Parameterization of words using Multivariable Calculus and Linear Algebra

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Abstract— This paper presents the use of Matlab and survey method for parameterizing a four-letter word, "COME" which is a single (broken) parametrized curve and animate plotter. This goes a long way to broaden our scope of vector-valued functions, parametric curves and some concept of vector calculus and integrals.

Keywords—parametric equations, multivariable calculus,

I. Introduction

Parametric equations have been of great importance to us. From the position of objects such as cars, tennis balls, athletes, tricycle tires, objects in motions and how fast these objects move, parametric equations form a formidable aspect of these objects in our environment. Beyond this, parameterization has many applications in the field of science and engineering, and they include CAD design, data fitting, surveys of spline surfaces, etc.[1] It is a fact that the concept of the parametric equation is mainly grasped when it is practically done and visualized. Curves that are parameterized are said to have their coordinates of (x, y) to be represented in a variable (t) and the variable (t) becomes an independent variable while the x and y become dependent variables.

Illustration:

$$x = f(t) y = g(t) \; where t \; R$$

The main objective of this project is to give an overview of the development of a four-letter word "come" using the concept of parameterization and a scientific software tool called Matlab.

II. METHODOLOGY

A. *Drawing a parameterized "C"*

In parameterizing the letter C, we envisaged 4 ways and they include:

1. a) Define the boundaries of the curve, that is, $0.13\pi \leq \theta \leq 1.87\pi$
2. b) Define the equations of a circle:
   i. $x = rcos\theta$
   ii. $y = rsin\theta$
3. c) Use the for loop to animate the drawing.
4. d) Plot the x and y values.

B. *Drawing of Parameterized "O"*

Similarly, we followed the steps of parameterizing C, just that our curve domain was bigger this time.

1. Define the boundaries of the curve, that is, $0 \leq \theta \leq 2\pi$
2. Define the equations of a circle:
   i. $x = rcos\theta$
   ii. $y = rsin\theta$
3. Use the for loop to animate the drawing.
4. Plot the x and y values.

C. *Drawing of Parameterized "M"*

1. Draw the vertical line, the two diagonals and the last vertical line.
2. Use the for loop to animate the drawing.

D. *Drawing of parameterized "E"*

1. Draw the vertical line and the three horizontal lines.
2. Use the for loop to animate the drawing.

III. PARAMETERIZED EQUATIONS AND COMPONENTS

Parameterized equations.

$$Velocity = \frac{d}{dt}(rcos(t)) = -rsin(t),$$

$$t = 0.1, -2\sin(0.1) = -0.00349$$

$$Acceleration = \frac{d}{dt}(-rsin(t)) = -rcos(t), t = 11, -2\cos(0.1) = -1.9999$$

$$Arc\ length = \int_{0.05}^{t} |r'(t)| dt = \int_{0.05}^{t} \left| \sqrt{(-rsin(t))^2} \right| dt = \int_{0.05}^{t} rsin(t) dt = [-rcos(t)]^2,$$

$$when\ r = 2, arc\ length = -2\cos(t) + 1.999$$

$$y = rsin\theta \; for \; 0 \le \theta \ge 2\pi r(t) = \cos(t) + \sin(t)$$

Parameterization of both x and y

$$r(t) = < \cos(t), \sin(t) > \quad 0 \le t \le 2\pi$$
$$r'(t) = < -\sin(t), \cos(t) >$$
$$|r'(t)| = \sqrt{(-\sin(t))^2 + (\cos(t))^2} = 1$$

$$L = \int_0^{2\pi} |r(t)| dt = 2\pi$$

$$S(t) = \int_{0.05}^{t} |r'(u)| du = s(t) = t$$

$$r(s) = < \cos(s), \sin(s) >, 0 \le s \le 2\pi$$

Curvature of letter O

Position = $r(t) = \cos(t)\, i + \sin(t)\, j + 0k$

Velocity = $r'(t) = -\sin(t)\, i + \cos(t)\, j + 0k$

Acceleration = $r''(t) = -\cos(t)\, i - \sin(t)\, j + 0k$

$$t \, x \, r''(t) = \begin{bmatrix} i & j & k \\ -\sin(t) & \cos(t) & 0 \\ -\cos(t) & -\sin(t) & 0 \end{bmatrix}$$

$$(sin_2(t) + cos_2(t))k - 0j + 0i$$

Letter C

When r =2

$$when \; r = 2, for \; 0.13 \le \theta \ge 1.87\pi$$

$$x = rcos\theta$$

Parameterized equations $x = r\cos(t), r = 2$

Letter M

Points (15, 0.5)and (15,5), $x = 5$

Points (25,5)and (20,2), $y = \frac{-3}{5}x - 10$

Points (15,5)and (20,2), $y = \frac{-3}{5}x + 10$

Parameterized equations for M.

