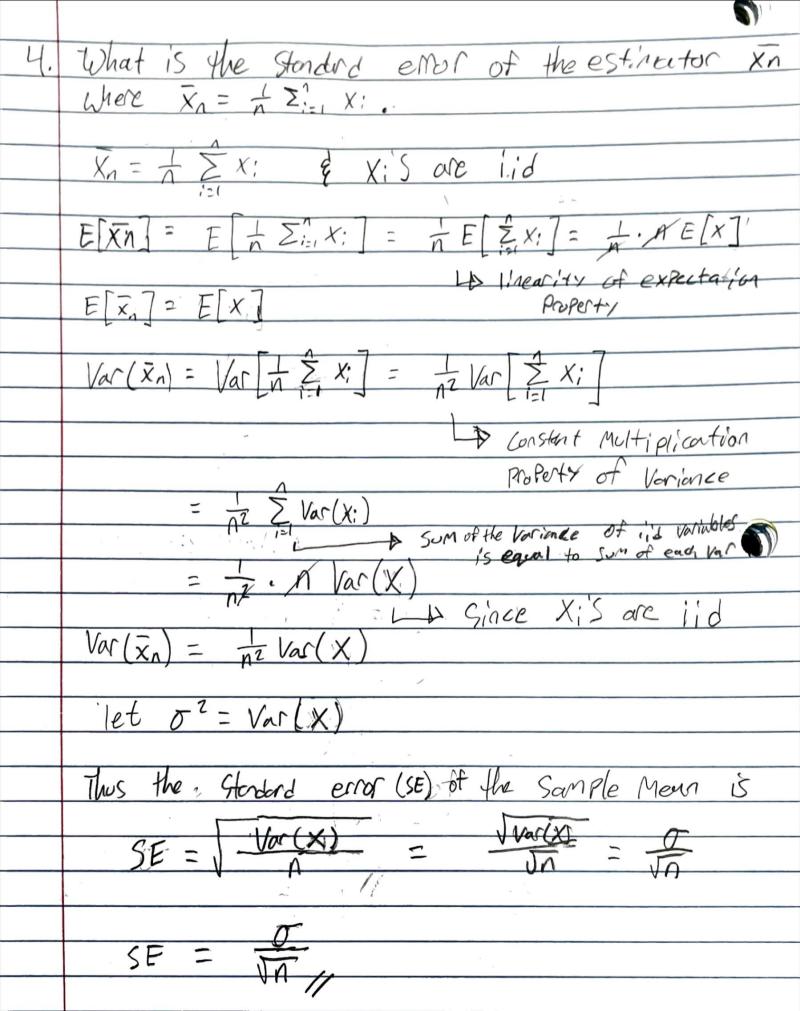
MIE 1613 HW1

William Hazen MENG - MIE 1009231225 2023-02-01

. ASSUME X is continuous & Unitormy distributed in [2,10] We are intrested in 0 = E[(x-5)+]. Note a = Max(a,0) a) compute 0 exactly using the definition of Expected Value. Hint f(x)= 5-a, x \in [a,b] \( \ext{o} \) otherwise  $X \in [2, 0] \rightarrow f(x) = \frac{1}{0-2} = \frac{1}{2}$  $\theta = E[(x-5)^{+}] = \int_{0}^{10} (x-5)^{+} f(x) dx$ (x-5) = 5 0 if 2 < x < 5  $\theta = \int_{1}^{10} (x-5)^{+} f(x) dx = \int_{1}^{10} (x-5)^{+} dx$  $= \frac{1}{8} \int_{0}^{5} (x-5)^{+} dx + \int_{-\infty}^{6} (x-5)^{+} dx$  $=\frac{1}{8}\left[\int_{1}^{5}0dx + \int_{1}^{6}(x-5)^{+}dx\right]$  $=\frac{1}{8}[0+\int_{0}^{10}(x-5)^{4}dx]$  $\frac{1}{8} \left[ \frac{1}{2} \times ^{2} \right]^{10} + 5 \times \left[ \frac{1}{5} \right]^{10}$ 

