MULTIVARIABLE CALCULUS AND LINEAR ALGEBRA PROJECT 1

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PROJECT 1

INTRODUCTION

Over the past few weeks, lectures on parametric equations have been of a great importance to us. From the position of objects such as cars, tennis ball, athletes, tricycle tires, objects in motions and how fast these objects move, parametric equations form a formidable aspect of these objects in our environment. It is a fact that the concept of parametric equation is mainly grasped when it is practically done and visualized.

Curves that are parameterized are said to have their coordinates of (x, y) to be represented in a variable (t) and the variable (t) becomes an independent variable while the x and y becomes dependent variables.

Illustration:

$$x = f(t)\, y = g(t) \quad where\ t \in R$$

The main objective of this project is to write a four letter word “come” which is a single (broken) parametrized curve and animate plotter with the use of the programming software called Matlab. This task goes a long way to broaden our scope of vector-valued functions, parametric curves and some concept of vector calculus and integrals to be able to achieve success in the multivariable calculus aspect of the course.

METHODOLOGY

Process

1. Drawing the C:

a) Define the boundaries of the curve, that is, $0.13\pi \leq \theta \leq 1.87\pi$

b) Define the equations of a circle:

i. $x = rcos\theta$

ii. $y = rsin\theta$

c) Use the for loop to animate the drawing.

d) Plot the x and y values.

2. Drawing the O

a. Define the boundaries of the curve, that is, $0\leq \theta \leq 2\pi$

b. Define the equations of a circle:

i. $x = rcos\theta$

ii. $y = rsin\theta$

c. Use the for loop to animate the drawing.

d. Plot the x and y values.

3. Drawing the M

a. Draw the vertical line, the two diagonals and the last vertical line.

b. Use the for loop to animate the drawing.

4. Drawing the E

a. Draw the vertical line and the three horizontal lines.

b. Use the for loop to animate the drawing.

EQUATION FOR THE WORD "come"

<u>Circle O $x = r\cos\theta$, r = 2 and y = $sin\theta$</u>

Parameterized equation;

$$Velocity = \frac{d}{dt}(rcos(t)) = -rsin(t), \ \ t = 0.1, \qquad -2\sin(0.1) = -0.00349$$

Acceleration $\frac{d}{dt}(-rsin(t)) = -rcos(t), \ \ t = 11, \quad -2\cos(0.1) = -1.999$

Arc length = $s = \int_{0.05}^{t} |\, r'(t) \,| \, dt = \int_{0.05}^{t} |\sqrt{(-rsin(t))^2}\,| dt = \int_{0.05}^{t} rsin(t)dt =$

$[-rcost]_{0.05}^{t}$

$r = 2, \qquad [-rcos(t)] - [-rcos(0.05)] = -2\cos(t) + 2\cos(0.05) = -2\cos(t) + 1.999$

$y = rsin\theta$ for $0 \leq \theta \geq 2\pi$

r(t) = cos(t) +sin(t)

<u>Curvature of letter O</u>

Position = $r(t) = \cos(t)\, i + \sin(t)\, j + 0k$

Velocity = $r'(t) = -\sin(t)\, i + \cos(t)\, j + 0k$

Acceleration = $r''(t) = -\cos(t)\, i - \sin(t)\, j + 0k$

$$r'(t)\,x\,r''(t) = \left|\begin{array}{ccc} i & j & k \\ -\sin(t) & \cos(t) & 0 \\ -\cos(t) & -\sin(t) & 0 \end{array}\right| = \left(sin^2(t) + cos^2(t)\right)k - 0j + 0i =$$

$$\left||r'(t) \times r''(t)|\right| = \sqrt{1^2} = 1\,, \;=\; |r'(t)|^3 = \left(\sqrt{sin^2(t) + cos^2(t)}\right)^3 \;=\; |r'(t)|^3 = 1^3 = 1$$

$$C = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3} = \frac{1\,x\,1}{1} = 1$$

<u>Letter C r =2 for 0.13 $\leq \theta \geq 1.87\pi$</u>

$X = rcos\theta$

Parameterized equation: x = rcos (t), r = 2

Velocity $\frac{dx}{dt}\,(rcos(t)) = -\mathrm{rsin}(t)\,, at\ t = 0.1, -2\sin(0.1) = -0.00349$

Acceleration $v'(t) = \frac{d}{dt}(-\mathrm{rsin}(t)) = \mathrm{rcos}(t)\,,\quad at\ t = 0.1,\ 2\cos(0.1)$ =1.999

Arc length = $s = \int_{0.05}^{t} |\,r'(t)\,|\,dt = \int_{0.05}^{t} |\sqrt{(-rsin(t))^2}\,|dt = \int_{0.05}^{t} rsin(t)dt =$

