

5

Scalars and Vectors

Used in Physics, Maths and Computer Science — Data Science

A scalar is a single numerical value.

It represents a magnitude or quantity and has no directions

EXAMPLES

Car speed - 45 km/h

Temperature - 25°C

Examples of magnitude

APPLICATION IN DATA SCIENCE

DATASET

f_1

f_2

f_3

Count of total records = 5

Average of feature f_1 = — Scalar

VECTORS

- A vector is a numerical value which has both magnitude and direction - in physics
- In Data Science is an ordered list of numbers it can represent a point in space or quantity with both magnitude and direction

EXAMPLE

- Speed of car is 45 km/h AND is heading East

- Student marks dataset

IQ

Hours Studied

Pass/Fail

90

3 hrs

Fail

100

3 hrs

Pass

A vector representing person's IQ and the number of hours studied



[90, 3HRS]

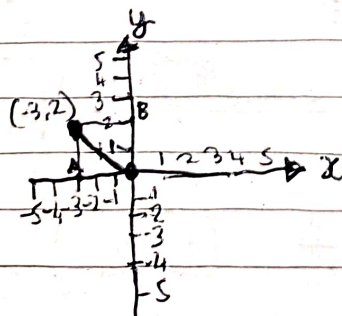
unit magnitude

A scalar - it has magnitude but no direction.

EXAMPLE

- A vector representing a person's weight over time [70, 72, 75, 73]
- When you say direction it doesn't necessarily mean physical direction

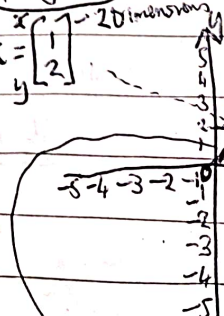
$$B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$



$$\text{Hyp} \sqrt{(OA)^2 + (OB)^2} = -3 + 2 = \sqrt{1}$$

PHYSICS

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



$$\text{Hyp} \sqrt{(OA)^2 + (OB)^2} = 1 + 4 = \sqrt{5}$$

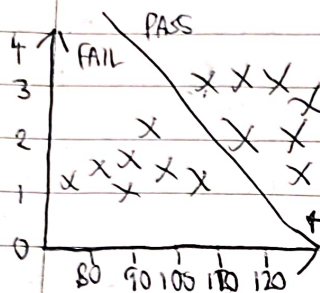
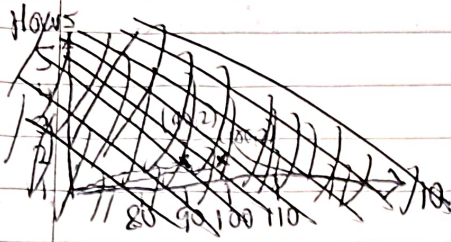
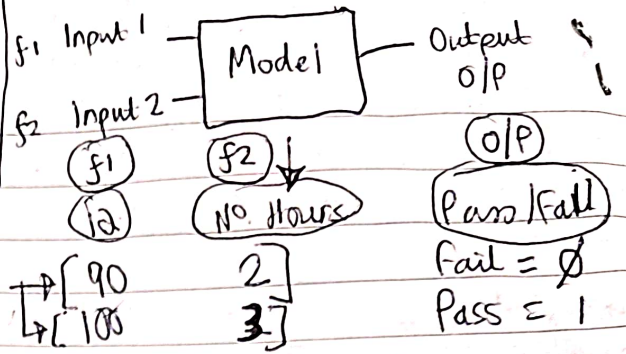
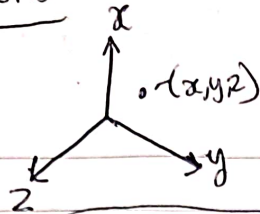
A collection of values

