Inverse Kinematics

When position & orientation of end effector are known and we want to calculate the joint parameters

2 ways to solve → Analytical / → Approx.

Jacobian method – approx.

What is jacobian ??

↳ matrix that gives relation b/w joint velocities ($\dot{q}$) & E.E velocities ($\dot{X}$)

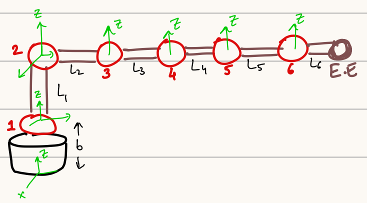
$$\dot{X} = J\dot{q}$$

→ Jacobian matrix : $J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$ → first 3 rows linear; → last 3 rows angular

→ No. of col. in jacobian matrix = no. of D.O.F of bot.

How to find Jacobian matrix ?

Ex. for a 6-DOF manipulator



1. Write transformation matrices from Forward Kinematics $T_{b1}, T_{12}, T_{23}, T_{34}, T_{45}, T_{56}, T_{6E}$
2. Multiply them all to find T_{bE} – pose & orientation of end effector

T_{bE} will be of form: $\begin{bmatrix} X & X & X & x \\ X & X & X & y \\ X & X & X & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ X = some value

3. Now that you know x, y, z,

Jacobian matrix will be of the form:

$$\begin{bmatrix} \partial x/\partial q_1 & \partial x/\partial q_2 & \partial x/\partial q_3 & \partial x/\partial q_4 & \partial x/\partial q_5 & \partial x/\partial q_6 \\ \partial y/\partial q_1 & \partial y/\partial q_2 & \partial y/\partial q_3 & \partial y/\partial q_4 & \partial y/\partial q_5 & \partial y/\partial q_6 \\ \partial z/\partial q_1 & \partial z/\partial q_2 & \partial z/\partial q_3 & \partial z/\partial q_4 & \partial z/\partial q_5 & \partial z/\partial q_6 \\ \hat{\omega}_1^b & \hat{\omega}_2^b & \hat{\omega}_3^b & \hat{\omega}_4^b & \hat{\omega}_5^b & \hat{\omega}_6^b \end{bmatrix}$$

→ compute the partial derivatives accordingly where $q_1, q_2 \ldots$ are joint positions

* This has 6 columns due to 6 DOF

How to find inverse kinematics using Jacobian?

X_g = goal position X_c = current transformation of EE

$[q_1, q_2 \text{---} q_n]$ = current joint positions

① For robots with velocity controlled joints :

a. $\Delta X = X_g - X_c$
b. $\dot{X} = P.\Delta X$ p = const. that decides speed of EE
c. Find Jacobian and then its inverse (J^{-1}) or pseudo inverse (J^{+})
d. $\dot{q} = (J^{-1}).\dot{X}$ so find $\dot{q}$ using this till $\Delta X = 0$

② For robots with position controlled joints:

a. $\Delta X = X_g - X_c$

b. $\delta X = f \cdot \Delta X$ where f = fractional value to make disp. small

c. Find Jacobian J using $[q_1, q_2 \dots q_n]$ & Jacobian inverse J^{-1} / J^{+}

d. Find $\delta q = J^{-1} \cdot \delta X$

e. New joint positions $q_{new} = q + \delta q$

Now this can be used to find new transformation

Limitations of Jacobian method:

- Doesn't work when Jacobian is not square matrix
  - ↳ for robots with no. of joints < 6 or > 6 → under-actuated or redundant
- When Jacobian is singular & can't be inverted

Fix: Use pseudo Jacobian $[J^{+}]$ ⟹ no need of manually calculating

⇓

calculated by libraries in python