Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.2.5: In an A.P., if the p-th term is $\frac{1}{q}$ and q-th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.(Using Z-transform of x(n)).

Symbols	Values	Description
$x(n)$	$\left(\frac{n}{pq}\right) u(n)$	general term of the series
$y(n)$	$\frac{(n+1)}{2}$	sum of n terms

Solution:

$$\frac{1}{q} = x(0) + pd \tag{1}$$

$$\frac{1}{p} = x(0) + qd \tag{2}$$

From equations (1) and (2), the argumented matrix is:

$$\begin{pmatrix} 1 & p & \frac{1}{q} \\ 1 & q & \frac{1}{p} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & p & \frac{1}{q} \\ 0 & q-p & \frac{q-p}{pq} \end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{q-p}} \begin{pmatrix} 1 & p & \frac{1}{q} \\ 0 & 1 & \frac{1}{pq} \end{pmatrix} \tag{4}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - pR_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{pq} \end{pmatrix} \tag{5}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{pq} \end{pmatrix} \tag{6}$$

$$x(n) = \left(\frac{n}{pq}\right) u(n) \tag{7}$$

$$X(z) = \frac{z^{-1}}{pq\left(1 - z^{-1}\right)^2} \qquad , |z| > 1 \tag{8}$$

Finding sum of pq terms,

$$y(n) = \frac{n}{2}\left(2x(0) + (n+1)d\right) \tag{9}$$

$$y(pq) = \frac{1}{2}(pq+1) \tag{10}$$