k dimensions

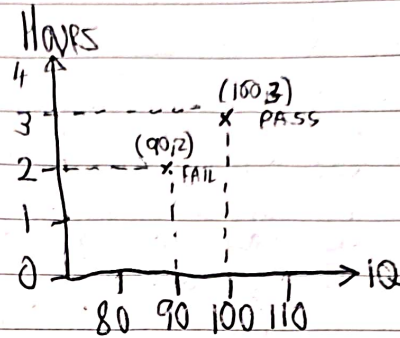
⑥

3 dimensions

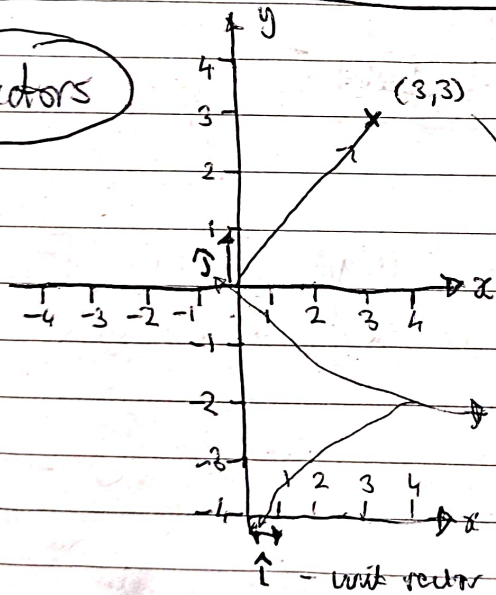
$$C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



This line can be represented as $y = mx + c$



Unit Vectors



unit vector

$$\hat{u} = \frac{u}{|u|}$$

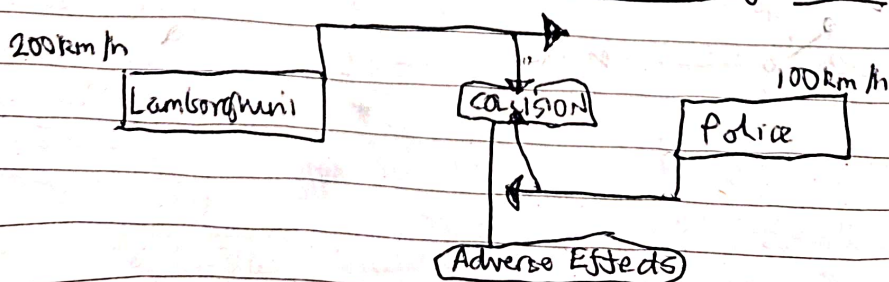
unit symbol or hat

$$|u| = 1 \rightarrow \text{unit vector}$$

$$(3, 3) = 3\hat{i} + 3\hat{j}$$

unit vectors of $\begin{matrix} x \\ y \end{matrix}$
= 1 magnitude

Example - Gaming Industry - Grand Theft Auto



Addition of vector

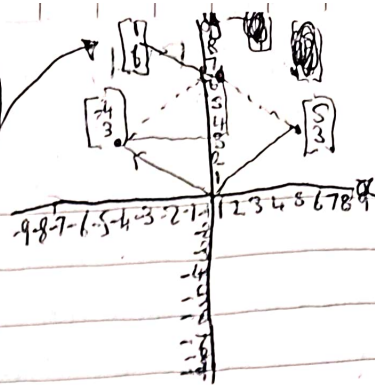
$P_1 = \begin{bmatrix} -4 \\ 3 \end{bmatrix} + P_2 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Addition of vector

$P_1 = \begin{bmatrix} -4 \\ 3 \end{bmatrix} + P_2 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$$\vec{p}_1 + \vec{p}_2 = \begin{bmatrix} -4 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[1]



$$A \begin{bmatrix} x^1 \\ y^1 \\ z^1 \end{bmatrix} + B \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix} = \begin{bmatrix} x^3 \\ y^3 \\ z^3 \end{bmatrix}$$

$$A \begin{bmatrix} -2 \\ -2 \end{bmatrix} + B \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

↓

-3

-3

Solving a use case

SENSOR

SENSOR

$$[3, 5, 7] + [2, 4, 6]$$

Final Sensor

$[5, 9, 13]$

FDA and FE

EXAMPLE - E-COMMERCE - WEBSITE

Reviews

Sentiment

Product is good
Product is bad

INPUTS

Model

OUTPUT

Text \rightarrow convert \rightarrow Vector

Using different techniques

DAE, TFIDF, BOW
WORD2VEC

change
into numbers

129, 149, 85

$$\begin{array}{r} 255, 128, 64 \\ 128, 255, 64 \\ 0, 0, 255 \end{array}$$

Convert
to grayscale

① Called word EMBEDDING

1. Data $[0.2, 0.1, 0.4]$

② Science [0.3, 0.7, 0.2]

③ Data source = [0.5, 0.8, 0.6]

(2) IMAGE PROCESSING

Colour image $[R, G, B]$

$-R - \text{red channel} = [255, 128, 0]$

- G - green channel = [128, 255, 0]

