Assignment I1

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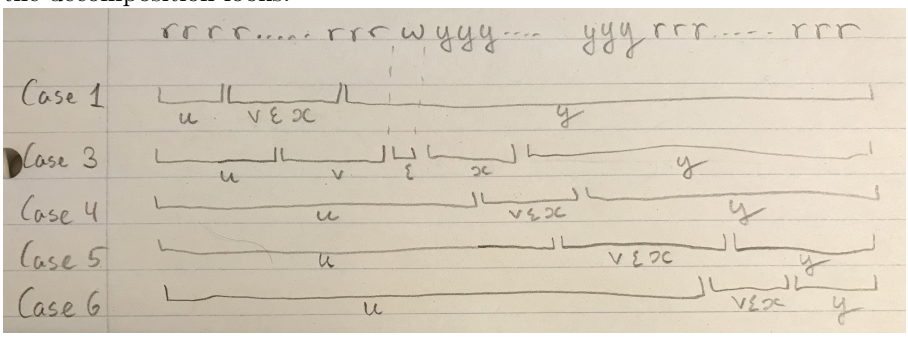
Question 1.

Claim: the language of the Spanish flag $L = \{r^n w y^{2n} r^n \mid n \geq 0\}$ is not context-free
Proof: we proceed by using the contraposition of the pumping lemma for context-free languages that states:

$$
\begin{aligned}
&X \subseteq \Sigma^* \text{ is not context free if:} \\
&\forall k \geq 1 \exists z \in X : |z| > k : \\
&\forall u, v, \epsilon, x, \zeta \in \Sigma^* : z = uv\epsilon x\zeta, |v\epsilon x| \leq k, |vx| > 0 : \\
&\exists i \geq 0 : uv^i\epsilon x^i\zeta \notin X
\end{aligned}
$$

So let $k \geq 1$ be given, we then choose $z = r^k w y^{2k} r^k$, and let a decomposition $u, v, \epsilon, x, \zeta \in \Sigma^* : z = uv\epsilon x\zeta, |v\epsilon x| \leq k, |vx| > 0$ be given. Now we proceed by looking at the possible cases of how the decomposition looks.



Case 1: $u = r^l, v\epsilon x = r^j, \zeta = r^{k-l-j} w y^{2k} r^k$ for $0 \leq l \leq k$ and $0 < j < k - l$
The condition $|vx| > 0$ forces atleast one of v and x to have atleast one r, so if we choose to pump with $i = 2$ we get atleast one more r in $v\epsilon x$ so $v^2\epsilon x^2 = r^p$ for $p > j$, consider then

$$uv^2\epsilon x^2\zeta = r^l r^p r^{k-l-j} w y^{2k} r^k = r^{k-j+p} w y^{2k} r^k \notin L$$

Because $p > j$ we get more r's in front of our expression than in the back, which is not allowed in L, concluding the case.
Case 2: If w is part of either v or x, simply pump $i = 2$ and we get two w's which our language doesnt support, completing the small case.
Case 3: $u = r^{k-l}, v = r^l, \epsilon = w, x = y^j, \zeta = y^{2k-j} r^k$ for $0 < l + 1 + j \leq k$
We have l and j not both zero, if
Case 4: $u = r^k w y^l, v\epsilon x = y^j, \zeta = y^{2k-j-l} r^k$ for $0 < j \leq k$ and $0 \leq l \leq k$
Case 5: $u = r^k w y^{2k-l}, v\epsilon x = y^l r^j, \zeta = r^{k-j}$ for $0 < l + j \leq k$

Case 6: $u = r^k w y^{2k} r^{k-l-j}, v\epsilon x = r^l, \zeta = r^j$ for $0 < l$ and $l + j \leq k$

Question 2.

Question 3.

Question 4.