$$r(t) = < (15,5) + t((15,5) - (15,0.5)) > = r(t) = < 15, 5 + 4.5t >$$
$$r(t) = < (20,2) + t((20,2) - (25,5)) > = r(t) = < 20 - 5t, 2 - 3t >$$
$$r(t) = < (20,2) + t((20,2) - (15,5)) > = r(t) = < 20 + 5t, 2 - 3t >$$

letter E

Points (27, 0.5) and (27,5) x = 27

Points (27, 5) and (35, 5) y = 5

Points (27, 2.7) and (35, 2.7) x =27

Points (27, 5) and (35,5) y =5

Points (27, 2.7) and (35, 2.7) y =2.7

Parameterized equations

$$r(t) = < 27, 5 + 4.5t >$$
$$r(t) = < 35 + 8t, 5 >$$
$$r(t) = < 35 + 32.3t, 2.7 >$$
$$r(t) = < 35 + 32.3t, 2.7 >$$
$$r(t) = < 35 + 8t, 0.5 >$$

A. Matlab Code

```
function come(x,y,radius)
% This function plots the functions
that spell out the word 'COME'.

%The
curve=animatedline('color','blue');
n=linspace(-3,3,100)
angle=0:0.01:2*pi;
xvalues=radius*cos(angle)+8;
yvalues=radius*sin(angle);
Fig = figure(1);
  width=
60; height=
25;
set(Fig, 'Position', [0 0 width height]);
  for
i=numel(n);
    addpoints(curve,xvalues,yvalues);
drawnow      grid on      pause(0.01)
end
% drawing the 'C'
angle1=0.130*pi:0.01:1.870*pi;
xvalues1=radius*cos(angle1);
yvalues1=radius*sin(angle1);
```


```
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yvalues1=radius*sin(angle1);
```

IV. RESULTS

From the parameterization performed in the above sections, the following graphs were obtained using the Matlab

.

A. Figures and Tables

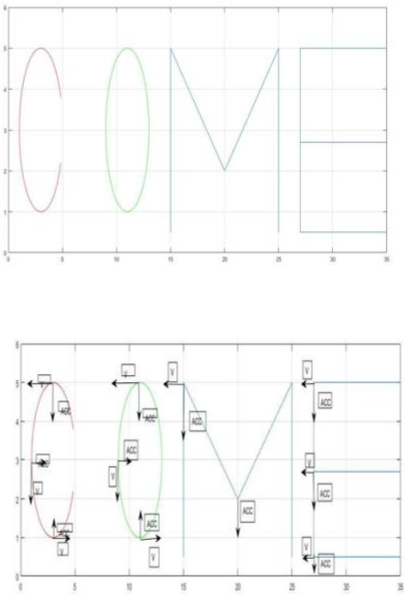

Fig. 1. Example of a figure caption. (*figure caption*)

V. CONCLUSION

There has been a lot of interest in parameterization recently and this paper has presented how everyday events in the classroom can be modeled using modern software, in this case, MATLAB. Through the implementation of various codes, we were able to apply classroom concepts such as vector-valued functions, parametric equations, curvature and the concept of curves in space. Because of this, we can clearly understand how parameterization relates to word display by LED screens and billboards that are seen along the roadside in town. In other complex devices such as modern-day phones like the new android and apple software, parameterizations and other applications from vector calculus are used to display letters and other figures to achieve a desiring effect. For some security systems in advanced virtual worlds, vector parameterization is used in lightening systems. The way a ray of light will travel in a given direction is highly dependent on how its path of travel was made [2]. This has been very significant in our understanding that objects such as photons (Light Packets) and other minute objects can be simulated by using advanced program software. For our level and convenience's sake, when it comes to simulation and implementing everyday events using the software, MATLAB will always be a good and vital tool.

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