# MVC

Hariq Hamid Ali
02-134191-071
BSCS-3A

$\bullet\ a_0 = \frac{1}{\pi}\int_a^b f(x)\,dx$

Q1

a) $f(x) \begin{cases} \pi + x & -\pi < x < \pi/2 \\ \pi/2 & -\pi/2 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$

$$a_0 \left[ \int_{-\pi}^{\pi/2} (\pi + x)\,dx + \int_{\pi/2}^{\pi/2} (\pi/2)\,dx + \int_{\pi/2}^{\pi} (\pi - x)\,dx \right]$$

$$a_0 = \frac{1}{\pi}\left[ \left(\pi x + \frac{x^2}{2}\right)^{-\pi/2} + \left(\frac{\pi x}{2}\right)_{-\pi/2}^{\pi/2} + \left(\pi x - \frac{x^2}{2}\right)_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi}\left( \pi^2 - \frac{7\pi^2}{8} + \frac{\pi^2}{2} + \pi^2 - \frac{7\pi^2}{8} \right) = \boxed{a_0 = \frac{3\pi}{4}}$$

for $a_n$:

$\bullet\ a_n = \frac{1}{\pi}\int_a^b f(x)\cos x$

$$a_n = \frac{1}{\pi}\left( \int_{-\pi}^{-\pi/2} \underbrace{(\pi + x)}_{u} \underbrace{(\cos x)}_{v}\,dx + \int_{-\pi/2}^{\pi/2} (\pi/2 \cos x)\,dx + \int_{\pi/2}^{\pi} \underbrace{(\pi - x)}_{u} \underbrace{(\cos x)}_{v}\,dx \right)$$

$\bullet\ u.v = u\int v - \int(u' \int v)$

$$= \frac{1}{\pi}\left[\left[-\pi + \frac{\pi}{2} + 1\right] + \left[\pi\right] + \left(1 - \pi + \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[-\cancel{\pi} + \frac{\pi}{2} + 1 + \cancel{\pi} + 1 - \pi + \frac{\pi}{3}\right]$$

$$\boxed{a_n = 2/\pi}$$

for $b_n$ $\qquad \cdot\ b_n = \frac{1}{\pi}\int_a^b f(x)\sin nx$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2}\left(\frac{\pi + x}{4}\right)\frac{\sin nx\,dx}{V} + \int_{-\pi/2}^{\pi/2}\frac{\pi}{2}\sin x\,dx + \right.$$

$$\left.\int_{\pi/2}^{\pi}\frac{\pi - x}{4}\,\frac{\sin nx\,dx}{V}\right]$$

$$= \frac{1}{\pi}\left[-1 + 0 + 1\right] = \frac{0}{\pi} \qquad \boxed{b_n = 0}$$

Substituting value in formula

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{3\pi}{4} + \frac{2}{\pi}\left(\cos x + \cos 2x + \cos 3x + \cos 4x + \cdots\right)$$

$$\boxed{f(x) = \frac{3\pi}{4} + \frac{2}{\pi}\left(\cos x + \cos 2x + \cos 3x + \cos 4x + \cdots\right)}$$

Q2

a) By divergence theorem

$$\iint f.\hat{n}\, ds = \iiint \text{div}\, \vec{F}\, dv$$

where S is the surface of tetrahedron $x=0, y=0, z=0, x+y+z=2$

$$= \iiint \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot (x^2+y^2+z^2)(x\hat{i}+\hat{j}+\hat{k})\, dv$$

$$= \iiint (2x+2y+2z)\, dv$$

$$= 2\iiint_V (x+y+z)\, dx\, dy\, dz$$

$$= 2\int_0^2 dx \int_0^{2-x} dy \int_0^{2-x-y} (x+y+z)\, dz$$

$$= 2\int_0^2 dx \int_0^{2-x} dy \left(x^2+y^2+\frac{z^2}{2}\right)^{2-x-y}$$

$$= 2\int_0^2 dx \int_0^{2-x} dy \left(2x-x^2-xy+2y-xy-y^2+\frac{(2-x-y)}{2}\right)$$

$$= 2\int_0^2 dx \left[2xy-x^2y-xy^2+y^2-y^3-\frac{(2-x-y)^2}{6}\right]_0^{2-x}$$

$$= 2\int_0^2 dx \left[2x(2-x)-x^2(2-x)-x(2-x)^2+(2-x)^2.\frac{(2-x)^3}{3}+\frac{(2-x)^3}{6}\right.$$

$$= 2\int_0^2 \left(4x - 2x^2 - 2x^2 + x^3 - 4x + 4x^2 - x^3 + (2-x)^2 - \frac{2-x^3}{3} + \frac{(2-x)^3}{6}\right)$$

