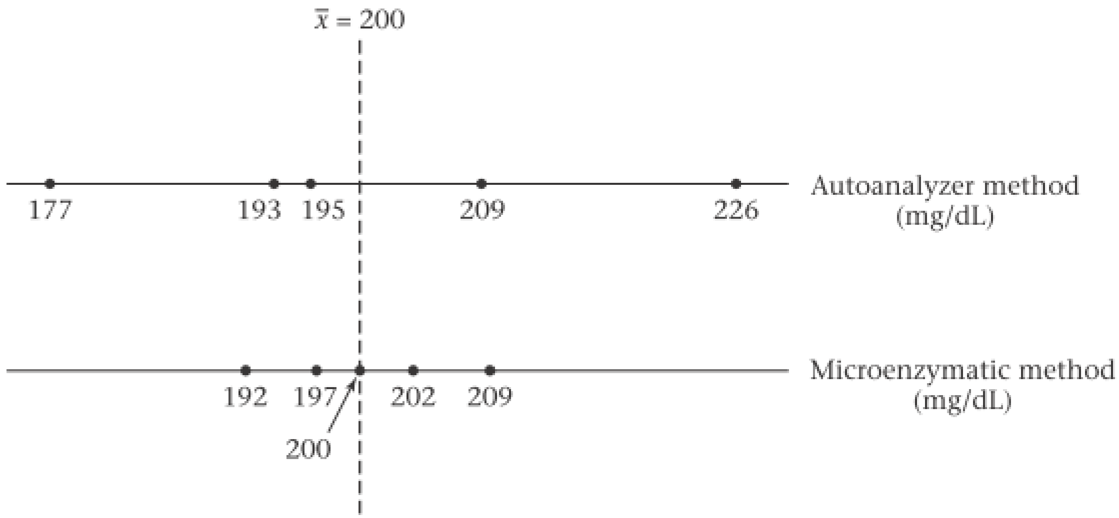
sample obtained by multiplying each original sample point by a factor c_1 and then shifting over by a constant c_2

If $y_i = c_1 x_i + c_2$ $\quad i = 1, \ldots, n$

then $\bar{y} = c_1 \bar{x} + c_2$

Measures of Spread

Figure 2.4 Two samples of cholesterol measurements on a given person using the Autoanalyzer and Microenzymatic measurement methods



The mean obtained by the two methods is the same. However, the **variability** or **spread** of the Autoanalyzer method appears to be greater.

Range or variability

- Range is the difference between the largest and smallest observations in a sample.
- Once the sample is ordered, it is very easy to compute the range.
- Range is very sensitive to extreme observations or outliers.
- Larger the sample size (n), the larger the range and the more difficult the comparison between ranges from data sets of varying sizes.

A better approach to quantifying the spread in data sets is percentiles or quantiles.

Percentiles are less sensitive to outliers and are not greatly affected by the sample size.

Quantiles or percentiles

The *p*th percentile is the value V_p such that *p* percent of the sample points are less than or equal to V_p.

Median is the 50th percentile, which is a special case of a quantile.

The ***p*th percentile** is defined by

- The (*k*+1)th largest sample point if *np*/100 is not an integer (where *k* is the largest integer less than *np*/100)
- The average of the (*np*/100)th and (*np*/100 +1)th largest observations if *np*/100 is an integer.

Frequently used percentiles are

- quartiles (25th, 50th, and 75th percentiles)
- quintiles (20th, 40th, 60th, and 80th percentiles)
- deciles (10th, 20th,…, 90th percentiles)

To compute percentiles, the sample points must be ordered.

If *n* is large, a stem-and-leaf plot or a computer program may be used.

Variance and Standard Deviation

If the center of the sample is defined as the arithmetic mean, then a measure that can summarize the difference (or deviations) between the individual sample points and the arithmetic mean can be expressed as

$$x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x}$$

that is, $$d = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n}$$

The sum of the deviations of the individual observations of a sample about the sample mean is always zero.

Standard deviation *d* is a reasonable measure of spread if the distribution is bell-shaped.

