Statistics Assignment 9

1. a) Let W_i be the event of winning the i^{th} game. By the law of total probability,
$$P(W_1) = (0.9 + 0.5 + 0.3)/3 = 17/30.$$

   b) We have $P(W_2|W_1) = P(W_2, W_1)/P(W_1)$. The denominator is known from (a), while the numerator can be found by conditioning on the skill level of the opponent:
$$P(W_1,\, W_2)$$
$$= \frac{1}{3}\, P(W_1, W_2|beginner) + \frac{1}{3}\, P(W_1, W_2|intermediate) + \frac{1}{3}\, P(W_1, W_2|expert).$$
   Since W_1 and W_2 are conditionally independent given the skill level of the opponent, this becomes
$$P(W_1, W_2) = \frac{0.9^2 + 0.5^2 + 0.3^2}{3} = \frac{23}{60}.$$
   So
$$\mathrm{P}(\mathrm{W}_2|\mathrm{W}_1) = \frac{23/60}{17/30} = 23/34$$

   c) Independence here means that knowing one game's outcome gives no information about the other game's outcome, while conditional independence is the same statement where all probabilities are conditional on the opponent's skill level. Conditional independence given the opponent's skill level is a more reasonable assumption here. This is because winning the first game gives information about the opponent's skill level, which in turn gives information about the result of the second game.
   That is, if the opponent's skill level is treated as fixed and known, then it may be reasonable to assume independence of games given this information; with the opponent's skill level random, earlier games can be used to help infer the opponent's skill level, which affects the probabilities for future games.