Identifiability Analysis of A Rigid Body Undergoing Frictional Contact With Its Environment

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***Abstract*— In this paper we addressed the question of the identifiability of inertial parameters of an object and contact forces imparted on the object as it makes and breaks contact with the environment. We assume that the object and environment are both rigid and we exploit the structure of the complementarity formulation for contact resolution to reason about inertial parameters and contact forces. Here we analyze cases of contact with and without known external forces, under sliding and sticking contact modes. Our identifiability analysis indicates that without known external forces the identifiable set of parameters are the ratio of mass moment of inertia to mass and the ratio of contact forces to the mass of the object whereas with known external forces the mass and mass moment of inertia can be deconvoluted, leading to identifiability of mass, mass moment of inertia, normal and tangential contact forces uniquely. We demonstrate the identifiability analysis on a 2D free-falling block and show that the results prove to be consistent with predictions. NF:Made this into active tense and changed the wording a little, but says the same thing, just tried to make it more clear and easier to read.**

***Index Terms*— System Identification, Identifiability, Frictional Contact, Rigid Body Impact NF:what do you think of the key words?**

I. Introduction

Accurate estimates of object parameters such as mass and stiffness are an important component of reliable and robust control strategies employed in robotic systems. Prior measurement of parameters or values reported by manufacturers of parts are often the first resource referred to when seeking values; however, typically the prior measurement of parameters is impractical due to factors such as lack of measurement equipment or difficulty in dissembling parts for the purposes of measurements and manufacturer information may be lost or properties of the part may change due to wear and tear.

Parameter estimation through measurements of the responses of systems, i.e. the displacements, velocities and accelerations, to external inputs such as forces and torques falls within the domain of system identification. System identification seeks to fit a model that is parameterized by the variables of interests to the inputs and outputs of the physical system. The advantage of system identification over more traditional parameter measurement techniques is that it can allow for online adaptation and refinement of parameter estimates without necessarily needing any disassembly of the system and can be integrated into the control loop or as a failsafe/error diagnosis mechanism to monitor the behavior of the system.

System identification techniques have successfully been applied to serial and parallel link robots (manipulators) and extensive literature exists on parameterization and formulation of such systems, prime examples of which can be found in [1] and [2]. These techniques succeed in finding a set of *base inertial parameters* given torques applied to joints and measurements of the generalized displacements, velocities and accelerations. This set of parameters is sufficient to control the manipulators considered and a demonstration of this can be found in [4] where an adaptive controller is designed such that as parameter estimates converge to the values of true base inertial parameters the controller essentially performs a full state feedback linearization of the dynamics of the manipulator and so arbitrary motions are achieved.

NF:I was trying to make the following part more prominent as Alberto suggested, so I gave it its own paragraph, is it better now?

The key idea behind such analysis is that though the dynamics are inherently nonlinear, they are linear in the base inertial parameter set and so a least squares formulation (or recursive least squares for online adaptation) can be achieved that would yield estimates of this set. An example of a dynamic system that is nonlinear but is linear in its inertial parameters is a fixed based serial link manipulator whose dynamics can be written as:

$$\tau(t) = \Phi(q, \dot{q}, \ddot{q}, t)\Theta \tag{1}$$

where τ is the vector of input torques (1 element for each actuator), $\Phi(q, \dot{q}, \ddot{q}, t)$ is a matrix comprised of joint positions (orientations) and their first and second derivatives as well as geometric lengths and Θ is a vector comprised of combinations of inertial parameters (mass, first and second mass moments) called the base inertial parameters of manipulator. Seminal papers such as [2] and [3] have paved the way for linear in parameter formulations of robotic manipulator dynamics used for system identification. Furthermore studies such as [1] seek to find and/or optimize trajectories such that

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the parameter estimates can be achieved with high accuracy and fidelity.

NF:I re-wrote the following paragraph to incorporate Alberto's comments. I re-worded it with the aim to make the distinction between fixed base and floating based robots and their literature more clear and better worded. In the original I made a comparison that was bad so I changed that to match more closely with Alberto's suggestion. It should overall be a better paragraph.

When compared to system identification literature pertaining to robots lacking a fixed base (such as walking robots) and robotic systems making and breaking contact with the environment, literature regarding robotic manipulators is more mature due, perhaps, to the prevalence and length of time that robotic manipulators have been used in the manufacturing and automation fields. The importance of system identification is further emphasized by the need to enable autonomous manipulation, were the robot needs to infer as much information as possible when interacting with unknown objects, and autonomous traversing of unknown environments for mobile robots. It is important to note that the interaction of the robot with the environment imparts additionally complexity to the analysis because unknown and spurious reaction forces due to frictional contact need to be accounted for.

In this paper we will exploit the *linear complementarity problem* (LCP) formulation for contact resolution to reason about the identifiability of inertial parameters of the object undergoing contact with **NF:Added the environment here** the environment as well as the forces imparted on the object during contact. The specific system we consider is an arbitrary object modeled as a single rigid body undergoing a rigid body impact after a period of free fall.

