THE ROLE OF MULTIVARIABLE CALCULUS IN ELECTRICITY AND HEAT EQUATIONS

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***Abstract*— Electricity plays an essential role in our everyday lives, right from domestic to industrial uses, electrical energy keeps our homes and business running. Although there are alternate sources of electricity such as windmill, solar panel, etc., electrical power from dams and nuclear plants have higher demands in our cities and towns today and as result, it is promoting urbanization. This paper presents the role that Multivariable Calculus plays to enable us to study the variables involved in the concept of electricity and heat equations**.

Keywords—multivariable calculus, heat diffusivity, an electric field

I. INTRODUCTION

Multivariable calculus has several applications in the finding electric fields where there is potential in a certain region of space. An electric field is the region in which an electrical force is exerted on point of charge by another charge. Before an electric field is created, a potential difference or electrical potential required. Electric potential is generated as a result of the movement of charges within a conductor or conducting wires. It is measured in Volts (V). It is also the property of the system of the surrounding charges. A critical look at our community reveals that there are high tension cables and transformers that help to transmit electricity to our homes. They are tagged "high tension" because electric charges that flow through create a high voltage and hence it is advisable no person goes close to it to prevent electrocution.

II. MATHEMATICAL FORMULATION

A. Electric Field

Whenever these charges move from one point to another, a potential difference is created and an electric field is generated. The electric field is always perpendicular to the surface hence it is directly proportional to the charge and inversely proportional to the square of the radius. In other words, it is directly proportional to the electric potential and inversely proportional to the distance. Mathematically,

$$E = \frac{kQ}{r^2} = \frac{Qv}{4\pi\varepsilon r^2} \; and \; F = qE$$

Where V- electric potential, F - electric force, E – electric field

Q – a charge,

$k - coulomb\ constant = 9.0\ x\ 10^9 Nm^2/C^2$

R – a distance from a single point charge,

$\varepsilon - permitivity\ of\ free\ space = 8.85X10^{-12}C^2/Nm^2$

In our case, we consider the formula $E = \frac{V}{R}$,

Independent variable: The independent variable is the r because it is the distance

(x, y).

Dependent variable: V is a dependent variable because it depends on the r and Q.

E is also a dependent variable because it depends on both V and R.

In multivariable calculus, the electric field vector is equal to the gradient of the electric potential for the considered electric field. Therefore, the components of the electric field vector at location R (x, y) can be found.

The electric potential (V) depends on the variable x and y.

$$\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Example: In a certain region of space, the electric potential is given by V $= (x^2 +$ xy). Find the electric field vector E. and the rate of change of the potential at P (1,1,) in the direction of the vector v = (3,4)

In multivariable calculus,

$$E_x = \frac{\partial V}{\partial x} = 2x + y, E_y = \frac{\partial V}{\partial x} = x$$

$$E = \big((2x + y)i + xj\big)N/C$$

Rate of change of the potential:

$$\nabla V = (x^2 + xy), \nabla V(1,1) = (3,1)$$

Directional derivative: $\frac{1}{\|V\|} V.\nabla V = \frac{1}{5}(3,4).(3,1) = \frac{13}{5}$

B. ANALYSIS

Concerning the graph below, the distance R which is the x-component and y-component indicates the flow of the potential from the right down towards the left in a slightly upward region, and the effect of the electric field is experienced or concentrated at the region with a hot or yellow color.

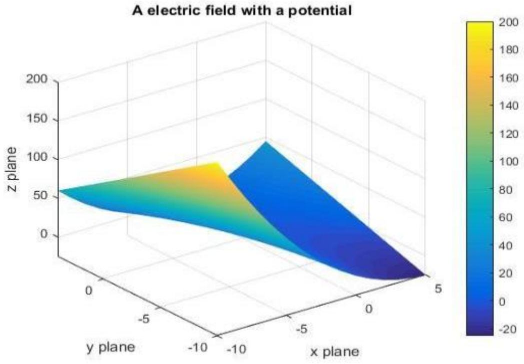

Figure 1. The 3D surface of an electric potential region with an electric field.

III. MATLAB CODE

```
%Initializing the coordinates x and y
x = [-10:1:5]; y = [-10:1:4];

%plot a meshgrid along x and y plane
[xx,yy] = meshgrid(x,y) zz
= xx.^2 + xx.*yy;

%the gradient function to
calculate the electric field
from the potential.
[px,py]=gradient(zz,.1,.1)

%Plotting a 3D surface the electric field.
figure surf(xx,yy,zz)
title('A electric field with
 a potential');
xlabel('x plane')
ylabel('y plane')
zlabel('z plane')
axis tight shading
interp colorbar
```

IV. HEAT EQUATIONS

Consider being given a thin metal rod in a physics lab, with an unsteady temperature supplied by candle or any heat source, it can be felt by the palms that heat travels through the atoms of the metal. And the direction of heat flow is from the region of higher temperature to a region of lower temperature. However, some equations govern the principle of heat transfer in material and this project seeks to discuss one of the most important heat equations: heat diffusivity equation. The significance of this study is to help observe and understand what happens at a particular region of material when the heat is applied.

