20XT56 - Scientific Computing Lab Package Vehicle Routing

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Introduction

One of the most important applications of optimization is *vehicle routing*, in which the goal is to find the best routes for a fleet of vehicles visiting a set of locations. Here are a few examples of routing problems:

- A package delivery company wants to assign routes for drivers to make deliveries.
- A cable TV company wants to assign routes for technicians to make residential service calls.
- A ride-sharing company wants to assign routes for drivers to pick up and drop off passengers.

A more general version of the TSP is the vehicle routing problem (VRP), in which there are multiple vehicles. In most cases, VRPs have constraints:

- Vehicle Routing Problem, a generalisation of the TSP with multiple vehicles.
- VRP with capacity constraints, in which vehicles have maximum capacities for the items they can carry.
- VRP with time windows, where the vehicles must visit the locations in specified time intervals.
- VRP with resource constraints, such as space or personnel to load and unload vehicles at the depot (the starting point for the routes).

Vehicle routing problems are inherently intractable: the length of time it takes to solve them grows exponentially with the size of the problem.

Vehicle Routing Problem

In the *Vehicle Routing Problem (VRP)*, the goal is to find optimal routes for multiple vehicles visiting a set of locations. When there's only one vehicle, it reduces to the Traveling Salesperson Problem.

Imagine a company that needs to visit its customers in a city made up of identical rectangular blocks. A diagram of the city is shown below, with the company location marked in black and the locations to visit in blue.