Mean deviation

The difference *d* does not help distinguish the difference in spreads between two methods.

Mean deviation, expressed as $\sum_{i=1}^{n} |x_i - \bar{x}| / n$ may be used.

Alternatively, sample variance or variance, which is the average of the squares of the deviations from the sample mean, may be used

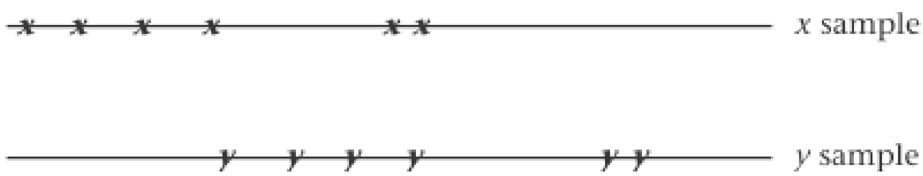
$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

Another commonly used measure of spread is the sample standard deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sqrt{\text{sample variance}}$$

Properties of Variance and Standard Deviation

Figure 2.5 **Comparison of the variances of two samples, where one sample has an origin shifted relative to the other**



Samples $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ where $y_i = x_i + c$ $\qquad i = 1, \ldots, n$

if respective sample variances are s_x^2 and s_y^2 then $s_y^2 = s_x^2$

Samples $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ where $y_i = cx_i$ $\qquad i = 1, \ldots, n$ *and* $c>0$

then $s_y^2 = c^2 s_x^2$ which is $s_v = cs_x$

$$s_y^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1} = \frac{\sum_{i=1}^{n}(cx_i-c\bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n}[c(x_i-\bar{x})]^2}{n-1} = \frac{\sum_{i=1}^{n}c^2(x_i-\bar{x})^2}{n-1}$$

$$= \frac{c^2\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = c^2 s_x^2$$

$$s_y = \sqrt{c^2 s_x^2} = cs_x$$

Coefficient of Variation (CV)

Defined as $100\% \times (s/\bar{x})$

Table 2.6 Reproducibility of cardiovascular risk factors in children, Bogalusa Heart Study, 1978–1979

	n	Mean	*sd*	*CV* (%)
Height (cm)	364	142.6	0.31	0.2
Weight (kg)	365	39.5	0.77	1.9
Triceps skin fold (mm)	362	15.2	0.51	3.4
Systolic blood pressure (mm Hg)	337	104.0	4.97	4.8
Diastolic blood pressure (mm Hg)	337	64.0	4.57	7.1
Total cholesterol (mg/dL)	395	160.4	3.44	2.1
HDL cholesterol (mg/dL)	349	56.9	5.89	10.4

Remains the same regardless of units used

Useful in comparing variability of different samples with different arithmetic means

Useful for comparing the reproducibility of different variables

Grouped Data

When sample size is too large to display all the raw data, data are frequently collected in grouped form.

Table 2.7 Sample of birthweights (oz) from 100 consecutive deliveries at a Boston hospital

58	118	92	108	132	32	140	138	96	161
120	86	115	118	95	83	112	128	127	124
123	134	94	67	124	155	105	100	112	141
104	132	98	146	132	93	85	94	116	113
121	68	107	122	126	88	89	108	115	85
111	121	124	104	125	102	122	137	110	101
91	122	138	99	115	104	98	89	119	109
104	115	138	105	144	87	88	103	108	109
128	106	125	108	98	133	104	122	124	110
133	115	127	135	89	121	112	135	115	64

The simplest way to display the data is to generate a frequency distribution using a statistical package.

A frequency distribution is an ordered display of each value in a data set together with its **frequency**, that is, the number of times that value occurs in the data set.

