Week 11: Comparing Two Groups

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Overview

- We will be introducing new forms of statistical testing that allows us to compare two groups
- Topics for this week
  - Matched Pairs
  - Comparing Two Proportions
  - Comparing Two Means
  - Sign Test for Matched Pairs
- This content corresponds with Module 10

2 Sets of Data: Matched vs Unmatched

Working with 2 sets of measurements

- Up until now, we have only been considering a single set of measurements that make up our data.
- However, there are many reasons why we may want to have two sets of measurements and talk about how they differ
- Some examples are:
  - comparing average midterm test scores between two sections of the same course
  - comparing proportion of students who got an A between two sections of the same course
  - comparing the change in test scores between the midterm and the final for students in one section of a course.
- Each of these are examples of different two sample tests we will be discussing this week

Are Samples Independent?

Grp. 1 | Grp 2
3 | 7
5 | 5
16 | 11

- The main difference between the three examples on the previous slide are whether or not the measurements are independent between the groups

unpaired/ unmatched

- **Independent samples**: each set of measurements that we are comparing have been made on different subjects/units
  - e.g. test scores in different sections of a class - the students are different in each section so the measurements/samples are independent.

paired/ matched

- **Dependent samples**: the two sets of measurements have been made on the same subjects/units but at different times, or natural **pairs** of units have been sampled and measurements are taken on both
  - e.g. midterm and final exam scores for the same group of students (students in the same section) - measurements made at different times but on same people
  - e.g. compare older spouse's and younger spouse's age - paired because we sample the couple and measure the difference in age.

Check your understanding

Which of the following are examples of paired samples?

a) number of pushups for students in two different gym classes unpaired

b) number of pushups at the beginning versus the end of term for students in one gym class paired

c) number of pushups for boys versus girls in one gym class, assuming equal number of each. unpaired

Matched Pairs

- Matched pairs are situations where the measurements themselves are not independent because there is something in how they have been collected that links them
  - obviously, two measurements on the same unit will be dependent
  - but any time we have some sort of link that ties together two measurements (before and after, spouse 1 and spouse 2, etc.), and we are specifically interested in comparing the groups, we are dealing with units that have been matched.
- One feature to look out for is that the number of measurements in each group should be the same for us to have matched pairs.
  - this is because we want to actually calculate the difference between each measurement in a pair

Testing for Two Groups

- Whether we have matched pairs or unmatched groups, we still want to compare the results between the two groups.
- In the one sample hypothesis test, we would find a sample statistic and compare with the hypothesized value.
- We want to do something similar in the two sample case: compare a sample statistic to a hypothesized value
  - but what sample statistic should we be using? Use a difference to compare with a hypothesized value
  - in matched pairs, we first find the difference for each pair, then find the average
  - in unmatched, we first find the sample statistic in each group, then take the difference and compare with a hypothesized value.

Paired Data: Paired T-test

Why does the order matter?

- When we have paired samples (matched data), why does it matter that we must first take the difference of each pair and then average them?
- It has to do with how the CLT works.
- Recall, we only found that the sample mean will be Normal, as long as the observations were independent. Independence of observation is a condition of CLT
- We will need to use a CLT result again for our two sample hypothesis test
  - the only way to have independent observations (so that the CLT will work) is to use the difference of each pair instead of the original data
  - then each individual difference will be independent and the CLT will work

Example: Comparing Grades

- Let's think about the situation where we want to know if students in this class do better on their midterm or their final exam.
  - we will be recording each student's midterm grade and final exam grade.
  - obviously we have matched pairs of data because each student has been measured twice.
  - this means, for one student, their midterm grade and final grade are <u>dependent</u> measurements!
  - If we were to take the average midterm grade and the average final grade and try to compare them, we would have that these average are not independent
  - this is because they represent the class average but the class didn't change between midterm and final exam - so the averages are still dependent.

