

---

# Mining Data Streams

**Panagiotis Karras**  
Aarhus U, Spring 2019

Implementation and Applications of Databases

1

## Today's Agenda

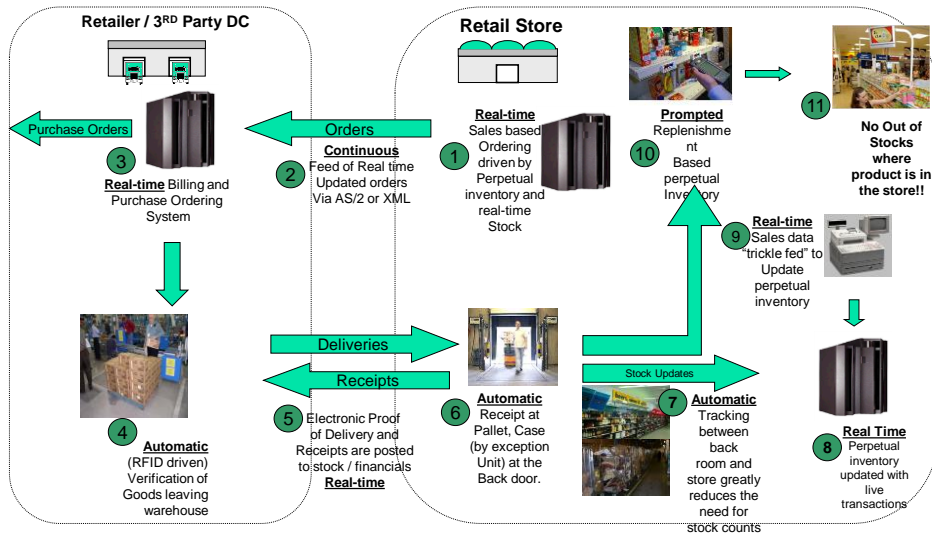
---

- Understand what a DSMS does.
- Queries on streams.
- Sliding Windows
- Counting on a stream:
  - The DGIM method.
  - How to derive an error bound.
- Counting distinct elements.
- Moments of a stream.

Implementation and Applications of Databases

2

## Motivating Example: Store Replenishment Process



Implementation and Applications of Databases

3

## Motivating Example: Production Control System



Implementation and Applications of Databases

4

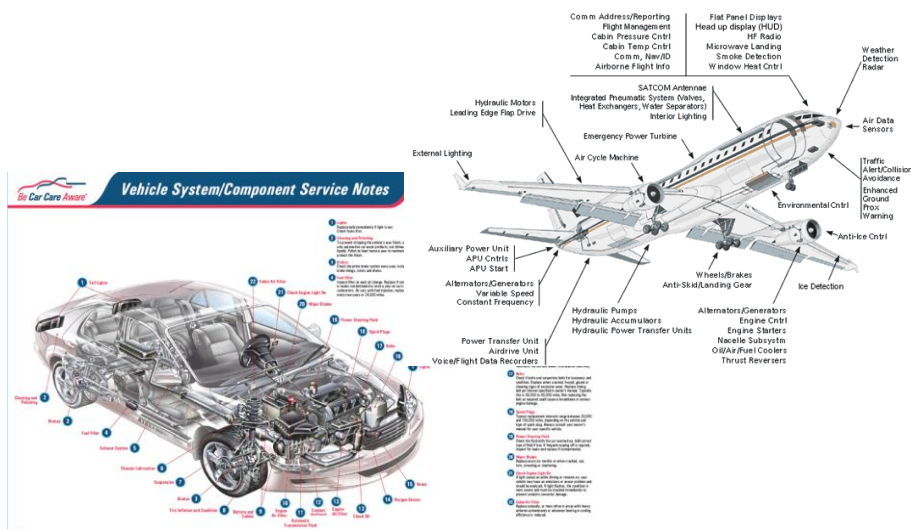
# Motivating Example: Production Control System