Table 2.8 Frequency distribution of the birthweight data in Table 2.7 using the MINITAB Tally program

Birthweight	Count	CumCnt	Percent	CumPct
32	1	1	1.00	1.00
58	1	2	1.00	2.00
64	1	3	1.00	3.00
67	1	4	1.00	4.00
68	1	5	1.00	5.00
83	1	6	1.00	6.00
85	2	8	2.00	8.00
86	1	9	1.00	9.00
87	1	10	1.00	10.00
88	2	12	2.00	12.00
89	3	15	3.00	15.00
91	1	16	1.00	16.00
92	1	17	1.00	17.00
93	1	18	1.00	18.00
94	2	20	2.00	20.00
95	1	21	1.00	21.00
96	1	22	1.00	22.00
98	3	25	3.00	25.00
99	1	26	1.00	26.00
100	1	27	1.00	27.00
101	1	28	1.00	28.00
102	1	29	1.00	29.00
103	1	30	1.00	30.00
104	5	35	5.00	35.00
105	2	37	2.00	37.00
106	1	38	1.00	38.00
107	1	39	1.00	39.00
108	4	43	4.00	43.00
109	2	45	2.00	45.00
110	2	47	2.00	47.00
111	1	48	1.00	48.00
112	3	51	3.00	51.00
113	1	52	1.00	52.00
115	6	58	6.00	58.00
116	1	59	1.00	59.00
118	2	61	2.00	61.00
119	1	62	1.00	62.00
120	1	63	1.00	63.00
121	3	66	3.00	66.00
122	4	70	4.00	70.00
123	1	71	1.00	71.00
124	4	75	4.00	75.00
125	2	77	2.00	77.00
126	1	78	1.00	78.00
127	2	80	2.00	80.00
128	2	82	2.00	82.00
132	3	85	3.00	85.00
133	2	87	2.00	87.00
134	1	88	1.00	88.00
135	2	90	2.00	90.00
137	1	91	1.00	91.00
138	3	94	3.00	94.00
140	1	95	1.00	95.00
141	1	96	1.00	96.00
144	1	97	1.00	97.00
146	1	98	1.00	98.00
155	1	99	1.00	99.00
161	1	100	1.00	100.00
N =	100			

If the number of unique sample values is large, then a frequency distribution may still be too detailed.

ble 2.9 General layout of grouped data

Group interval	Frequency
$y_1 \le x < y_2$	f_1
$y_2 \le x < y_3$	f_2
.	.
.	.
.	.
$y_i \le x < y_{i+1}$	f_i
.	.
.	.
.	.
$y_k \le x < y_{k+1}$	f_k

Table 2.10 Grouped frequency distribution of birthweight (oz) from 100 consecutive deliveries

Group interval	Frequency
$29.5 \le x < 69.5$	5
$69.5 \le x < 89.5$	10
$89.5 \le x < 99.5$	11
$99.5 \le x < 109.5$	19
$109.5 \le x < 119.5$	17
$119.5 \le x < 129.5$	20
$129.5 \le x < 139.5$	12
$139.5 \le x < 169.5$	6
	100

Note: If birthweight can only be measured to an accuracy of 0.1 oz, then a possible alternate representation of the group intervals in Table 2.10 could be 29.5–69.4, 69.5–89.4, to 139.5–169.5.

If the data is too large, then the data is categorized into broader groups.

Graphic Methods

Graphic methods of displaying data give a quick overall impression of data. The following are some graphic methods.

Bar graphs:

- used to display grouped data;
- difficult to construct;
- Identity of the sample points within the respective groups is lost

Stem-and-Leaf plots:

- easy to compute the median and other quantiles
- Each data point is converted into stem and leaf, e.g., 438 (stem: 43; leaf: 8)

Box plots:

- Uses the relationships among the median, upper quantile, and lower quantile to describe the skewness or symmetry of a distribution

Stem-and-leaf plots

Figure 2.6 Stem-and-leaf plot for the birthweight data (oz) in Table 2.7

Stem-and-leaf of birthwgt N = 100
Leaf Unit = 1.0

1	3	2
1	4	
2	5	8
5	6	478
5	7	
15	8	3556788999
26	9	12344568889
45	10	0123444445567888899
(17)	11	00122233555556889
38	12	01112222344445567788
18	13	222334557888
6	14	0146
2	15	5
1	16	1

