Handin G9

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1 Expressibility of Arithmetic

The goal is to show how the logic of arithmetic can express: There exists a cycle (of any length).

$$\forall n \exists v \exists p \text{ prime}(p) \land \forall i((i < n) \to v[i] \Rightarrow v[i+1]) \land v[n] = v[0])$$

We wish to express that the binary relation $\Rightarrow$ has cycles of any length, to do this we use a for all over n where n is supposed to be the length of the cycle. For each n we then need a "vector" v that is represented as seen in the lectures with a prime p, with the formula $v[i] = v \div p^i \mod p$. Now we say that for all i if it is strictly smaller than n we wish the vector element $v[i]$ to be in relation with the following element $v[i+1]$, and for the head of the vector to be equal to the end of the vector, so we get the length n cycle

$$v[0] \Rightarrow v[1] \Rightarrow v[2] \Rightarrow \cdots \Rightarrow v[n-1] \Rightarrow v[n] = v[0]$$

as wanted.
Instead of demanding $v[n] = v[0]$ we could say $v[n-1] \Rightarrow v[0]$ and reduce the $i < n$ criteria to $i < n - 1$, however this is equivalent.
Note also that this criterion is non trivial, since if the relation is reflexive such that there is an x such that $x \Rightarrow x$ you can append this onto itself as many times as you want, and achieve cycles of any length. If on the other hand the relation is always false you cannot make cycles of any length at all.