Implementation and Applications of Databases

5

# Motivating Examples: Monitoring Vehicle Operation



# Motivating Example: Financial Applications



Implementation and Applications of Databases

7

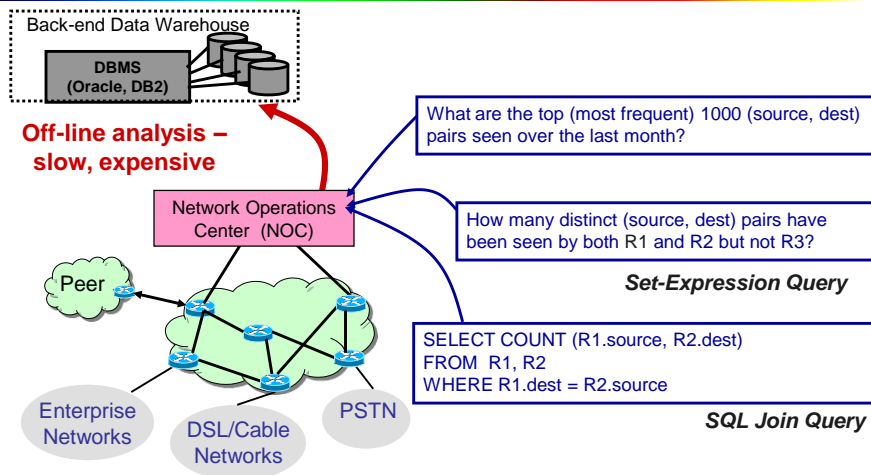
# Motivating Examples: Web Data Streams

- Mining query streams.
  - Google wants to know what queries are more frequent today than yesterday.
- Mining click streams.
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour.

Implementation and Applications of Databases

8

## Motivating Examples: Network Monitoring

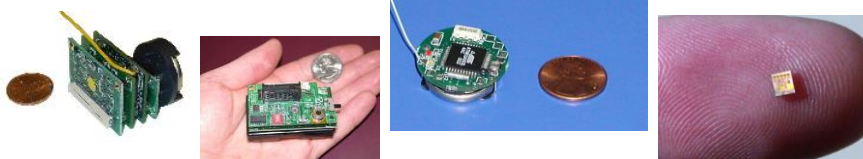


Implementation and Applications of Databases

9

## Motivating Examples: Sensor Networks

- the **sensors** era
  - ubiquitous, small, inexpensive sensors
  - bridging physical world and information technology

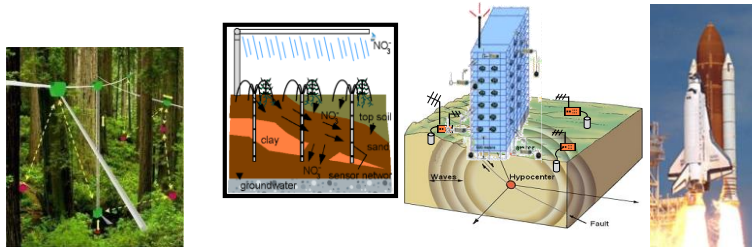


Implementation and Applications of Databases

10

## Motivating Examples: Sensor Networks

- the **sensors** era
  - ubiquitous, small, inexpensive sensors
  - bridging physical world to information technology
- unveil previously unobservable phenomena



Implementation and Applications of Databases

11

## What all this requires

- Efficient streaming algorithms
  - process this data online
  - allow approximate answers
  - operate in a distributed fashion (network as distributed database)
  - also usable as **one-pass** algorithms for massive datasets
- new **data mining** algorithms
  - help in data analysis in the above setting

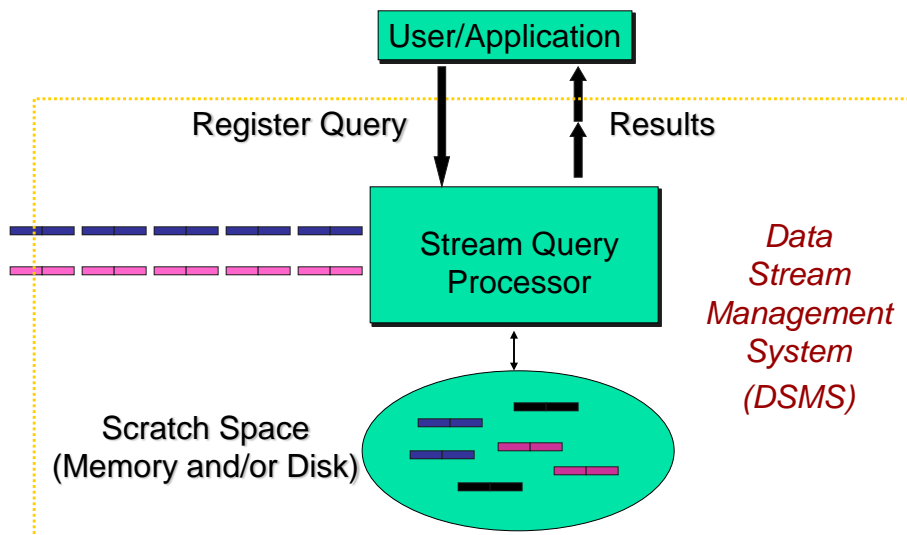
## Data Stream Management System?