Example: Comparing Grades

- We can break the dependence by instead realizing that the measurements are dependent but the students are independent.
- so instead we can find the difference between the midterm and final grade on each student.
  - our data now are differences, instead of two dependent grades.
- But because now we just have one measurement for each student, and students are independent, now our measurements are independent.
  - so the CLT will now work and we have removed dependence in the measurements.
- Now that we have only one measurement per unit (a difference), we are back to a one sample hypothesis test - and we already know how to do those!

Hypothesis Test for Matched Pairs

- As we have seen before, the procedure for a hypothesis test is standard.
- Once you have identified that you have matched pairs, just follow these steps to test your hypothesis:
  1. Write out your null and alternative hypothesis
  2. Calculate your test statistic
  3. Compute the p-value of your test statistic
  4. Conclude by comparing to a significance level.

Matched Pairs Test

x_{d_1} , x_{d_2} , ...

- Because we had to break the dependence by first computing the difference in measurements on each unit, our hypothesis must reflect that our data are now differences

- x_{d_i}= (first measurement) - (second measurement) for each unit
- We can now find our sample values:
  - The sample average difference $\bar{x}_d$

    # of pairs
  - The sample standard deviation of the differences $s_d = \sqrt{\frac{1}{n-1}\Sigma\left(x_{d_i} - \bar{x}_d\right)^2}$
- Since we are essentially now running a one sample test for a mean, our hypothesis must state a claim about the true average difference in the population
  - $H_0\colon \mu_d = 0$ vs. $H_A\colon \mu_d \neq 0$

Matched Pairs Test

- Even though we started with paired data, we reduced it down to one measurement on each unit (a difference)
- Since we are testing a mean, and only have one measurement on each unit now, we are running a one sample test for means
  - our data is a sample average difference, with a sample standard deviation
  - our test statistic is built in the same way as the one sample test: $T = \frac{\bar{x}_d - \mu_d}{s_d/\sqrt{n}}$
    - Notice that n is now the # of pairs
  - because we have to estimate s_d with our data, we must use a t-distribution to find the p-value
    - The degrees of freedom is (# of pairs) – 1 = n-1
- We call this procedure a **paired t-test.**

Example: Push-ups

We have paired data on students in a gym class on the number of pushups each boy-girls pair could do. Test whether boys and girls can do the same number of pushups (i.e. the average difference in number of pushups between boys and girls is 0).

Grade	1	2	3	4	5	6	7	8	9	10	11	12
Boys	17	27	31	17	25	32	28	23	25	16	11	34
Girls	24	7	14	16	2	15	19	25	10	27	31	8
Diff	-7	20	17	1	23	17	9	-2	15	-11	-20	26

$H_0: \mu_d = 0$ $H_A: \mu_d \neq 0$

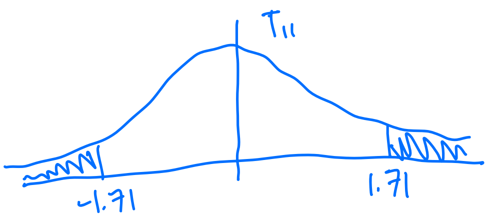
$\bar{x}_d = (-7 + 20 + \ldots + 26)/12 = 7.333$

$S_d = 14.83$

Test Statistic $= \dfrac{\bar{x}_d - \mu_d}{S_d/\sqrt{n}} = \dfrac{7.333 - 0}{14.83/\sqrt{12}} = 1.71$

We can either find the p-value using R, or use the critical value method

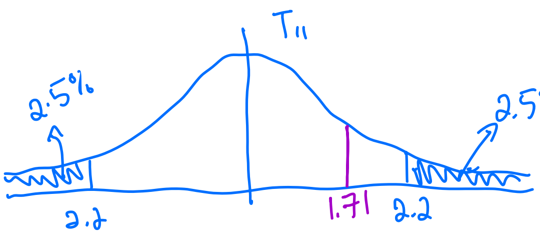
① Find p-value



pt(-1.71, df= 11) * 2

$\Rightarrow$ 0.115

② Use critical value



1.71 is in the non-rejection region

Conclusion: We do not reject the null hypothesis.

