CENG 280 HOMEWORK 1

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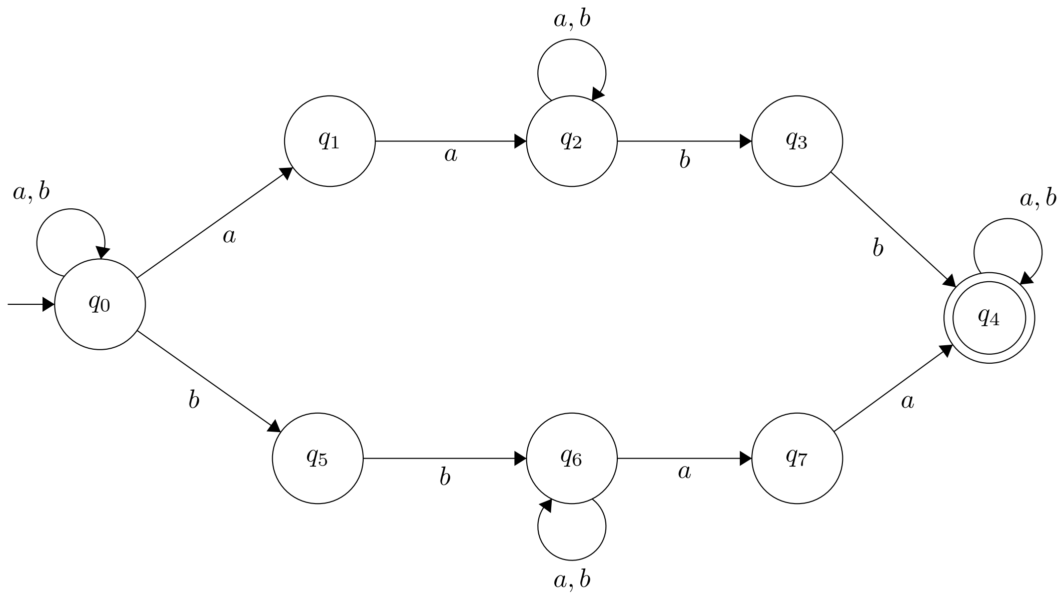
Question 1

a)

$(a \cup b)^* aa(a \cup b)^* bb(a \cup b)^* (aa \cup bb)^* \quad \cup \quad (a \cup b)^* bb(a \cup b)^* aa(a \cup b)^* (aa \cup bb)^*$

b)