Stem |Leaf

The collection of leaves indicates the shape of the data distribution

Figure 2.7 Stem-and-leaf plot for the birthweight data (g) in Table 2.1

20	69
21	
22	
23	
24	
25	81
26	
27	59
28	41 38 34
29	
30	31
31	01
32	65 60 45 00 48
33	23 14
34	84
35	41
36	49 09
37	
38	
39	
40	
41	46

Stem-and-leaf vs. Box Plot

Figure 2.8 Stem-and-leaf and box plots for the birthweight data (oz) in Table 2.7 as generated by the SAS UNIVARIATE procedure

```
Stem | Leaf                   #      Boxplot
  16 | 1                      1         |
  15 | 5                      1         |
  15 |                                  |
  14 | 6                      1         |
  14 | 014                    3         |
  13 | 557888                 6         |
  13 | 222334                 6         |
  12 | 5567788                7         |
  12 | 0111222234444         13      +-----+
  11 | 5555556889            10      |     |
  11 | 0012223                7      *--+--*
  10 | 5567888899            10      |     |
  10 | 012344444              9      |     |
   9 | 568889                 6      +-----+
   9 | 12344                  5         |
   8 | 556788999              9         |
   8 | 3                      1         |
   7 |                                  |
   7 |                                  |
   6 | 78                     2         |
   6 | 4                      1         |
   5 | 8                      1         0
   5 |
   4 |
   4 |
   3 |
   3 | 2                      1         0
       ----+----+----+----+
Multiply Stem.Leaf by 10**+1
```

Box plot

- If the distribution is symmetric, then upper and lower quartiles should be approximately equally spaced from the median

- If the upper quartile is farther from the median than the lower quartile, then the distribution is positively skewed

- If the lower quartile is farther from the median than the upper quartile, then the distribution is negatively skewed

An outlying value is a value x such that either

x > upper quartile + 1.5 × (upper quartile – lower quartile)

x < lower quartile – 1.5 × (upper quartile – lower quartile)

An extreme outlying value is a value x such that either

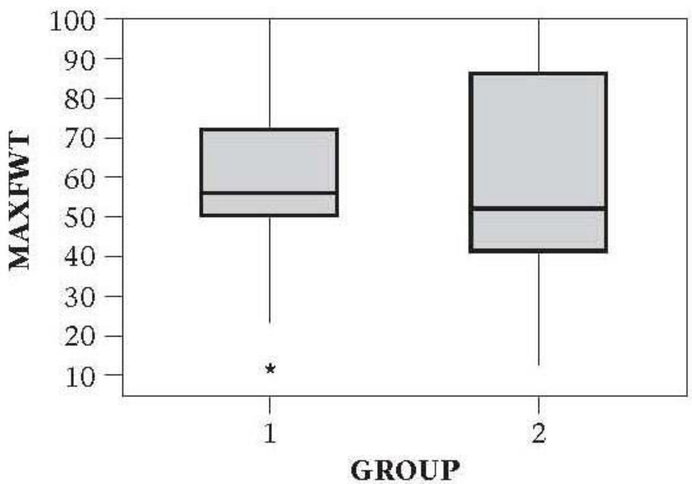
x > upper quartile + 3.0 × (upper quartile – lower quartile)

x < lower quartile – 3.0 × (upper quartile – lower quartile)

- A vertical bar connects the upper quartile to the largest nonoutlying value in the sample
- A vertical bar connects the lower quartile to the smallest nonoutlying value in the sample

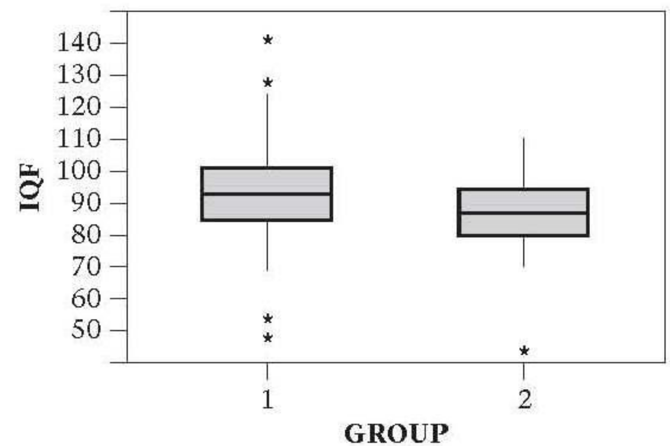
Case study 1: Effects of lead exposure on neurological and psychological function in children