NF:Insert figure here. An object just falling, Alberto has drawn a hand diagram of it

NF:This section has been bulleted as suggested.

The paper is organized as follows:

1) Section II: We present a brief review of contact resolution literature and a rationale for the use of LCP in our analysis.
2) Section III: We provide an introduction to the complementarity problem formulation for contact.
3) Section IV: We provide the identifiability analysis for the case where the rigid body undergoes a sliding single point of contact collision with the ground after a period of free-fall.
4) Section V: We provide the identifiability analysis under the condition that the collision involves no relative tangential sliding between the point of contact of the body and the fixed ground.
5) Section VI: We examine the effects of a known external force acting on the body during impact are considered on the identifiability analysis.
6) Section VII: We demonstrate the identifiability analysis applied to a 2D block as it dropped onto a rigid flat plane.
7) section VIII: We conclude the paper by reviewing the results and summing up the findings.

II. Background

NF:I feel like since I have separated LCP from background now it is a short section, is that okay or make it longer? I can but prefer not to unless its actually too short because of the whole idea of not writing more than I need too. In this section we provide a brief review of contact resolution literature and present the motivation for using the LCP formulation. **NF:In the following since I am quoting brogaliato I wasn't too sure if it is okay to change the names he gives the methods, is that okay? (this is regarding changing event-driven to penalty). Regarding existence of identifiability I have only found possibly 1 or 2 papers loosely related, really there is not much out there, at least as far as the way we are looking at it.** Brogliato in [7] has categorized rigid body numerical simulations into 3 main divisions: penalty methods, event-driven methods, and time-stepping methods. Event-driven methods penalize penetration by applying a force that is related to the amount of interpenetration of the two bodies, and so cannot often enforce non-penetration conditions. The event-driven methodologies rely on a listing of the possible events (collisions/contacts) and resolve each as it happens and discard scenarios that are no longer possible. Event-driven approaches can predict exceedingly high velocities when there should be multiple contacts as demonstrated by [8] and typically require some knowledge of contact time. The time-stepping methodologies integrate the equations of motion and check for collisions, should one (or multiple) be detected the algorithm steps back in time and resolve the collisions and continues to integrate the equations of motion at each step. The time-stepping approach in conjunction with the velocity-impulse resolution of contact which results in the CP formulation has been advocated by [5] and [6] and has been shown to be robust to phenomena such as Painleve's problem [6] as well as to always have a solution if linear approximations of the friction cone are made and the mass matrix is positive definite.

III. Complementarity Problems for Collision Resolution

NF:added this section so that we have plenty of room to talk about LCP formulation with contact, I was trying to hit the middle ground between being too detailed and being too brief, just trying to give intuition was the idea, let me know how you like it and how I can improve it

Given the favorable properties outlined in section II and its prevalence in modern contact simulators we chose to utilize the time-stepping technique and the LCP formulation for contact resolution and in this section we will provide the reader with the mathematical background and framework. For further details we refer the reader to [6].

A general (i.e. nonlinear) complementarity problem typically is defined as:

$$\begin{aligned}
\text{Find: } & z \\
\text{s.t. } & 0 \leq g(z) \\
& 0 \leq z \\
& 0 = zg(z)
\end{aligned} \tag{2}$$

And the linear complementarity problem is formed when $g(z) = Mz+q$. The benefit of utilizing the complementarity problem is that it allows us to write the equations of motion incorporating contact as a constraint. We can write the equations of motion using the Lagrange's formulation such that they incorporate the unilateral (contact) constraints as:

$$\begin{aligned}
\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - \frac{\partial L}{\partial q} &= Q + c(t)\nabla\phi(q(t)) \\
0 &\leq c(t) \\
0 &\leq \phi(q(t)) \\
0 &= \phi(q)c(t)
\end{aligned} \tag{3}$$

Here we note that $c(t)$ represents the contact forces and $\phi(q(t))$ represents the constraint (similar to a distance function in that $\phi(q) = 0$ implies contact) and Q is the vector of generalized forces not including contact. The constraints simply mean that either we have a positive distance from surface of contact ($\phi(q)$ greater than zero) which requires a zero contact force, therefore integration of the Lagrange equations continues without incorporation of the contact forces, or that $\phi(q) = 0$ which indicates collision therefore Lagrange equations must be integrated considering the contact forces.