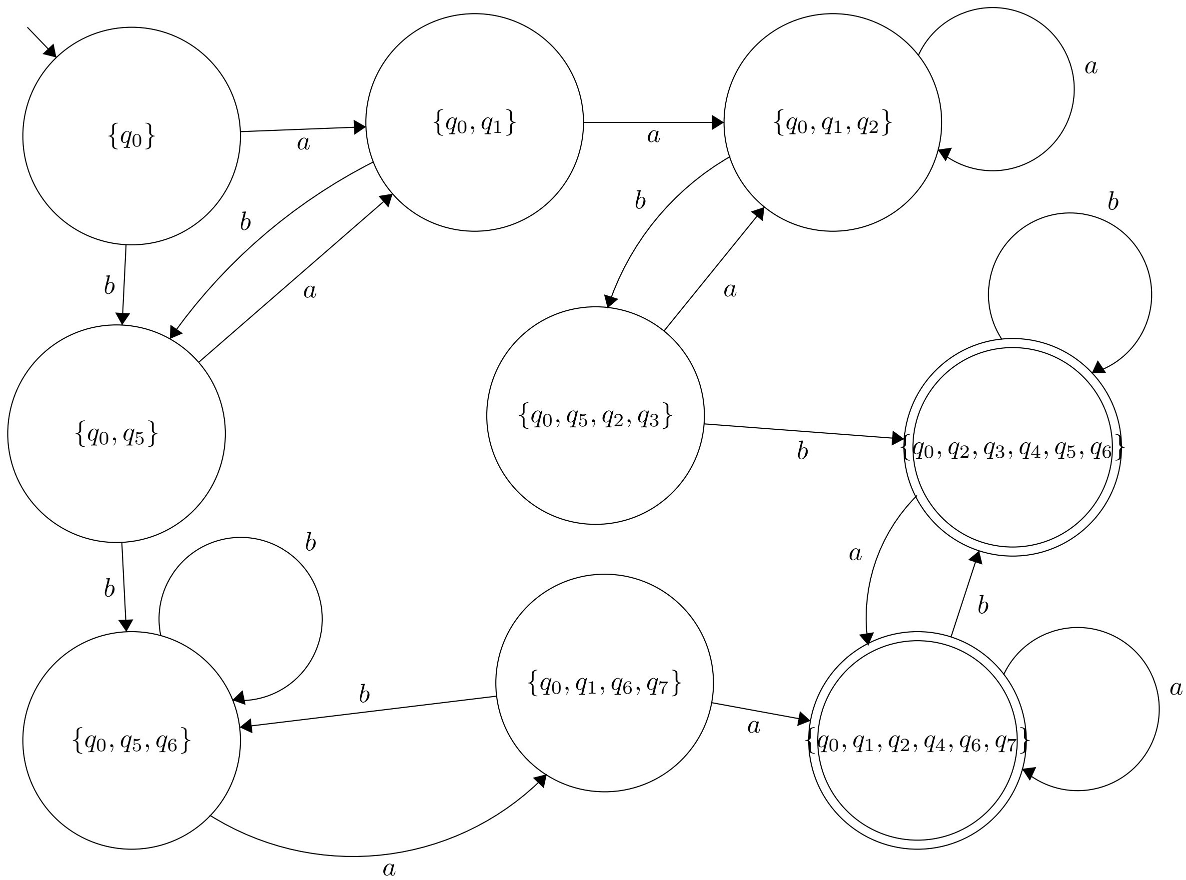
$M = (K, \Sigma, \Delta, s, F)$
$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$\Delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_0, b, q_5), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_2), (q_2, b, q_3), (q_3, b, q_4), (q_4, a, q_4)$
$, (q_4, b, q_4), (q_5, b, q_6)(q_6, a, q_6), (q_6, b, q_6), (q_6, a, q_7), (q_7, a, q_4) \ \}$
$s = \{q_0\}$
$F = \{q_4\}$



c)

$E(q_0) = q_0, E(q_1) = q_1$
$E(q_2) = q_2, E(q_3) = q_3, E(q_4) = q_4, E(q_5) = q_5$
$E(q_6) = q_6, E(q_7) = q_7$
$\delta(\{q_0\}, a) = E(q_0) \cup E(q_1) = \{q_0, q_1\}$
$\delta(\{q_0\}, b) = E(q_0) \cup E(q_5) = \{q_0, q_5\}$
$\delta(\{q_0, q_1\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_1\}, b) = E(q_0) \cup E(q_5) \cup \{\}) = \{q_0, q_5\}$
$\delta(\{q_0, q_5\}, a) = E(q_0) \cup E(q_1) \cup \{\}) = \{q_0, q_1\}$
$\delta(\{q_0, q_5\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_1, q_2\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_1, q_2\}, b) = E(q_0) \cup E(q_5) \cup E(q_2) \cup E(q_3) = \{q_0, q_5, q_2, q_3\}$
$\delta(\{q_0, q_5, q_2, q_3\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_5, q_2, q_3\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) \cup E(q_2) \cup E(q_3) \cup E(q_4) = \{q_0, q_2, q_3, q_4, q_5, q_6\}$
$\delta(\{q_0, q_5, q_6\}, a) = E(q_0) \cup E(q_1) \cup E(q_6) \cup E(q_7) = \{q_0, q_1, q_6, q_7\}$
$\delta(\{q_0, q_5, q_6\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_2, q_3, q_4, q_5, q_6\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_4) \cup E(q_6) \cup E(q_7) = \{q_0, q_1, q_2, q_4, q_6, q_7\}$
$\delta(\{q_0, q_2, q_3, q_4, q_5, q_6\}, b) = E(q_0) \cup E(q_5) \cup E(q_2) \cup E(q_3) \cup E(q_4) \cup E(q_6) = \{q_0, q_2, q_3, q_4, q_5, q_6\}$
$\delta(\{q_0, q_1, q_6, q_7\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_6) \cup E(q_7) \cup E(q_4) = \{q_0, q_1, q_2, q_4, q_6, q_7\}$
$\delta(\{q_0, q_1, q_6, q_7\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_1, q_2, q_4, q_6, q_7\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_6) \cup E(q_7) \cup E(q_4) = \{q_0, q_1, q_2, q_6, q_7, q_4\}$