Before heat is transferred through a material, it requires the mass of material (m), the specific heat capacity of the material (c) and a change in temperature (ΔT). In the same vein, simplest and the general form of heat equations mathematically is $H = mc\Delta T$

Heat diffusivity: Heat diffusivity measures the rate of heat transfer from one side (hot) to another (cold) side of the material. Also, its value describes how quickly a material reacts to a change in temperature. Advanced Physics broadens the scope of heat equation by introducing heat diffusion in solids. The formula for Schrodinger's equation and its meaning are illustrated below:

$$\rho c \frac{\partial T}{\partial x} = k\left(\frac{\partial T^2}{\partial x} + \frac{\partial T^2}{\partial x}\right)$$

$$\frac{\partial T}{\partial t} = k\nabla^2 T$$

ρ is the density of the material, c is the heat capactiy,

K is the thermal conductivity of the solid which explains how resistivity the material can be in terms of heat transfer. Since k is directly proportional to the ρc, a higher k means there is a large amount of heat flow through the material while a small k means the opposite. A substance with small k is likely to be insulators because they do lack free electrons.

A. Properties of Heat Equations

- The heat equations show that energy is conserved. This means the amount of energy generated cannot be destroyed, but be transferred or transformed.
- The amount of heat flowing out of the system is likened to be equal to the heat change of the system.
- Heat equations help determine that High thermal diffusivity materials conduct heat rapidly relative to their volumetric heat capacity.
- Heat equations take into account boundary conditions to determine the region with high and low heat. Heat travels in a small segment of a material, for instance, a thin metal rod. The change of heat in the segment in time (Δt) = heat energy of left boundary – heat out of right boundary.

  The heat energy of segment =

$$c\ x\ \rho A\Delta x\ x\ u = c\rho Au\Delta x\ (x,t)$$

Piecewise functions are not very good with heat equations since some boundary conditions cause oscillations at some transition points.

B. *Applications of Heat Equations*

- The equation can be applied in the cooling of rods in nuclear reactors and breeder reactors, which help in the production of electricity for consumption.
- In the manufacture of computers and other technological devices, heat equations are applied to emit a vast amount of heat generated by these appliances to prevent them from damage. For instance, Laptop adapters are designed with unique specifications to enhance the rate of heat flow, thereby dissipating the extra heat outside the cable through cooling. Lastly, heat equation equations help to select the type of electronic component for a particular electrical circuit.
- The heat equation is also applied in the manufacture of automobiles to help convey heat from movable parts such as the combustion chamber of the engine block to prevent overheating or explosion.

Considering the function of heat transfer from a region of higher concentration to a lower region of lower heat concentration: $f(x,y) = H(x,t) = e^{(-4\pi 2t)}\sin(2\pi x)$.

C. *Graph Representation*

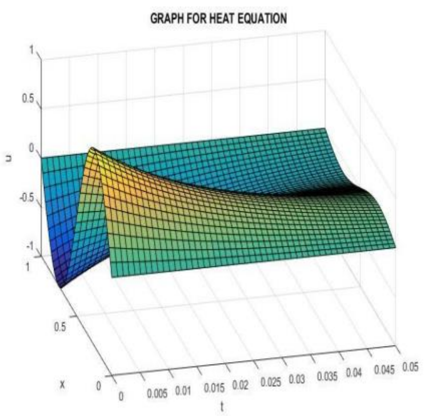

Figure 2 The graph for the heat equation showing the flow of heat from one side of the surface to the other.

The graph above illustrates how heat is diffused or transferred from a region of higher heat concentration to a region of lower heat concentration. The flow of heat is from the left side, thus, the surface with yellow or hot colors. The yellow or hot color surface gradually fades away as the surface spreads along the t-axis towards the green surface (cold) indicating the flow of heat to the lower region of heat.

MATLAB CODE

```
% Initializing the interval
for the x and t coordinates.
 x = linspace(0,1,50);
t = linspace(0,0.05,50);

%Plot the surface of the equation
[X,T]= meshgrid(x,t);
u = exp(-4.*pi.^2.*T).*sin(2.*pi.*X);
surf(T,X,u) view(-14,32)
  % Label the avrious axises.
xlabel('t'), ylabel('x'), zlabel('u')
title('GRAPH FOR HEAT EQUATION')
```

CONCLUSION

Multivariable Calculus as a concept under Mathematical sciences plays a major role in helping us study the different dimensions of heat and electricity. The scope allows us to see the independent and dependent variables and their relationship, and as such improvement could be made in the models to revamp the variables that constitute a heat or electrical system.

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