CSC210: Data Structures and Algorithms
Assignment 3

due: 9:30, on 6.9.2018
Please explain your answers in detail.

1. Given the recurrence
$$T(n) = 2T(\lfloor \frac{n}{2} \rfloor) + n$$

   show that the solution is:

   - $O(n \lg n)$
   - $\Omega(n \lg n)$

   Based on your answers, explain what is the exact asymptotic solution of this recurrence.

2. Use a recursion tree to determine a good asymptotic **upper bound** on the recurrence:
$$T(n) = 4T(\frac{n}{2} + 2) + n$$

   Draw a schema for the tree and then verify your answer using the substitution method.

3. Use the master theorem to solve the following recurrence.

$$T(n) = 3T(\frac{n}{2}) + n^2$$

4. Use indicator random variables to solve the following problem. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?

5. Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an **inversion** of A. Suppose that the elements of A for a uniform random permutation of $< 1, 2, \ldots, n >$. Use indicator random variables to compute the expected number of inversions.

6. Write a pseudocode for the procedures:

   - MIN-HEAPIFY(A, i).
   - BUILD-MIN-HEAP(A)