THE CURVE BALUSTRADE

If you have ever been to the Todd and Warren Library at Ashesi University College, then you know that your eyes catch the beautiful architectural design of a curve opening. This curve opening, which is surrounded by balustrades, separates the ground floor from the top floor. One of its significance is that, it makes ventilation possible between the two floors, and it also allows people at the top floor to see some interior part of the ground without having to descend the staircase. For instance, the curve opening allows students to see where the library desktop computers are. In the case, when you are on the top floor and you want to use one of the desktops, you can easily see the number of vacant desktops without having to waste time using the staircase. In this section, we will demonstrate the versatility of differentiation and integration to find out what the curve balustrade can also be used for. Firstly, several measurements (data) are taken from this curve design to derive a function. The slope and tangent of the curve are then found using differentiation technique. We continued to evaluate the area of the curve opening by integration.

METHODOLOGY

Apparatus:

- Geogebra software
- Tape measure
- Thread or string, pen and pencils, cello tape
- Calculators

Procedure:

The x and y axes were identified in the field, and the total length and width of the field was marked and measured. All these were done using the tape measure, cello tape, pen and paper. Cello tapes were stuck to the marked areas on the floor in order not to dirty the floor with the pens and markers. The pen and paper were then used to note down the data obtained from the measurements. All measurements taken in centimeters were converted to meters to help get standardized information. The data values were inputted into the Geogebra software to derive the function for the curve and the equation of the line.

The measurement taken from the curve

Spreadsheet

	A	B	C
1	-3.5	3.2	
2	-3	2.54	
3	-2.5	1.86	
4	-2	1.43	
5	-1.5	1.11	
6	-1	0.9	
7	-0.5	0.78	
8	0	0.73	
9	0.5	0.74	
10	1	0.87	
11	1.5	1.1	
12	2	1.4	
13	2.5	1.7	
14	3	2.11	
15	3.5	3.03	
16	4	4.1	
17			

To arrive at the formula for this curve, we inputted x-values and their corresponding y-values in the general quadratic formula of the form: $y = ax^2 + bx + c$

Values of X (m)	-3.5	0	4
Values of Y (m)	3.2	0.73	4.1

Supposing x = 0 and y = 0.73

$0.73 = a(0)^2 + b(0) + c$

$c = 0.73$

When x = 4 and y = 4.1

$$4.1 = a(4)^2 + b(4) + 0.73$$

$$4.1 = 16a + 4b + 0.7$$

$$3.37 = 16a + 4b - - - - - - eqn\ .1$$

When x = -3.5 and y = 3.2

$$3.2 = a(-3.5)^2 + b(-3.5) + 0.73$$

$$2.47 = 12.25a - 3.5b - - - - eqn. 2$$

Make "a" the subject of equation 1.

$$a = \frac{3.37 - 4b}{16}$$

We put "a" into equation 2 to find "b"