- **Traditional DBMS** – data stored in **finite, persistent data sets**
- **New Applications** – data input as **continuous, ordered data streams**
  - Network monitoring and traffic engineering
  - Telecom call records
  - Network security
  - Financial applications
  - Sensor networks
  - Manufacturing processes
  - Web logs and clickstreams
  - Massive data sets

Implementation and Applications of Databases

13

## Data Stream Management System!



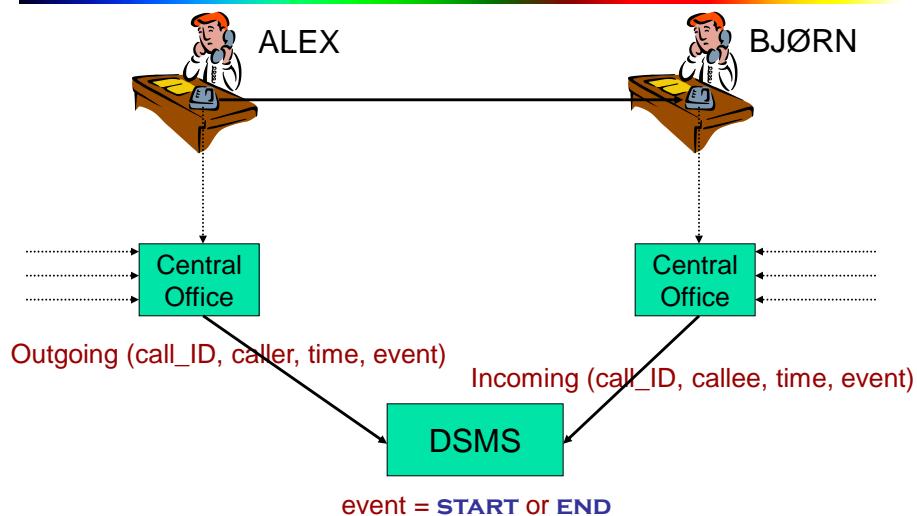
Implementation and Applications of Databases

14

# DBMS versus DSMS

- Persistent relations
- One-time queries
- Random access
- "Unbounded" disk store
- Only current state matters
- Passive repository
- Relatively low update rate
- No real-time services
- Precise answers
- Access plan determined by query processor, physical DB design
- Transient streams
- Continuous queries
- Sequential access
- Bounded main memory
- History/arrival-order is critical
- Active stores
- Possibly multi-GB arrival rate
- Real-time requirements
- Imprecise/approximate answers
- Access plan dependent on variable data arrival and data characteristics

## Making Things Concrete





## Query 1 (SELF-JOIN)

- Find all **outgoing calls** longer than **2 minutes**

```
SELECT O1.call_ID, O1.caller
FROM   Outgoing O1, Outgoing O2
WHERE  (O2.time - O1.time > 2
        AND O1.call_ID = O2.call_ID
        AND O1.event = START
        AND O2.event = END)
```

- Result requires **unbounded storage**
- Can provide **result as data stream**
- Can output after 2 min, **without seeing END**

## Query 2 (JOIN)

- Pair up **callers** and **callees**

```
SELECT O.caller, I.callee
FROM   Outgoing O, Incoming I
WHERE  O.call_ID = I.call_ID
```

- Can still provide **result as data stream**
- Requires **unbounded temporary storage ...**
- ... unless streams are **near-synchronized**

## Query 3 (group-by aggregation)

- **Total connection time** for each caller

```
SELECT      O1.caller, sum(O2.time - O1.time)
FROM        Outgoing O1, Outgoing O2
WHERE       (O1.call_ID = O2.call_ID
            AND O1.event = START
            AND O2.event = END)
GROUP BY    O1.caller
```

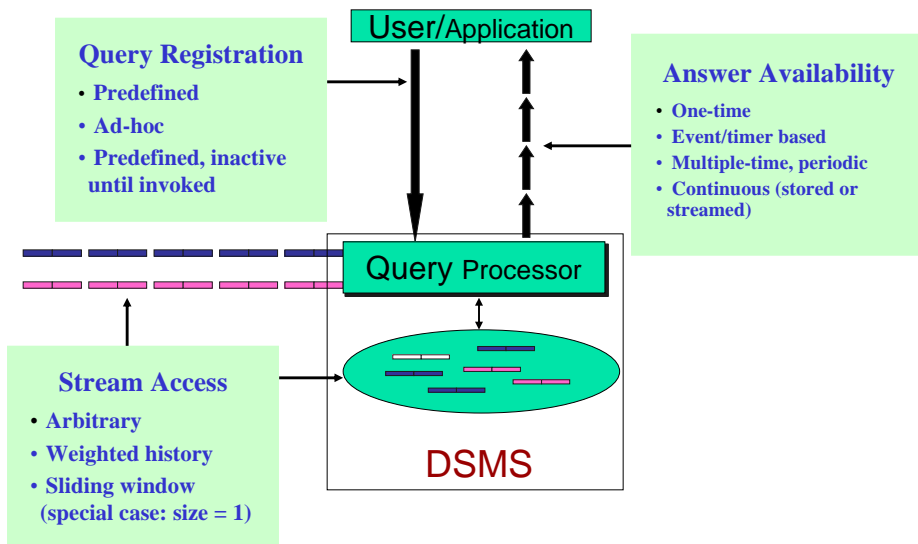
- **Cannot provide result in (append-only) stream**
  - Output **updates**?
  - Provide current value **on demand**?
  - **Memory**?

