CS F437: Generative Artificial Intelligence Assignment 1

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Part A: Principal Component Analysis

1. Introduction:

The image reconstruction task involves projecting an image into a latent vector space and reconstructing it to minimise information loss. This assignment explores three algorithms—PCA, PPCA, and VAEs—for image reconstruction and aims to analyse the differences in their reconstruction abilities using the MNIST dataset.

2. Methodology:**

Data Preparation:The dataset for this assignment is the 'MNIST' dataset, comprising 60,000 training images and 10,000 testing images of handwritten digits (0 to 9). The images are grayscale and have a resolution of 28x28 pixels. Labels associated with the pictures are not utilised for this assignment; only the pixel values of the images are considered.

Principal Component Analysis (PCA) Implementation: Scikit-learn is employed to implement Principal Component Analysis (PCA). The models are built with varying latent variable dimensions: 2, 4, 8, 16, 32, and 64. The process involves fitting the PCA model on the training data and transforming the test data to the latent space. The inverse transformation is then applied to reconstruct the images. Mean Squared Error (MSE) is calculated to quantify the difference between the original and reconstructed images.

Mean Squared Error (MSE) Calculation:
The Mean Squared Error (MSE) is computed as follows:

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

This methodology ensures a systematic approach to building and evaluating Principal Component Analysis models for image reconstruction. The selected latent dimensions provide a range of complexity for the analysis, allowing for a comprehensive assessment of the algorithm's reconstruction abilities. The calculated MSE values are a quantitative measure of the reconstruction accuracy for each latent variable dimension.

3 Results:

For Principal Component Analysis, models were built with 2, 4, 8, 16, 32, and 64 latent variable dimensions. The following results were obtained:

Latent dimension: 2, mean squared error: 0.05566943809390068
Latent dimension: 4, mean squared error: 0.04790342226624489
Latent dimension: 8, mean squared error: 0.03744113817811012
Latent dimension: 16, mean squared error: 0.026861028745770454
Latent dimension: 32, mean squared error: 0.01682826317846775
Latent dimension: 64, mean squared error: 0.009047850035130978

Part B: Probabilistic Principal Component Analysis

Certainly! Below is a detailed methodology section for Part B, Probabilistic Principal Component Analysis (PPCA), using the provided code:

Methodology

The mean of the data is calculated, and the data is centred by subtracting the mean from each data point.

The covariance matrix is computed from the centered data.

Eigenvalue Decomposition: Eigenvalues and eigenvectors are obtained through the eigenvalue decomposition of the covariance matrix.

Sorting and Selecting Principal Components: Eigenvalues and eigenvectors are sorted in descending order, and the top 'latent_dim' eigenvectors are selected as principal components.

Projection and Reconstruction: The data is projected into the latent space, and the original data is reconstructed using the selected principal components.

Mean Squared Error (MSE) Calculation:
The Mean Squared Error (MSE) is calculated to quantify the difference between the original and reconstructed images.

This methodology ensures a systematic approach to building and evaluating Probabilistic Principal Component Analysis models for image reconstruction. The selected latent dimensions offer a range of complexity for the analysis, allowing for a comprehensive assessment of the algorithm's reconstruction abilities. The calculated MSE values are a quantitative measure of the reconstruction accuracy for each latent variable dimension.

Result

Latent Dimension: 2, MSE: 0.5864296895937319
Latent Dimension: 4, MSE: 0.5045968802688311
Latent Dimension: 8, MSE: 0.39431396558727255
Latent Dimension: 16, MSE: 0.2829586660912335
Latent Dimension: 32, MSE: 0.1772455199492758
Latent Dimension: 64, MSE: 0.09532262452035184