 $=\frac{1}{8}\left[\left(\frac{75}{5}\right)-\left(25\right)\right]=\frac{25}{16}\approx1.5625$ 

0 = 40



From the following

Let 
$$M_x = E[x] \in O^2 = Var(x)$$

a)  $Var(ax+b) = a^2 Var(x)$ 

$$Var(ax+b) = E[(ax+b) - E[ax+b])^2$$

$$= E[(ax+b) - aE[x] - b)^2$$

$$= E[(ax+b) - aE[x] - b)^2$$

$$= E[(ax+a)^2]$$

$$= E[(ax+a)^2]$$

$$= E[(ax+a)^2]$$

$$= a^2 E[(x-M)^2] + (iii)$$

$$= a^2 Var(x) + (ii)$$

if the following

$$= A^2 = A^2 =$$

b) 
$$Var(X+Y) = Var(X) + Var(Y) = 2CoV(X,Y)$$
  
 $|ct Mx = E[X], M_Y = E[Y], CoV(X,Y) = E[(X-M_X)(Y-M_Y)]$   
 $|Var(X+Y) = E[((X+Y) - E(X+Y))^2] + (i)$   
 $= E[((X+Y - M_X - M_Y)^2]$   
 $= E[((X-M_X) + (Y-M_Y))^2 + 2(X-M_X)(Y-M_Y)]$   
 $= E[((X-M_X)^2 + (Y-M_Y)^2 + 2(X-M_X)(Y-M_Y)]$   
 $= E[((X-M_X)^2] + E[(Y-M_Y)^2] + 2E[(X-M_X)(Y-M_Y)]$   
 $= E[((X-M_X)^2] + [(Y-M_Y)^2] + 2E[((X-M_X)(Y-M_Y)]$   
 $= E[((X-M_X)^2] + [(Y-M_Y)^2] + 2E[((X-M_X)(Y-M_Y)]$ 

Name: William HazenStudent ID: 1009231225

Department: MIEProgram: M.Eng.

```
In [1]: import numpy as np
import matplotlib.pyplot as plt

# fix random number seed
np.random.seed(1)
```

## 1.

#### 1b)

```
In [2]: | # X is cont. U[2,10]
        # theta = E[(X-5)^+]
         \# a^+ = Max(a,0) \rightarrow a^+ = 0 \text{ if } a < 0
                          -> a^+ = a \text{ if } a >= 0
         ab value list = [] # initalize list
         a = 2
         b = 10
         n = 100000 # 100k samples
         for i in range(n):
             random_value = np.random.random() # create random float between 0.0 & 1.0
             # scale by difference between a & b and shift by a
             ab_value = (b-a) * random_value + a
             # output is uniformed list of values between 2 & 10
             ab value list.append(ab value)
         theta = []
         for x in ab_value_list:
             if x < 5: # if x is less than 5 then a^+ = 0
                 theta.append(0)
                          # if x >= 5 then a^+ = a
             else:
                 theta.append((x-5))
```

