SERIES RESONANT CIRCUITS

Ashok Ranade, SEAS, Ahmedabad University

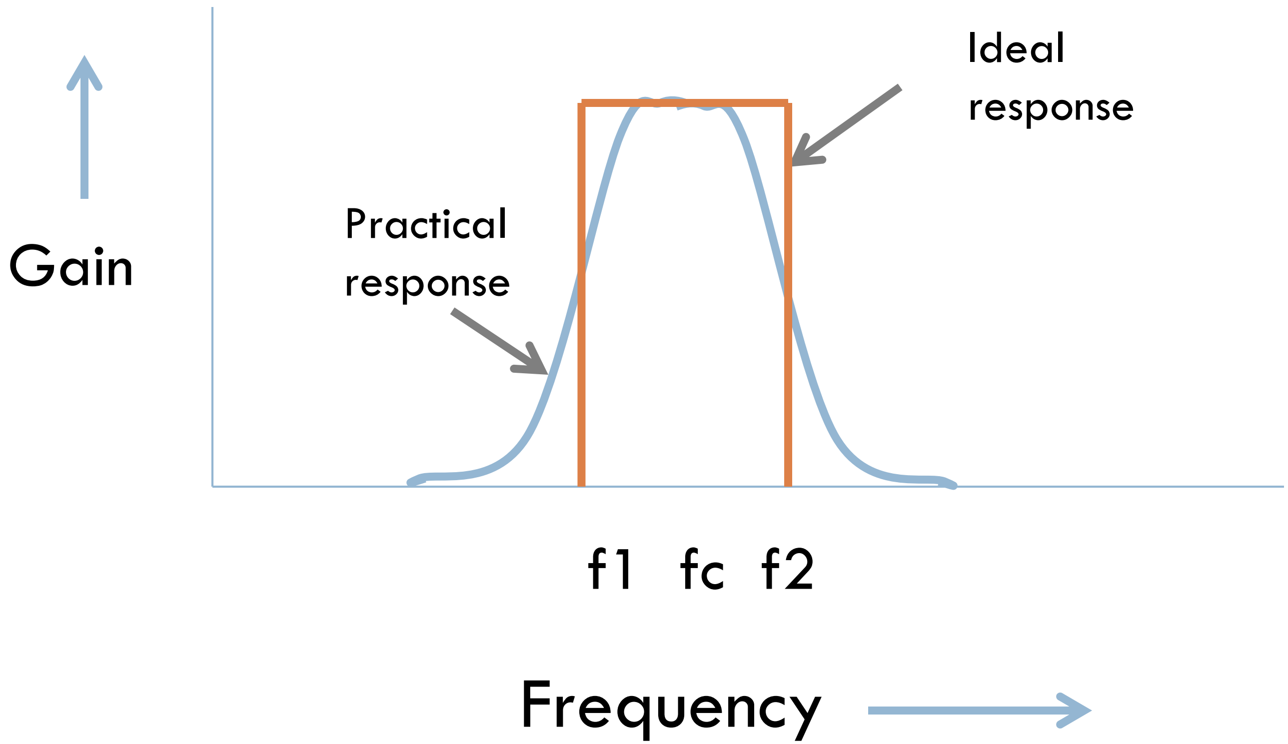
What is a resonant circuit?

- Contains L and C elements
- Either impedance or admittance becomes zero at a frequency called resonant frequency.

Why resonant circuits?

- Easy to obtain bandpass response with narrow bandwidth
- In narrowband circuits the bandwidth is a small fraction of center frequency
- Examples
- AM : fc = 1000 KHz : BW = 10kHz
- FM : fc = 100 MHz : BW = 200kHz
- GSM : fc = 900 MHz : BW = 5 MHz