Example: Push-ups

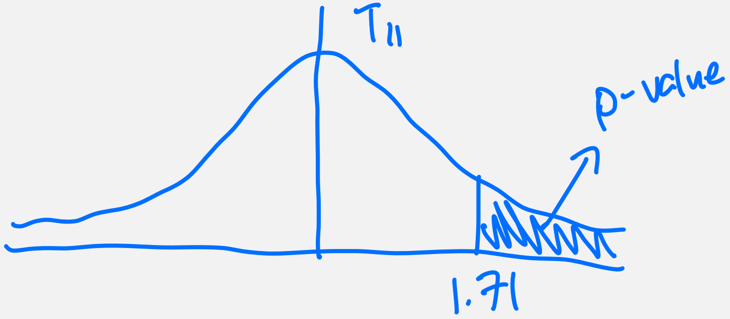
If we wanted to test that boys do more pushups than girls, our p-value would be

a) Larger than we found

b) Smaller than we found

$H_0: \mu_d = 0$ vs. $H_A: \mu_d > 0$

Test Stat = 1.71



p-value would be smaller

Confidence Intervals for Matched Pairs

- If we are testing a two-sided hypothesis, like we were in the example, we can of course use confidence intervals to come to the same conclusion as a test.
- We don't even need to do anything different, because when we are working with matched pairs, we are basically still doing a one-sample test.
- So the confidence interval will look the same as a CI for means, just the data we are working with are differences
- Therefore a $(1-\alpha)\%$ confidence interval for the true average difference between the pairs would be

$$\left(\bar{x}_d - t_{\frac{\alpha}{2},n-1} \times \frac{s_d}{\sqrt{n}}, \bar{x}_d + t_{\frac{\alpha}{2},n-1} \times \frac{s_d}{\sqrt{n}}\right)$$

Example: Push-ups

We have paired data on students in a gym class on the number of pushups each boy-girls pair could do. Test whether boys and girls can do the same number of pushups by building a 95% confidence interval.

Grade	1	2	3	4	5	6	7	8	9	10	11	12
Boys	17	27	31	17	25	32	28	23	25	16	11	34
Girls	24	7	14	16	2	15	19	25	10	27	31	8

From before: $\bar{x}_d = 7.333$

$s_d = 14.83$

$n = 12$

$t_{\alpha/2, n-1} = 2.2$

Putting it all together,

$$\left(7.333 - 2.2 \times \frac{14.83}{\sqrt{12}} \;,\; 7.333 + 2.2 \times \frac{14.83}{\sqrt{12}}\right)$$

$$\Rightarrow (-2.085 \;,\; 16.751)$$

Conditions for Paired T-test

- We do need certain conditions to be met in order for us to use a paired t-test.
  - we need pairs of observations to be independent of each other
  - need the differences to follow a Normal distribution in the population, or that the sample size is large enough so that the mean of the differences is approximately normally distributed.
- Whenever we doubt that the distribution of the differences is approximately normally distributed (i.e. extreme outliers or a bad skew), it is best if we avoid using the paired t-test.
- So what should we use instead?

Paired Data: Sign Test

Sign Test for Paired Differences

- The sign test is very different from the tests we have talked about so far.
- It is a nonparametric test, which means it doesn't require (approximate) Normality of the test statistic, so is useful when you don't feel that the condition is met.
  - All the other tests in this course are parametric as it requires a sampling distribution to hold
- If we are testing the null hypothesis that the difference is 0 (H_0: $\mu_d = 0$), we would expect that half of the differences would > 0 and half would be < 0 (if the null is true)
  - then when we average them, we should get an average difference of 0.
  - suppose we decide to count how many differences are positive
  - since the differences are independent, and we have N of them, this actually corresponds to a Bin(N, p)
  - here p is the probability of a positive difference, which if the average difference is 0, is the same as saying half of them should be positive (p = 0.5)

