Gate EE 2023
EE1205 Signals and Systems

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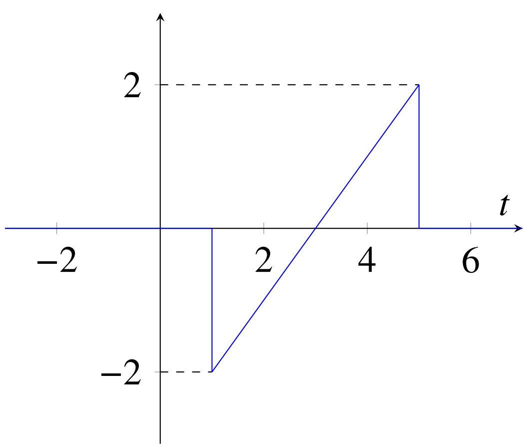
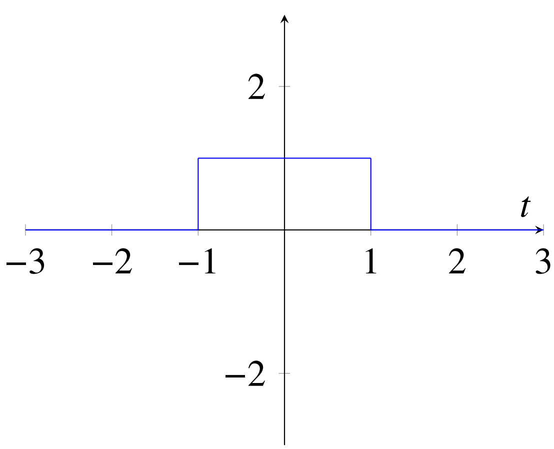
Question Gate 2023 EE: For the signals x(t) and y(t) shown in the figure, $z(t) = x(t) * y(t)$ is maximum at $t = T_1$. Then T_1 in seconds is .... (Round off to the nearest integer)

$y(t)$



$x(t)$



Solution:

Variable	values	Description
$x(t)$	$u(t+1) - u(t-1)$	signal 1
$y(t)$	$y(t) = \begin{cases} t-3 & ; 1 \le n \le 5 \\ 0 & ; otherwise \end{cases}$	signal 2
$X(s)$	$\int_0^\infty x(t)\, e^{-st} dt$	Laplace transform of $x(t)$
$Y(s)$	$\int_0^\infty y(t)\, e^{-st} dt$	Laplace transform of $y(t)$
$\mathscr{L}^{-}\{Z(s)\}$	$f(t-c)\,u(t-c) = \mathscr{L}^{-}(e^{-cs}F(s))$	Inverse Laplace transform

TABLE 1
INPUT PARAMETERS

Using laplace transform,

$$z(t) = x(t) * y(t) \tag{1}$$

$$Z(s) = X(s)\,Y(s) \tag{2}$$

$$X(s) = \frac{1}{s}\left(e^{s} - e^{-s}\right) \tag{3}$$

$$Y(s) = \frac{2s+1}{s^2}\left(e^{-s} - e^{-5s}\right) \tag{4}$$

$$Z(s) = \frac{2s+1}{s^3}\left(1 - e^{-4s} - e^{-2s} + e^{-6s}\right) \tag{5}$$

Now taking inverse laplace transform for each terms, $\mathscr{L}^{-}\{Z(s)\}$

$$\begin{aligned} z(t) = &\left(2t + \frac{t^2}{2}\right)u(t) \\ &- \left(2(t-4) + \frac{(t-4)^2}{2}\right)u(t-4) \\ &- \left(2(t-2) + \frac{(t-2)^2}{2}\right)u(t-2) \\ &+ \left(2(t-6) + \frac{(t-6)^2}{2}\right)u(t-6) \end{aligned}$$

From the plot it is clear that $T_1 = 4$.

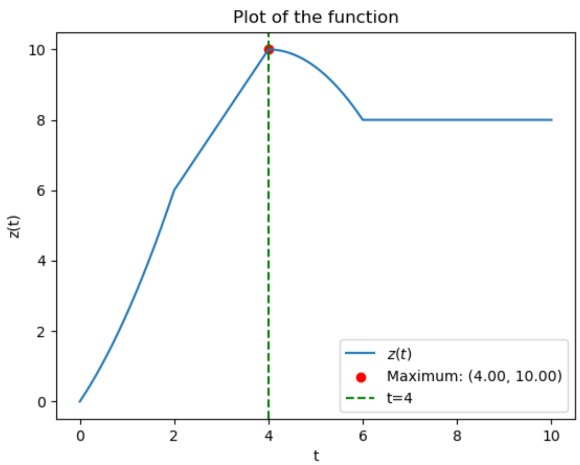


Fig. 1. z(t) vs. t