Ideal and practical Frequency response



Series Resonant circuit

- L-C-R are connected in series
- Output is taken across R
- Aims of analysis

Find the resonant frequency

Find the gain at resonant frequency

Find the frequencies at which the gain falls to 0.707

Of the gain at resonance

Hence obtain the expression for bandwidth in terms of Q

Quality factor

$$Coil\,Q = \frac{\omega\,L}{r}$$

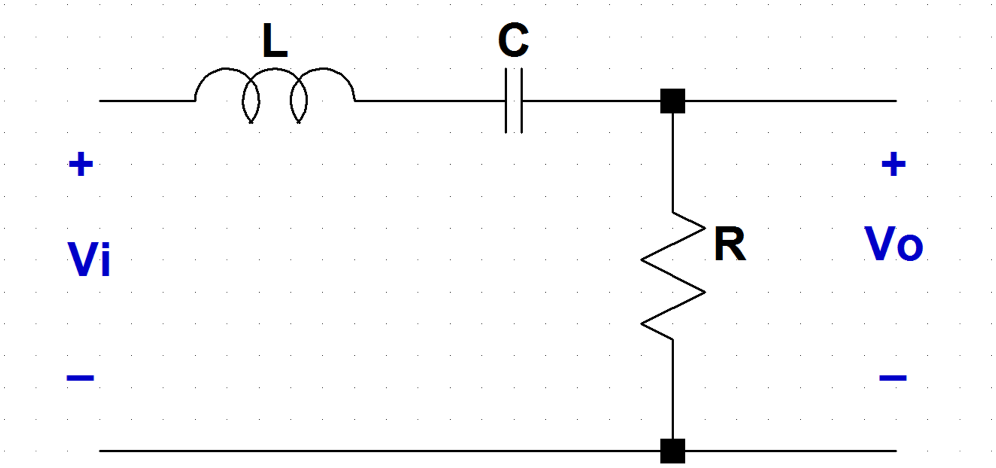
$$Coil\,Q = \frac{\omega_0 L}{r}$$

$$Circuit\;\;Q = \frac{\omega_0 L}{r+R}$$

$$For\;\;ideal\;\;coil\;\;r = 0$$

$$Circuit\;\;Q = \frac{\omega_0 L}{R}$$

Series resonant circuit



$$V_0 = \frac{V_i R}{R + j\omega L - j/\omega C}$$

$$A = \frac{V_0}{V_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

Magnitude response

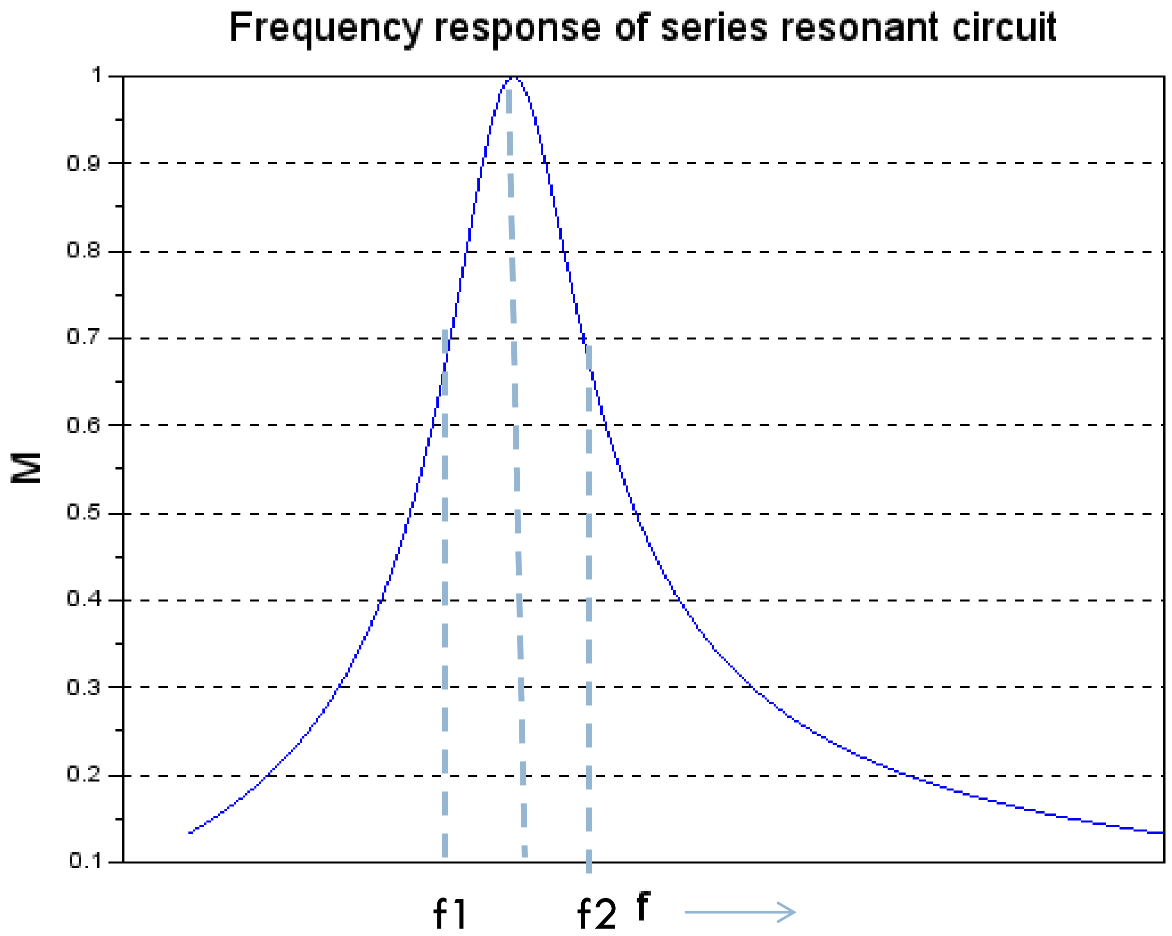
$$|A| = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$At \;\; \omega = \omega_0 \; : \; \omega_0 L = 1/\omega_0 C$$

$$|A| = |A|_{\max} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \; : \; f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Frequency Response



Bandwidth

$$BW = \omega_2 - \omega_1$$

$$BW(Hz) = f_2 - f_1$$