Sign Test

- So how can we use that the number of positive differences is Binomial to perform a test?
  - Well, first realize that H_0: $\mu_d = 0$ is the same as H_0: $p = 0.5$ by the reasoning on the previous slide
  - Also we are just counting *how many* pairs are positive differences, not how big the differences are.
  - therefore I can actually replace my differences by “+” if it was positive, and “-“ if it was negative, and count the number of “+”
  - unlike other tests, we don’t really need to find a test statistic, just a p-value
  - and how we find the p-value that uses the fact that we have Binomial data.

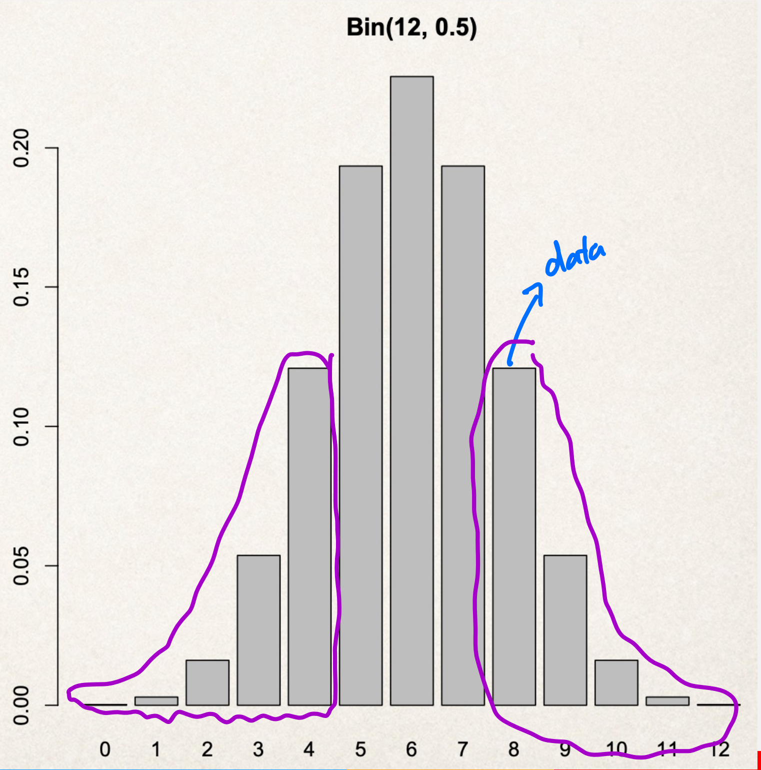
P-value meaning

- Let’s remind ourselves of what the p-value actually tells us in our usual hypothesis tests:
  - it represents the probability that we could have observed data that is as extreme or more (usually in both directions) than what we actually got.
- How do we find this probability for our sign test?
  - Remember we are counting the number of “+” and “-“ differences and these are Binomial(N, 0.5) under the assumption the null is true.
  - so more extreme corresponds to seeing more “+”s than what we got, and also fewer “-“s than we saw.
  - and this is something we can find with the Binomial distribution!

Example: Push-ups

We have paired data on students in a gym class on the number of pushups each boy-girl pair could do. Test whether boys and girls do the same number of pushups on average by using the sign test.

Grade	1	2	3	4	5	6	7	8	9	10	11	12
Boys	17	27	31	17	25	32	28	23	25	16	11	34
Girls	24	7	14	16	2	15	19	25	10	27	31	8
Sign	−	+	+	+	+	+	+	−	+	−	−	+



Let X be the number of positive differences

$X \sim Bin(12, p)$

$H_0: p = 0.5$ $\quad$ $H_A: p \neq 0.5$

$\hat{p} = 8/12$

Find the p-value:

Recall that the p-value is the probability of getting data that is as extreme or more extreme than 8 positives in 12 differences.