## Data Model

- **Append-only**
  - Call records
- **Updates**
  - Stock tickers
- **Deletes**
  - Transactional data
- **Meta-Data**
  - Control signals, punctuations

**System Internals** – probably need all of the above

# Query Model



Implementation and Applications of Databases

21

## Related Database Technology

- **DSMS must use ideas, but none is substitute**
  - Triggers, Materialized Views in Conventional DBMS
  - Main-Memory Databases
  - Distributed Databases
  - Pub/Sub Systems
  - Active Databases
  - Sequence/Temporal/Timeseries Databases
  - Realtime Databases
  - Adaptive, Online, Partial Results
- **Novelty in DSMS**
  - **Semantics:** input ordering, streaming output, ...
  - **State:** cannot store unending streams, yet need history
  - **Performance:** rate, variability, imprecision, ...

Implementation and Applications of Databases

22

## Unrestricted Window

---

- Queries refer to all data in a *window* that starts at the “beginning of time”, extends up to the current time, and expands with time (potentially infinite length).

## Unrestricted Window

---

qwertuiopasdfghjklzxcvbnm

← Past                      Future →

# Unrestricted Window

qwertyuioasdfghjklzxcvbnm

qwertyuioasdfghjklzxcvbnm

← Past Future →

Implementation and Applications of Databases

25

# Unrestricted Window

qwertyuioasdfghjklzxcvbnm

qwertyuioasdfghjklzxcvbnm

qwertyuioasdfghjklzxcvbnm

← Past Future →

Implementation and Applications of Databases

26

## Unrestricted Window

qwertyuioasdfghjklzxcvbnm

qwertyuioasdfghjklzxcvbnm

qwertyuioasdfghjklzxcvbnm

...

qwertyuioasdfghjklzxcvbnm

← Past Future →

Implementation and Applications of Databases

27

## Unrestricted Window

- Queries refer to all data in a *window* that starts at the “beginning of time”, extends up to the current time, and expands with time (potentially infinite length).
- What happens when we try to compute *joins* in this model?
  - Join results involving some piece of data may appear at any time in the future
  - In order to correctly compute the result, we need to store *all* values that have appeared in the past!

Implementation and Applications of Databases

28

## Shifting Window

- Queries are about a *window* of length  $N$ , which advances by  $N$ , where  $N$  are the most recent elements received, or the most recent time units.

## Shifting Window

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past                      Future →

## Shifting Window

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Implementation and Applications of Databases

31

## Shifting Window

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Implementation and Applications of Databases

32



## Shifting Window

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Implementation and Applications of Databases

33

## Shifting Window

- Queries are about a *window* of length  $N$ , which advances by  $N$ , where  $N$  are the most recent elements received, or the most recent time units.
- Useful queries within this model:
  - average number of calls every day
  - std deviation of packet losses every 10 minutes
  - etc.

Implementation and Applications of Databases

34

## Sliding Window

- Queries are about a *window* of length  $N$ , covering the  $N$  most recent elements received, or most recent time units.

## Sliding Window

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past                      Future →

## Sliding Window

---

- Queries are about a *window* of length  $N$ , covering the  $N$  most recent elements received, or most recent time units.
- **Interesting case:**  $N$  is so large it cannot be stored in memory, or even on disk.
  - Or, there are so many streams that we cannot store the values for all windows.

## Counting Bits --- (1)

---

- **Problem:** given a stream of 0's and 1's, be prepared to answer queries of the form "how many 1's in the last  $k$  bits?" where  $k \leq N$ .
- **Obvious solution:** store the most recent  $N$  bits.
  - When new bit comes in, discard the  $N+1^{\text{st}}$  bit.

## Counting Bits --- (2)

---

- You can't get an exact answer without storing the entire window.
- **Real Problem:** what if we cannot afford to store  $N$  bits?
  - E.g., we are processing 1 trillion streams and  $N = 1$  trillion, but we're happy with an approximate answer.

