Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.2.5: In an A.P., if the p-th term is $\frac{1}{q}$ and q-th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$. And also find Z-transform of x(n).

Solution:

$$\frac{1}{q} = x(0) + pd \tag{1}$$
$$\frac{1}{p} = x(0) + qd \tag{2}$$

Solving (1) and (2) gives

$$\frac{1}{pq} = d \tag{3}$$
$$x(0) = 0 \tag{4}$$

Sum of A.P.,

$$S(n) = \sum_{r=-\infty}^{n} x(r) \tag{5}$$

$$S(pq) = \sum_{r=0}^{pq} x(r) \tag{6}$$
$$S(pq) = \sum_{r=0}^{pq} (x(0) + rd) \tag{7}$$
$$= (pq+1)x(0) + d\sum_{r=0}^{pq} r \tag{8}$$
$$= (pq+1)x(0) + d\left(\frac{pq(pq+1)}{2}\right) \tag{9}$$

Using (3) and (4),

$$\boxed{S(pq) = \frac{1}{2}(pq+1)} \tag{10}$$

$$x(n) \overset{z}{\longleftrightarrow} X(z) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)z^{-n} \tag{12}$$
$$= x(0)\sum_{n=0}^{\infty} z^{-n} + d\sum_{n=0}^{\infty} nz^{-n} \tag{13}$$
$$= \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{14}$$

Using (3) and (4),

$$X(z) = \frac{z^{-1}}{pq(1-z^{-1})^2} \tag{15}$$

Symbols	Values	Description
$x(n)$	$x(0) + nd$	general term of the series
$S(n)$	$d\frac{n(n+1)}{2}$	sum of n terms
$X(z)$	$\frac{z^{-1}}{pq(1-z^{-1})^2}$	Z-transform of x(n)