

Discrete Assignment

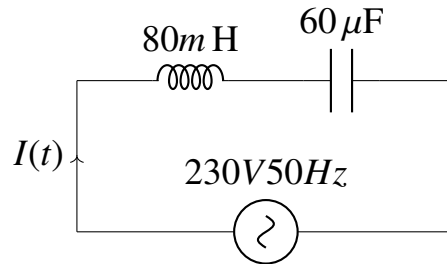
EE1205 Signals and Systems

Nimal Sreekumar
EE23BTECH11044

Question 12.7.18: A circuit containing a 80mH inductor and a 60 μ F capacitor in series is connected to a 230V, 50Hz supply. The resistance of the circuit is negligible.

- 1) Obtain the current amplitude and rms value.
 - 2) Obtain the rms value of potential drops across each element.
 - 3) What is the average power transferred to the inductor ?
 - 4) What is the average power transferred to the capacitor ?
 - 5) What is the total average power absorbed by the circuit ?
- ('Average' implies 'averaged over one cycle'.)

Solution:



Variable	values	Description
X_L	$\omega L = 8\pi$	inductive reactance
X_C	$\frac{1}{\omega C} = \frac{1000}{6\pi}$	capacitive reactance
I_0	?	peak current
I_{rms}	$I_{rms} = \frac{I_0}{\sqrt{2}}$	rms current
V_0	$230\sqrt{2}V$	peak voltage
V_{rms}	230V	rms voltage
Z	?	impedence
ω	100π	angular frequency

TABLE 1
INPUT PARAMETERS

1)

$$H(s) = \frac{V(s)}{I(s)} \quad (1)$$

$$= sL + \frac{1}{sC} \quad (2)$$

$$H(j\omega) = j\omega L + \frac{1}{j\omega C} \quad (3)$$

$$|H(j\omega)| = \sqrt{(X_L - X_C)^2} \quad (4)$$

$$= \omega L - \frac{1}{\omega C} \quad (5)$$

$$= 8\pi - \frac{1000}{6\pi} \quad (6)$$

$$= -27.958\Omega \quad (7)$$

Then,

$$I_0 = \frac{V_0}{Z} \quad (8)$$

$$= -11.63 \text{ A} \quad (9)$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = -8.22 \text{ A} \quad (10)$$

2) Voltage across the inductor:

$$|V_L| = I_{rms} \times X_L \quad (11)$$

$$= 206.48 \text{ V} \quad (12)$$

Voltage across the capacitor:

$$|V_C| = I_{rms} \times X_C \quad (13)$$

$$= 436.3 \text{ V} \quad (14)$$

3) Instantaneous power,

$$P = VI \quad (15)$$

$$P = V_0 \sin(\omega t) I_0 \sin(\omega t - \phi) \quad (16)$$

Average power,

$$P_{avg} = \frac{W}{T} \implies dW = P dt \quad (17)$$

$$W = V_0 I_0 \int_0^T \sin(\omega t) \sin(\omega t - \phi) dt \quad (18)$$

$$W = V_0 I_0 \int_0^T \sin(\omega t) (\sin(\omega t) \cos(\phi)) - \cos(\omega t) \sin(\omega t) dt \quad (19)$$

$$W = V_0 I_0 \int_0^T (\sin(\omega t))^2 \cos(\phi) dt - V_0 I_0 \int_0^T \sin(\omega t) \cos(\omega t) \sin(\phi) dt \quad (20)$$

$$W = V_0 I_0 \int_0^T \frac{1 - \cos 2\omega t}{2} \cos(\phi) dt - V_0 I_0 \int_0^T \sin(2\omega t) \sin(\phi) dt \quad (21)$$

After solving,

$$W = \frac{1}{2} V_0 I_0 T \cos \phi \quad (22)$$

$$P_{avg} = \frac{1}{2} V_0 I_0 \cos \phi \quad (23)$$

Converting to rms terms,

$$P_{avg} = V_{rms} I_{rms} \cos \phi \quad (24)$$

$$\tan(\phi) = \frac{\frac{1}{\omega C} - \omega L}{R} = \infty \quad (25)$$

$$\phi = \frac{\pi}{2} \quad (26)$$

voltage across the capacitor leads the current by 90° ,

$$\text{Average power across inductor}(P_L) = 0 \quad (27)$$

4) Similarly, since voltage across the capacitor lags the current by 90° ,

$$\text{Average power across capacitor}(P_C) = 0 \quad (28)$$

5)

$$\text{Total average power} = P_L + P_C \quad (29)$$

$$= 0 \quad (30)$$