Hypothesis Testing

Q1. Suppose a child psychologist claims that the average time working mothers spend talking to their children is at least 11 minutes daily. You conduct a random sample of 1000 working mother and find they spend an average of 11.5 minutes per day talking with their children. Assume prior research suggests the population Standard Deviation is 2.3 minutes. Conduct a test with a level significance of $\alpha = 0.05$.

$\mu_0 = 11$
$n = 1000$
$\bar{x} = 11.5$
$\sigma = 2.3$

$H_0: \mu = 11$
$H_1: \mu > 11$

Test Statistic z

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$\Rightarrow z = \frac{11.5 - 11}{2.3 / \sqrt{1000}} \sim N(0,1)$$
$$\Rightarrow z = \frac{0.5 * \sqrt{1000}}{2.3} \sim N(0,1)$$
$$\Rightarrow z = 6.87 \sim N(0,1)$$

$P(z > z_\alpha) = 0.05$
$\Rightarrow 1 - P(z \le z_\alpha) = 0.05$
$\Rightarrow 1 - [P(-\infty \le z \le 0) + P(0 \le z \le z_\alpha)] = 0.05$
$\Rightarrow 1 - [0.5 + P(0 \le z \le z_\alpha)] = 0.05$
$\Rightarrow 0.5 - P(0 \le z \le z_\alpha) = 0.05$
$\Rightarrow P(0 \le z \le z_\alpha) = 0.5 - 0.05$
$\Rightarrow P(0 \le z \le z_\alpha) = 0.45$

Table value $z_\alpha = 1.64$ when $P(0 \le z \le z_\alpha) = 0.45$
(Table: Area under standard Normal curve for given z values)

Since $z > z_\alpha$, we reject H_0 (i.e., z lies on critical region)

So, we can conclude that the time spent by working women with their children is less than 11 minutes per day.

Q2. A coffee shop claims their average wait time for customers is less than 5 minutes. A sample of 40 customers is taken to test this claim, and their wait times are recorded. The sample mean wait time is 4.6 minutes with a Standard Deviation of 0.8 minutes. Perform a hypothesis test at a significance level of 0.05 and determine whether there is enough evidence to support the coffee shop's claim.

$\mu_0 = 5$
$n = 40$
$\bar{x} = 4.6$
$s = 0.8$

$H_0: \mu \geq 5$
$H_1: \mu < 5$

Test Statistic z

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$$

$$\Rightarrow z = \frac{4.6 - 5}{0.8/\sqrt{40}} \sim N(0,1)$$
$$\Rightarrow z = \frac{-0.4 * \sqrt{40}}{0.8} \sim N(0,1)$$
$$\Rightarrow z = -3.16 \sim N(0,1)$$

$P(z < -z_\alpha) = 0.05$
$\Rightarrow P(z > z_\alpha) = 0.05$
$\Rightarrow 1 - P(z \leq z_\alpha) = 0.05$
$\Rightarrow 1 - [P(-\infty \leq z \leq 0) + P(0 \leq z \leq z_\alpha)] = 0.05$
$\Rightarrow 1 - [0.5 + P(0 \leq z \leq z_\alpha)] = 0.05$
$\Rightarrow 0.5 - P(0 \leq z \leq z_\alpha) = 0.05$
$\Rightarrow P(0 \leq z \leq z_\alpha) = 0.5 - 0.05$

$\Rightarrow P(0 \leq z \leq z_\alpha) = 0.45$

Table value $z_\alpha = 1.64$ when $P(0 \leq z \leq z_\alpha) = 0.45$
(Table: Area under standard Normal curve for given z values)

Since $z < -z_\alpha$, we reject H_0 (i.e., z lies on critical region)

So, the average wait time is less than 5 minutes.

Area under Standard Normal Curve for given z-values (for positive z values)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999

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