

Statistics Assignment 5

1. Consider two fair, independent coin tosses, and let A be the event that the first toss is Head, B be the event that the second toss is Head, and C be the event that the two tosses have the same result. Then A, B, C are dependent since $P(A \cap B \cap C) = P(A \cap B) = P(A)P(B) = 1/4 \neq 1/8 = P(A)P(B)P(C)$, but they are pairwise independent:

A and B are independent by definition;

A and C are independent since $P(A \cap C) = P(A \cap B) = 1/4 = P(A)P(C)$, and similarly B and C are independent.

2. Let A be the event that the initial marble is green, B be the event that the removed marble is green, and C be the event that the remaining marble is green. We need to find $P(C|B)$. There are several ways to find this;

One natural way is to condition on whether the initial marble is green:

$$P(C|B) = P(C|B, A)P(A|B) + P(C|B, A^c)P(A^c|B) = 1P(A|B) + 0P(A^c|B).$$

To find $P(A|B)$, use Bayes' Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{1/2}{P(B|A)P(A) + P(B|A^c)P(A^c)} = \frac{1/2}{1/2 + 1/4} = \frac{2}{3}.$$

So $P(C|B) = 2/3$.