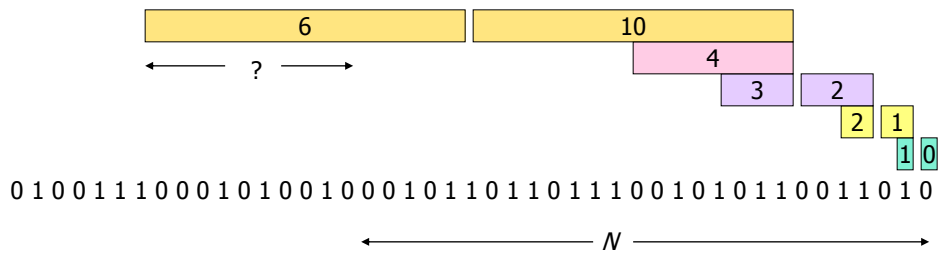
## Something That Doesn't (Quite) Work

---

- Summarize exponentially increasing regions of the stream, looking backward.
- Drop small regions if they begin at the same point as a larger (included) region.

## Example

We can construct the count of the last  $N$  bits, except we're not sure how many of the last 6 are included.



## What's Good?

- Stores only  $O(\log^2 N)$  bits.
  - $O(\log N)$  counts of  $\log_2 N$  bits each.
- Easy update as more bits enter.
- Error in count no greater than the number of 1's in the "unknown" area.

## What's Not So Good?

---

- As long as the 1's are fairly evenly distributed, the error due to the unknown region is small --- no more than 50%.
- But it could be that all the 1's are in the unknown area at the end.
- In that case, the (relative) error is unbounded! (we may miss them all and get 0 as an answer).

## Fixup

---

- Instead of summarizing fixed-**length** blocks, summarize blocks with a fixed **number** of 1's.
  - Let the block "sizes" (number of 1's) increase exponentially.
- By controlling the number of 1's in the window, we ensure errors are small.

## The DGIM\* Method

---

- Store  $O(\log^2 N)$  bits per stream.
- Gives approximate answer, *never off by more than 50%*.
  - Error factor can be reduced to any fraction  $> 0$ , with more complicated algorithm and proportionally more stored bits.

\*Datar, Gionis, Indyk, and Motwani

## Timestamps

---

- Each bit in the stream has a *timestamp*, starting 1, 2, ...
- Record timestamps modulo  $N$  (the window size), so we can represent any *relevant* timestamp in  $O(\log_2 N)$  bits.

# Buckets

- A *bucket* in the DGIM method is a record consisting of:
  1. The timestamp of its end [ $O(\log N)$  bits].
  2. The number of 1's between its beginning and end [ $O(\log \log N)$  bits].
- **Constraint:** number of 1's must be a power of 2.
  - That explains the  $\log \log N$  in (2). (**how?**)
  - It suffices to store its log only.

## Representing a Stream by Buckets

- 1 or 2 buckets with the same power-of-2 number of 1's.
- Buckets **do not** overlap in timestamps.
- Buckets are sorted by *size* (# of 1's).
  - Earlier buckets are not smaller than later buckets.
- Buckets disappear when their end-time is  $> N$  time units in the past.





## Updating Buckets --- (1)

- ## Implementation and Applications of Databases

25

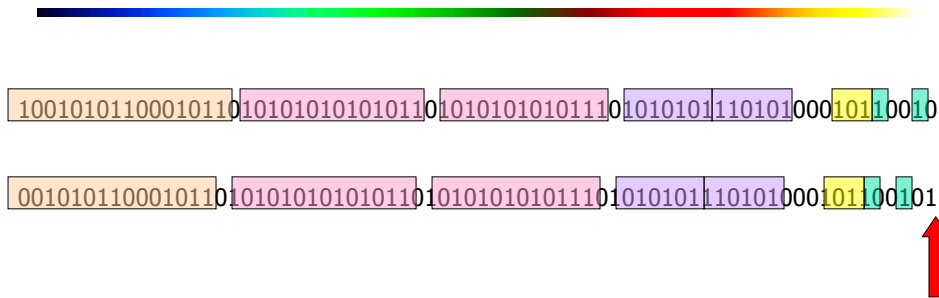
## Updating Buckets --- (2)

- If the current bit is 1:
  1. Create a new bucket of size 1, for just this bit.
    - ◆ End timestamp = current time.
  2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2.
  3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4.
  4. And so on...