$[-rcost]_{10}^{t}$

$r = 2, \qquad [-rcos(t)] - [-rcos(0.05)] = -2\cos(t) + 2\cos(0.05) = -2\cos(t) + 1.999$

Y = rsinθ r = 2 Parameterized equation: y= rsin (t),

Velocity: $\frac{d}{dt}(rsin(t)) = r\cos(t)\,, at\ t = 0.1, -\mathrm{rcos}(0.1) = 1.999$

Acceleration: $v'(t) = \frac{d}{dt}(rcos(t)) = -\mathrm{rsin}(t)\,,\quad at\ t = 0.1, -\mathrm{rsin(t)} = 2(\sin(0.1)) =$

0.00349

Parameterization both x and y

$r(t) =< \cos(t)\,, \sin(t) > \quad 0 \leq t \leq 2\pi \qquad r'(t) =< -\sin(t)\,, \cos(t) >$

$|\, r'(t)| = \sqrt{(-\sin(t))^2 + (\cos(t)^2)} \quad = 1$ since $sin^2 t + cos^2 t = 1$

$L = \int_0^{2\pi} |\, r'(t)| dt \quad = \; 2\pi$

$S(t) = \int_{0.05}^{t} |\, r'(u)|\, du \qquad => \; s(t) = t \qquad\qquad => t = s$

$r(s) =< \cos(s)\,, \sin(s) >, \quad 0 \leq s \leq 2\pi$

Letter M

| Points (15, 0.5) and (15,5)   x = 15

\ Points (25, 5) and (20,2)   y= -3/5x – 10

/ Points (15,5) and (20,2)   y = -3/5x +15

Parameterized equations for the points above.

r(t) = < (15,5) + t((15,5)-(15,0.5))> = r(t) = <15,5+4.5t>

r(t) = < (20,2) + t((20,2)-(25,5))> = r(t) = < 20 -5t,2-3t>

r(t) = < (20,2) + t((20,2)-(15,5))> = r(t) = < 20+5t, 2-3t>

Letter E

| Points (27, 0.5) and (27,5)   x = 27

_ Points (27, 5) and (35, 5)   y = 5

_ Points (27, 2.7) and (35, 2.7)   y = 2.7

_ Points (27, 0.5) and (35, 0.5)   y = 0.5

r(t) = <(27,5) + t((27,5) - ((27,0.5))>   = r(t) = <27, 5 + 4.5t>

r(t) = <(35,5) + t((35,5) – (27,5))>   = r(t) = <35+8t, 5>

r(t) = <(35,2.7) + t((35,27) – (2.7,27)>   = r(t) = <35 +32.3t, 2.7>

r(t) = <(35, 0.5) + t((35,0.5) - (27,0.5))   = r(t) = < 35+8t, 0.5>

MATLAB CODE FOR THE LETTER “COME”

```
function come(x,y,radius)
% This function plots the functions that spell out the word 'COME'.

%The
curve=animatedline('color','blue');
n=linspace(-3,3,100)
angle=0:0.01:2*pi;
xvalues=radius*cos(angle)+8;
yvalues=radius*sin(angle);
Fig = figure(1);

width= 60;
height= 25;
set(Fig, 'Position', [0 0 width height]);

for i=numel(n);
    addpoints(curve,xvalues,yvalues);
    drawnow
    grid on
    pause(0.01)
end

% drawing the 'C'
angle1=0.130*pi:0.01:1.870*pi;
xvalues1=radius*cos(angle1);
```

```
yvalues1=radius*sin(angle1);


% subplot(1,3,1);
plot(x+xvalues1,y+yvalues1,'r');
grid on
hold on

for i=numel(n);
    addpoints(curve,xvalues,yvalues);
    drawnow
    grid on
    pause(0.01)
end
% drawing the 'O'
% subplot(1,4,2);
plot(x+xvalues,y+yvalues,'g');

hold on

for i=numel(n);
    addpoints(curve,xvalues,yvalues);
    drawnow
    grid on
    pause(0.01)
end
%drawing the 'M'
% subplot(1,3,3)
line([15 15],[0.5 5]);
set(gca,'YLim',[0 6])
hold on

line([25 20],[5 2])
hold on

line([15 20],[5 2])
hold on

line([25 25],[0.5 5]);
set(gca,'YLim',[0 6])
hold on

for i=numel(n);
    addpoints(curve,xvalues,yvalues);
    drawnow
    grid on
    pause(0.01)
end
%drawing the 'E'
% g=subplot(1,3,4);

line([27 27],[0.5 5]);
set(gca,'YLim',[0 6])
hold on

line([27 35],[5 5])
hold on

line([27 35],[2.7 2.7])
hold on

line([27 35],[0.5 0.5])
```

```
hold on

end
```

