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Solution:

We are given Y1 = max (x1, x2, ---, xn)

Now for cdf of Y,

FY1 (x) = P(Y1 & x)

 $= P \left(\max \left(x_1, x_2, \dots, x_n \right) \leq x \right)$

 $\max(x_1, x_2, ---, x_n) \leq x$ we need to have each

one of Xi to be less than x,

So, FY, (x) = P(x1 ≤ x, x2 ≤ x, ---, xn ≤ x)

= P(x1 = x) . P(x2 = xy --- P(xn = xy

[: x1, x2, -- xn are independent]

SO FY, (X) = Fx1(X). Fx2(X) - F(X) [: P(x; < x) = Fx; (x)]

= Fx(x). Fx(x) --- Fx(x) [Fxi(x) = Fx(x) because

FY, (X) =[FX(X)]n

Xi are identically distributed

Now for pdf of Ex YI.

 $f_{Y}(x) = d [F_{Y}(x)] = d [F_{X}(x)]^{n} = n[F_{X}(x)]^{n-1} d F_{X}(x)$ dx dx dx

fy, (x) = n [fx(x)]n-1. Fx(x)

Now, for Yz, we are given Yz = min (X1, x2, ---, xn)

Now, for cdf of Yz

FY2(X)= P (Y2 = X)

= $P(\min(x_1, x_2, \ldots, x_n) \leq x)$

For $\min(X_1, X_2, -..., x_n) \leq x$, we need to have at least one

of Xi to be less than x, which is the negation of every

X: >x. So, Fy (x) can be written as

Fy (x) = 1 - p (x1 > x, x2 > x, ---, Xn > x)

= 1- P(X1>x). P(X2>x) --- P(Xn>x)

[: X1, X2, --, Xn are independent]

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	so Fy (x) = 1 - (1 - Fx1(x)). (1 - Fx2(x)) (1 - Fxx(x))
	[: P(X > x) = 1-Fx(xy) 1 (x)
	$F_{Y_2}(xy = 1 - (1 - F_X(xy)) \cdot (1 - F_X(xy)) \cdot (1 - F_X(xy))$
	Tag Xi axe identically distributed).
7	$FY_{1}(x) = 1 - (1 - F_{x}(x))^{n}$
`	Now, for pdf of Y2,
	$f_{Y_{L}}(xy) = d F_{Y_{L}}(xy) = d \left[1 - \left(1 - F_{X_{L}}(xy) \right)^{n} \right]$ $dx \qquad dx$
	The state of the s
	= -n (1-Fx(xy) n-1. [-d Fx(xy] dn
9	$f_{Y_2}(xy) = m \left(1 - F_{x}(xy)\right)^{n-1} F_{x}(xy)$
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$\tilde{\mathbb{T}}(f_{i},A_{i})_{i} \wedge_{i} F_{i}$	A provide the total and the second of the se
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Solution: According to the definition of GMM, we are selecting the ith distribution with probability pi, then we draw a value for a random variable with pdf as that of the selected distribution.

So, the equation of pdf for X becomes, $f_X(x) = \sum_{i=1}^{K} p_i f_{X_i}(x)$

Now, $f_{xi}(x) = \mathcal{N}(\mu i, \sigma i^2)$ so, $f_{x}(x) = \sum_{i=1}^{K} pi \mathcal{N}(\mu i, \sigma i^2)$

Mean :-ECX] = Sxfx(x)dx = Sx (E pi N(Hi, siy) dx = \(\frac{\times \(\rho_i\) \x \N(\rho_i^2) dx\\\

But, fx N(Mi, oi2) dx is nothing but Mi

 $E[x] = \sum_{i=1}^{k} (pi \mu i)$

Variance ! -

we have, $Var(x) = E[x^2] - (E[x])^2$

For $E[x^2]$ we have ω $E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(y) dx = \int_{-\infty}^{\infty} x^2 \left(\sum_{i=1}^{\infty} p_i \mathcal{N}(\mu_i, \sigma_i^2) \right) dx$ $= \sum_{i=1}^{\infty} \left(p_i \int_{-\infty}^{\infty} x^2 \mathcal{N}(\mu_i, \sigma_i^2) dx \right)$ $= \sum_{i=1}^{\infty} \left(p_i \int_{-\infty}^{\infty} x^2 \mathcal{N}(\mu_i, \sigma_i^2) dx \right)$