We note that the equations of motion can be written using Newton-Euler equations of motion, and we will from this point onwards rely on this formulation to build intuition for the workings of contact resolution with CPs:

$$\begin{aligned}
M(q)\frac{dv}{dt} = J_n c_n + D(q)c_t + k(q,v) \\
-\nabla V(q) + F_{ext}(t)
\end{aligned} \tag{4}$$

Here $J_n = \nabla\phi_n(q)$ were $0 \leq \phi_n(q)$ provides a description of the the contact constraint, i.e. the acceptable region of possible configurations. Further we can attribute $\phi_n(q)$ to a measure of distance between two contact surfaces and note that $\phi_n(q) \leq 0$ indicates penetration and c_n is the magnitude of the normal contact force. $D(q)$ denotes the positive span of direction vectors of the friction force and lies within the tangent plane of contact between objects and c_t is a column vector that is determined such that the product $D(q)c_t$ constitutes the tangential (frictional) component of the contact forces. $V(q)$ denotes the conservative forces (such as gravity), $k(q,v)$ denotes the centrifugal and Coriolis velocity components and F_{ext} denotes all external non-conservative forces excluding contact forces. Careful inspection of equation 4 indicates that while $\phi_n(q)$ is fairly straightforward to derive, further constraints are requires to find $D(q)$.

To find $D(q)$ we can invoke the maximal dissipation principal [5] which states simply that assuming contact has been established then the velocity of the object at an infinitesimal time ahead is in the opposing direction of friction. This statement can be reformulated into an optimization problem as:

$$\begin{aligned}
\min_{c_t}\ & (v^+)^T D(q)c_t \\
\text{s.t. } & \psi(c_t) \leq \mu c_n
\end{aligned} \tag{5}$$

NF:Need to fix this so that c_t goes directly under min

This optimization problem finds the components of the c_t vector such that the frictional force $D(q)c_t$ is in the opposing direction to velocity at an infinitesimal time ahead subject to the constraint that the frictional force lies within the friction surface defined by ψ. We can represent all possible frictional forces with $FC_0(q) = \{D(q)c_t \mid c_t \in R^d \text{ and } \psi(c_t) \leq \mu\}$. We note that ψ is convex, coercive and positively homogeneous which implies that $D(q)c_t \in c_n FC_0(q)$ is equivalent to $\psi(c_t) \leq \mu c_n$.

Just as we formulated the Lagrange equations of motion to incorporate contact with CPs, we can similarly convert the optimization problem stated into a CP constraint by noting that inequality constraints can be incorporated into the optimization function by using Lagrange multipliers such that $h(c_t, \lambda) = (v^+)^T D(q)c_t - \lambda(\mu c_n - \psi(c_t))$ and we note that:

$$\frac{\partial h}{\partial c_t} = 0 = \mu D(q)^T v^+ + \lambda\frac{\partial\psi(c_t)}{\partial c_t} \tag{6}$$

Furthermore we can write:

$$\begin{aligned}
& 0 \in \mu D(q)^T v^+ + \lambda\frac{\partial\psi(c_t)}{\partial\psi} \\
& 0 \leq \lambda \\
& 0 \leq \mu c_n - \psi(c_t) \\
& 0 = \lambda(\mu c_n - \psi(c_t))
\end{aligned} \tag{7}$$

which is simply the CP version of the optimization problem.

Given that now we have a full formulation for contact resolution using complementarity problem, we can combine the different components to write:

$$\begin{aligned}
M(q)\frac{dv}{dt} &= J_n c_n + D(q)c_t - \nabla V(q) + k(q,v) + F_{ext} \\
\frac{dq}{dt} &= v \\
& 0 \leq c_n \perp 0 \leq \phi_n \\
& 0 \in \mu D(q)^T v^+ + \lambda\frac{\partial\psi(c_t)}{\partial\psi} \\
& 0 \leq \lambda \perp 0 \leq \mu c_n - \psi(c_t) \\
& 0 = J_n^T v^+ \text{if} \phi_n(q) = 0
\end{aligned} \tag{8}$$

We note that the formulation presented so far is nonlinear with respect to the friction surface constraint (specifically the function ψ), so we will utilize a polyhedral approximation of the friction cone to convert the nonlinear CP to a linear CP. The polyhedral approximation to the friction cone is constructed by using $\{J_n + \mu d_i(q) | i = 1, 2, ..., m\}$ where d_i are direction vectors that positively span the tangent plane to the contact, are equiangular with respect to each other and

$d_i = -d_j$ for any non equal pair of i and j. Utilizing this approximation the frictional contact force is approximated by $\tilde{D}(q)\tilde{c}_t$ where $0 \leq c_t$ and $\Sigma c_{t,i} \leq \mu c_n$.

NF:Add a figure here for d_i and the friction cone

In the following we will drop the tilde from the notation but will assume that the polyhedral approximation holds for all further analysis.