Positive bound = 1.5717078094739845

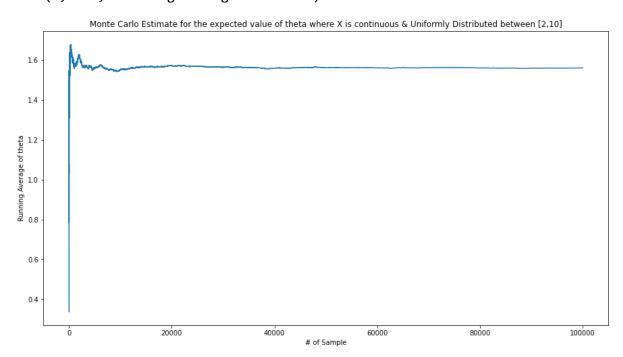
```
In [3]:
        average theta list = []
        n = 0
        for value in theta: # loop through the values in theta
            n += 1
            if n == 1:
                average_theta_list.append(value) # append the first value of theta
                average theta list.append(
                    (1/n) * ((n-1)*average theta list[-1] + value))
                # running average = adding the new number to the old average and divid
        ing by the total number of samples
        theta_mean = np.mean(theta) # mu of theta list
        theta_std = np.std(theta) # std of theta list
        print("Average theta =", theta_mean)
        # 95% confidence interval: alpha = mu +- 1.96 std/sqrt(n)
        print('95% Confidence interval theta:', np.mean(theta),
              "+/-", 1.96*np.std(theta, ddof=1)/np.sqrt(n))
        # 95% confidence interval: alpha = mu +- 1.96 std/sqrt(n)
        alpha neg = theta mean - (1.96*theta std)/np.sqrt(n)
        alpha plus = theta mean + (1.96*theta std)/np.sqrt(n)
        print(" negative bound =", alpha neg)
        print(" Positive bound =", alpha plus)
        Average theta = 1.5613895874744266
        95% Confidence interval theta: 1.5613895874744266 +/- 0.010318273591054833
         negative bound = 1.5510713654748687
```

1a) theta = 1.5625

# 1c)

```
In [4]: plt.figure(figsize=(15, 8))
   plt.plot(average_theta_list)
   plt.title(
        "Monte Carlo Estimate for the expected value of theta where X is continuou
   s & Uniformly Distributed between [2,10]")
   plt.xlabel("# of Sample")
   plt.ylabel("Running Average of theta")
```

Out[4]: Text(0, 0.5, 'Running Average of theta')



2.

```
In [5]: | def Q2_TTF(T):
            # pesudo random
            np.random.seed(1)
            # start with 2 functioning components at time 0
            clock = 0
            S = 2
            # initialize the time of events
            NextRepair = float('inf')
            NextFailure = np.ceil(6*np.random.random())
            # lists to keep the event times and the states
            EventTimes = [0]
            States = [S]
            A t = \{\}
             . . .
            Logic: While the runtime is less than the value of T, clock is stored as t
        he min value of either the repair or failure time.
                If the repair value is less than the failure, we initalize the NextRep
        air to inf, however if S = 1 then we reset and schedule the next repair and fa
        ilure time.
                Similarly if the repair value is greater than the failure, we have the
        same condition but initalize NextFailure to inf.
            while clock < T: # changed while loop from S>0 to clock<T
                # advance the time till next event
                clock = min(NextRepair, NextFailure, T) # Stops when clock = T
                if NextRepair < NextFailure: # next event is completion of a repair</pre>
                     S = S + 1
                     if S == 2:
                         # Can only be 1 or 2 thus NextRepair = inf if S!=1
                         NextRepair = float('inf')
                     elif S == 1: # When S = 1, schedule next repair and failure
                         NextRepair = clock + 2.5
                         NextFailure = clock + np.ceil(6*np.random.random())
                else: # next event is a failure
                     S = S - 1
                     if S == 0:
                         # Can only be 1 or 0 thus NextFailure = inf if S!=1
                         NextFailure = float('inf')
                     elif S == 1:
                         NextRepair = clock + 2.5
                         NextFailure = clock + np.ceil(6*np.random.random())
                # save the time and state
                EventTimes.append(clock)
                States.append(S)
                last_clock = clock
                # Stores value of 1 when State = 1 at time t
                for idx, state in enumerate(States):
```

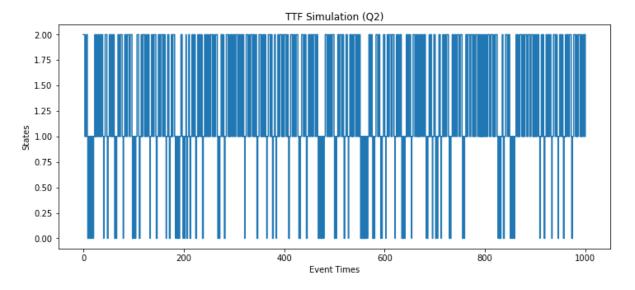
```
if state == 2:
    A_t[EventTimes[idx]] = 1
    else:
        A_t[EventTimes[idx]] = 0

# plot the sample path
print('T =', int(T))
print('Clock =', last_clock)
print('Event Time length =', len(EventTimes))
print('A_t =', sum(A_t.values())/len(EventTimes))
plt.figure(figsize=(12, 5))
plt.plot(EventTimes, States, drawstyle='steps-post')
plt.title("TTF Simulation (Q2)")
plt.xlabel("Event Times")
plt.ylabel("States")
plt.show()
```