They are stone

But,
$$\int_{-\infty}^{\infty} x^2 \mathcal{N}(\mu i, \sigma i^2) dx = \sigma i^2 + \mu i^2$$

$$E(x^2) = \sum_{i=1}^{k} p_i(\sigma i^2 + \mu i^2)$$

so, we get finally

MGF :-

$$\phi_{x}(t) = E \left[e^{tx} \right]$$

$$= \int_{-\infty}^{\infty} e^{tx} f_{x}(x) dx$$

$$= \int_{\infty}^{\infty} e^{tx} \left(\sum_{i=1}^{K} p_{i} \mathcal{N}(\mathcal{U}_{i}, \sigma_{i}^{2}) \right) dx$$

$$= \sum_{i=1}^{K} \left(p_{i} \int_{-\infty}^{\infty} e^{tx} \mathcal{N}(\mathcal{U}_{i}, \sigma_{i}^{2}) dx \right)$$

$$= \sum_{i=1}^{K} \left(p_{i} \int_{-\infty}^{\infty} e^{tx} \mathcal{N}(\mathcal{U}_{i}, \sigma_{i}^{2}) dx \right)$$

=
$$\sum_{i=1}^{k} (p_i \phi_{X_i}(t))$$
 [where $\phi_{X_i}(t)$ is MGF of normal distribution $N(\mu_i, \sigma_i^2)$]
= $\sum_{i=1}^{k} (p_i e^{\mu_i t + \sigma_i^2 t^2/2})$.

$$\begin{aligned} \text{Var}(Z) &= \not \in \text{Var}\left(\stackrel{E}{\Sigma} \text{ pi} \times i \right) \\ &= \stackrel{E}{\Sigma} \text{ Vor}\left(\text{pi} \times i \right) \qquad \text{[a]} \quad \text{Xi are independent]} \\ &= \stackrel{E}{\Sigma} \quad \text{pi}^2 \text{ Var}\left(\times i \right) = \stackrel{E}{\Sigma} \quad \text{pi}^2 \quad \text{si}^2 \end{aligned}$$

$$Vor(z) = \sum_{i=1}^{k} \rho_i^2 \sigma_i^2$$

$$\phi_{2}(t) = \exp\left\{ t \sum_{i=1}^{K} p_{i} \mu_{i} + t^{2} \sum_{i=1}^{K} p_{i}^{2} \sigma_{i}^{2} \right\}$$

PDF of Z

Consider a random variable Y, with pdf
$$(\stackrel{\cancel{\Sigma}}{\Sigma}pi\mathcal{U}i, \stackrel{\cancel{\Sigma}}{\Sigma}pi^2\sigma_i^2)$$

$$\phi_{Y}(t) = \exp\{t\stackrel{\cancel{\Sigma}}{\Sigma}pi\mathcal{U}i + t^2\stackrel{\cancel{\Sigma}}{\Sigma}pi^2\sigma_i^2\} = \phi_{Z}(t)$$

$$i=1$$

$$2 i=1$$

Now we know that for a discrete rand. continuous random variable pdf and MGF uniquely determine each other

$$f_{z}(x) = \frac{1}{\sqrt{\text{Var}(z).2\pi}} \frac{\exp\left(-1\left(x - \text{E}[z]\right)^{2}\right)}{2}$$

