Assignment 3:

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Q: Solve the following problem using Laplace Transform

$$y'' - 4y' + 4y = 64 \sin 2t$$

$$y(0) = 0 \quad ; \quad y'(0) = 1$$

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 64\,\mathcal{L}\{\sin 2t\}$$

$$\{s^2 Y(s) - s\,y(0) - y'(0)\} - 4\{s\,Y(s) - y(0)\} + 4Y(s) = \frac{64 \times 2}{s^2+4}$$

$$s^2 Y(s) - 4s\,Y(s) + 4Y(s) - 1 = \frac{128}{s^2+4}$$

$$Y(s)\left[s^2 - 4s + 4\right] - 1 = \frac{128}{s^2+4}$$

$$Y(s)\left[s^2 - 4s + 4\right] = \frac{128}{s^2+4} + 1$$

$$Y(s) = \frac{128 + s^2 + 4}{(s^2+4)(s-2)^2}$$

$$Y(s) = \frac{132 + s^2}{(s^2+4)(s-2)^2}$$

$$\frac{132+s^2}{(s^2+4)(s-2)^2} = \frac{A}{(s-2)} + \frac{B}{(s-2)^2} + \frac{Cs+D}{(s^2+4)}$$

$$s^2+132 = A(s-2)(s^2+4) + B(s^2+4) + (Cs+D)(s-2)^2$$

$$s^2+132 = A[s^3+4s-2s^2-8] + Bs^2+4B + Cs+D(s^2-4s+4)$$

$$s^2+132 = As^3+4As-2As^2-8A + Bs^2+4B + Cs^3-4Cs^2+4Cs+Ds^2-4Ds+4D$$

$$s^2+132 = s^3(A+C) + s^2(B-2A-4C+D) + s(4A+4C-4D) + 4B+4D-8A$$

Comparing coefficients

$A+C = 0$ —— (1)

$B-2A-4C+D = 1$ —— (2)

$4A+4C-4D = 0$

$A+C-D = 0$ —— (3)

$4B+4D-8A = 132$

$B+D-2A = 33$ —— (4)

$A+C-D=0$

$0-D=0$

$\boxed{D=0}$

$B-2A-4C+0=1$

$B-2A-4C = 1$ —— (2)

$$B + D - 2A = 33$$

$$B + 0 - 2A = 33$$

$$B - 2A = 33 \quad \text{———} \quad (5)$$

Subtract eq (5) and eq (2)

$$\cancel{B} - \cancel{2A} + 0 = 33$$

$$\cancel{B} - \cancel{2A} - 4C = 1$$

$$(-) \quad (+) \quad (+) \quad (-)$$

$$+4C = 33 + (-1)$$

$$C = \frac{32}{4}$$

$$\boxed{C = 8}$$

$$A + 8 = 0$$

$$\boxed{A = -8}$$

$$B + D - 2A = 33$$

$$B + 0 + 16 = 33$$

$$\boxed{B = 17}$$

$$Y(s) = \frac{A}{(s-2)} + \frac{B}{(s-2)^2} + \frac{Cs + D}{s^2 + 4}$$

$$\mathcal{L}\{y(t)\} = \frac{-8}{s-2} + \frac{17}{(s-2)^2} + \frac{8s}{(s^2+4)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{-8}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{17}{(s-2)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{8s}{s^2+4}\right\}$$

$$y(t) = -8e^{2t} + 17te^{2t} + 8\cos 2t$$

Ans

Q2 Solve to find inverse laplace transfoom of the following.

i) $\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\}$

$$= \frac{1}{s^2}\Bigg|_{s \to s-(+3)}$$

$$= te^{3t}$$

Ans

ii) $\frac{s}{2s^2-1}$

$$= \mathcal{L}^{-1}\left\{\frac{s}{2s^2-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{2\left(s^2-\left(\frac{1}{\sqrt{2}}\right)^2\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{2\left[s^2-\frac{1}{(\sqrt{2})^2}\right]}\right\}$$

$$= \frac{1}{2}\cosh\frac{1}{\sqrt{2}}t$$

Ans

Department of Computer Science

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Multivariable Calculus

Semester 03 (Fall 2021)

QUIZ 3B

Marks: 2.5

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Problem 01 [PLO 2, CLO 2]

1(a) Solve the value of $y'' + 2y' + 2y = 5sint$. Where y(0)=y'(0)=0.