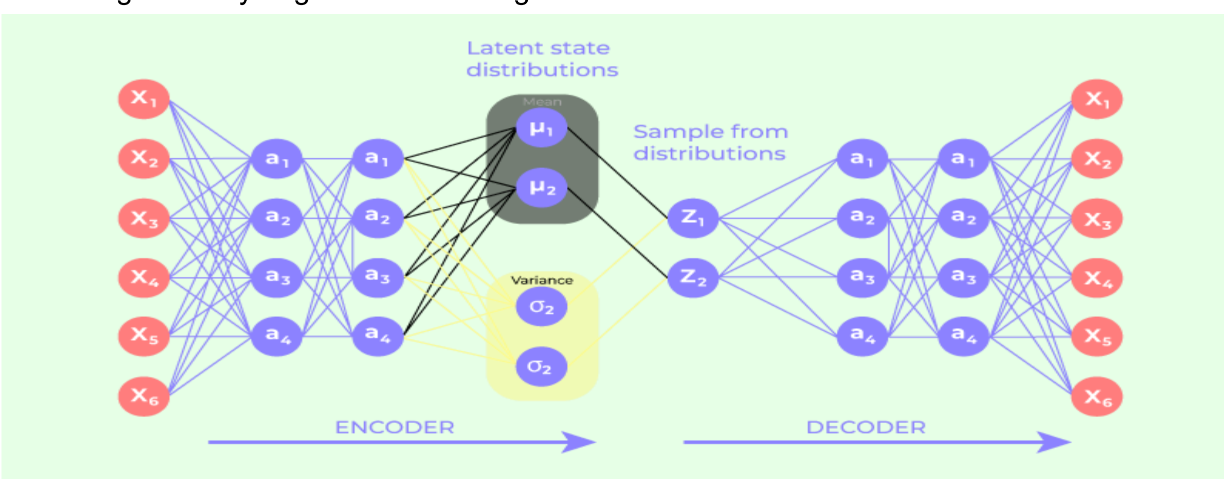
Part C: Variational Autoencoders

Methodology

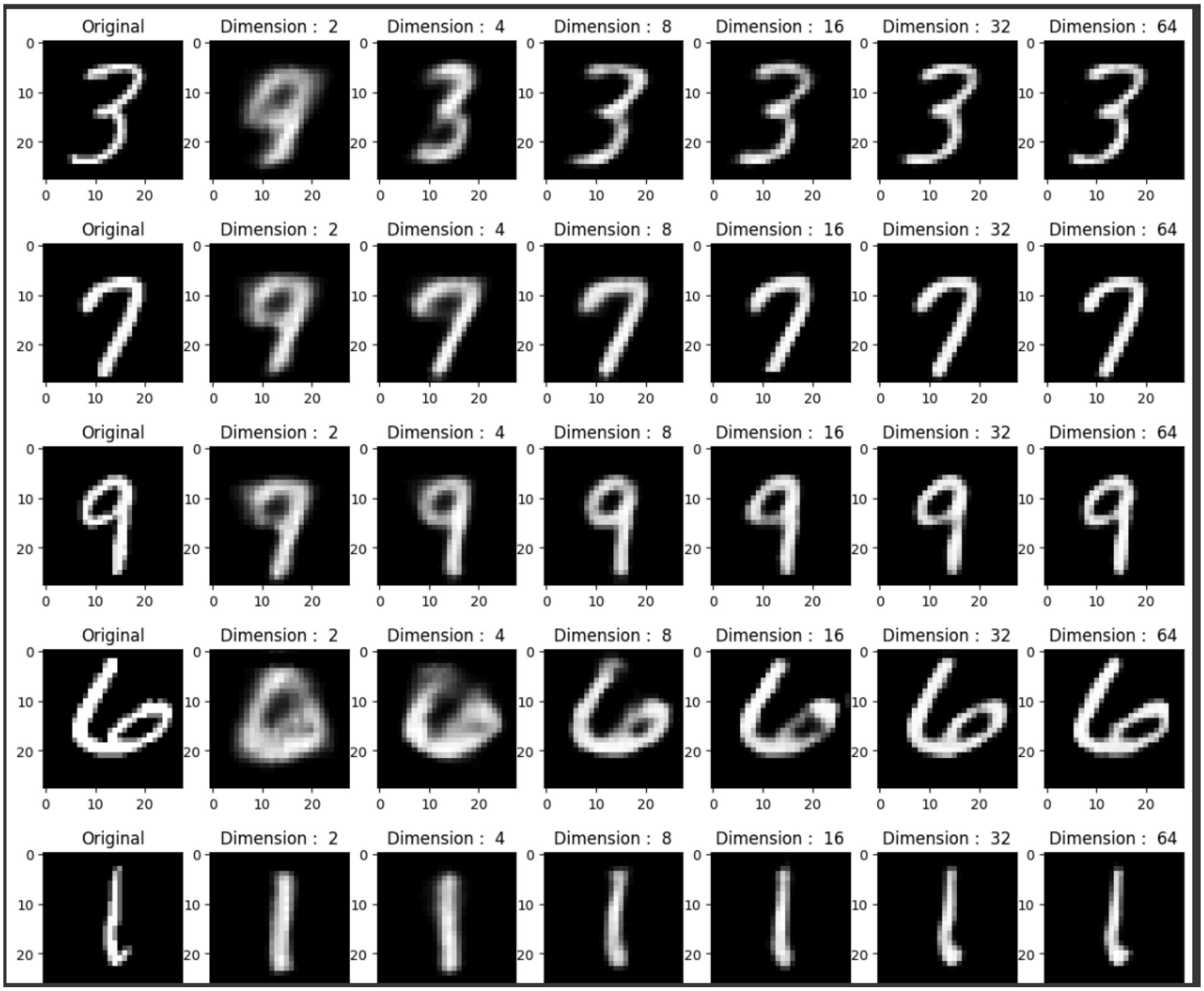
Variational Autoencoder (VAE) Implementation: Variational Autoencoders are constructed using a combination of convolutional and dense layers. The encoder network takes input images of size (28, 28, 1) and compresses them into a latent space of specified dimensionality. The decoder network then reconstructs the input images from the latent representation.