Therefore, p-value $= P(X \geq 8) + P(X \leq 4)$

$= P(X=0) + P(X=1) + \ldots + P(X=4) + P(X=8) + \ldots + P(X=12)$

$= 0.39$

We do not reject H_0.

Unpaired Data: T-test for Means

Unmatched Tests

- Previously we were talking about tests to be used for when we have matched pairs of data
  - this is when the measurements in the pairs are not independent.
- Now we will talk about tests for when we no longer have matched data.
  - In this case, we have data collected from two groups
  - the two groups are independent because we don’t have matching
  - the observations within each group (assuming we sampled correctly) are also independent.
- This means that we don’t need to worry about treating the two groups as if they are related - we can treat them as two groups!

Comparing Two Means - Hypotheses

- When we want to run a test to determine if the means are the same between two groups, we treat each group as its own sample
- This means we can compute sample statistics for the data in each group.
  - e.g. if we want to compare midterm scores between two sections of this class, we can calculate the sample mean and standard deviation for the test in each section
  - Class 1: $n_1, \bar{x}_1, s_1^2$
  - Class 2: $n_2, \bar{x}_2, s_2^2$
  - The hypotheses we would be testing are $H_0: \underbrace{\mu_1 - \mu_2 = 0}_{}$ vs. $H_A: \mu_1 - \mu_2 \neq 0$

    $\mu_1 = \mu_2$ $\quad$ $\mu_1 \neq \mu_2$
- It turns out that CLT still works even when we are looking at a difference in means so we can use a test that is very similar to the one sample hypothesis test.

Comparing Two Means – Test Statistic

- Usually the test statistic for a test of means will look like $T = \frac{\bar{X} - \mu}{s/\sqrt{n}}$
- More generally, test statistics are of the form

$$\frac{Estimate\ - Null\ Value}{SE(Estimate)}$$

- All parametric hypothesis tests follow this formula for the test statistic
    - The sign test we just did is the only non-parametric test we will cover
- Now we are looking at a difference in sample means $\bar{x}_1 - \bar{x}_2$ and comparing it to a hypothesized value for this difference in the population, e.g. $\mu_1 - \mu_2 = 0$

Comparing Two Means – Test Statistic

- When comparing two groups, the null value for the difference between the groups is almost always 0. We test the null hypothesis that there is no difference between the groups.
- The estimate for the difference is $\bar{x}_1 - \bar{x}_2$
- This gives a test statistic of: $\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE(\bar{x}_1 - \bar{x}_2)} = \frac{(\bar{x}_1 - \bar{x}_2)}{SE(\bar{x}_1 - \bar{x}_2)}$
- The biggest change between one-sample and two-sample tests of means is the standard error in the denominator. We have two options for this:
  - **Unpooled** version for when we think the population variances of the two groups are different
    - $SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
  - **Pooled** version for when we think the population variances are equal
    - $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$ and $SE(\bar{x}_1 - \bar{x}_2) = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Comparing Two Means – T distribution

- Regardless of which standard error we use, we will still be using a T distribution to help us calculate our p-value.
- However, depending on whether or not we use a pooled standard error for our test statistic, we will need to use different degrees of freedom.
  - Unpooled: $df = \dfrac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$ which is an approximation for the degrees of freedom, and will not necessarily be an integer. In this class, you can round.
  - Pooled: $df = n_1 + n_2 - 2$ which is a much simpler expression
- P-values can then be calculated in the usual way

Pooled versus Unpooled

- We need to be careful with when we decide to use a pooled standard error versus an unpooled standard error.
- If it is true that the population standard deviations are the same, then the test using the pooled standard error is slightly better because it is slightly more powerful
- However, if we were wrong to assume the population variances are the same, then we are in big trouble...
    - in this case, the pooled results are not valid and will give you misleading p-values and conclusions.
- Further, it turns out that we use the pooled version mainly to have more degrees of freedom to work with (because we are merging the samples into one big population)
    - But as the samples themselves get bigger, the difference in df between pooled and unpooled becomes so small that it isn't really helpful anymore.
- So unless you have a really good reason not to, just use the unpooled.

