Discrete Assignment
EE1205 Signals and Systems

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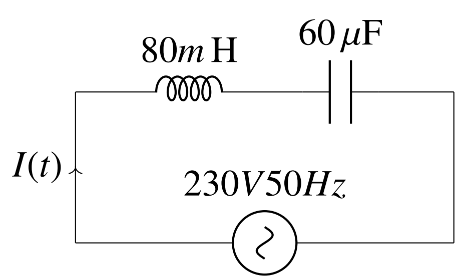
EE23BTECH11044

Question 12.7.18: A circuit containing a 80mH inductor and a 60μF capacitor in series is connected to a 230V, 50Hz supply. The resistance of the circuit is negligible.

1) Obtain the current amplitude and rms value.
2) Obtain the rms value of potential drops across each element.
3) What is the average power transferred to the inductor ?
4) What is the average power transferred to the capacitor ?
5) What is the total average power absorbed by the circuit ?

('Average' implies 'averaged over one cycle'.)

Solution:



Variable	values	Description
X_L	$\omega L = 8\pi$	inductive reactance
X_C	$\frac{1}{\omega C} = \frac{1000}{6\pi}$	capacitive reactance
I_0	?	peak current
I_{rms}	$I_{rms} = \frac{I_0}{\sqrt{2}}$	rms current
V_0	$230\sqrt{2}V$	peak voltage
V_{rms}	$230V$	rms voltage
Z	?	impedence
ω	100π	angular frequency

TABLE 1
Input Parameters

1)

$$Z(s) = \frac{V(s)}{I(s)} \tag{1}$$

$$= sL + \frac{1}{sC} \tag{2}$$

$$Z(j\omega) = j\omega L + \frac{1}{j\omega C} \tag{3}$$

$$|Z(j\omega)| = \sqrt{(X_L - X_C)^2} \tag{4}$$

$$= \omega L - \frac{1}{\omega C} \tag{5}$$

$$= 8\pi - \frac{1000}{6\pi} \tag{6}$$

$$= -27.958\Omega \tag{7}$$

Then,

$$I_0 = \frac{V_0}{Z} \tag{8}$$

$$= -11.63\,\text{A} \tag{9}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = -8.22\,\text{A} \tag{10}$$

2) Voltage across the inductor:

$$|V_L| = I_{rms} \times X_L \tag{11}$$

$$= 206.48V \tag{12}$$

Voltage across the capacitor:

$$|V_C| = I_{rms} \times X_C \tag{13}$$

$$= 436.3V \tag{14}$$

3) Instantaneous power,

$$P = VI \tag{15}$$

$$P = V_0 sin(\omega t) I_0 sin(\omega t - \phi) \tag{16}$$

Average power,

$$P_{avg} = \frac{W}{T} \implies dW = Pdt \tag{17}$$

$$W = V_0 I_0 \int_0^T sin\,(\omega t)\, sin\,(\omega t - \phi)\, dt \tag{18}$$

$$W = V_0 I_0 \int_0^T sin\,(\omega t)\,(sin\,(\omega t)\, cos\,(\phi)) - cos\,(\omega t)\, sin\,(\omega t)\, dt \tag{19}$$

$$W = V_0 I_0 \int_0^T (sin\,(\omega t))^2\, cos\,(\phi)\, dt - V_0 I_0 \int_0^T sin\,(\omega t)\, cos\,(\omega t)\, sin\,(\phi)\, dt \tag{20}$$

$$W = V_0 I_0 \int_0^T \frac{1 - cos2\omega t}{2} cos\,(\phi)\, dt - V_0 I_0 \int_0^T sin\,(2\omega t)\, sin\,(\phi)\, dt \tag{21}$$

After solving,

$$W = \frac{1}{2} V_0 I_0 T cos\phi \tag{22}$$

$$P_{avg} = \frac{1}{2} V_0 I_0 cos\phi \tag{23}$$

Converting to rms terms,

$$P_{avg} = V_{rms} I_{rms} cos\phi \tag{24}$$

$$tan\,(\phi) = \frac{\frac{1}{\omega C} - \omega L}{R} = \infty \tag{25}$$

$$\phi = \frac{\pi}{2} \tag{26}$$

voltage across the capacitor leads the current by 90°,

$$\text{Average power across inductor}(P_L) = 0 \tag{27}$$

4) Similarly, since voltage across the capacitor lags the current by 90°,

$$\text{Average power across capacitor}(P_C) = 0 \tag{28}$$

5)

$$\text{Total average power} = P_L + P_C \tag{29}$$

$$= 0 \tag{30}$$