Handin G1

Thomas Vinther & Morten Fausing & Jens Kristian Refsgaard Nielsen

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Kleene star case:
The following will become useful, consider for $i > 0$

$$\begin{aligned}
\nabla : \mathrm{Pre}(\mathcal{L}(E)^i) &= \mathrm{Pre}(\mathcal{L}(E)^{i-1}\mathcal{L}(E)) \\
&\overset{1)}{=} \mathrm{Pre}(\mathcal{L}(E)^{i-1}) \cup \mathcal{L}(E)^{i-1} \cdot \mathrm{Pre}(\mathcal{L}(E)) \\
&= \left(\mathrm{Pre}(\mathcal{L}(E)^{i-2}) \cup \mathcal{L}(E)^{i-2} \cdot \mathrm{Pre}(\mathcal{L}(E))\right) \cup \mathcal{L}(E)^{i-1} \cdot \mathrm{Pre}(\mathcal{L}(E)) \\
&\vdots \\
&= \bigcup_{0 \leq j < i} \mathcal{L}(E)^j \cdot \mathrm{Pre}(\mathcal{L}(E)) \\
&= \left(\bigcup_{0 \leq j < i} \mathcal{L}(E)^j\right) \cdot \mathrm{Pre}(\mathcal{L}(E))
\end{aligned}$$

By applying part 1 of the assignment a finite number of times or a short induction omitted here, we see that taking powers of prefix languages it is sufficient to concatenate on the right by the prefix language of the base expression.
Let $E_1 = E^*$ in the name of readability, and consider

$$\begin{aligned}
\mathrm{Pre}\mathcal{L}(E_1) = \mathrm{Pre}\mathcal{L}(E^*) &= \mathrm{Pre}\left(\bigcup_{i \geq 0} \mathcal{L}(E)^i\right) \\
&= \bigcup_{i \geq 0} \mathrm{Pre}(\mathcal{L}(E)^i) \overset{\nabla}{=} \bigcup_{i \geq 0} \mathcal{L}(E)^i \mathrm{Pre}(\mathcal{L}(E)) \\
&= \mathcal{L}(E^*)\mathrm{Pre}(\mathcal{L}(E)) = \mathcal{L}(E^*)\mathcal{L}(E') \\
&= \mathcal{L}(E^* E')
\end{aligned}$$

By applying the induction hypthesis on the base expression E we obtain a regular expression E' such that $\mathrm{Pre}(\mathcal{L}(E)) = \mathcal{L}(E')$, which we use in the last line of the proof.