$\delta(\{q_0, q_1, q_2, q_4, q_6, q_7\}, b) = E(q_0) \cup E(q_5) \cup E(q_2) \cup E(q_3) \cup E(q_6) \cup E(q_4) = \{q_0, q_2, q_3, q_4, q_5, q_6\}$



d)

$w \in L(M)\ if\ and\ only\ if\ (s, w) \vdash_M^* (f, e)\ where\ f \in F$
Hence , starting with NFA :
$(q_0, bbabb) \vdash_M (q_5, babb) \vdash_M (q_6, abb) \vdash_M (q_7, bb)$
$Since\ bb \neq e\ and\ q_7 \notin F\ , where\ \ F\ contains\ final\ states\ of\ NFA,\ the\ word\ is\ not\ in\ the\ language.$
We can try another path since in NFA ,multiple steps can give different results.
$(q_0, bbabb) \vdash_M (q_0, babb) \vdash_M (q_0, abb) \vdash_M (q_1, bb)$
$Since\ bb \neq e\ and\ q_1 \notin F\ , the\ word\ is\ again\ not\ in\ the\ language$
For the DFA :
$(\{q_0\}, bbabb) \vdash_M (\{q_0, q_5\}, babb) \vdash_M (\{q_0, q_5, q_6\}, abb) \vdash_M (\{, q_0, q_1, q_6, q_7\}, bb) \vdash_M$

$(\{, q_0, q_5, q_6\}, b) \vdash_M (\{q_0, q_5, q_6\}, e)$
$Since \{q_0, q_5, q_6\} \notin F_D$ $where$ F_D $contains$ the $final$ $states$ of $DFA,$ the $word$ is not in the $language.$

Question 2)

a)

$Assume\ L_1\ is\ regular\ and\ let\ n\ be\ the\ pumping\ length.$
$Choose\ a\ string\ w = a^m b^n\ where\ |w| \geq n$
$w = xyz\ and\ |xy| \leq n\ Our\ chosen\ string\ looks\ similar\ to\ the\ following\ :$
$aaa......abbb....b\ where\ number\ of\ as\ is\ bigger\ than\ number\ of\ bs.$
$Firstly\ y\ part\ can\ not\ be\ at\ bs\ since\ |xy| \leq n$
$Hence\ y\ part\ must\ be\ at\ as.$
$We\ can\ write\ x, yz as x = a^k y = x = a^l, z = a^{n-k-l} b^n$
$For\ i = 0\ , xy^i z = a^{n-k} b^n \notin L_1\ , so\ L_1\ is\ not\ regular.$
$Since\ L_2 = \overline{L_1},\quad L_2\ is\ not\ regular\ by\ closure\ properties.$

b)

$Firstly\ let's\ consider\ L_4\ and\ L_5.$
$L_4\ excludes\ natural\ number\ 0\ and\ L_5\ does\ not\ .\ Hence\ their\ union\ is\ :$
$L_4 \cup L_5 = a^* b^*,\ since\ we\ can\ write\ a\ regular\ expression\ out\ of\ their\ union\ ,\ L_4 \cup$
$L_5\ is\ regular$
$L_4 \cup L_5 \cup L_6 = a^* b^* \cup b^* a(ab^* a)$
$Since\ we\ can\ write\ a\ regular\ expression\ out\ of\ their\ union\ L_4 \cup L_5 \cup L_6\ is\ regular.$