

# 1 Supplementary for Lemma 3.5

The following is the list of values  $p_1, p_2, q_1, q_2$  after the specified change of coordinates, as well as the domain of the new variables  $u, v$ .

Case 0:  $0 \leq i < b - \tau - 1$ . Change the variables with  $u(i) = \frac{i}{b}$  and  $v = \frac{1}{b}$ .  $0 \leq u \leq \frac{1}{2}, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} 150u(i) + 865v - 95 & -450u(i) - 855v + 285 \\ 50u(i) + 675v - 225 & -150u(i) - 1445v + 675 \end{pmatrix}.$$

Case 1:  $i = b - \tau - 1$ . Change the variables with  $u = \frac{\tau}{b}$  and  $v = \frac{1}{b}$ .  $\frac{5}{11} \leq u \leq 1, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} -150u(i) - 635v + 55 & 450u(i) + 165v - 165 \\ -50u(i) + 175v - 175 & 150u(i) - 1105v + 525 \end{pmatrix}.$$

Case 2:  $b - \tau \leq i < 2b - 2\tau - 2$ . Change the variables with  $u(i) = \frac{i}{b}$  and  $v = \frac{1}{b}$ .  $0 \leq u \leq 1, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} 150u(i) + 565v - 95 & -450u(i) + 45v + 285 \\ 50u(i) + 575v - 225 & -150u(i) - 1145v + 675 \end{pmatrix}.$$

Case 3:  $i = 2b - 2\tau - 2$ . Change the variables with  $u = \frac{\tau}{b}$  and  $v = \frac{1}{b}$ .  $\frac{5}{11} \leq u \leq 1, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} -600u(i) + 1025v + 410 & 1800u(i) + 1681v - 1230 \\ -200u(i) - 625v - 250 & 600u(i) - 1025v + 750 \end{pmatrix}.$$

Case 4:  $i = 2b - 2\tau - 1$ . Change the variables with  $u = \frac{\tau}{b}$  and  $v = \frac{1}{b}$ .  $\frac{11}{24} \leq u \leq 1, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} -600u(i) + 445v + 410 & 1800u(i) + 869v - 1230 \\ -200u(i) + 2275v - 250 & 600u(i) - 1605v + 750 \end{pmatrix}.$$

Case 5:  $2b - 2\tau - 1 < i \leq b - 1$ . Change the variables with  $u(i) = \frac{i}{b}$  and  $v = \frac{1}{b}$ .  $0 \leq u \leq 1, 0 \leq v \leq \frac{1}{12}$  and

$$\begin{pmatrix} p_1(i) & p_2(i) \\ q_1(i) & q_2(i) \end{pmatrix} = \frac{80}{29} \begin{pmatrix} 150u(i) + 565v - 95 & -450u(i) + 45v + 285 \\ 50u(i) + 575v - 225 & -150u(i) - 1145v + 675 \end{pmatrix}.$$

*E-mail:* alibabaei.nima.28c@st.kyoto-u.ac.jp

DEPARTMENT OF MATHEMATICS, KYOTO UNIVERSITY, KYOTO 606-8501, JAPAN