

Discrete Assignment

EE1205 Signals and Systems

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Question 11.9.2.5: In an A.P., if the p -th term is $\frac{1}{q}$ and q -th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq + 1)$, where $p \neq q$. (Using Z-transform of $x(n)$).

Solution:

$$\frac{1}{q} = x(0) + pd \quad (1)$$

$$\frac{1}{p} = x(0) + qd \quad (2)$$

Solving (1) and (2) gives

$$\frac{1}{pq} = d \quad (3)$$

$$x(0) = 0 \quad (4)$$

$$x(n) \leftrightarrow X(z) \quad (5)$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd) z^{-n} \quad (6)$$

$$= x(0) \sum_{n=0}^{\infty} z^{-n} + d \sum_{n=0}^{\infty} n z^{-n} \quad (7)$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (8)$$

Using (3) and (4),

$$X(z) = \frac{z^{-1}}{pq(1 - z^{-1})^2}, \quad |z| > 1 \quad (9)$$

$$Y(z) = X(z) U(z) \quad (10)$$

$$= \frac{z^{-1}}{pq(1 - z^{-1})^2} \times \frac{1}{(1 - z^{-1})} \quad (11)$$

$$= \frac{z^2}{pq(z - 1)^3} \quad (12)$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (13)$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{n+1} dz}{pq(z - 1)^3} \quad (14)$$

$$= \frac{1}{(m - 1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (15)$$

Where m is the number of repeated poles (here $m = 3$),

$$= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{z^{n+1}}{pq(z - 1)^3} \right) \quad (16)$$

$$= \frac{1}{2} \lim_{z \rightarrow 1} (n + 1) \times n \times (z)^{n-1} \quad (17)$$

$$y(n) = \frac{n(n + 1)}{2} \quad (18)$$

$$y(pq) = \frac{pq(pq + 1)}{2} \quad (19)$$

Symbols	Values	Description
$x(n)$	$(x(0) + nd)(u(n))$	general term of the series
$y(n)$	$\frac{n(n+1)}{2}$	sum of n terms
$y(n)$	$x(n) * u(n)$	-