$$2.47 = 12.25a\left(\frac{3.37-4b}{16}\right) - 3.5b$$

$$39.52 = 41.2825 - 49b - 3.5b$$

$$-1.7625 = -52.5b, \qquad b = 0.033$$

$$a = \frac{3.37-4(0.03357)}{16} \qquad , \ a = 0.20$$

From the formula: $ax^2 + bx + c$

The function for the curve**:** $\mathbf{0.2x^2 + 0.03x + 0.73}$

THE SLOPE AND LINE OF TANGENT OF THE CURVE

$$g(x) = 0.4x + 0.03$$

$$\frac{dg}{dx} = 0.4x + 0.03$$

The slope at point x = -2.5 was determined by

At $x = -2.5$,

$$\frac{dg}{dx} = 0.4(-2.5) + 0.03 \ = -0.97$$

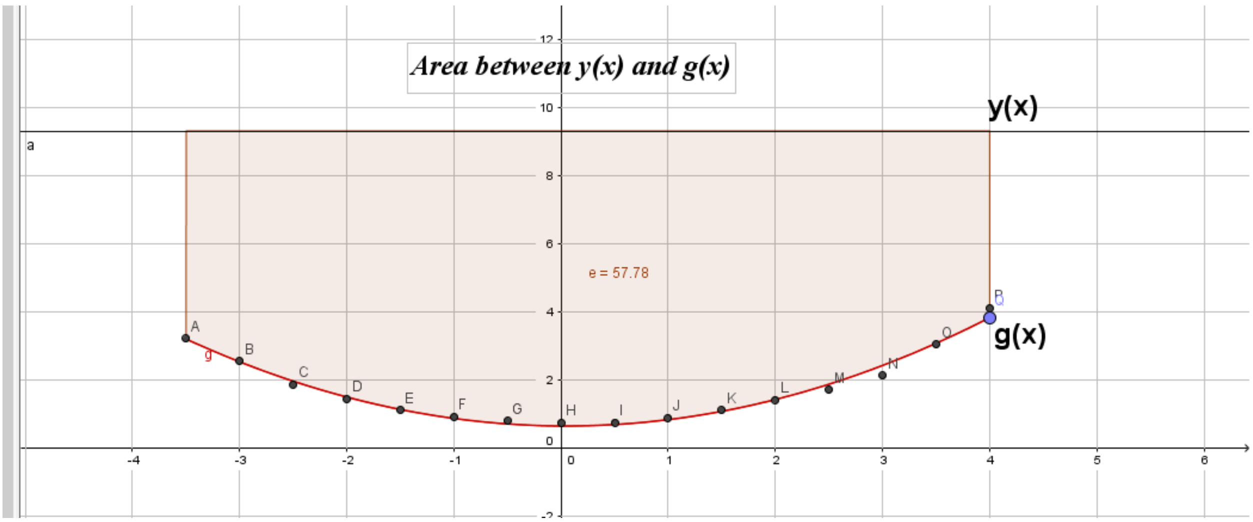
Again, we can calculate the equation of the line which tangent to the curve at the same point (-2.5, 1.86).

From the Point- slope form equation: $y - y_1 = m\,(x - x_1)$

= $y - 1.86 = -0.97(x + 2.5)$

$= y \quad = -0.97x - 0.565$

FINDING THE AREA OF THE CURVE BALUSTRADE



The graph above represents the curve obtained when the data were computed in Geogebra software.

Finding the Area of the curve balustrade

$= \int_a^b \big(f(x) - (gx)\big)dx$

$= \quad \int_{-3.5}^{4} (\ f(x) - g(x)\ )dx$

$= \qquad \int_{-3.5}^{4} (9.3 \ - (0.2x^2 + 0.03x \ + 0.73)\ )dx$.

$$= \left[9.3x \ - \ \frac{0.2x^3}{3} - \frac{0.03x^2}{2} - 0.73x\right]_{-3.5}^{4}$$

$= \left[9.3(4) - \frac{0.2(4)^3}{3} - \frac{0.03(4)^2}{2} - 0.73(4)\right] - \left[9.3(-3.5) - \frac{0.2(-3.5)^3}{3} - \frac{0.03(-3.5)^2}{2} - 0.73(-3.5)\right]$

$= \ 29.7733 - (-27.3204)$

$= 57.0m^2$

VOLUME OF THE CURVED BALUSTRADE

=$Volume = Area\ of\ the\ base\ x\ Height,\qquad Height\ of\ the\ curve\ balustrade = 120cm = 1.2m$

You should notice that the area of the base has been found already so we input it here directly.

Volume = $57.0m^2\ x\ \ 1.20m$

Volume = $68.4m^3$

VOLUME OF THE CURVED HOLE

Volume $= Area\ of\ the\ curved\ hole\ \ x\ its\ thickness\ (height),\quad thichkness = 0.95m$

Volume $= 57.0m^2\ x\ 0.95m$

Volume = $54.15m^3$

The area occupied by a sample Library desk with four chairs.

Area = $Length\ x\ width$

Length = $400.5cm = 4.005m$

Width of the table = $100cm = 1m$

Area = $4.005m\ x\ 1m = 4.005m^2$

The volume of the desk space = $Length\ x\ width\ \ x\ height$, where height = 102.5cm = 1.205m