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3.	Solution!-
	Variance 52
	We define a random variable $Y = X - \mu$.
	It is clear that $E[Y] = E[X] - E[X] = 0$.
	and Variance of $Y = E[LY - 0]^2] = E[X^2 - 2\mu X + \mu^2]$
	$= E[x^{2}] - 2\mu^{2} + \mu^{2} = E[(x-\mu)^{2}] = \sigma^{2}$ Now for any $\mu > 0$
	We have tor T >0
	$P(X-M \ge \tau) = P(Y \ge \tau) = P(Y+u \ge \tau+u)$ $\neq P((Y+u \ge \tau+u) + (Y+u \ge \tau+u)$
	≤ ET (Y+u) ² 7
	$\leq E[(Y+u)^2]$ [By Markov's inequality]
	$= \sigma^{2} + u^{2} [: E[(Y+u)^{2}] = E[(Y^{2} + 2uY + u^{2})] = \sigma^{2} + u^{2}]$
	(T+u)2
	This is true for any u >0. So choose u= 02.
	T A T T T T T T T T T T T T T T T T T T
	we get,
	$P(x-\mu \geqslant \tau) \leq \sigma^2 + \sigma^4/\tau^2 = \sigma^2$
	$(\tau + \sigma^2/\tau)^2 \qquad \sigma^2 + \tau^2$
	so, $P((x-\mu) \geq \tau) \leq \sigma^2$ for $\tau > 0$ -(1) $\sigma^2 + \tau^2$
	Now for any u <0, we have for T <0,
,	$P(X \cup X \geq T) = P(Y \geq T) = P(Y + U \geq T + U)$
	$\leq P((Y+u)T+u)U(Y+u \leq -(T+u)))$
	∠ P ((Y+u)² ≤ (T+u)²) [:: T+u <o] <="" p=""></o]>
	= 1 -
	P(X-M > T) = P(Y>T) = P(Y+U>T+U)
	> p((Y+u)2 < (T+u)2) [: T+u < 0]
	> 1 - P((Y+u)2 > (T+u)2) [:P(E) = 1-P(E)]

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	so, P(X-M ≥T) ≥ 1-P((Y+u)2 > (T+4)2)
	Now we have by Markov's inequality
	$P((Y+u)^2 \ge (T+u)^2) \le E[(Y+u)^2]$
	U-Y-Y (T+u)2
	SO P(X-12 ≥ T) ≥ 1- E[(Y+4) -]
	[u : xu - x] = 1 [(T+w) 2
5 1	= 1- (E[Y2+2Y4+42])
	11 (2+4), 2 100 1 3V11
	(v=) = 1-1{ \signa^2+u^2} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
7.1	(+1) - > (+x) U (++ = U+ (t+w)2
To	since this is true for any $u \leq 0$, choose $u = \sigma^2 (T < 0)$
Ţ.	The Bollow Part [(MILL)] I am the state of
	so we get P(X-4>T)> 1- { 52+54/T2}
Trans- 1 100	715+ 17] = [1mm] = 1 (7+ 52/t)2
	(=+1,- 62
	- = -15 week & 5 & 18 your fat + Turt or will
	so, $P(X-\mu \geq \tau) \geq 1-\sigma^2$ for $\tau < 0$.
	σ2+τ2 to the
	-19 = 27/20 + 27 = (1 = M-1) 3
	(24,0,42)
	(1) - 0 = 1 for 7 = 0 - (1)
	- 1+2D
	may fix any at sic me have for it is
	(n-1 = n+1, 10 = (1 = 1) 1 = (2 < 1 - 1) 1
	11+21-311+1) A(N+2 (N+2)) 15
1	DSUTT 1.] ("10+1) = "(10+1) 9 =
	(n-11 & 11 + 1) d = (2 & 1) d = (2 & 1 - 1) .
	F ((3+10) = (1+10)) 7: 7:
1,314-1	= 1- P ((pid) = (1-u) =) [.P(r)]

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4.	Solution:
	N00 L
	$ \frac{\Phi_{x}(t) = \int_{-\infty}^{\infty} e^{tz} f_{x}(z) dz}{-\infty} > \int_{x}^{\infty} e^{tz} f_{x}(z) dz $
	-co m
	> etx fx(z)dz [as etx is minimum value of etz
	when $\mathbf{z} \in [x, \infty)$
	so, $\phi_{x}(t) \geq e^{tx} P(x \geq x)$
	so, $P(x \ge x) \le e^{tx} \phi(t)$
	Now, we prove it for discrete random variables.
	$\phi(t) = \sum_{z} e^{tz} f_{z}(z) \geq \sum_{z} e^{tz} f_{z}(z)$
, ($\phi_{x}(t) = \sum_{z} e^{tz} f_{x}(z) \geqslant \sum_{z > x} e^{tz} f_{x}(z)$
4	> etx. \(\int fx(z) \) [as etx is minimum value of etz
	$\geq e^{tx}$. $\sum f_{x}(z)$
	so, ox lt) > etx P(x > x)
	$P(x \ge x) \le e^{tx} \Phi_{x}(t)$
76-7	THE STATE OF THE POLY OF THE LEAST SEE MA SEE
11.50	Now given n independent Bernoulli random variables X1, X2,, Xn,
	where ELXi]=pi and M= \(\Sigma\)
1744	out = t les t viets in the est alley overrining and = ?
	We have by using the inequality proved above
150	the appropriate value of t = log (1+5)
	$P(X > (1+8) \mu) \leq e^{t(1+8)\mu} \Phi_{x}(t)$
=	≥ \$x\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	e(1+8)+M
	Given xi are independent,
	$P(X>(1+8), M) \leq \frac{\phi_{X_1}(1)\phi_{X_2}(1)\phi_{X_n}(1)}{e^{(1+s)tM}}$
	elitsitu
	We know that as Xi's are Bernoulli random variables,
	So prilt = 1-p+pet = 1+p(et-1)