#### 2a)

```
In [6]: Q2_TTF(1000)

T = 1000
Clock = 1000
Event Time length = 513
A_t = 0.32748538011695905
```



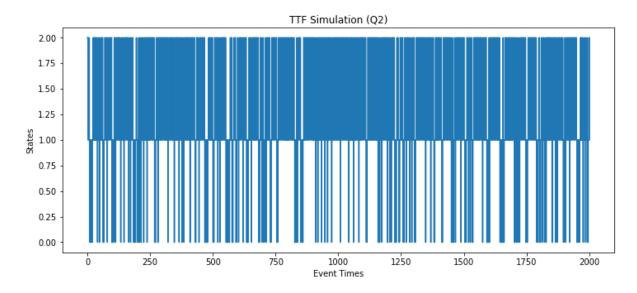
# 2b)

In [7]: Q2\_TTF(2000)

T = 2000Clock = 2000

Event Time length = 1031

 $A_t = 0.3326867119301649$ 



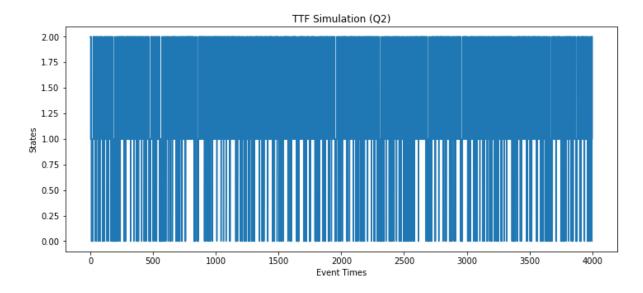
In [8]: Q2\_TTF(4000)

T = 4000

Clock = 4000.0

Event Time length = 2083

 $A_t = 0.33461353816610656$ 



T=1k: 0.32748538011695905
T=3K: 0.3326867119301649
T=4k: 0.33461353816610656

### **Observations**

• The fraction of the system being fully functional increases with time.