RESULT

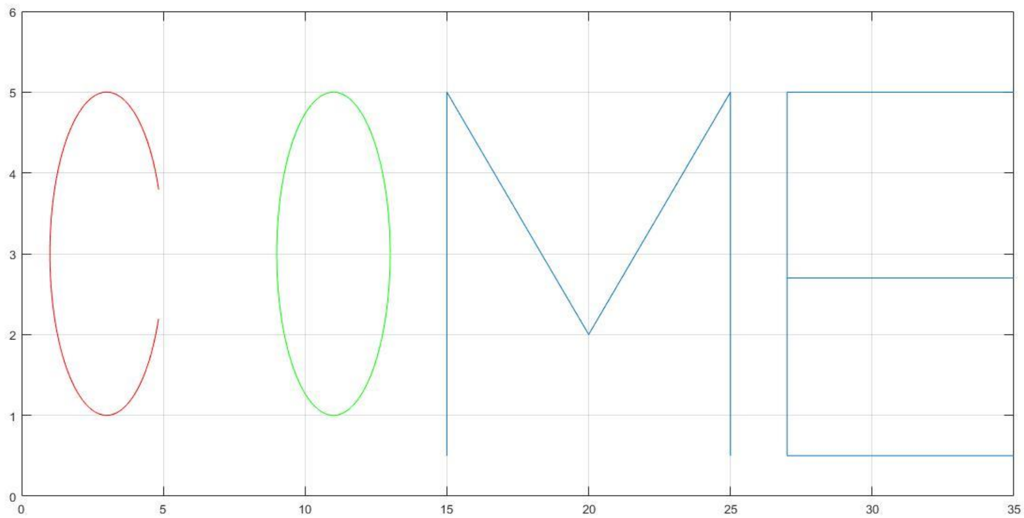

Figure 1. The graph above shows the outcome of the Matlab code.

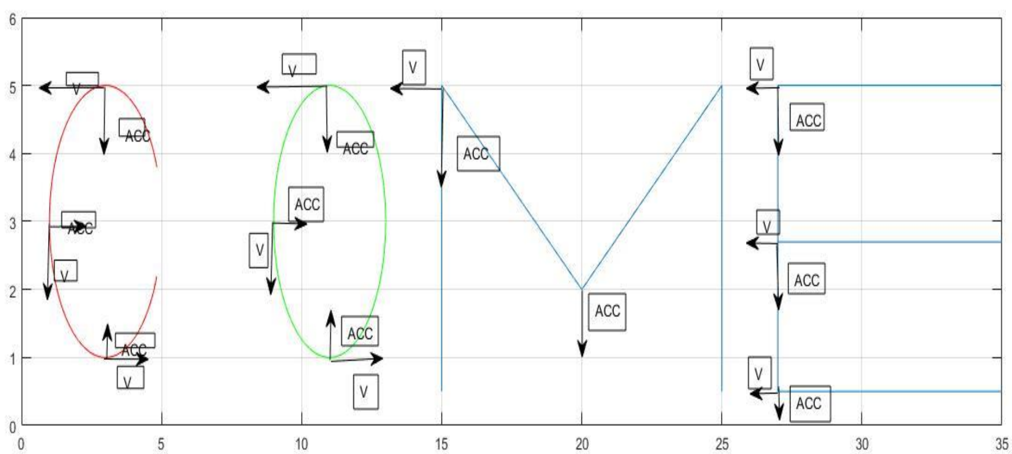

Figure 2. The graph above shows the letter "come" with its velocity and acceleration direction indicated with arrows from the Matlab software.

CONCLUSION

This project has taught us how everyday events in the classroom can be modeled using modern software in this case MATLAB. Through implementation of various codes, we were able to apply classroom concepts such as vector-valued functions, parametric equations, curvature and the concept of curves in space. In view of this, we can clearly understand how parameterization relates to word display by LED screens and billboards that are seen along the roadside in town. In other complex devices such as modern day phones like the new android and apple software, parameterizations and other applications from vector calculus are used to display letters and other figures to achieve a desiring effect. For some security systems in advanced virtual worlds, vector parameterization is used in lightening systems. The way a ray of light will travel in a given direction is highly dependent on how its path of travel was made. It has come to our notice that for objects such as photons (Light Packets) and other minute objects can be simulated by using advanced program software. For our level and convenience sake, when it comes to simulation and implementing everyday events using software, Matlab will always be a good and vital tool.

REFERENCE

Parveen, N. (n.d) Parametric Equations. Department of Mathematics and Science Education. Univeristy of Georgia. Retrieved on the 20th September, 2016 from http://jwilson.coe.uga.edu/EMAT6680Fa05/Parveen/Assignment%2010/parametric_equations.html