X-Y COORDINATES TO THE LOCATIONS SHOWN IN THE CITY DIAGRAM

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[(456, 320), # LOCATION 0 - THE DEPOT
 (228, 0),   # LOCATION 1
 (912, 0),   # LOCATION 2
 (0, 80),    # LOCATION 3
 (114, 80),  # LOCATION 4
 (570, 160), # LOCATION 5
 (798, 160), # LOCATION 6
 (342, 240), # LOCATION 7
 (684, 240), # LOCATION 8
 (570, 400), # LOCATION 9
 (912, 400), # LOCATION 10
 (114, 480), # LOCATION 11
 (228, 480), # LOCATION 12
 (342, 560), # LOCATION 13
 (684, 560), # LOCATION 14
 (0, 640),   # LOCATION 15
 (798, 640)] # LOCATION 16
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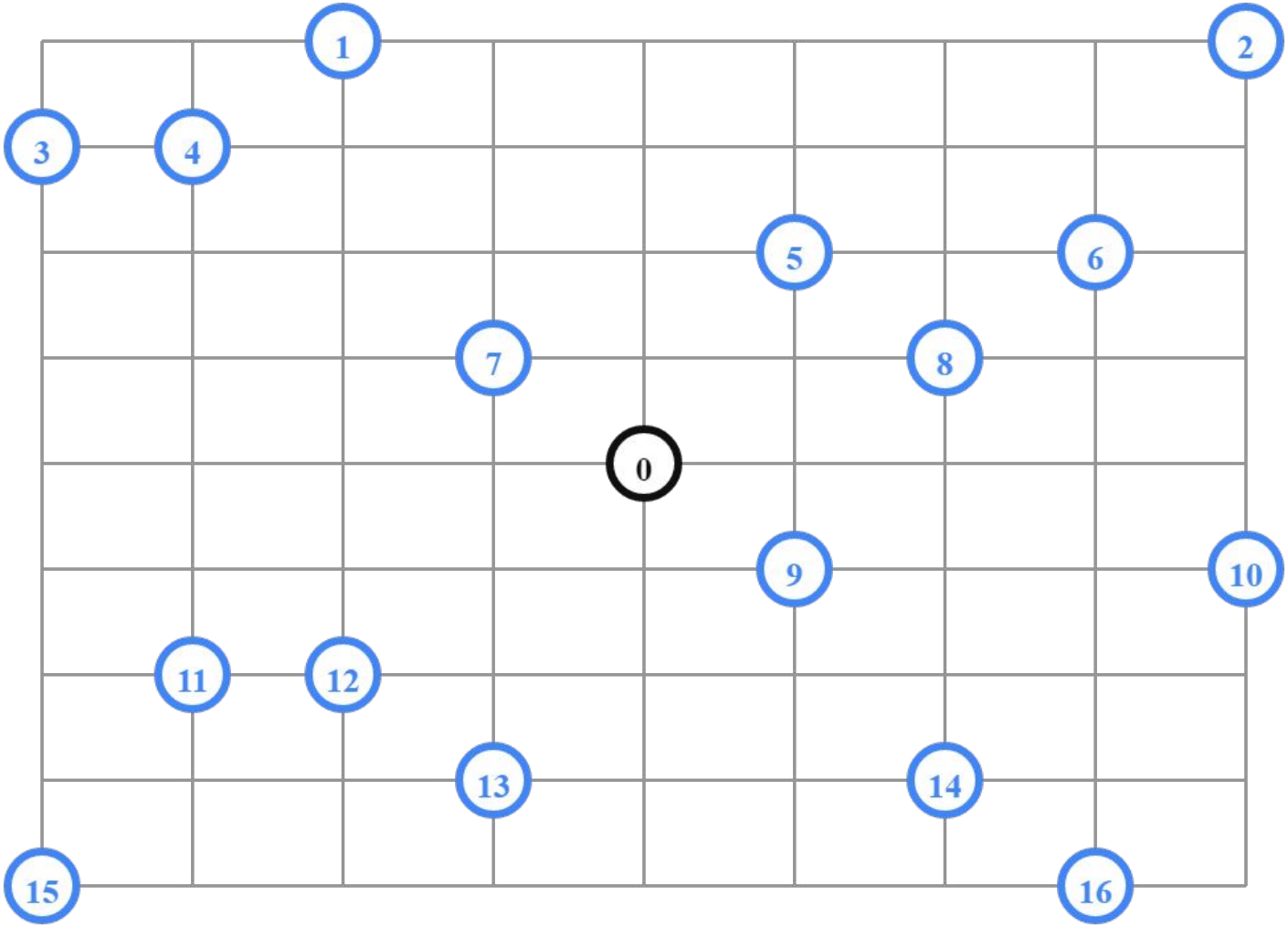
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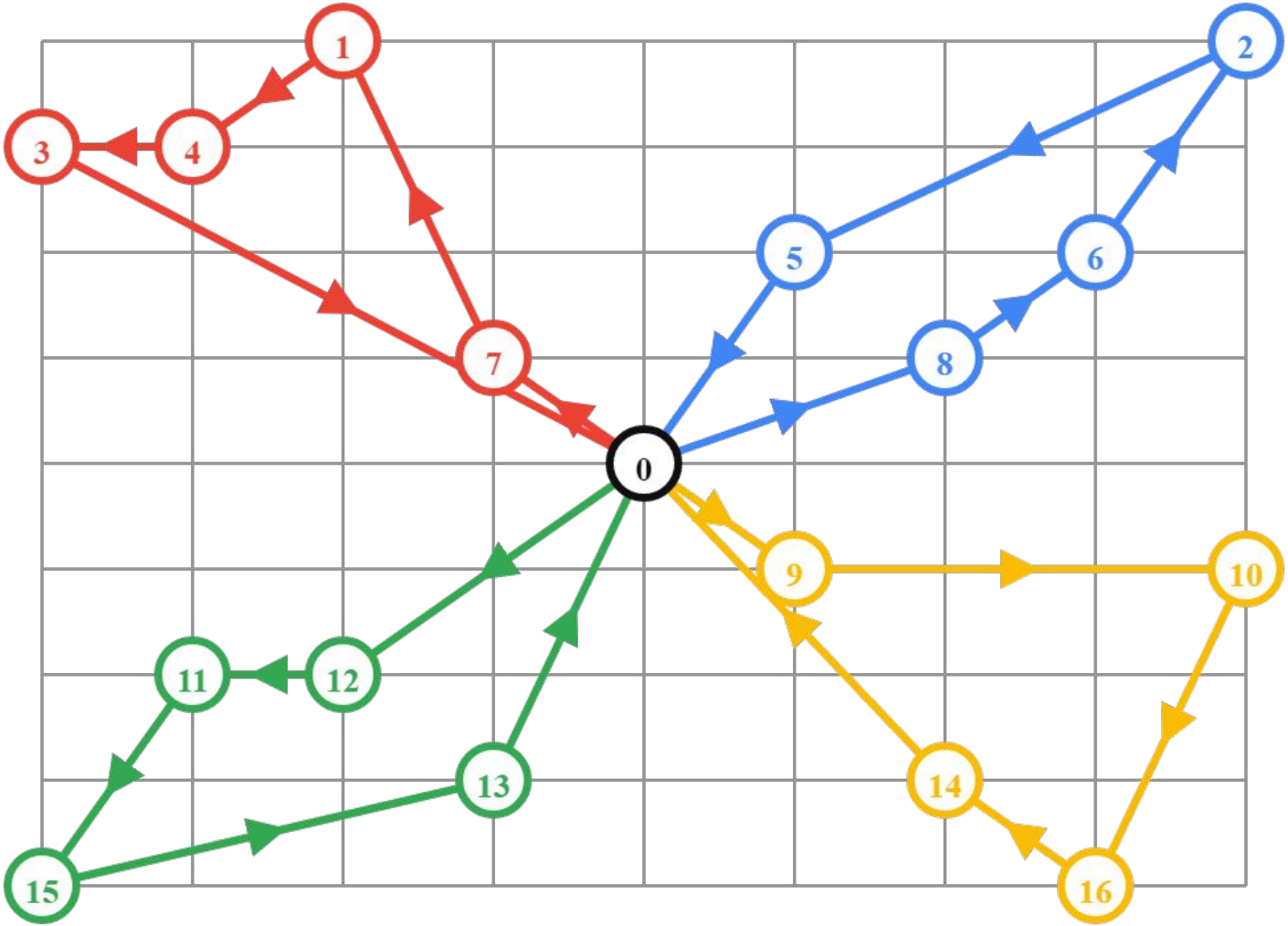
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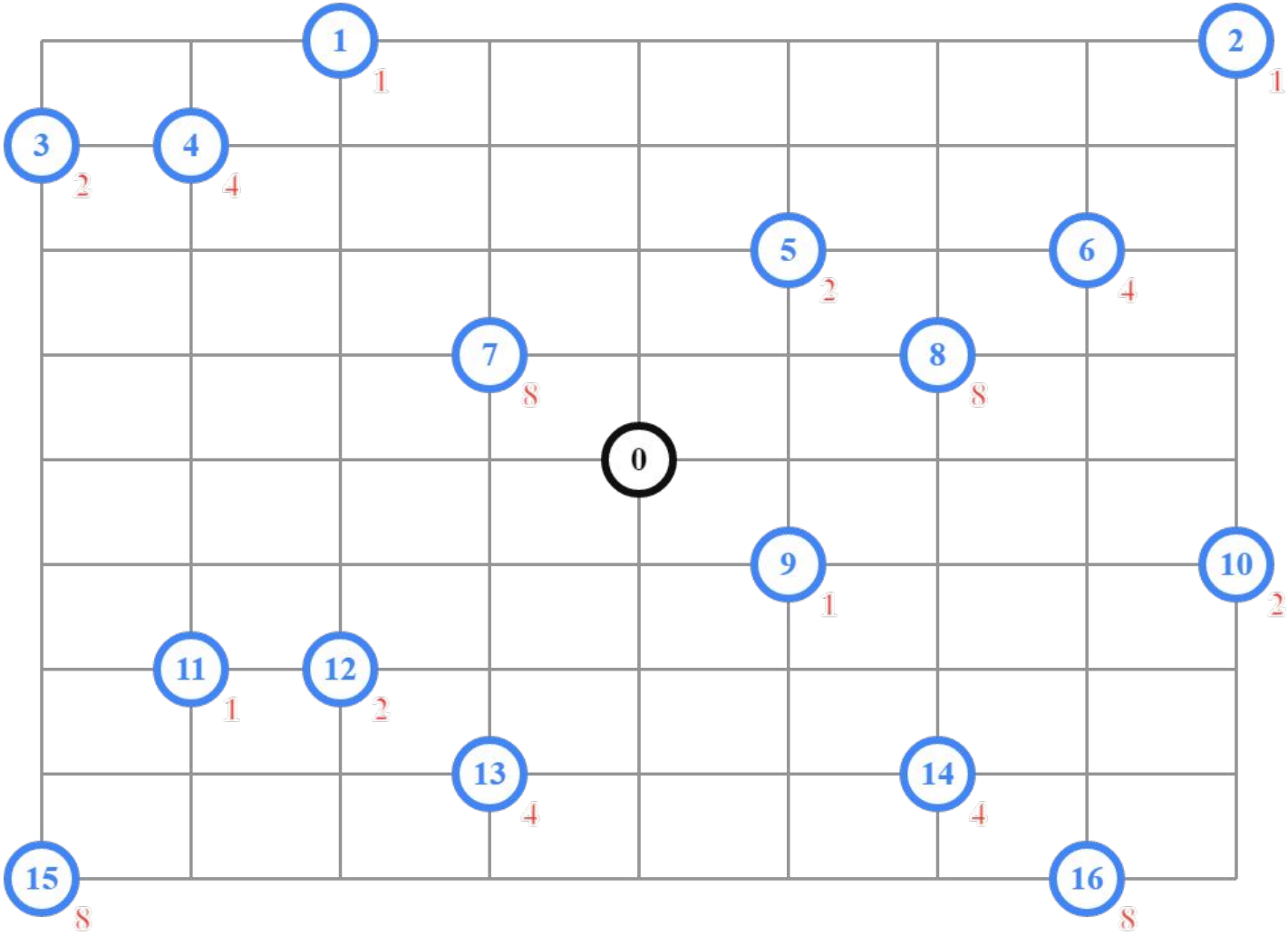
Vehicle Routing with Capacity Constraints