Next, to be able to simulate the evolution of the dynamics we will convert the formulation to its discrete time equivalent to be able to use a time-stepping technique therefore we use the integrals of the contact forces over the time steps. The resulting equations of motion and constraints were derived by [6] and are given by:

$$\begin{aligned} M(q_k)(v_{k+1} - v_k) &= c_n J_n + D c_t \\ &\quad -hk(q_k, v_k) - h\nabla V(q_k) + hF_{ext} \\ q_{k+1} &= q_k + hv_{k+1} \\ 0 \leq c_n &\perp 0 \leq J_n^T(v_{k+1} + \epsilon v_k) \\ 0 \leq c_t &\perp 0 \leq \lambda e + D^T v_{k+1} \\ 0 \leq \lambda &\perp 0 \leq \mu c_n - e^T c_t \end{aligned} \tag{9}$$

We emphasis several important properties of equation 9:

1) We made a small abuse of notation in that now c_n and Dc_t are the value of the forces multiplied by the length of the time step, and their units are force multiplied by time.
2) The first complementarity constraint governs the existence of the normal contact force, to understand this we consider the next time step velocity at contact, if this velocity has no component along the contact normal or points in the same direction then a force (impulse) must be applied to induce such a change during the time-step that contact occurs.
3) The second complementarity constraint is the restatement of the principle of maximum dissipation and tangential force.
4) The third complementarity constraint is related to the saturation of the frictional force and relative sliding between the contact surfaces.

The equations in 9 are integrated forward in time using an Euler approximation and contact is acknowledged using a measure defining distance between the closest point of a rigid body to the surface of contact given by:

$$\Phi = \begin{bmatrix} \phi_n \\ \phi_t \end{bmatrix} \tag{10}$$

NF:Insert drawing here

such that if this measure is positive in the first element then contact has not been established, if it is equal to zero contact occurs and if the measure is negative then interpenetration has occurred. It is also noted that:

$$\frac{\partial \Phi}{\partial q} = \begin{bmatrix} J_n \\ J_t \end{bmatrix} \tag{11}$$

NF:I changed the following paragraph to clarify the purpose of LCP, why we use it and why contact resolution is hard but important (this was a comment made by Alberto)

It is important to note that the key idea behind using LCP formulation is that it solves simultaneously for forces at contact and velocities at the next time step. We feel it is important to emphasize that the difficulty with contact resolution is the simultaneous determination of the contact forces and post contact velocities which complicates the matter of identifiability. Another helpful intuition we can provide is that this formulation tells us which mode of contact the object will be post impact given pre-impact kinematic measurements, coefficient of restitution and externally applied forces. In other words if the mode of contact is known then there is no need to solve the LCP and the impulses during impact and velocities/positions post impact can be solved for strictly as functions of these states and external influences prior to impact. This is the key idea that is used section IV to VI in conjunction with the assumption that kinematic measurements and external forces are available during the trajectory of the object as it makes and breaks contact with the environment.

IV. Identifiability Analysis: Sliding Contact Mode

NF:Changed the introduction to this section to be more clear about what we assume we have and what we are interested in.

In this section we will consider the identifiability of the inertial parameters (mass and second moment of inertia) of an object starting in a phase of free-fall and undergoing sliding contact with a rigid and fixed flat surface. Our assumption is that we have the positions, orientations and velocities (linear and angular) of the object and are interested in reasoning about the inertial parameters of the object as it interacts with the environment. We further assume that only gravity acts on the object, and in section VI we will remove this restriction. The second moment of inertia is expressed with respect to a reference frame attached to the center of mass and aligned with the principle axes. Given that the kinematic measurements of the trajectory are available, we can derive the direction of friction by invoking the *principle of maximum dissipation* [6] (outlined in section III) and denote the direction by J_t. Since in this section we assume that the mode of contact is sliding then considering the CP formulation from equation 9:

$$\begin{aligned} 0 &< c_n \\ 0 &< c_t \\ 0 &< \lambda \end{aligned} \tag{12}$$

NF:I wrote an explanation regarding the inequalities to explain better why there hold and where they come from

The first inequality comes from the fact that at contact the distance constraint is equal to zero so its dual must be greater than zero. To understand the second and third inequalities we emphasize that since we have sliding contact then v_{k+1} must have at least one component that is not perpendicular to the tangent plane spanned by D and so $\max -d_i v_{k+1} \leq \lambda$ where d_i are the columns of D. This directly implies that $0 < \lambda$ and the second inequality comes from the fact that since $0 < \lambda$ then $\mu c_n = e^T c_t$ which simply means that $0 < c_t$.

NF:add a figure to demonstrate the second and third inequalities. Also the next paragraph is more clear than before and says where the equation comes from (incorporate Alberto's comment)

Taking into consideration the fact that the contact mode is sliding and the inequalities from equation 12 we revisit equation 9 and re-write it with c_n, c_t and λ as variables:

$$\begin{bmatrix} J_n^T M^{-1} J_n & J_n^T M^{-1} J_t & 0 \\ J_t^T M^{-1} J_n & J_t^T M^{-1} J_t & 1 \\ \mu & -1 & 0 \end{bmatrix} \begin{bmatrix} c_n \\ c_t \\ \lambda \end{bmatrix} = - \begin{bmatrix} J_n^T b \\ J_t^T b \\ 0 \end{bmatrix} \tag{13}$$

where:

$$b = v_k + hM^{-1}(-\nabla V - k(q, v)) \tag{14}$$

$$k = \begin{bmatrix} 0_{3\times 1} \\ \tilde{\omega} R I_0 R^T \omega \end{bmatrix} \tag{15}$$

