

# List of decision problems

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## 1 The theory

A decision problem is something that takes an input and asks a question, we wish to determine whether it is possible to make a general algorithm that can answer this question.

### 1.1 Reduces to, $P_1 \leq P_2$

We say a decision problem  $P_1 = (I_1, Q_1)$  reduces to another problem  $P_2 = (I_2, Q_2)$  iff  $\exists F : I_1 \rightarrow I_2$  such that  $\forall i \in I_1 : i$  is a yes instance of  $Q_1$  iff  $F(i)$  is a yes instance of  $Q_2$ .

### 1.2 Reduction theorem

Given two problems and a reduction  $P_1 \leq P_2$  we get two very important results:

**Decidable** if  $P_2$  is decidable then so is  $P_1$ ,

*"if you live inside something solveable, then you too are solveable"*

**Undecidable** if  $P_1$  is undecidable then so is  $P_2$ ,

*"if something unsolveable lives inside of you, there is no salvation in more data"*

### 1.3 Language property

A property  $R$  on TMs is a language property if

$$\forall M, T \in \text{TM} : \mathcal{L}(M) = \mathcal{L}(T) \Rightarrow (R(M) \iff R(T))$$

And non trivial if there exists two TM's such that  $R(M)$  and  $\neg R(T)$ , i.e. there is a language with the property and one without it.

#### 1.3.1 Rice theorem

Every non-trivial language property of Turing machines is undecidable.

#### 1.3.2 Examples

## 2 The list

In order to decide whether something is decidable or not, it is very important to have a good list of results to use for our deductions. What follows are lists of problems handled in the lectures

and exercise classes of "Beregnelighed & Logik" at Aarhus university in the spring of 2019.  
Each will be of the form: Name I: input Q: question  
Notation:

TM = set of turing machines  
CFG = set of context free grammars  
REC = set of recursive languages  
RECE = set of recursive enumerable languages  
REG = set of regular languages

## 2.1 Decidable problems

$$\text{I: } G \in \text{CFG}, w \in \Sigma_G^* \quad \text{Q: } w \in \mathcal{L}(G)? \quad (2.1)$$

$$\text{I: } G \in \text{CFG} \quad \text{Q: } \mathcal{L}(G) = \emptyset? \quad (2.2)$$

$$\text{I: } G_1, G_2 \in \text{CFG} \quad \text{Q: } \mathcal{L}(G_1) \cup \mathcal{L}(G_2) = \emptyset? \quad (2.3)$$

$$\text{I: } \quad \text{Q: } \quad (2.4)$$

## 2.2 Undecidable problems

$$\text{Halting} \quad \text{I: } T \in \text{TM}, w \in \Sigma_T, \quad \text{Q: does } T \text{ halt on } w? \quad (2.5)$$

$$\text{Self-Accept} \quad \text{I: } M \in \text{TM} \quad \text{Q: Does } M \text{ halt on } e(M)? \quad (2.6)$$

$$\text{Accepts} \quad \text{I: } M \in \text{TM}, w \in \Sigma_M^* \quad \text{Q: Does } M \text{ accept } w? \quad (2.7)$$

$$\text{Halts} \quad \text{I: } M \in \text{TM}, w \in \Sigma_M^* \quad \text{Q: Does } M \text{ halt on } w? \quad (2.8)$$

$$\text{Accept-}\Lambda \quad \text{I: } M \in \text{TM} \quad \text{Q: } \Lambda \in \mathcal{L}(M)? \quad (2.9)$$

$$\text{Accepts All} \quad \text{I: } M \in \text{TM} \quad \text{Q: } \mathcal{L}(M) = \Sigma_M^*? \quad (2.10)$$

$$\text{Subset} \quad \text{I: } M_1, M_2 \in \text{TM} \quad \text{Q: } \text{in } \mathcal{L}(M_1) \subset \mathcal{L}(M_2)? \quad (2.11)$$

$$\text{Equivalent } \Lambda \quad \text{I: } M_1, M_2 \in \text{TM} \quad \text{Q: } \mathcal{L}(M_1) = \mathcal{L}(M_2) \quad (2.12)$$

$$\text{WriteSym} \quad \text{I: } M \in \text{TM}, a \in \Sigma_M^* \quad \text{Q: will } M \text{ write } a? \quad (2.13)$$

$$\text{PCP} \quad \text{I: } A, B : |A| = |B| \quad \text{Q: } \exists J : \Pi_{j \in J} A_{i_j} = \Pi_{j \in i} A_{i_j}? \quad (2.14)$$

$$\text{MPCP} \quad \text{I: } A, B : |A| = |B| \quad \text{Q: } \exists J : A_1 \Pi_{j \in J} A_{i_j} = B_1 \Pi_{j \in i} A_{i_j}? \quad (2.15)$$

$$\text{AmbiCFG} \quad \text{I: } G \in \text{CFG} \quad \text{Q: is } G \text{ ambiguous?} \quad (2.16)$$

$$\text{KitchenTiling} \quad \text{I: set of tiles} \quad \text{Q: can we tile ANY square with matching sides?} \quad (2.17)$$

## 2.3 Reductions

$$\text{Accepts} \leq \text{MPCP} \leq \text{PCP} \quad (2.18)$$

$$\text{SelfAccept} \leq \text{Accept} \leq \text{Halts} \quad (2.19)$$

$$\text{Accept} \leq \text{Accept-}\Lambda \leq \text{writeSym} \quad (2.20)$$

$$\text{Accept-}\Lambda \leq \text{AcceptAll} \leq \text{Subset} \leq \text{Equivalent} \quad (2.21)$$

$$\text{PCP} \leq \text{AmbiCFG} \quad (2.22)$$

$$\begin{aligned}
\int_a^b f(x)dx &= \lim_{y \rightarrow b} \lim_{z \rightarrow a} \int_z^y f(x)dx \\
&= (\lim_{y \rightarrow b} F(y)) - (\lim_{z \rightarrow a} F(z))
\end{aligned}$$