Example: Comparing midterm grades

Suppose we want to test whether the average grades between two section's midterms are different. From section 1, we take a sample of 50 exam papers and find a sample mean of 85 and standard deviation of 5. From section 2, we take a sample of 40 papers and get a sample mean of 80 and standard deviation of 4. Use unpooled approach.

$H_0: \mu_1 = \mu_2$ OR $\mu_1 - \mu_2 = 0$ OR No difference between means in each section

$H_A: \mu_1 \neq \mu_2$ OR $\mu_1 - \mu_2 \neq 0$ OR Means are not equal

Given: $n_1 = 50$ $\bar{x}_1 = 85$ $s_1 = 5$

$n_2 = 40$ $\bar{x}_2 = 80$ $s_2 = 4$

Test Statistic $= \dfrac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)} = \dfrac{85-80}{\sqrt{\frac{5^2}{50} + \frac{4^2}{40}}} = 5.27$

$$df = \frac{\left(\frac{5^2}{50} + \frac{4^2}{40}\right)^2}{\frac{(5^2/50)^2}{49} + \frac{(4^2/40)^2}{39}} = 87.9 \longrightarrow \text{Round to } 88$$

Compare T_{88}

Then p-value $= P(T_{88} > 5.27) + P(T_{88} < -5.27)$ which we can find using R to be very close to 0.

$\Rightarrow$ Reject the null hypothesis that there's no difference between the sections

Example: Comparing midterm grades

Repeat the test using a pooled approach.

H_0 and H_A are the same

$$S_p = \sqrt{\frac{49 \times 5^2 + 39 \times 4^2}{49 + 39}} = 4.58$$

$$SE(\bar{x}_1 - \bar{x}_2) = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 4.58 \sqrt{\frac{1}{50} + \frac{1}{40}} = 0.97$$

$$\text{Test Statistic} = \frac{85 - 80}{0.97} = 5.14$$

$df = n_1 + n_2 - 2 = 88$ Compare to T_{88}

p-value = $P(T_{88} > 5.14) + P(T_{88} < -5.14)$ which again is very close to 0

$\Rightarrow$ Reject H_0

Confidence Intervals for Two Means

- As before, we can also use a confidence interval to make the same conclusion as a hypothesis test.
- For the case of comparing two means, this course only talks about confidence intervals when we are working with the unpooled variance situation.
- It will have the same format as all other confidence intervals we have talked about: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df} \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- again, we need to approximate the degrees of freedom using the formula seen earlier.

Unpaired Data: T-test for Proportions

Two Sample Proportions

- Lastly, we can do the same kinds of tests regarding the proportions of two independent samples.
- We have a very similar setup as with unmatched sample means.
  - The hypotheses are usually that the difference in proportions is equal to a value, $H_0 : p_1 - p_2 = 0$, versus some reasonable alternative (usually not equal)
  - The test statistic is built similarly, but now we don't need to worry about whether the population variances are the same, but rather choose a reasonable choice for p_1, p_2 when making confidence intervals or running tests.
  - if we want a CI, use $p_1 = \hat{p}_1$and $p_2 = \hat{p}_2$ in the standard error
  - if we want a test, work under the assumption that the null is true and use a pooled $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$ in the standard error

CI for Two Proportions

- The confidence interval always takes the same format: $\hat{p} \pm z_{\frac{\alpha}{2}} \times SE(\hat{p})$
- Now we are working with differences so use $\hat{p}_1 - \hat{p}_2$ as our sample value and the standard error will be

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- So the interval will be $(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \times SE(\hat{p}_1 - \hat{p}_2)$
- Again since the CLT still applies even though we have two samples, we are using critical values from the Normal table based on the confidence level we want.

