Handin 8

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1 Reachability

Let $G = (V, E)$ be a directed graph in which each vertex $u \in V$ is labeled with a unique integer $L(u)$ from the set $\{1, 2, \ldots, |V|\}$. For each vertex $u \in V$, let $R(u) = \{v \in V : u \rightsquigarrow v\}$ be the set of vertices that are reacheable from u. Define $\min(u)$ to be the vertex in $R(U)$ whose label is minimum. Give an $O(V + E)$-time algorithm that computes $\min(u)$ for all vertices $v \in V\mathcal{L}$.

1.1 Solution

Consider the following algorithm.

Time	Line nr	Pseudocode
$\Theta(V)$	1	for $v \in V$
1	2	$\min(u) = \infty$
O$(V + E)$	3	SCC(G)
O$(V + E)$	4	Topological-Sort(G^{SCC})
O(V^{SCC})	5	for $v \in V^{SCC}$ in reverse Topological order
O(V^{SCC})	6	$m = \min\{\mathcal{L}(z) \mid z \in V^{SCC}, (v, z) \in E^{SCC}\}$, under convention $\min(\varnothing) = \infty$
	7	if the SCC v represents conatains a loop in G
O(V)	8	$\mathcal{L}(v) = \min\left\{m, \min\{L(u) \mid u \in v \cap V\}\right\}$
1	9	else $\mathcal{L}(v) = m$
O(V)	10	for $u \in v \cap V$
1	11	$\min(u) = \mathcal{L}(v)$

Correctness: Note that this problem has optimal substructure, when we calculate the L function we need only consider nodes that are greater in the topological sort of the DAG G^{SCC}, because these are the only nodes we can communicate with