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{16}$$

and we can solve for c_n and c_t to arrive at:

$$\begin{aligned} c_n &= \frac{J_n^T v_k + hJ_n^T M^{-1}(-\nabla V - k)}{J_n^T M^{-1} J_n + \mu J_n^T M^{-1} J_t} \\ c_t &= \mu c_n \end{aligned} \tag{17}$$

At this stage we have explicitly solved for the contact forces as functions of the inertial properties, geometry of contact and pre-contact kinematic measurements. To perform identifiability analysis we require an equation that is strictly a function of kinematic measurements and inertial properties so we replace the values derived for c_n and β into the dynamic equations of motion:

$$\begin{aligned} v_{k+1} =& v_k + hM^{-1}\{-\nabla V - k\} + \\ & M^{-1} \frac{J_n^T v_k + hJ_n^T M^{-1}(-\nabla V - k)}{J_n^T M^{-1} J_n + \mu J_n^T M^{-1} J_t} \\ & \{J_n + \mu J_t\} \end{aligned} \tag{18}$$

We need to further exploit the structure of the matrices in this equation to be able to reason about the identifiability of the inertial parameters therefore to this end we write:

$$M^{-1} = \frac{1}{m} \begin{bmatrix} I_3 & 0 \\ 0 & RmI_0^{-1}R^T \end{bmatrix} \tag{19}$$

$$\nabla V = m \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} \frac{\partial H}{\partial q} \tag{20}$$

$$k = m \begin{bmatrix} 0 \\ \tilde{\omega} R (I_0/m) R^T \omega \end{bmatrix} \tag{21}$$

Where H denotes the vertical distance of the center of mass to some reference multiplied by the gravitational constant. In order to make the replacement of these expressions into equation 18 tractable we make the following definitions:

$$a_{6\times 1} = \begin{bmatrix} \hat{a}_{3\times 1} \\ \check{a}_{3\times 1} \end{bmatrix} \tag{22}$$

$$\Xi_1 = v_{k+1} - v_k + h \begin{bmatrix} \frac{\partial \hat{H}}{\partial q} \\ 0 \end{bmatrix} \tag{23}$$

$$\xi_2 = \hat{J}_n^T \hat{J}_n + \mu \hat{J}_n^T \hat{J}_t \tag{24}$$

$$\xi_3 = J_n^T v_k - hJ_n^T \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} \frac{\partial H}{\partial q} \tag{25}$$

NF:hopefully now the notation is more clear why the following paragraph

The hat notation ($\hat{a}$) is used when we refer to the first 3 elements of vector a and the check notation ($\check{a}$) is used to refer to the second 3 elements of vector a. Though abstract definitions can be attributed to Ξ_1, ξ_2 and ξ_3 they are primarily defined for mathematical manipulation. With this notation we have:

$$\Xi_1 = \begin{bmatrix} \{\hat{J}_n + \mu \hat{J}_t\} \\ R(mI_0^{-1})R^T\{\check{J}_n + \mu \check{J}_t\} \end{bmatrix} \tag{26}$$
$$\frac{\xi_3 - hJ_n^T \begin{bmatrix} 0 \\ R(mI_0^{-1})R^T \tilde{\omega} R(I_0/m)R^T \omega \end{bmatrix}}{\xi_2 + \check{J}_n^T(R(mI_0^{-1})R^T)\check{J}_n + \mu \check{J}_n^T(R(mI_0^{-1})R^T)\check{J}_t}$$

This equation is uniquely parameterized by the quantity mI_0^{-1}. **NF:This bit is new to explain why mI_0^{-1} is actually uniquely identifiable.** To demonstrate this we define:

$$\Pi = R(mI_0^{-1})R^T \tag{27}$$

and note that Π is a function of the parameter we are interested in and orientations of the object which we already know. Now consider $\Xi = \left[\hat{\Xi}_1 \check{\Xi}_1\right]^T$ we can write:

$$\begin{aligned} \hat{\Xi}_1 &= \{\hat{J}_n + \mu \hat{J}_t\} \frac{\xi_3 - h\check{J}_n^T \Pi \tilde{\omega} \Pi^{-1} \omega}{\xi_2 + \check{J}_n^T \Pi \check{J}_n + \mu \check{J}_n^T \Pi \check{J}_t} \\ \check{\Xi}_1 &= \Pi\{\check{J}_n + \mu \check{J}_t\} \frac{\xi_3 - h\check{J}_n^T \Pi \tilde{\omega} \Pi^{-1} \omega}{\xi_2 + \check{J}_n^T \Pi \check{J}_n + \mu \check{J}_n^T \Pi \check{J}_t} \end{aligned} \tag{28}$$

