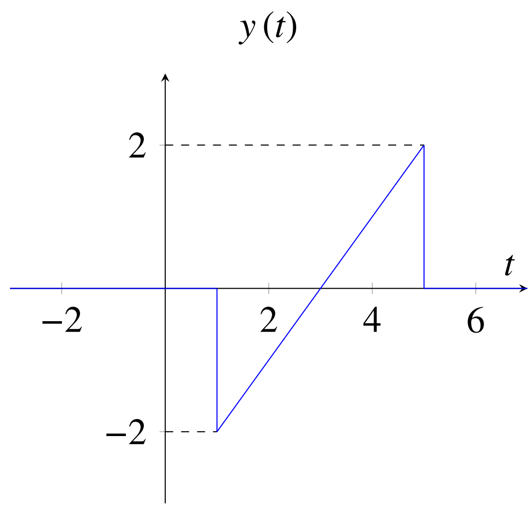
Gate EE 2023
EE1205 Signals and Systems

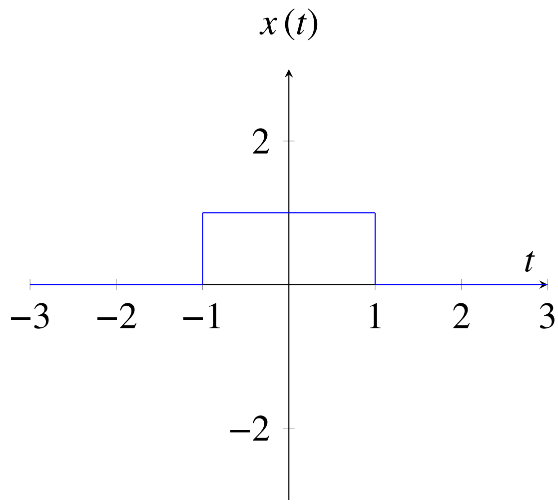
Nimal Sreekumar
EE23BTECH11044

Question Gate 2023 EE: For the signals x(t) and y(t) shown in the figure, $z(t) = x(t) * y(t)$ is maximum at $t = T_1$. Then T_1 in seconds is .... (Round off to the nearest integer)

Variable	values	Description
$x(t)$	$u(t+1) - u(t-1)$	signal 1
$y(t)$	$y(t) = \begin{cases} t-3 & ; 1 \leq n \leq 5 \\ 0 & ; otherwise \end{cases}$	signal 2
$X(s)$	$\int_0^\infty x(t) e^{-st} dt$	Laplace transform of $x(t)$
$Y(s)$	$\int_0^\infty y(t) e^{-st} dt$	Laplace transform of $y(t)$
$\mathcal{L}^{-}\{Z(s)\}$	$f(t-c) u(t-c) = \mathcal{L}^{-}(e^{-cs} F(s))$	Inverse Laplace transform

TABLE 1
INPUT PARAMETERS





Solution:

Using laplace transform,

$$z(t) = x(t) * y(t) \tag{1}$$

$$Z(s) = X(s) Y(s) \tag{2}$$

$$X(s) = \frac{1}{s}\left(e^{s} - e^{-s}\right) \tag{3}$$

$$Y(s) = \frac{2s+1}{s^2}\left(e^{-s} - e^{-5s}\right) \tag{4}$$

$$Z(s) = \frac{2s+1}{s^3}\left(1 - e^{-4s} - e^{-2s} + e^{-6s}\right) \tag{5}$$

Now taking inverse laplace transform for each terms, $\mathcal{L}^{-}\{Z(s)\}$

$$\begin{aligned} z(t) = &\left(2t + \frac{t^2}{2}\right) u(t) \\ &- \left(2(t-4) + \frac{(t-4)^2}{2}\right) u(t-4) \\ &- \left(2(t-2) + \frac{(t-2)^2}{2}\right) u(t-2) \\ &+ \left(2(t-6) + \frac{(t-6)^2}{2}\right) u(t-6) \end{aligned} \tag{6}$$

From the plot it is clear that $T_1 = 4$.

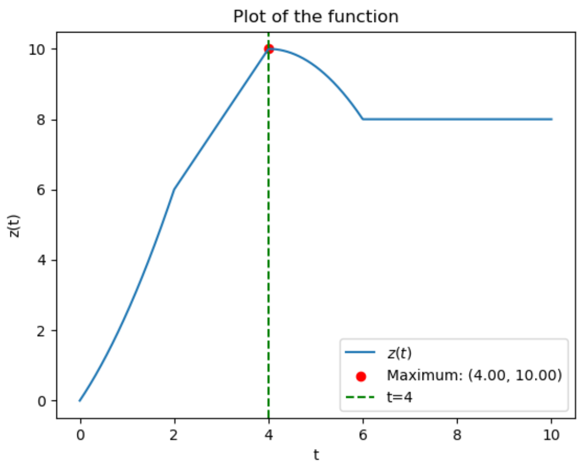


Fig. 1. z(t) vs. t

Now in time domain,

$$z(t) = x(t) * y(t) = y(t) * x(t) \tag{7}$$

$$z(t) = \int_{-\infty}^{\infty} y(\tau) x(t-\tau) d\tau \tag{8}$$

$x(\tau)$ is an even signal,

$$x(\tau) = x(-\tau) \tag{9}$$

$$x(-\tau) = \begin{cases} 1 & ; -1 \leq -\tau \leq 1 \\ 0 & ; \text{otherwise} \end{cases} \tag{10}$$

$$x(-\tau) \xleftrightarrow{\text{Time shifting}} x(t-\tau) \tag{11}$$

$$x(t-\tau) = \begin{cases} 1 & ; t-1 \leq t-\tau \leq t+1 \\ 0 & ; \text{otherwise} \end{cases} \tag{12}$$

For $z(t)$ to be maximum both $y(\tau)$ and $x(t-\tau)$ must be maximum,

$$\implies t-1 = 3 \quad \text{or} \quad t+1 = 5$$
$$t = T_1 = 4$$