Statistics Assignment 9

1. Marginally we have $X \sim Bin(n, ps)$, as shown on a previous homework problem using a story proof (the eggs can be thought of as independent Bernoulli trials with probability ps of success for each). Here X and Y are not independent, unlike in the chicken-egg problem from class (where N was Poisson). This follows immediately from thinking about an extreme case: if $X = n$, then clearly Y = 0. So they are not independent: P(Y = 0) < 1, while P(Y = 0|X = n) = 1.
   To find the joint distribution, condition on N and note that only the $N = i + j$ term is nonzero: for any nonnegative integers i, j with $i + j \leq n$,
$$\begin{aligned} P(X = i, Y = j) &= P(X = i, Y = j | N = i + j) P(N = i + j) \\ &= P(X = i | N = i + j) P(N = i + j) \\ &= \binom{i + j}{i} s^i (1 - s)^j \binom{n}{i + j} p^{i+j} (1 - p)^{n-i-j} \\ &= \frac{n!}{i!\, j!\, (n - i - j)!} (ps)^i \left(p(1 - s)\right)^j (1 - p)^{n-i-j}. \end{aligned}$$
   If we let Z be the number of eggs which don't hatch, then from the above we have that (X, Y, Z) has a Multinomial(n,(ps, p(1 -s),(1 -p)) distribution, which makes sense intuitively since each egg independently falls into 1 of 3 categories: hatch-and-survive, hatch-and-don't-survive, and don't-hatch, with probabilities ps, p(1 -s), 1- p respectively.