$$A = \frac{1}{1 + j\left(\frac{\omega Q}{\omega_0} - \frac{\omega_0 Q}{\omega}\right)} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$Q = \frac{\omega_0 L}{R} \quad : \quad \frac{L}{R} = \frac{Q}{\omega_0}$$

$$Q = \frac{1}{\omega_0 CR} \quad : \quad \frac{1}{CR} = \omega_0 Q$$

$$A = \frac{R}{R + j(\omega L - 1/\omega C)} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)}$$

BW calculation

$$A = \frac{1}{1 + j\left(\frac{\omega Q}{\omega_0} - \frac{\omega_0 Q}{\omega}\right)} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$At \quad \omega = \omega_2 \quad : |A| = \frac{1}{\sqrt{2}}$$

$$Also \;\; At \quad \omega = \omega_1 \quad : |A| = \frac{1}{\sqrt{2}}$$

$$Q\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right) = 1$$

$$Q\left(\frac{\omega_0}{\omega_1} - \frac{\omega_1}{\omega_0}\right) = 1$$

BW in terms of ω_0 and Q

$$\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = \frac{\omega_0}{\omega_1} - \frac{\omega_1}{\omega_0} = \frac{1}{Q}$$

$$\frac{\omega_2^2 - \omega_0^2}{\omega_2 \omega_0} = \frac{1}{Q}$$

$$\frac{\omega_2^2 - \omega_0^2}{\omega_2 \omega_0} = \frac{\omega_0^2 - \omega_1^2}{\omega_1 \omega_0}$$

