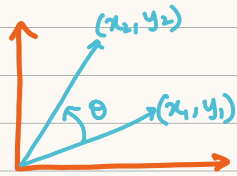
Rotation Matrices

→ 2D Rotation Matrix:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = R(\theta)\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$



Properties of rotation matrix:

→ Inverse = Transpose

→ Determinant = 1

→ Rotation × Rotation = Rotation

How to write rotation matrices:

3D:

Ex. for $R_x(\theta)$:

$R_x[x,x] = 1$

Set the rest of the diagonals to be $\cos\theta$

Set rest of the row & col. with 0

Set the empty space in the row below 1 as $-\sin\theta$. The rest with $\sin\theta$.

$$R_x(\theta) = \begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix} \rightarrow \begin{bmatrix} 1 & & \\ & \cos\theta & \\ & & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \\ 0 & & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation Matrices for X, Y and Z:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

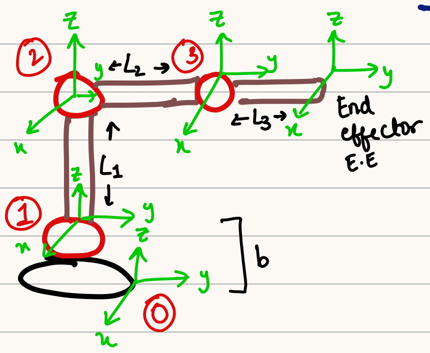
$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In 4-element:

$$\begin{bmatrix} \begin{bmatrix} & R_x(\theta) & \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

Ex. $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R_x(\theta)$$

Forward Kinematics



— Links
— joints
— frames

Translation: $\begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Transformation of Link 1 w.r.t Base 0:

Transformation = Translation × Rotation

→ Translation: Link 1 is b units +z wrt Base 0

→ Rotation: Rotates about joint1 around z-axis.

So, T.F = Trans(b, z) × Trotation(θ_1, z)

$$T.F_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T.F_{01} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Similarly, we find T_{12}, T_{23}, $T_{3,E}$

T.F of end effector $TF_{0E} = T_{01} \times T_{12} \times T_{23} \times T_{3E}$

$$TF_{0E} = \begin{bmatrix} C_1 & -S_1C_2C_3 + S_1S_2S_3 & S_1C_2S_3 + S_1S_2C_3 & L_3(-S_1C_2C_3 + S_1S_2S_3) - S_1C_2L_2 \\ S_1 & C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2C_3 & L_3(C_1C_2C_3 - C_1S_2S_3) + C_1C_2L_2 \\ 0 & S_2C_3 + C_2S_3 & -S_2S_3 + C_2C_3 & L_3(S_2C_3 + C_2S_3 + S_2L_2 + L_1 + b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orientation of EE — Position of EE

Inspired by:
rosroboticslearning.com