$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 5\mathcal{L}\{\sin t\}$

$[s^2Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + 2Y(s) = 5 \times \frac{1}{s^2+1}$

$s^2Y(s) - 0 - 0 + 2sY(s) - 0 + 2Y(s) = \frac{5}{s^2+1}$

$Y(s)\{s^2+2s+2\} = \frac{5}{s^2+1}$

$Y(s) = \frac{5}{(s^2+1)(s^2+2s+2)}$

2.5/2.5

s^2+2s+2

$s^2+2s+1+1$

$(s+1)^2+1$

$\frac{5}{(s^2+1)(s^2+2s+2)} = \frac{As+B}{(s^2+1)} + \frac{Cs+D}{(s^2+2s+2)}$

$5 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$

$5 = As^3 + 2As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + Cs + Ds^2 + D$

$5 = s^3(A+C) + s^2(2A+B+D) + s(2A+2B+C) + 2B + D$

Comparing Coefficients

$A + C = 0$ —— ①

$2A + B + D = 0$ —— ②

$2A + 2B + C = 0$ —— ③

$2B + D = 5$ —— ④

$2A + B + D = 0$

$2A + 2B + C = 0$

$A + C + A + 2B = 0$

$0 + A + 2B = 0$

$\boxed{A = -2B}$

substitute $A = -2B$ in eq ②

$$2A + B + D = 0$$
$$-4B + B + D = 0$$
$$-3B + \cancel{D} = 0$$
$$2B + \cancel{D} = 5$$
$$\ominus \quad \ominus \quad \ominus$$

$$-5B = -5$$
$$\boxed{B = 1}$$
$$\boxed{A = -2}$$

$$A + C = 0$$
$$\boxed{C = 2}$$

$$2B + D = 5$$
$$2(1) + D = 5$$
$$\boxed{D = 3}$$

$$\mathcal{L}\{y(t)\} = \frac{-2s+1}{(s^2+1)} + \frac{2s+3}{(s^2+2s+2)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{-2s}{(s^2+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2s+3}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-2s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2s}{(s+1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-2s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2(s+1-1)}{(s+1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-2s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+1}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$
$$+ 3\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-2s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{s}{s+1}\right\}_{s \to s-(-1)} - 2\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}_{s \to s-(-1)}$$
$$+ 3\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}_{s \to s-(-1)}$$

$$= -2\cos t + \sin t + 2e^{-t}\cos t - 2e^{-t}\sin t + 3e^{-t}\sin t$$

$$y(t) = -2\cos t + \sin t + 2e^{-t}\cos t + e^{-t}\sin t$$

Ans

Quiz 3A

Solve value of $\sin t * t^2$

$$(f*g)(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

$f(t) = \sin t$ $\qquad g(t) = t^2$

$f(\tau) = \sin\tau$ $\qquad g(t-\tau) = (t-\tau)^2$

$$(f*g)(t) = \int_0^t (\sin\tau)(t-\tau)^2\, d\tau$$

$$= \int_0^t (t^2 - 2t\tau + \tau^2)\sin\tau\, d\tau$$

$$= \int_0^t t^2\sin\tau - 2t\tau\sin\tau + \tau^2\sin\tau\, d\tau$$

$$= t^2\int_0^t \sin\tau\, d\tau - 2t\int_0^t \underbrace{\tau\sin\tau}_{\mathrm{I}} + \int_0^t \underbrace{\tau^2\sin\tau\, d\tau}_{\mathrm{II}}$$

$u = \tau$ $\qquad \int v' = \int \sin\tau$

$du = d\tau$ $\qquad v = -\cos\tau$

$= -\tau\cos\tau - \int -\cos\tau\, d\tau$

I $= -\tau\cos\tau + \sin\tau$

$u = \tau^2 \qquad \int v' = \int \sin\tau$

$du = 2\tau\, d\tau \qquad v = -\cos\tau$

(II) $= -\tau^2 \cos\tau - \int -\cos\tau \times 2\tau\, d\tau$

(II) $= -\tau^2 \cos\tau + 2 \int \underbrace{\tau \cos\tau\, d\tau}_{\text{III}}$

$u = \tau \qquad \int v' = \int \cos\tau$

$du = d\tau \qquad v = \sin\tau$

$= \tau \sin\tau - \int \sin\tau\, d\tau$

(III) $= \tau \sin\tau + \cos\tau$

(II) $= -\tau^2 \cos\tau + 2\left[\tau \sin\tau + \cos\tau\right]$

(II) $= -\tau^2 \cos\tau + 2\tau \sin\tau + 2\cos\tau$

$= t^2 \left[\int_0^t \sin\tau\, d\tau\right] - 2t \int_0^t \tau \sin\tau\, d\tau + \int_0^t \tau^2 \sin\tau\, d\tau$