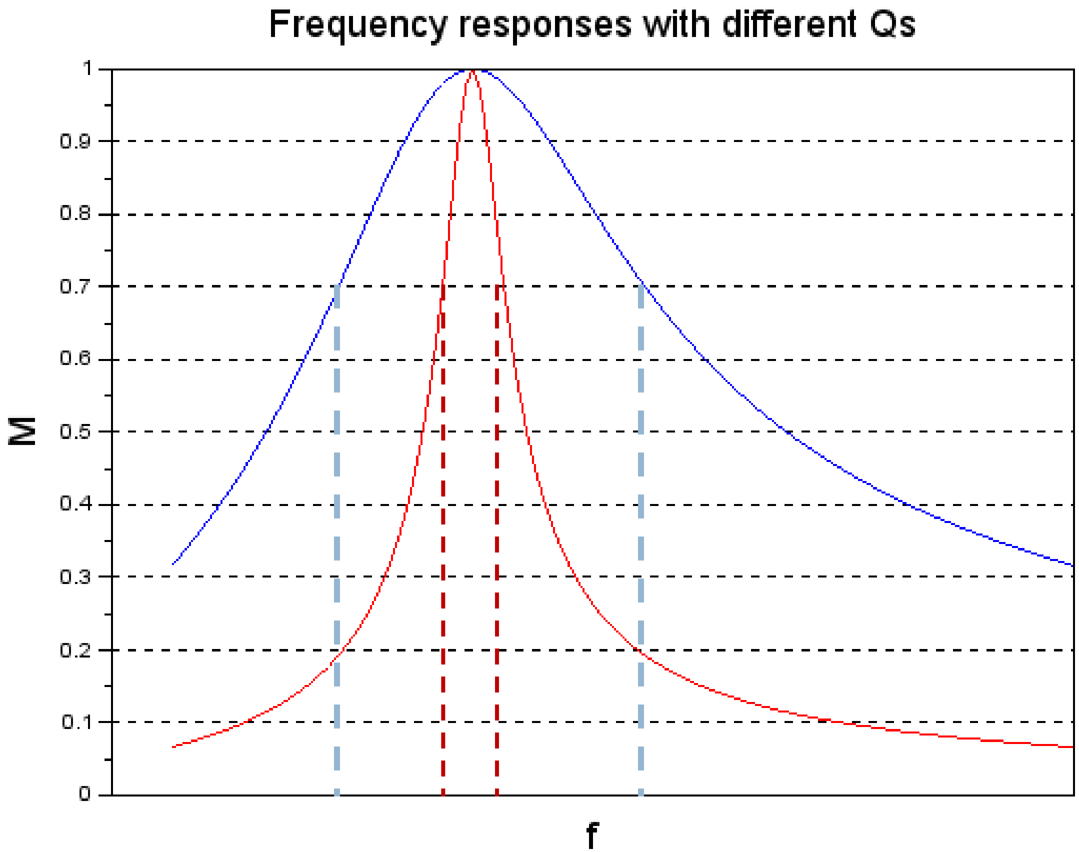
$$\frac{\omega_2^2 - \omega_1 \omega_2}{\omega_2 \omega_0} = \frac{1}{Q}$$