3.

```
In [9]: def Q3 TTF(N):
            # Set number of replications
            Rep = 1000
            # Define lists to keep samples of the outputs across replications
            TTF list = []
            # fix random number seed
            np.random.seed(1)
            for rep in range(0, Rep):
                clock = 0
                S = N
                Spare = N - 1 # number of spare components
                # initialize the time of events
                NextRepair = float('inf')
                NextFailure = np.ceil(6*np.random.random())
                EventTimes = [0]
                States = [S]
                 . . .
                 Logic: For each replication, while S > 0, if the repair value is less
        than the failure, if S = N then there is no need to schedule a repair.
                     However if not then the NextRepair will be scheduled if there is a
        spare avaliable.
                     If NextRepair >= NextFailure then the next failure is scheduled in
        itally unless S = 0.
                     If S!=0 then we schedule the next repair [The reason why the NextF
        ailure is outside is
                      because scheduling nextfailure does not depend on the spare comp
        onents]
                     If the repair value is greater than the failure, we have the same
        condition but initalize NextFailure to inf.
                     I accounted for each value of S in the event that Repair<Fail,
                     But In order to make sure only one repair can happen at a time
                    when Fail<Repair, a NextRepair is scheduled when S = N-1
                while S > 0:
                     # advance the time till next event
                     clock = min(NextRepair, NextFailure)
                     if NextRepair < NextFailure: # next event is completion of a repa</pre>
        ir
                         S = S + 1
                         Spare = Spare + 1 # Spare components increases with S
                         if Spare == N: # Max spare components = N-1
                             Spare = N-1
                         if S == N: # Max components -> no repairs
                             NextRepair = float('inf')
                         else:
                             if Spare > 0: # If there are spare components available
```

```
Spare = Spare
                                  NextRepair = clock + 2.5
                      else: # next event is a failure
                          S = S - 1
                          NextFailure = clock + np.ceil(6*np.random.random())
                          if S == 0:
                              NextFailure = float('inf')
                          else:
                              if Spare > 0:
                                  Spare = Spare - 1
                                  if S == (N-1):
                                      NextRepair = clock + 2.5
                      # save the time and state
                      EventTimes.append(clock)
                      States.append(S)
                 # save the TTF and average # of func. components
                 TTF_list.append(clock)
                 #From Q1
                 average_TTF_list = []
                 n = 0
                 for value in TTF list:
                      n += 1
                      if n == 1:
                          average TTF list.append(value)
                      else:
                          average_TTF_list.append((1/n) * ((n-1)*average_TTF_list[-1] +
         value))
             print("For N = ", N)
             print('TTF List Length =', len(TTF_list))
             print('Estimated expected TTF:', np.mean(TTF_list))
             print('95% CI for TTF:', np.mean(TTF_list), "+/-", 1.96*np.std(TTF_list, d
         dof = 1)/np.sqrt(Rep)
In [10]: | Q3_TTF(2)
         For N = 2
         TTF List Length = 1000
         Estimated expected TTF: 14.193
         95% CI for TTF: 14.193 +/- 0.7212458114694061
In [11]: Q3_TTF(3)
         For N = 3
         TTF List Length = 1000
         Estimated expected TTF: 87.648
         95% CI for TTF: 87.648 +/- 5.523633399450503
```

```
In [12]: Q3_TTF(4)

For N = 4
    TTF List Length = 1000
    Estimated expected TTF: 648.805
    95% CI for TTF: 648.805 +/- 42.26686864295534
```

## 6.

```
In [13]: \# X_n = 1/n sum(X_i)
         def X_n(n): # create a X_n function
             sum xn = 0
             for i in range(n):
                 # Random uniformed values between 0 & 1
                 X = np.random.uniform(0, 1, 1000)
                 sum_xn += X # sum
             X_n = (1/n)*sum_xn
             return X n
         x1 = X_n(1)
         x2 = X_n(2)
         x30 = X_n(30)
         x500 = X_n(500)
         x_s = {"x1": x1, "x2": x2, "x30": x30, "x500": x500}
         for x in (x_s):
             print("Length of " + x + ":", len(x_s[x]))
```

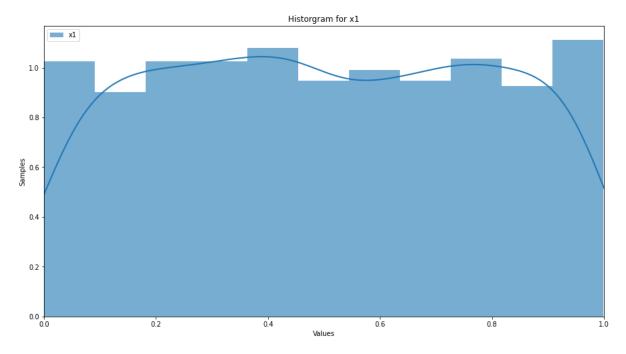
Length of x1: 1000 Length of x2: 1000 Length of x30: 1000 Length of x500: 1000

```
In [14]: import seaborn as sns
    kwargs = dict(hist_kws={'alpha': .6}, kde_kws={'linewidth': 2})

for key, value in x_s.items():
    plt.figure(figsize=(15, 8))
    sns.distplot(value, **kwargs, label=key)
    plt.xlim([0, 1])
    plt.title(f"Historgram for {key}")
    plt.xlabel('Values')
    plt.ylabel('Samples')
    plt.legend()
    plt.show()
```