→ blue channel = $[64, 64, 255]$

MULTIPLICATION OF VECTORS

Element wise Multiplication

THREE TYPES

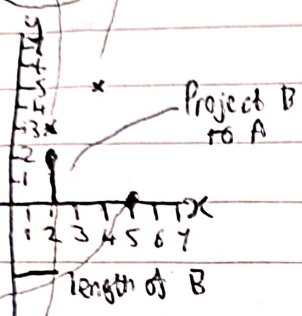
① **DOT PRODUCT** - The dot product of two vectors results in a scalar and is calculated as the sum of products of their corresponding components

Inner Product

$$A = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \quad A \cdot B = 2 \times 4 + 3 \times 5 = 8 + 15 = 23$$

$A \cdot B$ - dot product
 T = Transpose
 $23 = \text{scalar value}$

$$A \cdot B^T = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 \end{bmatrix}$$



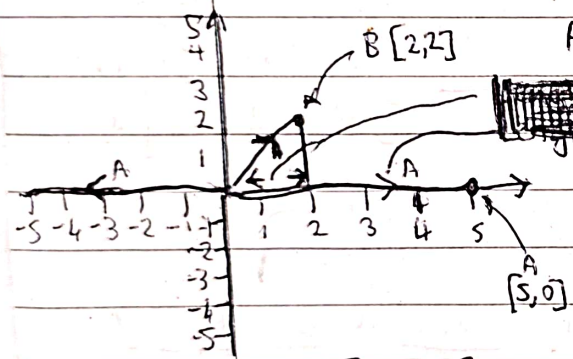
AVENGERS

$$\frac{A \cdot B}{\|A\| \cdot \|B\|} = \frac{6}{3.872 \times 2.646} = 0.586$$

MOVIE B

$$A = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} = 10$$

LENGTH of A = 2
 LENGTH of B = 5



$$A = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix} = -10$$

length of A = 5
 length of B = 2

$A \cdot B = 0$ = Project the vector to the origin

Step 1 - Dot product of $A \cdot B$

$$A \cdot B = 1 \cdot 2 + 2 \cdot 0 + 0 \cdot 1 + 3 \cdot 1 + 1 \cdot 0 = 2 + 3 + 1 = 6$$

Step 2 - $\|A\| \cdot \|B\|$

$$\|A\| = \sqrt{1^2 + 2^2 + 0^2 + 3^2 + 1^2} = \sqrt{15} = 3.872$$

$$\|B\| = \sqrt{2^2 + 0^2 + 1^2 + 1^2 + 1^2} = \sqrt{7} = 2.646$$

called EUCLIDIAN DISTANCE

$$\cos \theta = \frac{A \cdot B}{\|A\| \cdot \|B\|}$$

magnitudes

Recommendation System

Netflix - Action Movie

Recommendation of other movies.

$$A = \begin{bmatrix} 1, 2, 0, 3, 1 \end{bmatrix}$$

Action | Drama | Comedy | Romance

RECOMMEND ? MOVIE $B = \begin{bmatrix} 2, 0, 1, 1, 2 \end{bmatrix}$

APPLICATION OF DOT PRODUCT IN DATA SCIENCE

Gen AI App = RAG System(AI)

① Cosine Similarity

It is a measured used to determine how similar two vectors are. It calculates the cosine of the angle between two vectors providing similarity score that ranges from +1 (dissimilar) to -1 (completely similar).

APPLICATION OF DOT PRODUCT IN DATA SCIENCE

COSINE SIMILARITY

MOVIE
A PULP FICTION

MOVIE
B ?

$$\cos \theta = \frac{A \cdot B}{\|A\| \cdot \|B\|}$$

$[1, 2, 0, 3, 1]$ ← Characteristics → $[2, 0, 1, 1, 1]$

STEP 1 $A \cdot B = 1 \cdot 2 + 2 \cdot 0 + 0 \cdot 1 + 3 \cdot 1 + 1 \cdot 1 = 6$

STEP 2 $\|A\| \cdot \|B\| = \sqrt{1^2 + 2^2 + 0^2 + 3^2 + 1^2} = \sqrt{15}$
 $\sqrt{2^2 + 0^2 + 1^2 + 1^2 + 1^2} = \sqrt{7}$

$= 3.872$
 $= 2.646$

$$\frac{A \cdot B}{\|A\| \cdot \|B\|} = \frac{6}{3.872 \times 2.646} = +0.586$$

Calculates the cosine of the angle between two vectors providing similarity score that ranges from
 -1 = dissimilar to
 $+1$ = completely similar

Movie A is 58.6% similar to Movie B

Vector Databases RAG SYSTEM