Hypothesis Test for Proportions

- As we mentioned earlier, the standard error will be different when we run a test for proportions from when we compute a CI
- Since tests are built under the null hypothesis, we can actually find a pooled estimate of $\hat{p}$ that is essentially an average of $\hat{p}_1$ and $\hat{p}_2$:

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

  - This gives us an estimate for $\hat{p}$ if we were to pool the groups into a single sample
- We use this $\hat{p}$ to find the standard error

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

  - The rest of the test statistic remains the same

Hypothesis Test for Proportions

- The test statistic becomes:

$$\frac{\hat{p}_1 - \hat{p}_2}{SE(\hat{p}_1 - \hat{p}_2)} \sim N(0,1)$$

- Where

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Example: News Sources

A Journalism study wanted to investigate where people are getting their news. In the study, 34% of 60 responders said they get their news online, as opposed to 31% of 50 responders favouring newspapers. Use a 95% confidence interval to determine if there is a difference in the proportion of readers using these two sources.

$n_1 = 60 \qquad \hat{p}_1 = 0.34$

$n_2 = 50 \qquad \hat{p}_2 = 0.31$

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{0.34(1-0.34)}{60} + \frac{0.31(1-0.31)}{50}} = 0.0895$$

Critical value is from $N(0,1)$; We get 1.96

$\hat{p}_1 - \hat{p}_2 = 0.34 - 0.31 = 0.03$

95% CI: $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \times SE(\hat{p}_1 - \hat{p}_2)$

$\Rightarrow 0.03 \pm 1.96 \times 0.0895$

$\Rightarrow (-0.145, 0.205)$

Since 0 is within this interval, there is no difference between the two proportions.

Example: News sources

A Journalism study wanted to investigate where people are getting their news. In the study, 34% of 60 responders said they get their news online, as opposed to 31% of 50 responders favouring newspapers. Test whether there is a difference in the proportion of people using each news source.

$H_0: p_1 = p_2$ or $p_1 - p_2 = 0$

$H_A: p_1 \neq p_2$ or $p_1 - p_2 \neq 0$

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} = \frac{60 \times 0.34 + 50 \times 0.31}{60 + 50} = 0.326$$

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{0.326(1 - 0.326)\left(\frac{1}{50} + \frac{1}{60}\right)} = 0.090$$

Test Statistic $= \dfrac{\hat{p}_1 - \hat{p}_2}{SE(\hat{p}_1 - \hat{p}_2)} = \dfrac{0.34 - 0.31}{0.090} = 0.334$

Compare to $N(0,1)$

p-value $= P(Z > 0.334) + P(Z < -0.334) = 0.738$

$\Rightarrow$ Do not reject H_0

- One sample data, means
  Week 9
  - σ is known
    - Critical value $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ follows a N(0,1) distribution under the Null Hypothesis
  - σ is unknown
    - Critical value $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ follows student t distribution with n-1 degrees of freedom under the Null Hypothesis
- Two sample data
  Week 11
  - σ1 and σ2 are unknown
    - σ1 =/= σ2
      - Unpooled (Welch) T-test
    - σ1 = σ2
      - Pooled Variance T-Test
  - Two proportions
    - Similar to pooled variance t-test but use Z instead of T, and calculate a pooled p for the denominator
  - σ1 and σ2 are known
    Not covered
    - Same setup as σ1 =/= σ2 case but use Z instead of T
  - Paired Samples ("before" vs "after" on the same units)
    - Parametric method: Treat the differences as a single sample and do a one-sample test
    - Nonparametric method: Nonparametric sign test
      This is the only test here that does NOT follow the regular formula for test statistic.
- Linear Regression
  Week 12
  - T-test for B1
- One sample data, proportions
  Week 9
  - Z-test with test statistic $Z = \frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ follows a N(0,1) distribution under the Null Hypothesis

Practice Problems

- In this lecture, we covered all of Module 10
- Practice problems are posted

Coming up

- Next week is our last class
- Please see exam schedule for official final exam information

Next & last quiz due Tues. April 2