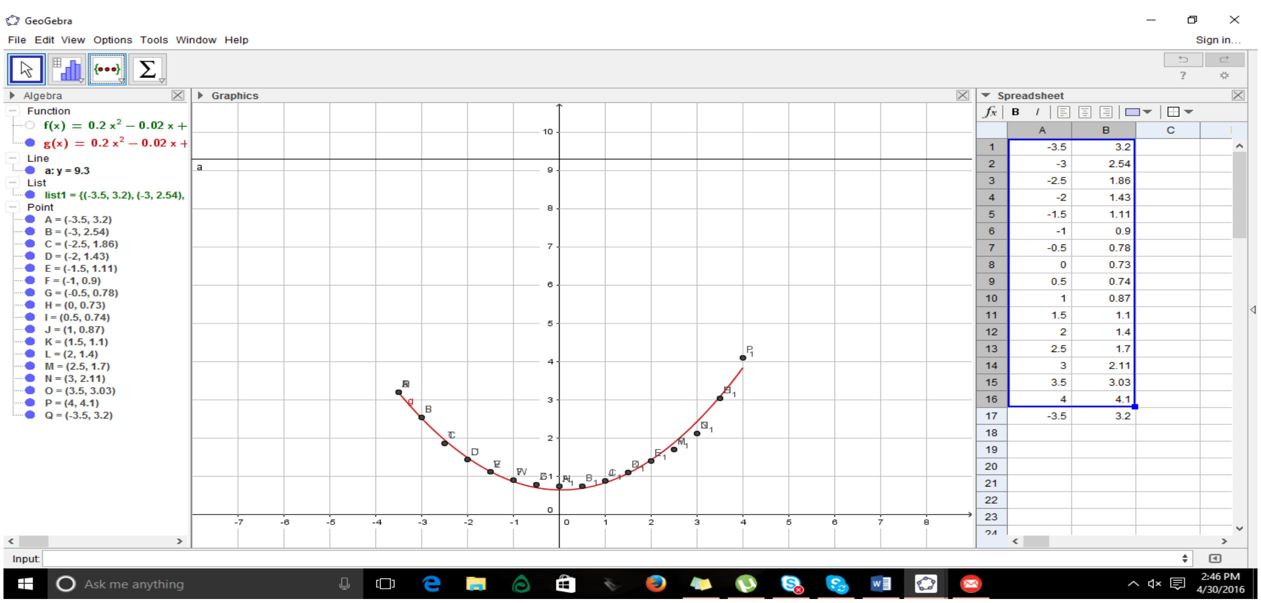
Volume = $4.005m\ x\ 1m\ x\ 1.205m = 4.83m^3$

APPLICATION

The above information shows that the area and the volume of the curve balustrade can occupy an extra number of about five library desks with about twenty chairs. This can increase the number of available spaces in the library for students to study. Ashesi University College is well known for its beautiful architectural designs. Hence, supposing the college wants to build a very big aquarium for aesthetic purposes, the library will be one of the most suitable places to host one because of its large curved balustrade. All that will be required is that the engineers have to fill the curve hole with a volume of about 54.2m^3 of building materials. And they will raise a glass tank to the height of the balustrade so that it can contain a volume of about 68m^3 of water in an aquarium of that size.

Geogebra Process

1. Select all the x and y values in the spreadsheet.
2. Go to list of point, the icon with three dots, select list of point and rename your list.