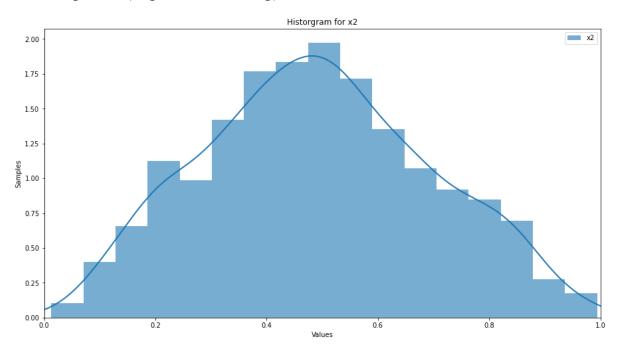
/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)



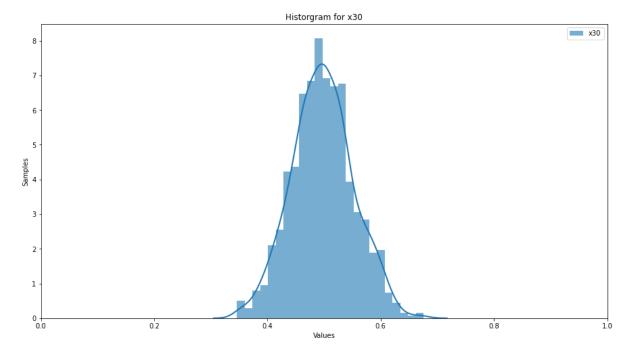
/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)



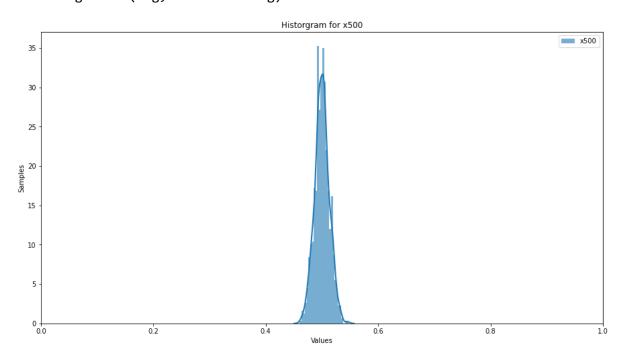
/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)



/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)



```
In [15]: plt.figure(figsize=(15, 8))
    for key, value in x_s.items():
        sns.distplot(value, **kwargs)
    plt.title("Historgram for All")
    plt.xlabel('Values')
    plt.ylabel('Samples')
    plt.legend()
    plt.show()
```

/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)

/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

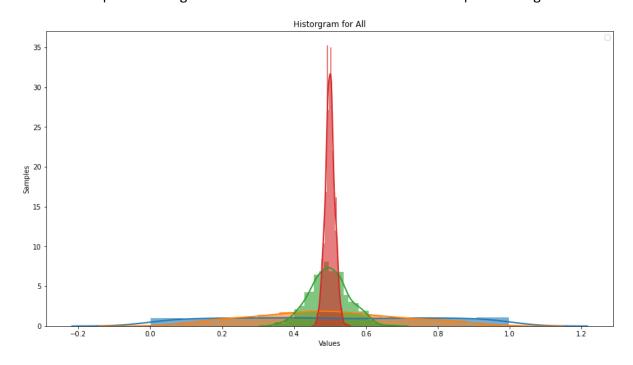
warnings.warn(msg, FutureWarning)

/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)

/usr/local/lib/python3.8/dist-packages/seaborn/distributions.py:2619: FutureW arning: `distplot` is a deprecated function and will be removed in a future v ersion. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histog rams).

warnings.warn(msg, FutureWarning)
WARNING:matplotlib.legend:No handles with labels found to put in legend.



What happens to the mean and variability of the sample means as n increases? Relate your observations to the Central Limit Theorem discussed in the class.