Which we can further manipulate to yield:

$$\begin{aligned} \{\hat{J}_n + \mu \hat{J}_t\}\xi_3 - \xi_2 \hat{\Xi}_1 =& \hat{\Xi}_1(\check{J}_n^T \Pi \check{J}_n + \mu \check{J}_n^T \Pi \check{J}_t) + \\ & h(\hat{J}_n + \mu \hat{J}_t)(\check{J}_n^T \Pi \tilde{\omega} \Pi^{-1} \omega) \\ -\xi_2 \check{\Xi}_1 =& \check{\Xi}_1(\check{J}_n^T \Pi \check{J}_n + \mu \check{J}_n^T \Pi \check{J}_t) + \\ & h\Pi(\check{J}_n + \mu \check{J}_t)(\check{J}_n^T \Pi \tilde{\omega} \Pi^{-1} \omega) - \\ & \Pi\{\check{J}_n + \mu \check{J}_t\}\xi_3 \end{aligned} \tag{29}$$

We note that the left hand sides of the equalities are known and the right hand sides are functions of Π so an optimization problem can be formulated to solve for Π and it can be identified uniquely. Furthermore the values of the normal (c_n) and tangential (c_t) impulses normalized by the mass (m) are identifiable, not their absolute values. This is due to the fact that we derived both quantities as functions of Π so for example from equation 17 and using the notation already defined we can write::

$$\frac{c_n}{m} = \frac{\xi_3 - h\check{J}_n \Pi \tilde{\omega} \Pi^{-1} \omega}{\xi_2 + \check{J}_n^T \Pi \check{J}_n + \mu \check{J}_n^T \Pi \check{J}_t} \tag{30}$$

$$\frac{c_t}{m} = \mu \frac{c_n}{m} \tag{31}$$

In conclusion under the assumptions made we can identify mI_0^{-1}, c_t/m and c_n/m uniquely.

V. Identifiability Analysis: Sticking Contact Mode

In this section we will perform identifiability analysis assuming that the relative tangential velocity of the body making contact with the ground is zero. Just as in section IV our assumption is that we have the positions, orientations and velocities (linear and angular) of the object and are interested in reasoning about the inertial parameters of the object as it interacts with the environment. We further assume that only gravity acts on the object, and in section VI we will remove this restriction. Under the assumption that the contact is sticking we note that:

$$\begin{aligned} 0 &< c_n \\ 0 &< c_t \\ 0 &\leq \lambda \end{aligned} \tag{32}$$

To understand why these inequalities hold we note that due to contact since the contact constraint is zero, its compliment (c_n) must then be greater than zero and since we have sticking contact then a tangential force must exist to eliminate sliding therefore c_t must be greater than zero. A less intuitive justification can be garnered by considering the complementarity constraints of equation 9 and noting that since the velocity post contact will not have a component within the tangential plane of contact then $D^T v_{k+1} = 0$, then in this scenario either $e^T c_t = \mu c_n$ which implies that frictional force is at its boundary and the analysis will follow as section IV or that $e^T c_t \leq \mu c_n$ which implies that the frictional force lies within the boundary of the base of the friction cone which requires further investigation. We note that simply requiring that $D^T v_{k+1} = 0$ will result in $\lambda = 0$. For the case where the frictional force lies within the boundary of its maximum, the LCP formulation from equation 9 can be simplified to:

$$\begin{bmatrix} J_n^T M^{-1} J_n & J_n^T M^{-1} J_t \\ J_t^T M^{-1} J_n & J_t^T M^{-1} J_t \end{bmatrix} \begin{bmatrix} c_n \\ c_t \end{bmatrix} + \begin{bmatrix} J_n^T b \\ J_t^T b \end{bmatrix} = 0 \tag{33}$$

Next we will solve for c_n and β:

$$\begin{bmatrix} c_n \\ c_t \end{bmatrix} = -\frac{1}{\Delta} \begin{bmatrix} J_t^T M^{-1} J_t & -J_n^T M^{-1} J_t \\ -J_t^T M^{-1} J_n & J_n^T M^{-1} J_n \end{bmatrix} \times \begin{bmatrix} J_n^T b \\ J_t^T b \end{bmatrix} \tag{34}$$

where $\Delta = (J_n^T M^{-1} J_n)(J_t^T M^{-1} J_t) - (J_n^T M^{-1} J_t)(J_t^T M^{-1} J_n)$ is the determinant of the matrix on the left hand side of the equality. Since we are interested in deriving an equation that is strictly a function of inertial properties and contact forces we replace the derived expressions for c_n and c_t into equation 18:

$$\begin{aligned} v_{k+1} = v_k + h \begin{bmatrix} \frac{\partial \check{H}}{\partial q} \\ 0 \end{bmatrix} - \frac{1}{\Delta} m \begin{bmatrix} I_3 & 0 \\ 0 & R^T m I_0^{-1} R \end{bmatrix} \\ \times \{(J_t^T M^{-1} J_t) J_n^T b J_n - (J_n^T M^{-1} J_t) J_t^T b J_n - \\ (J_t^T M^{-1} J_n) J_n^T b J_t + (J_n^T M^{-1} J_n) J_t^T b J_t\} \end{aligned} \tag{35}$$

where as before:

$$\begin{aligned} b = & v_k - h \begin{bmatrix} \frac{\partial H}{\partial q} \\ 0 \end{bmatrix} - \\ & h \begin{bmatrix} 0 \\ (R^T m I_0^{-1} R) \tilde{\omega} (R I_0 / m R^T) \omega \end{bmatrix} \end{aligned} \tag{36}$$