$= t^2 \left[-\cos\tau \Big|_0^t\right] - 2t\left[-\tau\cos\tau + \sin\tau\right]\Big|_0^t$

$+ \left[-\tau^2 \cos\tau + 2\tau \sin\tau + 2\cos\tau\right]\Big|_0^t$

$$= t^2\left[-\cos t + 1\right] - 2t\left[-t\cos t + \sin t - (0+0)\right] + \left[-t^2\cos t + 2t\sin t + 2\cos t - \left(0 + 0 + 2(1)\right)\right]$$

$$\Rightarrow -t^2\cos t - t^2 + 2t^2\cos t - 2t\sin t - t^2\cos t + 2t\sin t + 2\cos t - 2$$

$$\Rightarrow \cancel{-2t^2\cos t} + \cancel{2t^2\cos t} - \cancel{2t\sin t} + \cancel{2t\sin t} + 2\cos t - t^2 - 2$$

$$(f * g)t \Rightarrow 2\cos t - t^2 - 2$$

Ans

Quiz 3B

$$y'' + y = \sin 3t$$
$$y(0) = 0 \qquad y'(0) = 0$$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin 3t\}$$

$$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] + Y(s) = \frac{3}{s^2 + 3^2}$$

$$s^2 Y(s) - 0 - 0 + Y(s) = \frac{3}{s^2 + 3^2}$$

$$Y(s)\left[s^2 + 1\right] = \frac{3}{(s^2 + 9)}$$

$$Y(s) = \frac{3}{(s^2+9)(s^2+1)}$$

$$\frac{3}{(s^2+9)(s^2+1)} \Rightarrow \frac{As+B}{(s^2+9)} + \frac{Cs+D}{s^2+1}$$

$$3 = (As+B)(s^2+1) + (Cs+D)(s^2+9)$$

$$3 = As^3 + As + Bs^2 + B + Cs^3 + 9Cs + Ds^2 + 9D$$

$$3 = s^3(A+C) + s^2(B+D) + s(A+9C) + 9D + B$$

$$A + C = 0$$
$$B + D = 0$$
$$A + 9C = 0$$
$$9D + B = 3$$

$$9D + \cancel{B} = 3$$
$$D + \cancel{B} = 0$$
$$\ominus \quad \ominus \quad \ominus$$
$$8D = 3$$
$$D = \frac{3}{8}$$

$$\boxed{B = \frac{-3}{8}} \qquad \boxed{C = 0} \qquad \boxed{A = 0}$$

$$= \frac{0 + \left(\frac{-3}{8}\right)}{s^2+9} + \frac{0+3}{8(s^2+1)}$$

$$\mathcal{L}(y(t)) = \frac{3}{8(s^2+1)} - \frac{3}{8(s^2+9)}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{3}{8(s^2+1)}\right] - \mathcal{L}^{-1}\left[\frac{3}{8(s^2+9)}\right]$$

$$= \frac{3}{8}\left[\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)\right] - \frac{1}{8}\mathcal{L}^{-1}\left(\frac{3}{s^2+3^2}\right)$$

$$y(t) = \frac{3}{8}\sin t - \frac{\sin 3t}{8}$$ Ans

(b) $$\mathcal{L}^{-1}\left\{\frac{1}{s^2-5s+6}\right\}$$

$= s^2 - 5s + 6$

$= s^2 - 3s - 2s + 6$

$= s(s-3) - 2(s-3)$

$= (s-2)(s-3)$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-2)(s-3)}\right]$$

$$\frac{1}{(s-2)(s-3)} = \frac{A}{(s-2)} + \frac{B}{s-3}$$

$$1 = A(s-2) + B(s-3)$$
$$= As - 2A + Bs - 3B$$
$$1 = s(A+B) - 2A - 3B$$
$$A + B = 0$$
$$-2A - 3B = 1$$
$$+2\cancel{A} + 2B = 0$$
$$-\cancel{2}A - 3B = 1$$
$$-B = 1$$
$$\boxed{B = -1}$$
$$\boxed{A = 1}$$

$$\frac{1}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3} \qquad L\{e^{at}\} = \frac{1}{s-a}$$