• As n increases the mean and variability converge to a central point. In X\_1, we can see high variability and as n increases we can see the variability decrease and the mean hovers around 0.5 at X\_500. Thus as the sample size increases the distributation closely resembles a normal distribution and becomes tightly clustered around the mean, which is a behaviour that agrees with the Central Limit Theorem.

[NbConvertApp] WARNING | pattern '/content/HW1.ipynb' matched no files This application is used to convert notebook files (\*.ipynb) to various other formats. WARNING: THE COMMANDLINE INTERFACE MAY CHANGE IN FUTURE RELEASES. **Options** ====== The options below are convenience aliases to configurable class-options, as listed in the "Equivalent to" description-line of the aliases. To see all configurable class-options for some <cmd>, use: <cmd> --help-all --debug set log level to logging.DEBUG (maximize logging output) Equivalent to: [--Application.log level=10] --show-config Show the application's configuration (human-readable format) Equivalent to: [--Application.show config=True] --show-config-json Show the application's configuration (json format) Equivalent to: [--Application.show config json=True] --generate-config generate default config file Equivalent to: [--JupyterApp.generate\_config=True] -у Answer yes to any questions instead of prompting. Equivalent to: [--JupyterApp.answer\_yes=True] --execute Execute the notebook prior to export. Equivalent to: [--ExecutePreprocessor.enabled=True] --allow-errors Continue notebook execution even if one of the cells throws an error and include the error message in the cell output (the default behaviour is to abo rt conversion). This flag is only relevant if '--execute' was specified, too. Equivalent to: [--ExecutePreprocessor.allow errors=True] --stdin read a single notebook file from stdin. Write the resulting notebook with default basename 'notebook.\*' Equivalent to: [--NbConvertApp.from stdin=True] --stdout Write notebook output to stdout instead of files. Equivalent to: [--NbConvertApp.writer\_class=StdoutWriter] --inplace Run nbconvert in place, overwriting the existing notebook (only relevant when converting to notebook format) Equivalent to: [--NbConvertApp.use\_output\_suffix=False --NbConvertApp.exp ort format=notebook --FilesWriter.build directory=] --clear-output Clear output of current file and save in place, overwriting the existing notebook. Equivalent to: [--NbConvertApp.use\_output\_suffix=False --NbConvertApp.exp ort\_format=notebook --FilesWriter.build\_directory= --ClearOutputPreprocessor. enabled=True] --no-prompt Exclude input and output prompts from converted document.