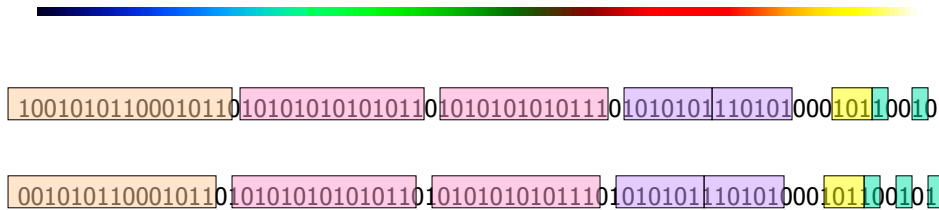
## Example

10010101100010110 101010101010110 101010101010110 1010101110101 00010110010

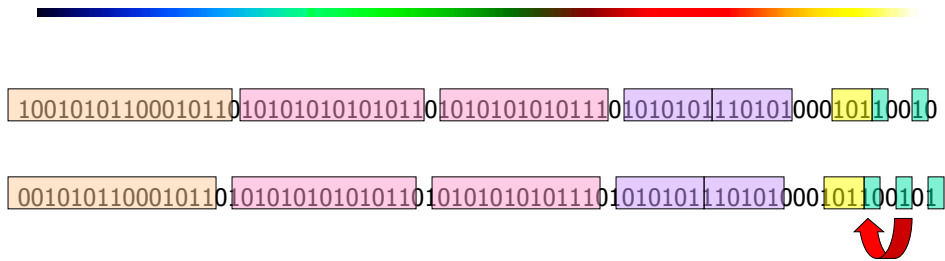
## Example



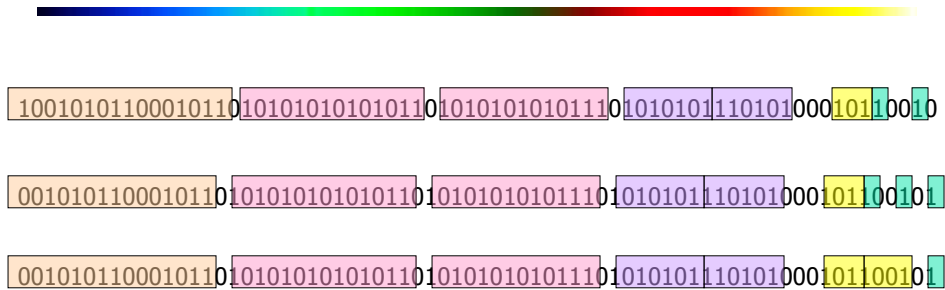
## Example



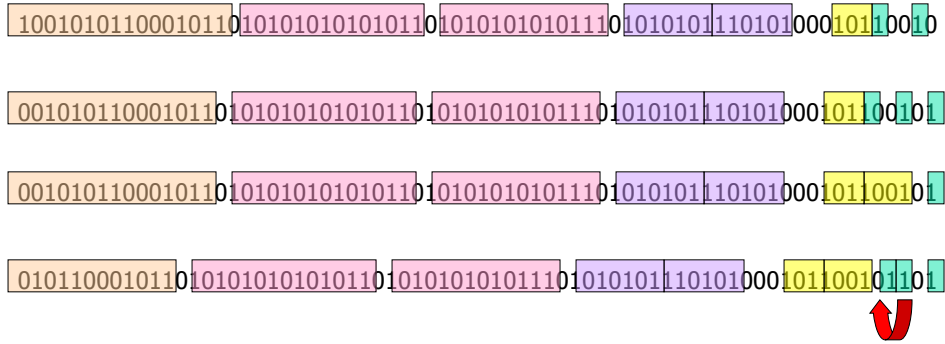
## Example



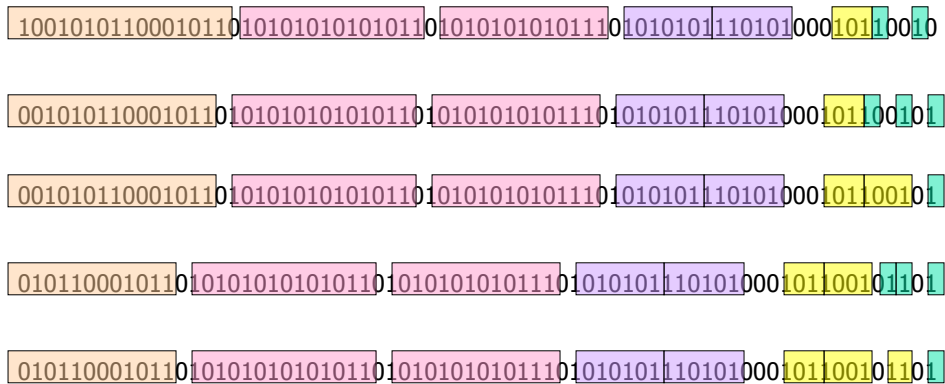
## Example



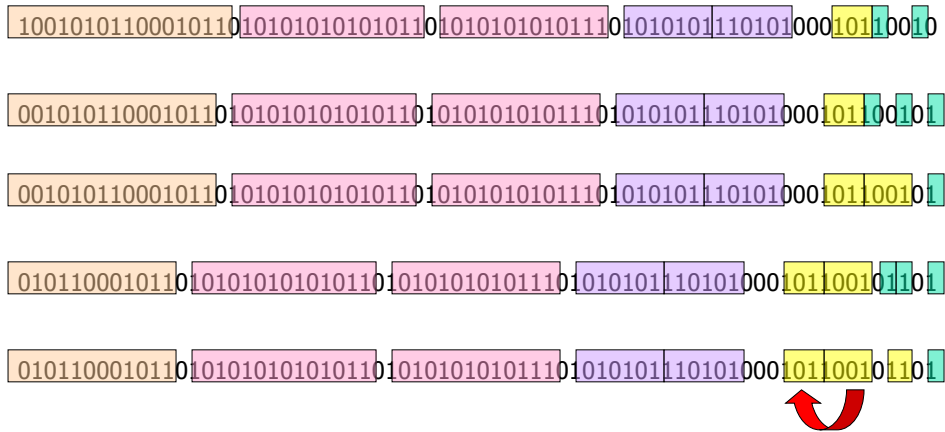
## Example



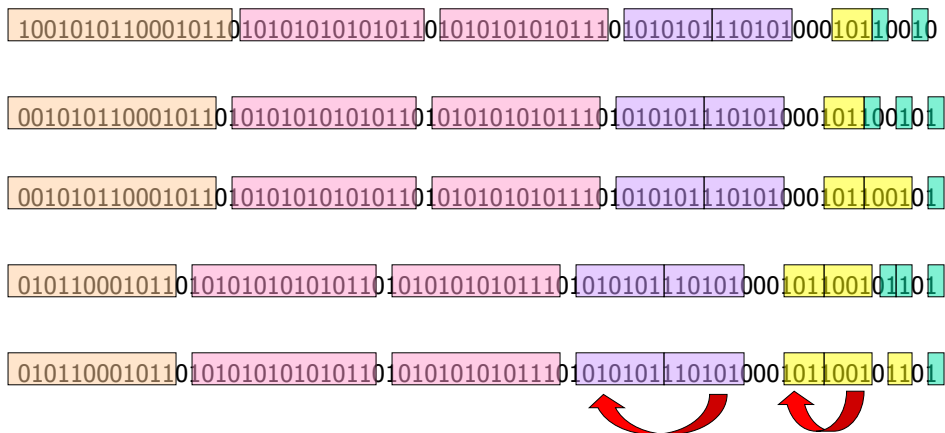
## Example



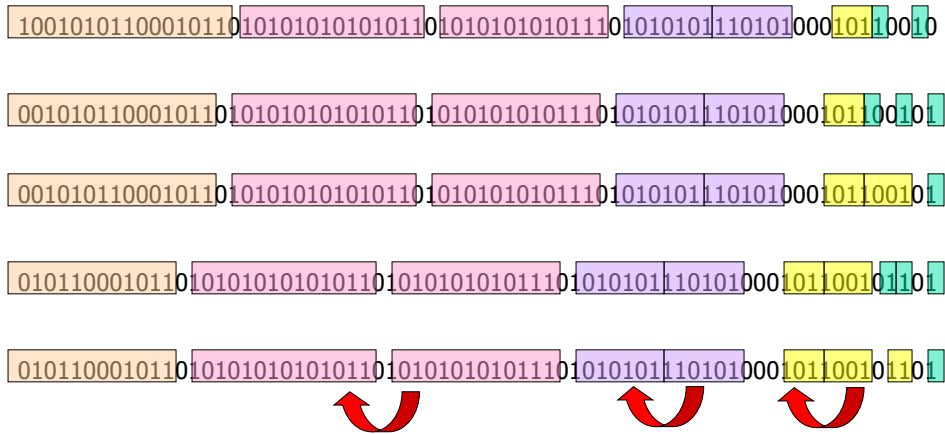
## Example



## Example



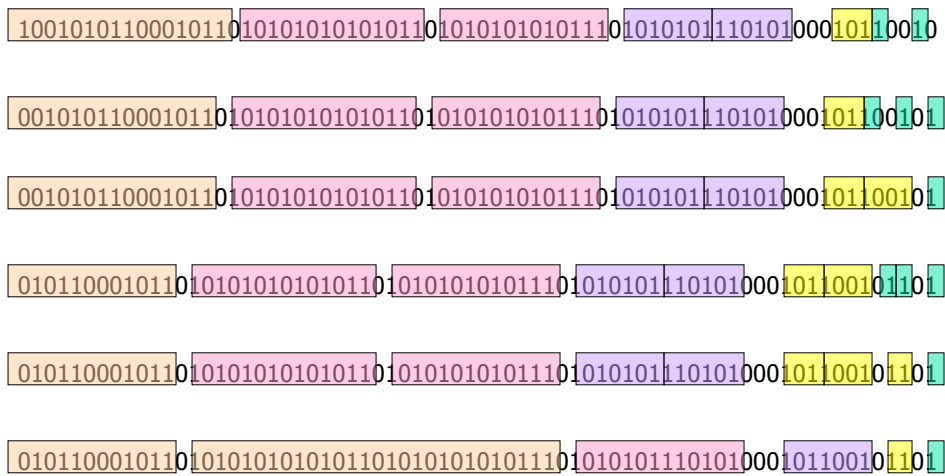
## Example



Implementation and Applications of Databases

61

## Example



Implementation and Applications of Databases

62

## Answering Queries

- To estimate the number of 1's in the most recent  $N$  bits:
  1. Sum the sizes of all buckets but the last.
  2. Add in half the size of the last bucket.
- Remember, we don't know how many 1's of the last bucket are still within the window.

## Error Bound

- Suppose the last bucket has size  $2^k$ .
- Then, when we assume  $2^{k-1}$  of its 1's are still within the window, we make an error of **at most**  $2^{k-1}$ .
- Since there is at least one bucket for each size from 1 to  $2^{k-1}$ , the true sum is **no less** than  $2^{k-1}$ .
- Thus, the relative error is at most 50%.



## Counting Distinct Elements

---

- **Problem:** a data stream consists of elements chosen from a set of size  $n$ . Maintain a count of the number of distinct elements seen so far.
- **Obvious approach:** maintain the set of elements seen.