The *capacitated vehicle routing problem* (CVRP) is a VRP in which vehicles with limited carrying capacity need to pick up or deliver items at various locations. The items have a quantity, such as weight or volume, and the vehicles have a maximum *capacity* that they can carry. The problem is to pick up or deliver the items for the least cost, while never exceeding the capacity of the vehicles.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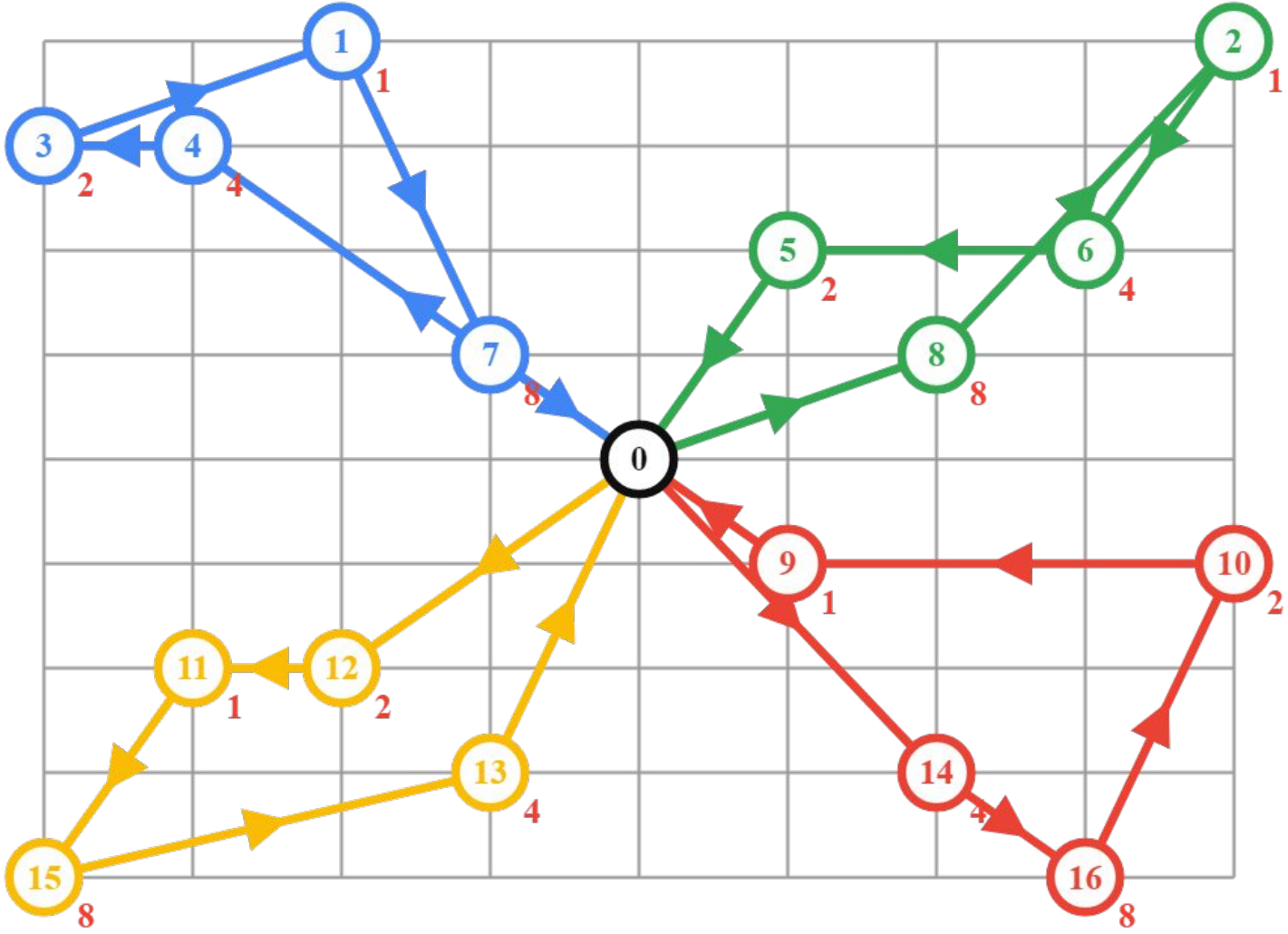
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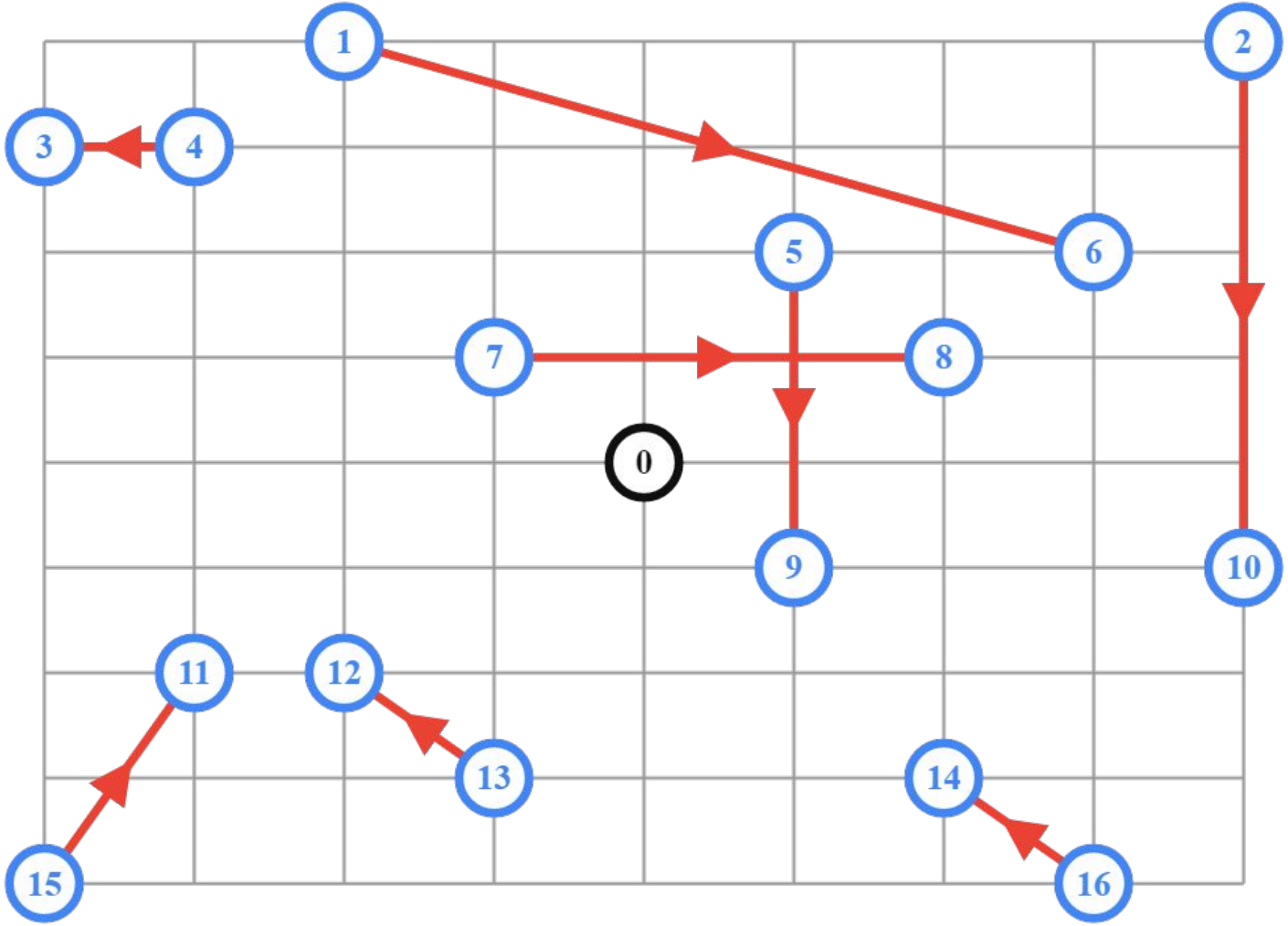
Vehicle Routing with Pickups and Deliveries