Equivalent to: [--TemplateExporter.exclude\_input prompt=True --TemplateEx

```
porter.exclude output prompt=True]
--no-input
    Exclude input cells and output prompts from converted document.
            This mode is ideal for generating code-free reports.
    Equivalent to: [--TemplateExporter.exclude output prompt=True --TemplateE
xporter.exclude input=True]
--log-level=<Enum>
    Set the log level by value or name.
    Choices: any of [0, 10, 20, 30, 40, 50, 'DEBUG', 'INFO', 'WARN', 'ERROR',
'CRITICAL']
    Default: 30
    Equivalent to: [--Application.log_level]
--config=<Unicode>
    Full path of a config file.
    Default: ''
    Equivalent to: [--JupyterApp.config file]
--to=<Unicode>
    The export format to be used, either one of the built-in formats
            ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook',
'pdf', 'python', 'rst', 'script', 'slides']
            or a dotted object name that represents the import path for an
            `Exporter` class
    Default: 'html'
    Equivalent to: [--NbConvertApp.export format]
--template=<Unicode>
    Name of the template file to use
    Default: ''
    Equivalent to: [--TemplateExporter.template file]
--writer=<DottedObjectName>
   Writer class used to write the
                                        results of the conversion
    Default: 'FilesWriter'
    Equivalent to: [--NbConvertApp.writer class]
--post=<DottedOrNone>
    PostProcessor class used to write the
                                        results of the conversion
    Default: ''
    Equivalent to: [--NbConvertApp.postprocessor class]
--output=<Unicode>
    overwrite base name use for output files.
                can only be used when converting one notebook at a time.
    Default: ''
    Equivalent to: [--NbConvertApp.output_base]
--output-dir=<Unicode>
    Directory to write output(s) to. Defaults
                                  to output to the directory of each noteboo
k. To recover
                                  previous default behaviour (outputting to t
he current
                                  working directory) use . as the flag value.
    Equivalent to: [--FilesWriter.build directory]
--reveal-prefix=<Unicode>
    The URL prefix for reveal.js (version 3.x).
            This defaults to the reveal CDN, but can be any url pointing to a
copy
            of reveal.is.
```

```
For speaker notes to work, this must be a relative path to a loca
1
            copy of reveal.js: e.g., "reveal.js".
            If a relative path is given, it must be a subdirectory of the
            current directory (from which the server is run).
            See the usage documentation
            (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-js-
html-slideshow)
            for more details.
    Default: ''
    Equivalent to: [--SlidesExporter.reveal url prefix]
--nbformat=<Enum>
    The nbformat version to write.
            Use this to downgrade notebooks.
    Choices: any of [1, 2, 3, 4]
    Default: 4
    Equivalent to: [--NotebookExporter.nbformat version]
Examples
   The simplest way to use nbconvert is
            > jupyter nbconvert mynotebook.ipynb
            which will convert mynotebook.ipynb to the default format (probab
ly HTML).
            You can specify the export format with `--to`.
            Options include ['asciidoc', 'custom', 'html', 'latex', 'markdow
n', 'notebook', 'pdf', 'python', 'rst', 'script', 'slides'].
            > jupyter nbconvert --to latex mynotebook.ipynb
            Both HTML and LaTeX support multiple output templates. LaTeX incl
udes
            'base', 'article' and 'report'. HTML includes 'basic' and 'ful
1'. You
            can specify the flavor of the format used.
            > jupyter nbconvert --to html --template basic mynotebook.ipynb
            You can also pipe the output to stdout, rather than a file
            > jupyter nbconvert mynotebook.ipynb --stdout
            PDF is generated via latex
            > jupyter nbconvert mynotebook.ipynb --to pdf
            You can get (and serve) a Reveal.js-powered slideshow
            > jupyter nbconvert myslides.ipynb --to slides --post serve
            Multiple notebooks can be given at the command line in a couple o
f
            different ways:
```

```
> jupyter nbconvert notebook*.ipynb
            > jupyter nbconvert notebook1.ipynb notebook2.ipynb
            or you can specify the notebooks list in a config file, containin
g::
                c.NbConvertApp.notebooks = ["my notebook.ipynb"]
            > jupyter nbconvert --config mycfg.py
To see all available configurables, use `--help-all`.
CalledProcessError
                                          Traceback (most recent call last)
<ipython-input-16-ede384e11680> in <module>
---> 1 get ipython().run cell magic('shell', '', 'jupyter nbconvert --to htm
1 /content/HW1.ipynb\n')
/usr/local/lib/python3.8/dist-packages/IPython/core/interactiveshell.py in ru
n_cell_magic(self, magic_name, line, cell)
   2357
                    with self.builtin trap:
   2358
                        args = (magic arg s, cell)
-> 2359
                        result = fn(*args, **kwargs)
   2360
                    return result
   2361
/usr/local/lib/python3.8/dist-packages/google/colab/_system_commands.py in _s
hell cell magic(args, cmd)
        result = run command(cmd, clear streamed output=False)
    107
    108
          if not parsed args.ignore errors:
            result.check returncode()
--> 109
    110
        return result
    111
/usr/local/lib/python3.8/dist-packages/google/colab/ system commands.py in ch
eck returncode(self)
        def check returncode(self):
    132
          if self.returncode:
    133
--> 134
             raise subprocess.CalledProcessError(
    135
                  returncode=self.returncode, cmd=self.args, output=self.outp
ut)
    136
CalledProcessError: Command 'jupyter nbconvert --to html /content/HW1.ipynb
' returned non-zero exit status 255.
```