$$= 2\left[2x^2 - \frac{4x^3}{3} + \frac{x^4}{4} - 2x^2 + \frac{4x^3}{3} - \frac{x^4}{4} - \frac{(2-x)^3}{3} + \frac{(2-x)^4}{12} - \frac{(2-x)^4}{24}\right]_0^2$$

$$= 2\left[-\frac{(2-x)^3}{3} + \frac{(2-x)^4}{12} - \frac{(2-x)^4}{24}\right]_0^2$$

$$= 2\left[\frac{8}{3} - \frac{16}{12} + \frac{16}{24}\right]$$

$$\boxed{= 4}$$

Q2

b) $F = 2xi + 3yj + 4zk$

$$\nabla \cdot F = \frac{\delta}{\delta x} 2xi + \frac{\delta}{\delta y} 3j + \frac{\delta}{\delta z} 4z$$

$$\nabla \cdot F = (2+3+4)$$

$$\text{flux} = \iiint (2+3+4)\, dx\, dy\, dz = 9 \iiint dx\, dy\, dz$$

$$\frac{x}{a} + \frac{y}{b} \le 1 \quad x \ge 0,\ y \ge 0$$

$$\frac{y}{b} \le 1 - \frac{x}{a} \quad \text{or} \quad \boxed{y \le b - \frac{b}{a}x}$$

So, $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} < 1$$

$$\frac{z}{c} < 1 - \frac{x}{a} - \frac{y}{b} \quad \text{or} \quad \boxed{1 - \frac{x}{a} - \frac{y}{b}}$$

Now

$$\iiint dx\, dy\, dz = \int^{a} dx \int^{b(1-\frac{x}{a})} dy \int^{1-\frac{x}{a}-\frac{y}{b}} dz$$

$$= \int_0^a dx \int_0^{b(1-\frac{x}{a})} c\left(1 - \frac{x}{a} - \frac{y}{b}\right) dy$$

$$= bc \int_0^a \left(\left(1-\frac{x}{a}\right)\left(1-\frac{x}{a}\right) - \left(\frac{(1-x(a))^2}{2}\right]\right) dx$$

$$= \frac{bc}{2a^2} \int_0^a (a-x)^2\, dx$$

$$= \frac{bc}{2a^2} \int_0^a (a^2 - 2ax + x^2)\, dx = \frac{bc}{2a^2} \left[\left(a^2x - ax^2 + \frac{x^3}{3}\right)\right]_0^a$$

$$= \frac{bc}{2ac^2} \left(\cancel{a^3} - \cancel{a^3} + \frac{a^3}{3}\right) = \boxed{\frac{abc}{6}}$$

Q3 a.

i) Sine Series

Assume $f(x)$ is odd func. so $a_0 = 0$, $a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^\pi (x+1)\sin nx \, dx$$

$$= \frac{2}{\pi}\left[(x+1)\left(\frac{-\cos nx}{n}\right)\Big|_0^\pi - \int_0^\pi \frac{-\cos nx}{n}\,dx\right]$$

$$= \frac{2}{\pi}\left[\left(\frac{\pi+1}{n}\right) - (-1)^n - (1)(-1)\right] + \frac{1}{n^2}\left(\sin nx\right)\Big]_0^\pi$$

$$b_n = \frac{2}{\pi}\left[-\left(\frac{\pi+1}{n}\right)(-1)^n + 1\right]$$

$$b_1 = \frac{2}{\pi}\left[-\frac{\pi-1}{1}(-1)+1\right]$$

$$b_1 = \frac{2}{\pi}(\pi+1+1) = \frac{2}{\pi}(\pi+2)$$

$$\boxed{b_1 = 2 + \frac{4}{\pi}}$$

$$b_2 = \frac{2}{\pi}\left[\frac{-\pi-1}{2} + 1\right]$$

$$= \frac{2}{\pi}\left[\frac{-\pi-1+2}{2}\right]$$

$$\boxed{b_2 = -1 + \frac{1}{\pi}}$$

$$b_3 = \frac{2}{\pi}\left[\frac{\pi+1+1}{3}\right]$$

$$b_3 = \frac{2}{\pi}\left(\frac{\pi+1+3}{3}\right) = \frac{2}{\pi}\left(\frac{\pi+4}{3}\right)$$

$$b_3 = \frac{2}{3} + \frac{24}{\pi}$$

ii) Cosine Series

Assume $f(x)$ is even

so, $b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}(x+1)\,dx = \frac{2}{\pi}\left(\frac{x^2}{2}+x\right)_0^{\pi}$$

$$\boxed{a_0 = \pi + 2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(x+1)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(x+1)\cdot\frac{\sin nx}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{\sin nx}{n}\,dx$$

$$= \frac{2}{\pi}\left(\frac{1}{n^2}\left[\cos n\pi - \cos 0\right]\right.$$

$$a_n = \frac{2}{\pi n^2}\left\{(-1)^n - 1\right\}$$

$$a_1 = \frac{2}{\pi}\left\{-1-1\right\}$$

$$a_3 = \frac{2}{\pi 9}\left\{-1-1\right\}$$

$$a_1 = \frac{-4}{\pi}$$

$$a_3 = \frac{-4}{\pi 9}$$

$$a_2 - \frac{2}{\pi 4}\left\{-1-1\right\} \Rightarrow a_2 = 0$$

Q3

b). Evaluate the surface integral

According to Divergence Theorem we have

$$\iint_V x^2\hat{i} + z\hat{j} + yz\hat{k} \cdot dx = \iiint \operatorname{div}(x^2\hat{i} + z\hat{j} + yz\hat{k})\, dv$$

where V is the volume of the cuboid enclosing the surface.

