Discrete Assignment
EE1205 Signals and Systems

Nimal Sreekumar

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Question 11.9.2.5: In an A.P., if the p-th term is $\frac{1}{q}$ and q-th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$. And also find Z-transform of x(n).

Solution:

$$\frac{1}{q} = x(0) + pd \tag{1}$$

$$\frac{1}{p} = x(0) + qd \tag{2}$$

Solving (1) and (2) gives

$$\frac{1}{pq} = d \tag{3}$$

$$x(0) = 0 \tag{4}$$

$$x(n) \overset{z}{\longleftrightarrow} X(z) \tag{5}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)z^{-n} \tag{6}$$

$$= x(0) \sum_{n=0}^{\infty} z^{-n} + d \sum_{n=0}^{\infty} nz^{-n} \tag{7}$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{8}$$

Using (3) and (4),

$$X(z) = \frac{z^{-1}}{pq(1 - z^{-1})^2} \tag{9}$$

$$Y(z) = X(z).U(z) \tag{10}$$

$$= \frac{z^{-1}}{pq(1 - z^{-1})^2} \times \frac{1}{(1 - z^{-1})} \tag{11}$$

$$= \frac{z^2}{pq(z - 1)^3} \tag{12}$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C X(z)z^{n-1}dz \tag{13}$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{n+1}dz}{pq(z - 1)^3} \tag{14}$$

$$= \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \tag{15}$$

$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{z^{n+1}}{pq(z - 1)^3} \right) \tag{16}$$

$$= \frac{1}{2} \lim_{z \to 1} (n + 1).n.(z)^{n-1} \tag{17}$$

$$y(n) = \frac{n(n + 1)}{2} \tag{18}$$

$$y(pq) = \frac{pq(pq + 1)}{2} \tag{19}$$

Symbols	Values	Description
$x(n)$	$(x(0) + nd)(u(n))$	general term of the series
$y(n)$	$\frac{n(n+1)}{2}$	sum of n terms
$y(n)$	$x(n) * u(n)$	-