In order to be able to reason further about identifiability and make the formulation more tractable we introduce a new notation:

$$\psi_{ij} = J_i^T b J_j \tag{37}$$

Additionally using the notation introduced in the previous section regarding contact in sliding:

$$\begin{aligned} \Xi_1 = & -\frac{1}{\Delta} \begin{bmatrix} I_3 & 0 \\ 0 & R^T m I_0^{-1} R \end{bmatrix} \{(\hat{J}_t^T \hat{J}_t \\ & + J_t^T (R^T m I_0^{-1} R) J_t) \psi_{nn} \\ & - (\hat{J}_n^T \hat{J}_t + J_n^T (R^T m I_0^{-1} R) J_t) \psi_{tn} \\ & - (\hat{J}_t^T \hat{J}_n + J_t^T (R^T m I_0^{-1} R) J_n) \psi_{nt} \\ & + (\hat{J}_n^T \hat{J}_n + J_n^T (R^T m I_0^{-1} R) J_n) \psi_{tt}\} \end{aligned} \tag{38}$$

Careful examination of equation 38 leads us to the same conclusion derived from equation 26 for the contact in sliding mode case, both equations are characterized uniquely by mI_0^{-1}. Furthermore the values of the normal (c_n) and tangential (β) impulses normalized by the mass (m) are identifiable, not their absolute values.

VI. Identifiability Analysis: External Force During Contact

In this section it is of interest to incorporate the effects of an external but known force on the identifiability analysis of the object. The effects of the known external force can be analyzed for both sliding and non-sliding contact but would yield the same result, therefore for the sake of brevity we will consider sliding contact. Following the derivation from section IV with the known external forces:

$$\begin{aligned} \Xi_1 = & h \begin{bmatrix} F_{ext}/m \\ R^T m I_0^{-1} R \tau_{ext} \end{bmatrix} \\ & + \begin{bmatrix} \{\hat{J}_n + \mu \hat{J}_t\} \\ R^T m I_0^{-1} R \{\check{J}_n + \mu \check{J}_t\} \end{bmatrix} \\ & \left\{ \frac{\xi_3 + J_n^T h \begin{bmatrix} F_{ext}/m \\ R^T m I_0^{-1} R \tau_{ext} \end{bmatrix}}{\xi_2 + \check{J}_n^T (R^T m I_0^{-1} R) \check{J}_n + \mu \check{J}_n^T (R^T m I_0^{-1} R) \check{J}_t} \right. \\ & \left. - \frac{h J_n^T \begin{bmatrix} 0 \\ R^T (m I_0^{-1}) R \tilde{\omega} R (I_0/m) R^T \omega \end{bmatrix}}{\xi_2 + \check{J}_n^T (R^T m I_0^{-1} R) \check{J}_n + \mu \check{J}_n^T (R^T m I_0^{-1} R) \check{J}_t} \right\} \end{aligned} \tag{39}$$

From this equation it can be inferred that given a known external force applied to the object, the mass and consequently the mass moment of inertia of the object can be derived uniquely. Using knowledge of the mass, the normal and tangential forces can be found in absolute terms. This understanding, though perhaps not formally proved elsewhere, is commonly used in estimation of parameters characterizing manipulated objects within grasp or unknown objects undergoing external provocation by instrumentation.

VII. Identifiability Analysis: Example of 2-D Block

In this section we will demonstrate the identifiability analysis conducted in sections IV to VI by applying it to a 2-D Block undergoing free-fall and colliding with a fix flat surface that lies perpendicular to the direction of the gravitational field. We measure the position, orientation and velocities of the block as it interacts with the environment and seek to identify the inertial parameters and contact forces. Regarding the block itself we denote the length of its side by a, the angle of rotation by θ, its center of mass vertical and horizontal distances to a reference frame attached to the ground by y and x, the mass of the block by m and the mass moment of inertia by J. The block can be seen in Fig. 1

