Example 1 (Question 9.62 on page 359)

In a random sample of 150 business graduates 50 agreed or strongly agreed that businesses should focus their efforts on innovative e-commerce strategies. Test at the 5% level the null hypothesis that at most 25% of all business graduates would be in agreement with this assertion.

Solution

We will let P denote the population proportion of business graduates who are in agreement with the idea that businesses should focus their efforts on innovative e-commerce strategies. We should test the following hypothesis;

$$H_0: P \leq 0.25 \quad \text{(at most 25\% of all business ...)}$$
$$H_1: P > 0.25$$

The decision rule is to reject H_0 in favor of H_1 if

$$\frac{\hat{p} - P_0}{\sqrt{P_0(1-P_0)/n}} > Z_{1-\alpha}$$

For this random sample;

$n = 150 \qquad \hat{p} = \frac{50}{150} = 0.33 \qquad \alpha = 0.05 \qquad 1-\alpha = 0.95$

Table value:

$Z_{1-\alpha} = Z_{0.95} = 1.645$

Test statistic:

$$\frac{0.33 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{150}}} = 2.26$$

Decision:

Since $2.26 > 1.645$. Thus, we reject the null hypothesis at the 5% level and conclude that more than 25% of all business graduates are in agreement with the idea that businesses should focus their efforts on innovative e-commerce strategies.

Example 2 (Question 9.57 on page 359)

An insurance company employs agents on a commission basis. It claims that in their first-year agents will earn a mean commission of at least $40,000 and that the population standard deviation is no more than $6,000. A random sample of nine agents found for commission in the first year,

$$\sum_{i=1}^{9} x_i = 333 \quad \text{and} \quad \sum_{i=1}^{9}(x_i - \bar{x})^2 = 312$$

where x_i is measured in thousands of dollars and the population distribution can be assumed to be normal. Test, at the 5% level, the null hypothesis that the population mean is at least $40,000.

Solution

In this example we want to test a hypothesis about a population mean. Here population variance is unknown. Actually population standard deviation is known to be no more than $6,000 but exact value is unknown.

We will test the following hypothesis;

$H_0: \mu \geq 40{,}000$ (population mean is atleast 40,000)

$H_1: \mu < 40{,}000$

The decision rule is to reject H_0 in favor of H_1 if

$$\frac{\bar{X} - \mu_0}{S/\sqrt{n}} < -t_{n-1,\alpha}$$

From the random sample;

$n = 9 \quad \bar{X} = \frac{\sum_{i=1}^{9} x_i}{n} = \frac{333000}{9} = 37000 \quad S^2 = \frac{\sum_{i=1}^{9}(x_i - \bar{x})^2}{n-1} = \frac{312000000}{8}$

$S^2 = 39000000 \quad S = \sqrt{39000000} = 6245. \quad \alpha = 0.05$

Table value:

$t_{8,0.05} = 1.86 \quad -t_{8,0.05} = -1.86$

Test Statistic: $\frac{37000 - 40000}{6245/\sqrt{9}} = -1.44$

Decision: Since $-1.44 > -1.86$. Thus, we accept H_0 at the 5% level and conclude that population mean is at least 40 000.

Example 3 (Question 9.71 on page 360)

An insurance company employs agents on a commission basis. It claims that, in their first year, agents will earn a mean commission of at least \$40,000 and that the population standard deviation is no more than \$6,000. A random sample of nine agents found for commission in the first year,

$$\sum_{i=1}^{9} x_i = 333{,}000 \quad \text{and} \quad \sum_{i=1}^{9} (x_i - \bar{x})^2 = 312{,}000{,}000$$

The population distribution can be assumed to be normal. Test, at the 10% level, the null hypothesis that the population standard deviation is at most \$6,000.

Solution

In this example we want to test a hypothesis about a population standard deviation (variance). We will test the following hypothesis;

$$H_0: \sigma^2 \leq 36{,}000{,}000 \quad \text{(std. deviation is at most 6000)}$$
$$H_1: \sigma^2 > 36{,}000{,}000$$

The decision rule is to reject H_0 in favor of H_1 if

$$\frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{n-1,\alpha}$$

From this random sample;

$$n = 9 \qquad s^2 = \frac{\sum_{i=1}^{9}(x_i - \bar{x})^2}{n-1} = \frac{312{,}000{,}000}{8} = 39{,}000{,}000$$

$\alpha = 0.10$

Table value:

$$\chi^2_{n-1,\alpha} = \chi^2_{8,0.10} = 13.362$$

Test statistic: $\frac{(n-1)s^2}{\sigma_0^2} = \frac{8 \times 39000000}{36000000} = 8.67$

Decision: Since $8.67 < 13.362$. Thus, we accept H_0 at the 10% level and conclude that the population standard deviation is at most \$6,000.

Example 4 (Question 9.58 on page 359)

Supporters claim that a new windmill can generate an average of at least 800 kilowatts of power per day. Daily power generation for the windmill is assumed to be normally distributed with a standard deviation of 120 kilowatts. A random sample of 100 days is taken to test this claim against the alternative hypothesis that the true mean is less than 800 kilowatts. The claim will not be rejected if the sample mean is 776 kilowatts or more and rejected otherwise. What is the probability α of a Type I error using the decision rule if the population mean is, in fact, 800 kilowatts per day?

Solution

In this example we want to find the α value for a hypothesis about a population mean. Here population variance is known (standard deviation = 120). We will find the α value (or p value) for the following hypothesis;

$$H_0: \mu \geq 800 \quad \text{(at least 800 kilowatts)}$$
$$H_1: \mu < 800$$

The decision rule is to reject H_0 in favor of H_1 if

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} < -z_{1-\alpha} \rightarrow \left(\text{This rule is given in the question}\right)$$

From this random sample;

$n = 100 \quad \bar{X} = 776 \quad \sigma = 120$

We should find α value such that;

$$\frac{776 - 800}{120/\sqrt{100}} < -z_{1-\alpha}$$

$$-2 < -z_{1-\alpha}$$

from the Z Table if $z_{1-\alpha} = 2$ then $1-\alpha = 0.9772$

So, $\alpha = 1 - 0.9772 \Rightarrow \underline{\underline{\alpha = 0.0228}}$ (Actually this is the p value of this test)