## Applications

---

- How many different words are found among the Web pages being crawled at a site?
  - Unusually low or high numbers could indicate artificial pages (spam).
- How many different Web pages does each customer request in a week?

## Using Small Storage

- **Real Problem:** what if we do not have space to store the complete set?
- Estimate the count in an unbiased way.
- Accept that the count may be in error, but limit the probability that the error is large.

## Flajolet-Martin\* Approach

- Pick a hash function  $h$  that maps each of the  $n$  elements to at least  $\log_2 n$  bits.
- For each stream element  $a$ , let  $r(a)$  be the number of trailing 0's in  $h(a)$ .
- Record  $R =$  the maximum  $r(a)$  seen.
- Estimate  $= 2^R$ .

\* Really based on a variant due to AMS (Alon, Matias, and Szegedy)

## Why It Works

- The probability that a given  $h(a)$  ends in **at least**  $r$  0's is  $2^{-r}$ .
- If there are  $m$  different elements, the probability that  $R \geq r$  is:

$$1 - (1 - 2^{-r})^m$$

Prob. all  $h(a)$ 's end in **fewer** than  $r$  0's.      Prob. a given  $h(a)$  ends in **fewer** than  $r$  0's.

Implementation and Applications of Databases

69

## Why It Works – (2)

- Since  $2^{-r}$  is small,  $1 - (1 - 2^{-r})^m \approx 1 - e^{-m2^{-r}}$ .
- If  $2^r \gg m$ ,  $1 - (1 - 2^{-r})^m \approx 1 - (1 - m2^{-r}) \approx m/2^r \approx 0$ .  
 First 2 terms of the Taylor expansion of  $e^x$
- If  $2^r \ll m$ ,  $1 - (1 - 2^{-r})^m \approx 1 - e^{-m2^{-r}} \approx 1$ .