Training and Evaluation: For each specified latent dimension (2, 4, 8, 16, 32, 64), a VAE model is built and trained using the Adam optimiser and Mean Squared Error (MSE) loss function. The training is conducted for 10 epochs with the training set, and the model is evaluated on the validation set (test set). The reconstructed images and MSE values are recorded.

This methodology ensures a systematic approach to constructing Variational Autoencoder models, training them on the MNIST dataset, and evaluating their performance using reconstructed images and Mean Squared Error values for different latent dimensions. Convolutional layers enable the VAE to capture hierarchical features in the input images, enhancing its ability to generate meaningful reconstructions.

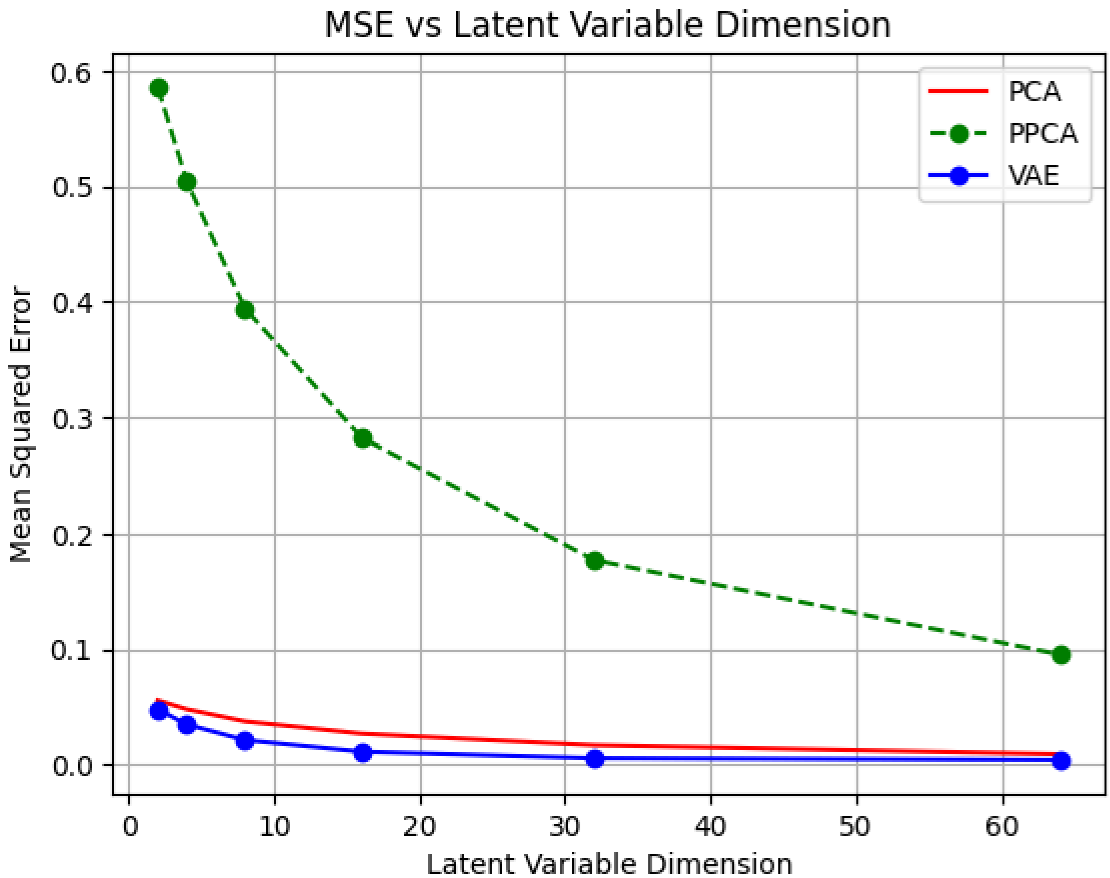


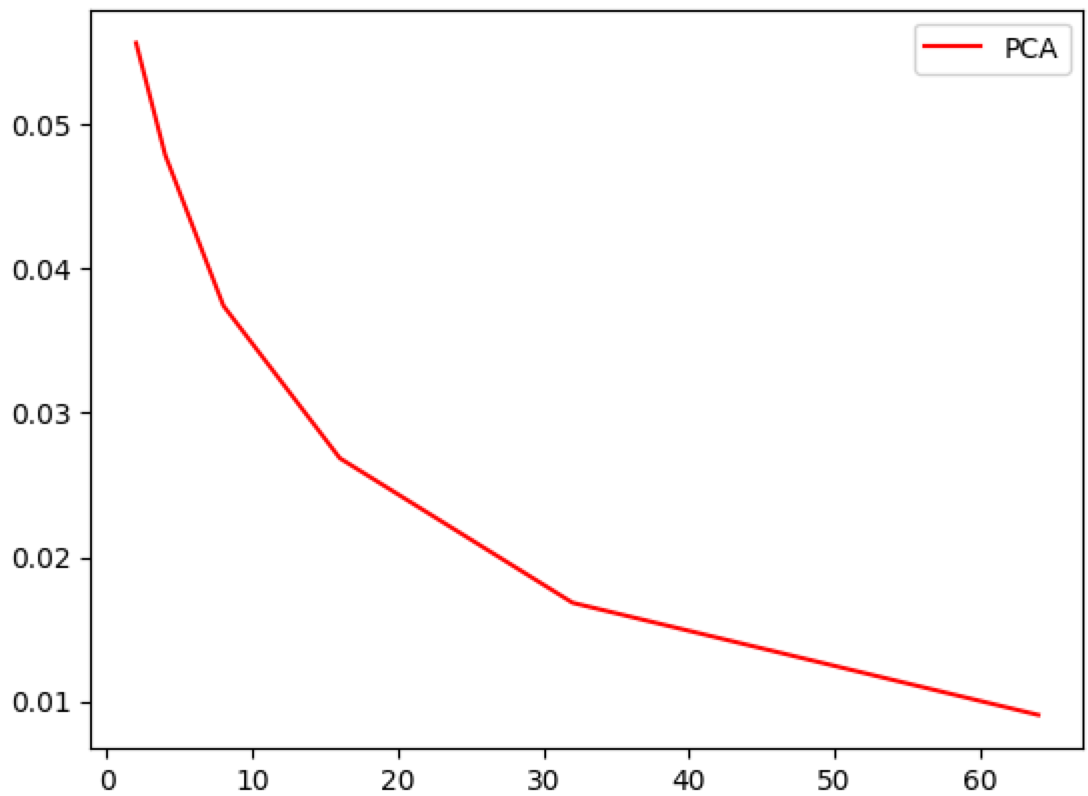
Results



Latent Dimension 2: MSE = 0.049321193248033524
Latent Dimension 4: MSE = 0.03566211089491844
Latent Dimension 8: MSE = 0.020942172035574913
Latent Dimension 16: MSE = 0.011992661282420158
Latent Dimension 32: MSE = 0.0057846251875162125
Latent Dimension 64: MSE = 0.0042060441337525845

Part D: Graph Plotting





PCA
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60