$$= L^{-1}\left(\frac{1}{s-2}\right) - L^{-1}\left(\frac{1}{s-3}\right)$$

$$= e^{2t} - e^{3t}$$

$$L^{-1} \frac{1}{(s-2)(s-3)} = e^{2t} - e^{3t}$$

Ans

Quiz 3C

$$\mathcal{L}^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$$

$$s^2-4s+13$$

$$=(s)^2-2(2)(s)+13$$

$$=(s)^2-2(2)(s)+4+9$$

$$=(s)^2-4s+(2)^2+9$$

$$=(s-2)^2+9$$

$$=\mathcal{L}^{-1}\left\{\frac{s+2}{(s-2)^2+9}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{(s-2)+2+2}{(s-2)^2+9}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{(s-2)}{(s-2)^2+9}+\frac{4}{(s-2)^2+9}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}_{s\to s-(2)}+\frac{4}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}_{s\to s-(2)}$$

$$=e^{2t}\cos 3t+\frac{4}{3}e^{2t}\sin 3t$$

Ans

Quiz 3A

$$y'' + 25y = 10\cos 5t$$

$$y(0) = 2 \; ; \; y'(0) = 0$$

$$\mathcal{L}\{y''\} + 25\mathcal{L}\{y\} = 10\mathcal{L}\{\cos 5t\}$$

$$\{s^2Y(s) - sy(0) - y'(0)\} + 25\{\cancel{s}Y(s) - \cancel{y(0)}\}$$

$$= 10\left\{\frac{s}{s^2+25}\right\}$$

$$Y(s)[s^2+2s] = \frac{10s}{s^2+2s} + 2s$$

$$= \frac{10s + 2s(s^2+2s)}{s^2+2s}$$

$$Y(s) = \frac{10s + 2s^3 + 50s}{(s^2+2s)^2}$$

$$= \frac{60s + 2s^3}{(s^2+2s)^2}$$

$$\frac{\cancel{10s} + 2s^3 + 60s}{(s^2+25)^2} = \frac{As+B}{s^2+25} + \frac{Cs+D}{(s^2+25)^2}$$

$$\cancel{10s} + 2s^3 + 60s = (As+B)(s^2+25) + Cs + D$$

$$\cancel{10s} + 2s^3 + 60s = As^3 + 25As + Bs^2 + 25B + Cs + D$$

$$= As^3 + s(25A + C) + Bs^2 + D + 25B$$

$$\boxed{A = 2}$$

$$25A + C = 60$$

$$B = 0$$

$$D + 25B = 60$$

$$25(2) + C = 60$$

$$C = 60 - 50$$

$$\boxed{C = 10}$$

$$\boxed{B = 0}$$

$$D + 25B = 0$$

$$\boxed{D = 0}$$

$$\frac{2s^3 + 60s}{(s^2+25)^2} = \frac{2s+0}{s^2+25} + \frac{10s+0}{(s^2+25)^2}$$

$$\mathcal{L}\{y(t)\} = \frac{2s}{s^2+25} + \frac{10s}{(s^2+25)^2}$$

$$= 2\mathcal{L}^{-1}\left[\frac{s}{s^2+5^2}\right] + 10\mathcal{L}^{-1}\left[\frac{s}{(s^2+5)^2}\right]$$

$$= 2\cos 5t + \mathcal{L}^{-1}\left\{\frac{2(5)s}{(s^2+5)^2}\right\}$$

$$= 2\cos 5t + t\sin at$$

Laplace Inverse of $\frac{5s-10}{9s^2-16}$

$$= \mathcal{L}^{-1}\left\{\frac{5s-10}{9s^2-16}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{5s}{9s^2-16}\right\} - \mathcal{L}^{-1}\left\{\frac{10}{9s^2-16}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{5s}{9\left[s^2-\frac{16}{9}\right]}\right\} - \mathcal{L}^{-1}\left\{\frac{10}{9\left[s^2-\frac{16}{9}\right]}\right\}$$

$$= \frac{5}{9}\mathcal{L}^{-1}\left[\frac{s}{s^2-\left(\frac{4}{3}\right)^2}\right] - \mathcal{L}^{-1}\frac{10}{9}\left\{\frac{1}{s^2-\left(\frac{4}{3}\right)^2}\right\}$$

$$= \frac{5}{9}\mathcal{L}^{-1}\left[\frac{s}{s^2-\left(\frac{4}{3}\right)^2}\right] - \frac{\cancel{10}^{5}}{\cancel{9}_{3}}\times\frac{\cancel{3}^{1}}{\cancel{4}_{2}}\mathcal{L}^{-1}\left\{\frac{4/3}{s^2-\left(\frac{4}{3}\right)^2}\right\}$$

$$= \frac{5}{9}\cosh\frac{4}{3}t - \frac{5}{6}\sinh\frac{4}{3}t$$