Excersise 1

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0.1 Calculus

0.1.1 Differentiation

The curve $y = f(x)$ has a slope at point $x = a$ given by the derivative of f with respect to x at a:

$$f^{'}(a) = \frac{df}{dx}|_a = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}. \tag{1}$$

A few particular derivatives are:

$$\begin{array}{ll} f(x) = ax^n & f^{'}(x) = anx^{n-1} \\ f(x) = e^{\lambda x} & f^{'}(x) = \lambda e^{\lambda x} \\ f(x) = u(x)v(x) & f^{'}(x) = u^{'}(x)v(x)|u(x)v^{'}(x) \\ f(x) - u(x)/v(x) & f^{'}(x) - [u^{'}(x)v(x) - u(x)v^{'}(x)]/v^2(x). \end{array}$$

0.1.2 Integration

Integration is the opposite of differentiation:

$$\int_a^b f'(x)dx = [f(x)]_a^b = f(b) - f(a) \quad or \quad \int f'(x)dx = f(x) \tag{2}$$

and it may also be seen as giving the area under a curve - so the integral $\int_a^b f(x)dx$ gives the area under the curve $y = f(x)$ between $x = a$ and $x = b$. Some specific integrals are:

$$\begin{array}{cc} \int x^n dx = x^{n+1}/(n+1) \text{ for } n \neq -1 & \int x^{-1}dx = lnx \\ \int e^{\lambda x}dx = e^{\lambda x}/\lambda & \int u(x)v'(x)dx = [u(x)v(x)] - \int u'(x)v(x)dx \end{array}$$

0.2 Lines and Circles

The vector equation of a straight line in three-dimensional space is

$$\underline{x} \quad \underline{a} + v\underline{b} \text{ with } u \in (-\infty, \infty) \text{ a real scalar.} \tag{3}$$

1. Identify the set of variables, the objective function, the set of constraints, the sign confitions and the solution set of the following Mathematical Programming problem.

   (a) Min x + y
   s.t. $2x + y \leq 3$
   $x, y \geq 0$

   (b) Max $x^2 + y^2 + z^2$
   s.t. $x + y \leq 6$
   $x + 3y \leq 8$
   $x + 2y + z \leq 20$
   $x, y \geq 0$

2. Draw two level curves for each of the following functions.
   (a) $f(x, y) = 3x - 2y$ (b) $g(x, y) = x^2 + y^2$ (c) $h(x, y) = x + 5y$

3. Draw the following sets and say if they are closed and/or bounded sets.

   (a) $S = \{(x, y) \in \mathbb{R}^2 / x + y \leq 6, 2x + y \geq 3, x \geq 0, y \geq 0\}$
   (b) $S = \{(x, y) \in \mathbb{R}^2 / x + y \leq 6, x + 2y \geq 3, y \geq 0\}$
   (c) $S = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 9\}$

4. Solve the following problems graphically.

$I) Max.\ \ x + y$	$II) Max.\ \ 2x + 3y$	$III) Max.\ \ x - y$
$s.t.\ \ 2x + y \geq 3$	$s.t.\ \ x + 2y \leq 10$	$s.t.\ \ 3x + 2y \leq 5$
$x + 2y \geq 3$	$x \geq 20$	$x \geq 0, y \geq 0$
$x + y \leq 6$	$x \geq 0, y \geq 0$	
$x \geq 0, y \geq 0$		

$IV) Max.\ \ x^2 + y^2$	$V) Max.\ \ x^2 + y^2$	$VI) Max.\ \ x + y$
$s.t.\ \ x + y = 1$	$s.t.\ \ x + y = 1$	$s.t.\ \ x + 4y \geq 5$
		$x \geq 0, y \geq 0$

Function: IsThisNumberPrime