- Thus,  $2^R$  will almost always be around  $m$ .

Implementation and Applications of Databases

70

## Why It Does not Work

---

- $E(2^R)$  is actually infinite.
  - When  $R$  becomes  $R + 1$ , probability halves, but value doubles.
- Workaround: use many hash functions, get many samples.
- How are samples combined?
  - **Average**? What if one very large value?
  - **Median**? All values are a power of 2.

## Solution

---

- Partition your samples into small groups.
- Take the **average** of groups.
- Then take the **median** of the averages.

## Generalization: Moments

---

- Suppose a stream has elements chosen from a set of  $n$  values.
- Let  $m_i$  be the number of times value  $i$  occurs.
- The  $k^{\text{th}}$  *moment* is the sum of  $(m_i)^k$  over all  $i$ .

## Special Cases

---

- 0<sup>th</sup> moment = number of different elements in the stream.
  - The problem just considered.
- 1<sup>st</sup> moment = count of the numbers of elements = length of the stream.
  - Easy to compute.
- 2<sup>nd</sup> moment = *surprise number* = a measure of how uneven the distribution is.

## Example: Surprise Number

---

- Stream of length 100; 11 values appear.
- **Unsurprising**: 10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9. Surprise # = 910.
- **Surprising**: 90, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1. Surprise # = 8,110.

## Thanks for slides to:

---

- Themis Palpanas
- Minos Garofalakis
- Divesh Srivastava
- Nick Koudas
- Jiawei Han
- Jeffrey Ullman
- Anand Rajaraman