$$\omega_0^2 = \omega_1 \omega_2$$

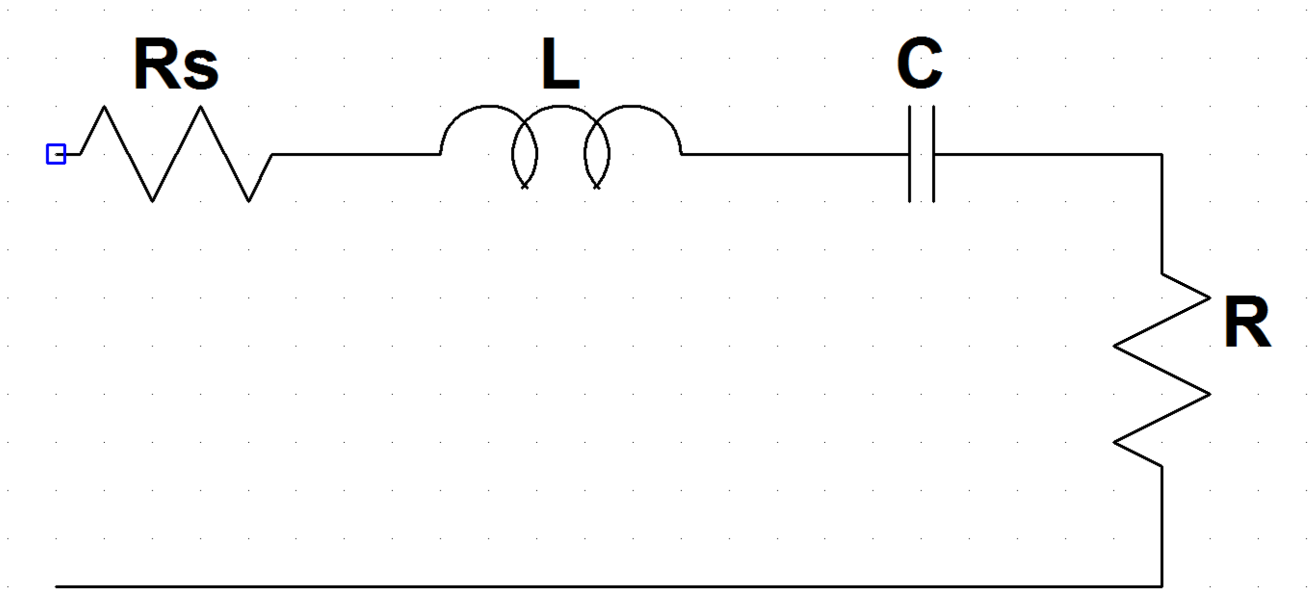
$$\boxed{f_2 - f_1 = \frac{f_0}{Q}}$$

$$\omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

Effect of Q on bandwidth



Effect of series resistance



$$A_0 = \frac{R}{R + R_s}$$

$$Effective\ \ Q_{eff} = \frac{\omega_0 L}{R_s + R}$$

Problem 1

A series RLC circuit has fo = 5 MHz , BW = 100 kHz and R = 50 ohms. Find out L and C

L = 79.577 microhenries
C = 12.732 pF

LTspice

- Simulate circuit of problem 1 in LTspice and test its frequency response

Problem 2

- For a series resonant circuit prove that

$$\frac{|A(n\omega_0)|}{|A(\omega_0)|} \approx \frac{n}{Q(n^2-1)}$$

$$When\ Q >> 1$$

Solution to Problem 2

$$A = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$A(n\omega_0) = \frac{1}{1 + jQ\left(\frac{n\omega_0}{\omega_0} - \frac{\omega_0}{n\omega_0}\right)}$$

$$A(n\omega_0) = \frac{1}{1 + jQ\left(n - \frac{1}{n}\right)}$$

$$|A(n\omega_0)| = \frac{1}{\sqrt{1 + Q^2\left(n - \frac{1}{n}\right)^2}}$$

When Q is large

$$|A(n\omega_0)| \approx \frac{1}{Q\left(n - \frac{1}{n}\right)} = \frac{n}{Q\left(n^2 - 1\right)}$$

Solution to Problem 2 continued

$$|A(n\omega_0)| \approx \frac{1}{Q\left(n - \frac{1}{n}\right)} = \frac{n}{Q\left(n^2 - 1\right)}$$

Now $|A(\omega_0)| = 1$

$$\frac{|A(n\omega_0)|}{|A(\omega_0)|} = \frac{n}{Q\left(n^2 - 1\right)}$$

Problem 3

- In a series resonant circuit what must be the minimum Q for the amplitude of the 5th harmonic to be 40 dB below the amplitude of the fundamental frequency

$$Q = 20.83$$

Solution to Problem 3

$$\frac{|A(jn\omega_0)|}{|A(j\omega_0)|} = \frac{n}{Q\left(n^2-1\right)} \ : \ 20\log\frac{n}{Q\left(n^2-1\right)} = -40$$

$$\log\frac{5}{Q\left(25-1\right)} = -2 \ : \ \log\frac{24Q}{5} = 2$$

$$\frac{24Q}{5} = 100 \ : Q = \frac{500}{24} = 20.83$$