The problem is to assign routes for the vehicles to pick up and deliver all the items, while minimizing the length of the longest route. For each item, there is a directed edge from the pickup location to the delivery location.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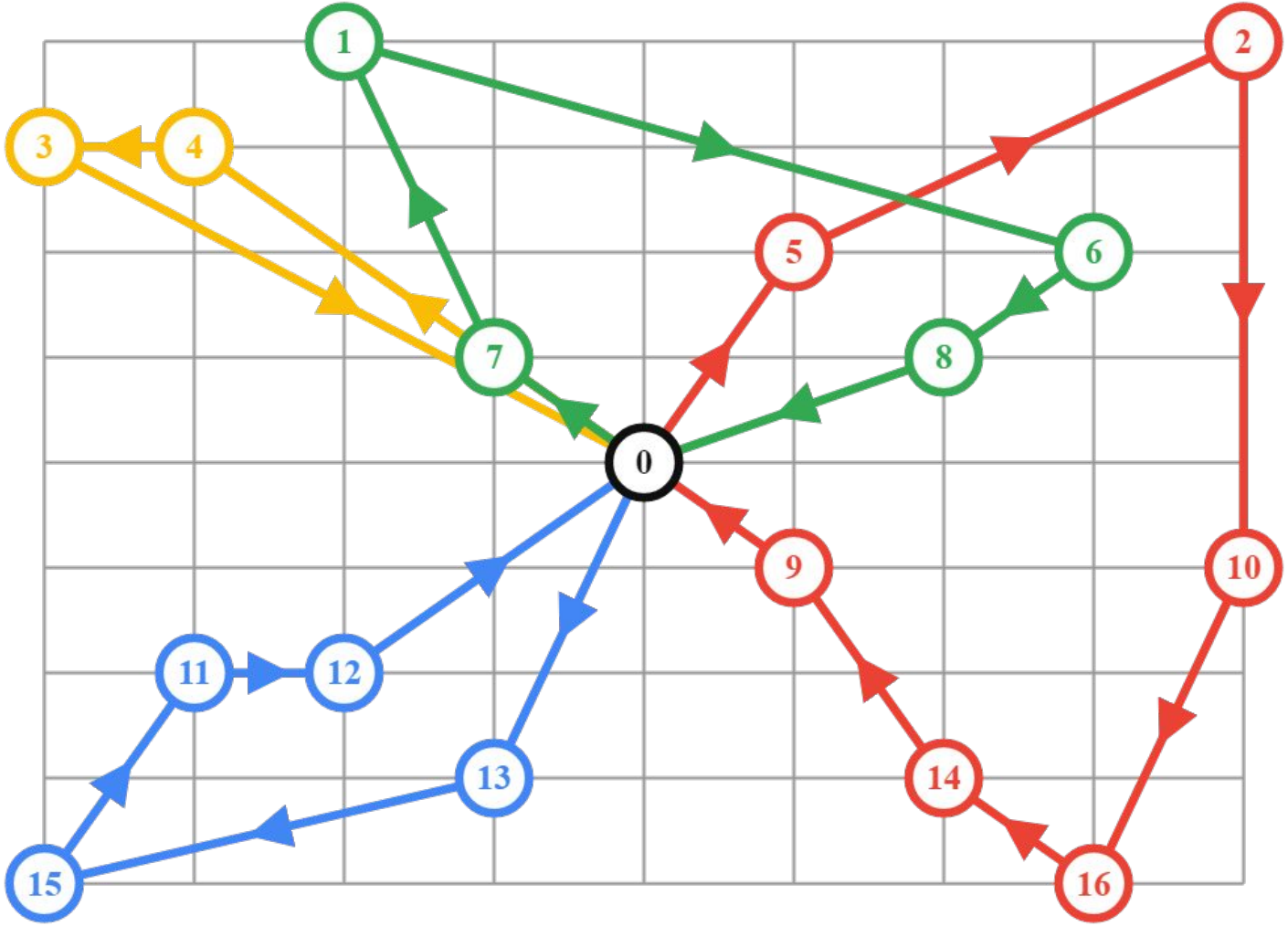
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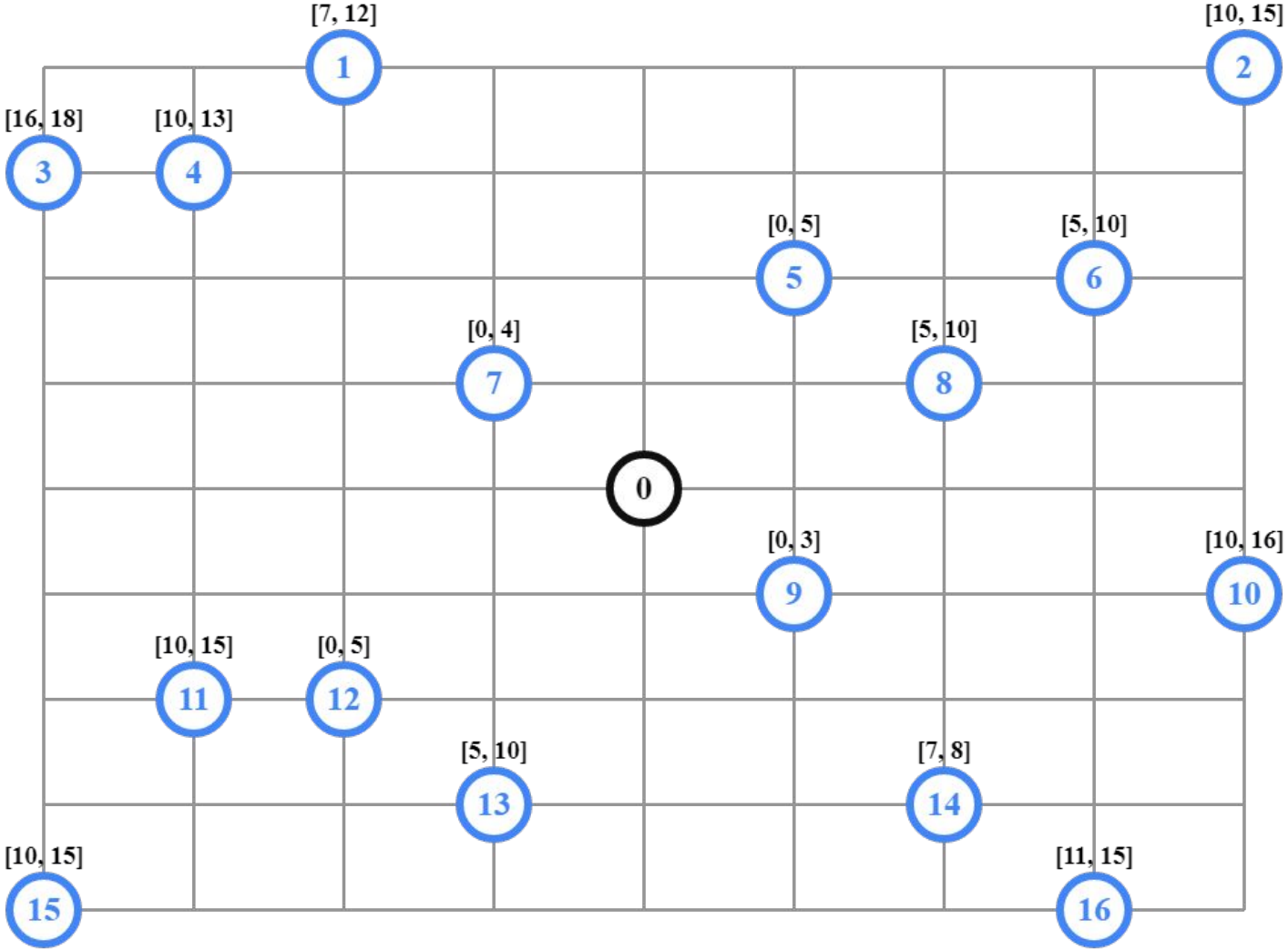
Vehicle Routing Problem with Time Windows

Many vehicle routing problems involve scheduling visits to customers who are only available during specific time windows. These problems are known as *vehicle routing problems with time windows*. The goal is to minimize the total travel time of the vehicles.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