Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.2.5: In an A.P., if the p-th term is $\frac{1}{q}$ and q-th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.

Solution:

symbols	expression	Description
a_n	$a_n = a_1 + (n-1)d$	General equation for the n^{th} term, where d is the common d
S_n	$S_n = \frac{n}{2}(2a_1 + (n-1)d)$	General equation for the sum of first n terms in the A

Using the above formulas, we can also write a_p and a_q as

$$a_p = a_1 + (p-1)d \tag{1}$$

$$a_q = a_1 + (q-1)d \tag{2}$$

Now substituting $a_p = \frac{1}{q}$ and $a_q = \frac{1}{p}$ in equations (1) and (2) respectively gives

$$\frac{1}{q} = a_1 + (p-1)d \tag{3}$$

$$\frac{1}{p} = a_1 + (q-1)d \tag{4}$$

Subtracting (3) and (4) gives

$$\frac{1}{q} - \frac{1}{p} = d((p-1) - (q-1)) \tag{5}$$

$$\frac{1}{pq} = d \tag{6}$$

Now dividing (3) and (4) gives

$$\frac{p}{q} = \frac{a_1 + (p-1)d}{a_1 + (q-1)d} \tag{7}$$

After solving, we get

$$\frac{1}{pq} = a_1 \tag{8}$$

From equations (6) and (8), we get $a_1 = d = \frac{1}{pq}$.

Now we just have to find the sum of the first pq terms,

$$S_{pq} = \frac{pq}{2}(2a_1 + (pq - 1)d) \tag{9}$$

Using (6) and (8), we get

$$S_{pq} = \frac{pq}{2}\left(2\left(\frac{1}{pq}\right) + (pq - 1)\frac{1}{pq}\right) \tag{10}$$

$$S_{pq} = 1 + \frac{pq}{2} - \frac{1}{2} \tag{11}$$

Finally,

$$\boxed{S_{pq} = \frac{1}{2}(pq + 1)} \tag{12}$$

Hence proved.