Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.2.5: In an A.P., if the p-th term is $\frac{1}{q}$ and q-th term is $\frac{1}{p}$, prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.(Using Z-transform of x(n)).

Solution:

$$\frac{1}{q} = x(0) + pd \tag{1}$$
$$\frac{1}{p} = x(0) + qd \tag{2}$$

Solving (1) and (2) gives

$$\frac{1}{pq} = d \tag{3}$$
$$x(0) = 0 \tag{4}$$

$$x(n) \overset{z}{\longleftrightarrow} X(z) \tag{5}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd) z^{-n} \tag{6}$$
$$= x(0) \sum_{n=0}^{\infty} z^{-n} + d \sum_{n=0}^{\infty} nz^{-n} \tag{7}$$
$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{8}$$

Using (3) and (4),

$$X(z) = \frac{z^{-1}}{pq(1 - z^{-1})^2}, \qquad |z| > 1 \tag{9}$$
$$Y(z) = X(z) U(z) \tag{10}$$
$$= \frac{z^{-1}}{pq(1 - z^{-1})^2} \times \frac{1}{(1 - z^{-1})} \tag{11}$$
$$= \frac{z^2}{pq(z-1)^3} \tag{12}$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \tag{13}$$
$$= \frac{1}{2\pi j} \oint_C \frac{z^{n+1} dz}{pq(z-1)^3} \tag{14}$$
$$= \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \tag{15}$$

Where m is the number of repeated poles (here m =3),

$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{z^{n+1}}{pq(z-1)^3} \right) \tag{16}$$
$$= \frac{1}{2} \lim_{z \to 1} (n+1) \times n \times (z)^{n-1} \tag{17}$$
$$y(n) = \frac{n(n+1)}{2} \tag{18}$$
$$\boxed{y(pq) = \frac{pq(pq+1)}{2}} \tag{19}$$

Symbols	**Values**	**Description**
$x\left(n\right)$	$\left(x\left(o\right)+nd\right)\left(u\left(n\right)\right)$	general term of the series
$y\left(n\right)$	$\frac{n(n+1)}{2}$	sum of n terms
$y\left(n\right)$	$x\left(n\right)*u\left(n\right)$	-