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	so $\phi_{xilt} = 1 + p_i(e^t - 1) \le e^{p_i(e^t - 1)}$ [given $1 + x \le e^x$]
	so we get
	P(x>(1+8)u) < ep1(et-1) ep2(et-1) Ph(et-1)
	o(1+8) tu
	e(1+8)tμ ≤ e(5pi) (et-1)
	p (1+8) th
	= eucet-1) [given \(\frac{n}{2} \) pi = u] eucet-1) [given \(\frac{n}{2} \) pi = u]
	eci+s)tu i=1
	Hence $P(X > (1+8)\mu) \leq e^{\mu(et-1)}$ $e^{(1+s)t\mu}$
	To tighten this bound, we should make the right hand side
71	
	of the inequality minimum. So let glt) = eu(et-1) = eu(et-1)-(1+8)th
	e(1+8) + m
	g'(t) = eu(et-1)-(1+8)+u. (net-c1+8) u)
	we can see that for t< loge(1+8), gl(+) is negative
	and for +> loge (L+8) g'lt is positive. So, gets decreases
	for t < log c1+8) and increases for t> loge (1+8).
	So the minimum value for gett is attained at t=loge(1+8)
	where having the manufactured and the
-	50 the appropriate value of t = loge (1+8)
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6.	1) Non-inverted:
	The correlation coefficient is positive, which implies that
	the images are not inverted relative to each other.
	2) Inverted:
	The correlation coefficient will be minimum when the two
	images overlap each other and the shift is 0; this can be
	seen in the figure. Also, we can see from the graphs that
	the quadratic mutual information and correlation coefficient
	are inversely related. Also, the curves are not symmetric,
	so we can infer that the images are also not symmetric
	about Y axis.
	when the joint histogram is more concentrated in a small
,	number of bins, then the QMI is also higher.
1	
	*

	The D
	$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_$
7.	solution: $Cov(Xi,Xj) = E[Xi,Xj] - E[Xi]E[Xj]$ $(i\neq j)$
	" m// C / m./h. mia/ II
	We know that, MGF of multinomial a $\phi_x(\vec{t}) = (p_1e^{t_1} + p_2e^{t_2} + + p_ke^{t_k})^n$
4	Now E Exixi] = 22
	$E \left[x_i \cdot x_j \right] = \partial^2 \left(\phi_x \left(\overrightarrow{t} \right) \right) \left(\begin{array}{c} (i \neq j) \\ \text{tistje0 } \overrightarrow{t} = 0 \end{array} \right)$
	atiati tietjeo teo
	= 2 m(p_et1+p_et2++puetx)n-1 pieti
	Oti + -0
	= m(n-1) (pieti+pretz+-+ pretu)n-2 pieti.pjetj ==0
	= m(n-1) (xpi) n-2. pipi
	= n(n+). (1) pipj
	So
	E [xi xj] = m(n-1) pipj
	so, cov(xi, xj) = m(n-1)pipj - mpi. mpj
	The state of the s
	so Gv(xi, xi) = -npipi
	so the covariance matrix is
5.2	WE Stray E. J. Try May V. Manga peters of a con-
	$Cov(x_i, x_j) = /-npip_i$ if $i \neq j$
	$\int var(x_i) if i=j$
-	So, n rend and stand of the first than I say it is
	$cov(x_i, x_i) = /-npip_i$ (if $i \neq i$)
	mpi(1-pi) (if i=j)
(1)	