BAHRIA UNIVERSITY (KARACHI CAMPUS)

FINAL EXAMINATION –SPRING SEMESTER – 2020

(Multivariable Calculus: GSC-211)

Class: **BS (CS) – 3A and 3B** **(Morning)**

Course Instructor: **Miss Kaniz Fatima** Time Allowed: **08 Hours**

Date: **10th July 2020; Session: I** Max Marks: **50**

Student Name: ________________ **Enrolment #____________________**

Note:

1) "Take Home Assessment Assignment" for Final Spring-2020 is uploaded on Friday, 10th July 2020 at 08:00 a.m. on LMS of CMS (BUKC).
2) Students will return their individual solved assessment assignment on Friday, 10th July 2020 till 04:00 pm. on same LMS of CMS (BUKC).
3) Students are required to mention necessary mathematical steps in their solution.
4) Over writing will not be encouraged and every step of mathematical solution should be readably clearly.
5) Students are strictly restricted to take any help/assistance from human intellectual support such as class fellows, seniors, teachers, or others human source of help.
6) Caution: Plagiarism and sharing-similarity comes underuse of unfair means if found in response assignment by the students, it shall be dealt with in accordance with BU examination rules.
7) It will be appreciated, if students submit their assessment assignment solution in typing form in Microsoft MS-word. Otherwise hand written solved assignment also accepted for assessment.
8) Front page should be included, Name of student, Registration/Enrollment No., Name of Assessment Assignment, Date of Submission, Class with Section, Semester, Instructor Name etc.
9) In addition to upload the solution on LMS, please also email it to course instructor.
10) In order to avoid any run time electricity and internet unavailability situation, it is suggested that keep your laptop charged. Also activate 3G/4G connection as an alternative of Wifi/internet option to upload your solution.
11) Scientific Calculator is allowed.
12) Attempt ALL questions.

Question No 1: **[4+4=8 Marks]**

a) Expand as a Fourier series, the function f(x) defined as

$$f(x) = \left\{ \begin{array}{lll} \pi + x & \text{for} & -\pi < x < -\frac{\pi}{2} \\ \frac{\pi}{2} & \text{for} & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{for} & \frac{\pi}{2} < x < \pi \end{array} \right\}$$

b) Show that the Fourier series for $f(x) = e^{-x}$ in the interval $0 \leq x \leq 2\pi$ is

$$\frac{1 - e^{-2\pi}}{\pi}\left(\frac{1}{2} + \frac{1}{2}\cos x + \frac{1}{5}\cos 2x + \frac{1}{10}\cos 3x + \frac{1}{2}\sin x + \frac{2}{5}\sin 2x + \frac{3}{10}\sin 3x + \cdots\right)$$

Question No 2: **[4+4=8 Marks]**

a) Evaluate the surface integral by using divergence theorem of

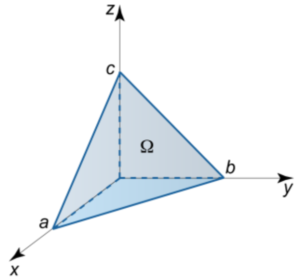
$$\mathbf{F} = (x^2 + y^2 + z^2)(\mathbf{i} + \mathbf{j} + \mathbf{k}),$$

where S is the surface of the tetrahedron $x = 0,\ y = 0,\ z = 0,\ x + y + z = 2$ and n is the unit normal in the outward direction to the closed surface S.

b) Show that the surface integral by using divergence theorem of

$$\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$$

where S is the surface of the pyramid $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \le 1,\ x \ge 0, y \ge 0,\ z \ge 0$ is $\frac{abc}{6}$.



Question No 3: **[4+4=8 Marks]**

a) Find the (i) Fourier sine series and (ii) Fourier cosine series of

$$f(x) = x + 1, \quad \text{for } 0 < x < \pi.$$

Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

b) The vector field $\mathbf{F} = x^2\mathbf{i} + z\mathbf{j} + yz\mathbf{k}$ is defined over the volume of the cuboid given by

$$0 \le x \le a, \qquad 0 \le y \le b, \qquad 0 \le z \le c.$$

Evaluate the surface integral, where S is the surface of the cuboid

Question No 4: **[4+4=8 Marks]**

Using Laplace transform technique solve the initial value problem.

a) $y'' + 2y' + 5y = e^{-t}\sin t,$ where $y(0) = 0$ and $y'(0) = 1$

b) $y'' + ay' - 2a^2y = 0,$ where $y(0) = 6$ and $y'(0) = 0$

Question No 5: **[3+3=6 Marks]**

a) Find the z transform of $\{f(k)\}$ where,

$$f(k) = \begin{Bmatrix} 5^k, & k < 0 \\ 3^k, & k \ge 0 \end{Bmatrix}$$

b) What sequence is generated when $f(k) = \begin{cases} 0, & k < 0 \\ \cos\frac{k}{2}, & k \geq 0 \end{cases}$. Also find the z-transform of $\{f(k)\}$.

Question No 6: **[6+6=12 Marks]**

a) Show that the Laplace transformation of the given functions are equal to corresponding given functions.

i) $\cos^2 t = \frac{1}{2}\left[\frac{s}{s^2+4} + \frac{1}{s}\right]$

ii) $\sin 2t \cos 3t = \frac{2(s^2-5)}{(s^2+1)\,(s^2+25)}$

iii) $\frac{1}{t}(1 - \cos t) = \frac{1}{2}\log(s^2 + 1) - \log s$

b) Show that the Inverse Laplace transformation of the given functions are equal to corresponding given functions.

i) $\frac{s-4}{4(s-3)^2+16} = \frac{1}{4}e^{3t}\cos 2t - \frac{1}{8}e^{3t}\sin 2t$

ii) $\frac{3s-8}{4s^2+25} = \frac{3}{4}\cos\frac{5t}{2} - \frac{4}{5}\sin\frac{5t}{2}$

iii) $\frac{s+4}{s(s-1)(s^2+4)} = -1 + e^t - \frac{1}{2}\sin 2t$

***********GOOD LUCK*************