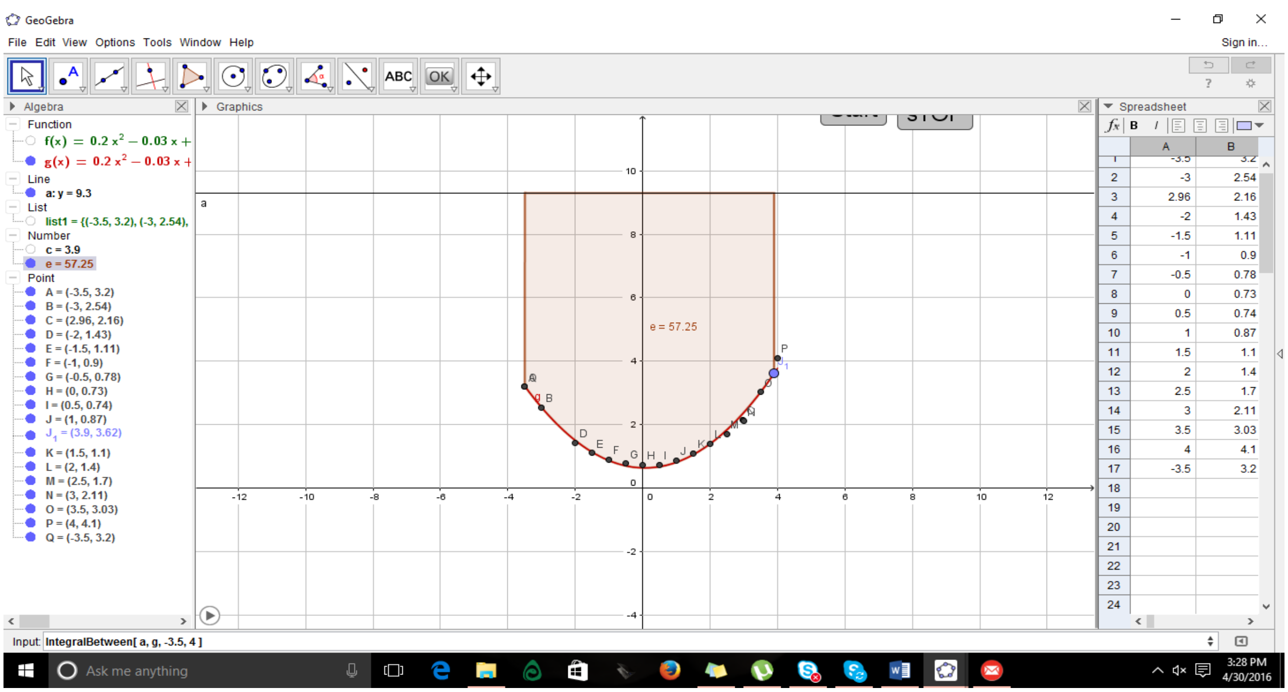


3. Type **fit** in the command button or input, select FitPoly[<List of Points>, <Degree of Polynomial>] and enter the details. For eg. FitPoly[list1, 2] to get the equation of the curve.

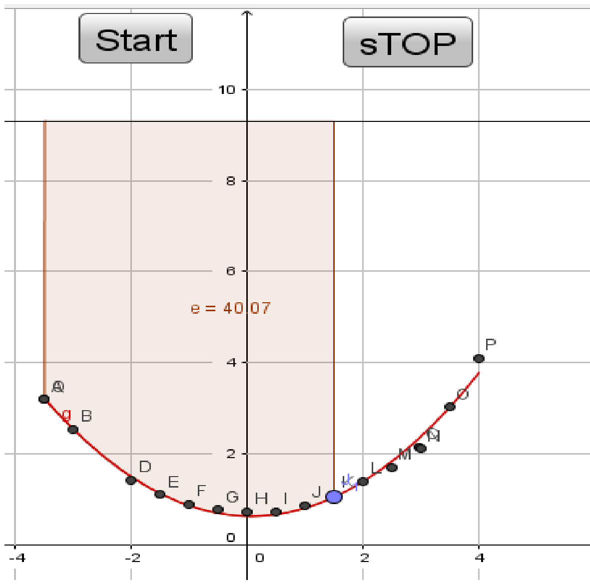
4. Enter If[<Condition>, <Then>] and fill the condition. For eg. If[-3.5≤x≤4, f] in order to get the curve with the specified interval .

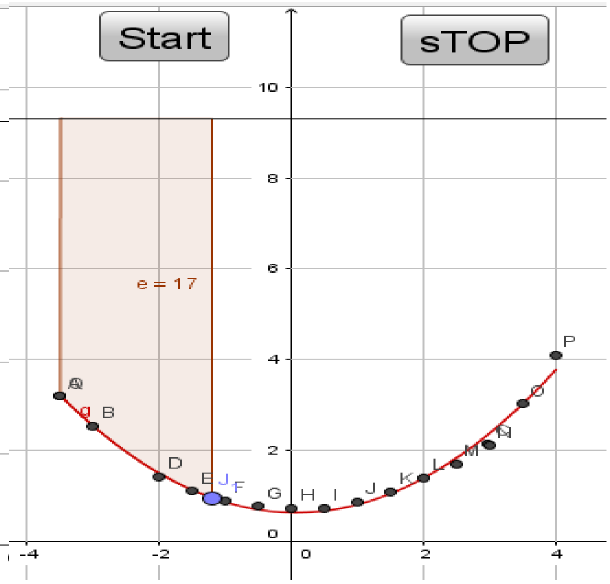
5. Type y= 9.3 in the input button as the upper function.

6. Enter IntegralBetween[<Function>, <Function>, <Start x-Value>, <End x-Value>]. And fill the details. For instance, IntegralBetween[a, g, -3.5, 4] to get the area of between the two functions.



7. Create a slider by clicking on the OK icon and redefine with e, which shows the area of the field.
8. In this place, Create start and stop button using the Ok button to control the slider in showing the area of the field within the specified interval.





CONCLUSION

This project helped us learn the usefulness of some aspect of calculus, that is, differentiation and integration, in calculating for space in our environment. Although the purpose of the project was achieved, through the research as well as experience, we learned that the curve was not smoother on its surface, and some parts of the balustrade were not properly straightened or curved, thereby making the collected data difficult to work with. We promise to inculcate the lessons learned in our future project to get accurate results.