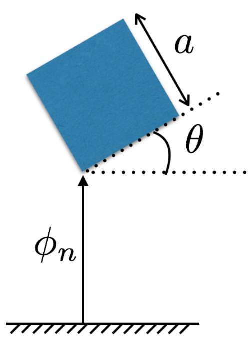

Fig. 1. 2D Block in Free-Fall

Given the geometry of the block and the sharp corners, the normal (vertical) distance of the lowest point on the block to the ground can be found from:

$$\phi_n = \min\left(\begin{bmatrix} f_1(q) = y - \frac{a}{\sqrt{2}}\cos(\pi/4 - \theta) \\ f_2(q) = y - \frac{a}{\sqrt{2}}\cos(\pi/4 + \theta) \\ f_3(q) = y + \frac{a}{\sqrt{2}}\cos(\pi/4 - \theta) \\ f_4(q) = y + \frac{a}{\sqrt{2}}\cos(\pi/4 + \theta) \end{bmatrix} \right) \quad (40)$$

where each $f_i(q)$ denotes the vertical distance of vertex i as a function of vertical distance of the center of mass and orientation of the block. For demonstration purposes, assume that the lowest vertex is found to be $f_1(q)$, with this the normal Jacobian is:

$$J_n = \frac{\partial f_1(q)}{\partial q} = \begin{bmatrix} 0 \\ 1 \\ -\frac{a}{\sqrt{2}}\sin(\pi/4 - \theta) \end{bmatrix} \quad (41)$$

and the tangential Jacobian is:

$$J_t = \pm \begin{bmatrix} 1 \\ 0 \\ \frac{a}{\sqrt{2}}\cos(\pi/4 - \theta) \end{bmatrix} \quad (42)$$

where the sign is dependent on the sign of x and θ, here both are assumed to be non-negative thus the sign for the tangential Jacobian is positive. Replacing the Jacobian definitions into the dynamic equations we arrive at:

$$\begin{aligned} v_{k+1} = v_k + h \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} \\ + c_n \begin{bmatrix} 0 \\ 1/m \\ -\frac{a}{J\sqrt{2}}\sin(\pi/4 - \theta) \end{bmatrix} \\ + c_t \begin{bmatrix} 1/m \\ 0 \\ \frac{a}{J\sqrt{2}}\cos(\pi/4 - \theta) \end{bmatrix} \end{aligned} \quad (43)$$

To be able to reason about identifiability we observe that the velocity vector has the 3 elements v_x, v_y and v_θ, using this fact we can write:

$$\begin{aligned} v_{x,k+1} &= v_{x,k} + \frac{c_t}{m} \\ v_{y,k+1} &= v_{y,k} - hg + \frac{c_n}{m} \\ v_{\theta,k+1} &= v_{\theta,k}\frac{m}{J}\frac{a}{\sqrt{2}}\left(\frac{c_t}{m}\cos(\pi/4 - \theta) - \frac{c_n}{m}\sin(\pi/4 - \theta)\right) \end{aligned} \quad (44)$$

Therefore we can derive the values of c_t/m and c_n/m from the first two equations and replace these values in the final expression to find m/J. Based on this observation we can say that the vector:

$$\Theta = \left[\frac{c_n}{m} \; \frac{\beta}{m} \; \frac{m}{J}\right]^T \quad (45)$$

can be found uniquely to satisfy equation 43 and this observation is consistent with predictions made from identifiability analysis performed in previous sections.

VIII. Conclusions and Summary

In this paper we tackled the problem of identifiability of inertial parameters and contact forces for a rigid body as it interacts with the environment through contact. The problem was broken down into the scenarios of sticking and sliding contact, with and without the presence of known external forces acting on the body (other than gravity) and we found that given a time history of the kinematic measurements of the object, i.e. its positions, orientations and derivatives of these quantities, without external force the parameters identifiable are the ratio of mass moment of inertia to mass of the object, and the ratio of the tangential and normal forces to mass. We also found that given a known external force (other than gravity) acting on the object during the contact phase, that mass and mass moment of inertia could be decoupled and so both could be found uniquely, consequently the tangential and normal forces could also be found in absolute terms. We further demonstrated the applicability of the identifiability analysis on a 2-D free-falling rigid block that undergoes rigid body impact with the environment and showed that the results proved to be consistent with the predictions made by the identifiability analysis.

IX. Limitations and Future Work

In this paper we addressed the question of identifiability of inertial parameters and contact forces of an object making and breaking contact with the environment with or without known external forces. The analysis was performed under assumptions which constitute the limitations of the work and serve as possible motivation for future efforts in this type of analysis. Future work could address issues such as:

1) The analysis involved only a single rigid body undergoing impact and not a system of bodies. Future analysis can identify how the addition of more rigid bodies can affect identifiability. The authors preliminary work shows that if the interconnection of the bodies is with passive elements such as springs or dampers or revolute/linear joints then masses or mass moments of inertias of individual bodies are not identifiable, only combinations of these elements are available without external forces applied.
2) The body undergoing impact was assumed to be rigid, if this assumption is lifted more parameters should be involved in characterizing the contact mechanics and the analysis would need to change to incorporate the additional parameters characterizing deformations. Removal of the rigid body assumption will result in a *nonlinear complementarity problem* for which fewer results and guarantees exist when compared to the LCP formulation presented here.
3) It was implicitly assumed that the geometry of the body is well known, in fact the key component to the calculation of the Jacobians (normal and tangential) at least locally was knowledge of the geometry of the contact point with respect to the center of mass. Further work needs to be done to understand how the analysis can be extended to incorporate unknown geometry.

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