Figure 2.9 Number of finger–wrist taps in the dominant hand for exposed and control groups, El Paso Lead Study



1 = control
2 = exposed

Figure 2.10 Wechsler full-scale IQ scores for exposed and control groups, El Paso Lead Study

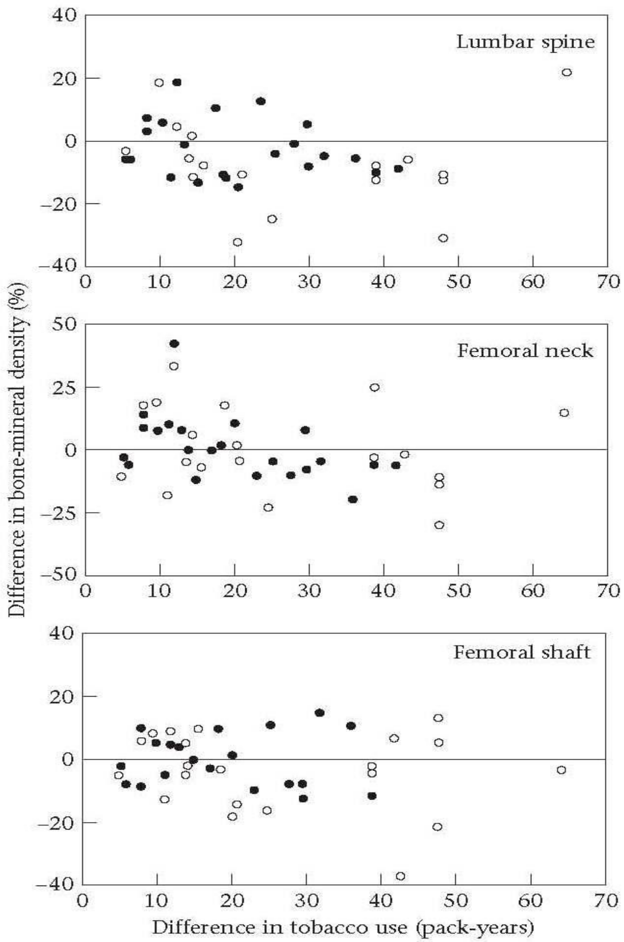


1 = control
2 = exposed

The finger-wrist tapping scores (MAXFWT) and full-scale IQ scores (IQF) seem slightly lower in the exposed group than in the control group.

Scatter plot

re 2.11 Within-pair differences in bone density at the lumbar spine, femoral neck, and femoral shaft as a function of within-pair differences in pack-years of tobacco use in 41 pairs of female twins. Monozygotic (identical) twins are represented by solid circles and dizygotic (fraternal) twins by open circles. The difference in bone density between members of a pair is expressed as the percentage of the mean bone density for the pair.



Matched-pair study

Exposed and control are matched on other characteristics related to the outcome BMD.

In this case, matching is based on having similar genes.

An inverse relationship is seen between difference in BMD and difference in tobacco use.

Obtaining descriptive statistics using a computer

- Numerous statistical packages may be used.

- Excel may be used to compute average (for the arithmetic mean), median (for the median), Stdev (for the standard deviation), Var (for the variance), GeoMean (for the geometric mean) and Percentile (for obtaining arbitrary percentiles from a sample).

Summary

Numeric or graphic methods for displaying data help in

- quickly summarizing a data set
- And/or presenting results to others

A data set can be described numerically in terms of measure of location and a measure of spread

Measure of location	Measure of spread
Arithmetic mean	Standard deviation
Median	Quantiles
Mode	Range
Geometric mean	

Graphic methods include bar graphs and more exploratory methods such as stem-and-leaf plots and box plots.

The End