$$\iiint_V \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(x^2\hat{i} + z\hat{j} + yz\hat{k})\, dv$$

$$\iiint_V \left(\frac{\delta}{\delta x}(x^2) + \frac{\delta}{\delta y} z + \frac{\delta}{\delta z} yz\right) dx\, dy\, dz$$

$$= \int_0^a dx \int_0^b dy \int_0^c (2x+y)\, dz$$

$$= \int_0^a dx \int_0^b dy\; 2xz + yz \Big|_0^c = \int_0^a dx \int_0^b (2xc+yc)\, dy$$

$$= c\int_0^a dx \int_0^b (2x+y)\, dy = c\int_0^a 2xy + \frac{y^2}{2}\Big|_0^b$$

$$= c\int_0^a \left(2xb + \frac{b^2}{2}\right) dx$$

$$= c\left\{\frac{\cancel{2}x^2 b}{\cancel{2}} + \frac{b^2 x}{2}\Big|_0^a\right\}$$

$$= c\left[a^2 b + \frac{ab^2}{2}\right] = abc\left(a + \frac{b}{2}\right)$$

$$= abc(2a+b)/2 \quad \text{OR} \quad abc\frac{(2a+b)}{2}$$

Q4

a) $y'' + 2y' + 5y = e^{-t} \sin t$ where $y(0) = 0$ and $y'(0) = 1$

$y'' + 2y' + 5y = e^{-t} \sin t$

Taking Laplace Transformation on both sides

$L[y'' + 2y' + 5y] = L[e^{-t} \sin t]$

$L[y''] + 2[y'] + 5[y] = L[e^{-t} \sin t]$

$[s^2 F(s) - s y(0) - y'(0)] + 2[sF(s) - y(0)] + 5[F(s)] = 1/(s+1)^2 + 1$

$s^2 F(s) - 0 - 1 + 2sF(s) + 5F(s) = \frac{1}{s^2 + 2s + 2}$

$F(s)\{s^2 + 2s + 5\} = \frac{1}{s^2 + 2s + 2} + 1$

$F(s)\{s^2 + 2s + 5\} = \frac{1 + s^2 + 2s + 2}{s^2 + 2s + 2}$

$F(s) = \frac{3 + s^2 + 2s}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$

$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} = \frac{As + B}{(s^2 + 2s + 2)} + \frac{Cs + D}{(s^2 + 2s + 5)}$ — (A)

$s^2 + 2s + 3 = (As + B)(s^2 + 2s + 5) + 1(s + D)(s^2 + 2s + 2)$

$s^2 + 2s + 3 = As^3 + 2As^2 + 5As + Bs^2 + 2Bs + 5B + Cs^3 + 2Cs^2 + 2Cs + Ds^2 + 2Ds + 2D$

$0 = A + C$ (i) $\qquad 2 = 5A + 2B + 2C + 2D$ (iii)

$1 = 2A + B + 2C + D$ (ii) $\qquad 4 = 5B + 2D$ (iv)

By solving above questions we get
$A = 0$, $B = 1/3$, $C = 0$, $D = 2/3$

Putting all values in eq (A)

$$= \frac{1/3}{(s^2+2s+2)} + \frac{2/3}{(s^2+2s+5)}$$

$$= \frac{1/3}{(s+1)^2 + 1^2} + \frac{2/3}{(s+1)^2 + 2^2}$$

$$= \frac{1}{3} e^{-t} \sin t + \frac{1}{3} e^{-t} \sin 2t$$

$$= \frac{e^{-t}}{3} (\sin t + \sin 2t$$

Scanned with CamScanner

Q4

b) $y'' + ay' - 2a^2y = 0$ where $y(0) = 6$ and $y'(0) = 0$

$L(y'') + L(ay') - L(2a^2y) = 0$

$s^2F(s) - sy(0) - y'(0) + a\{sF(s) - y(0)\} - 2a^2F(s) = 0$

$F(s)\ (s^2 + as - 2a^2) - y(0)(s+a) - y'(0)$

$F(s)\ (s^2 + as - 2a^2) - 6(s+a) - 0$

$F(s)\ (s^2 + as - 2a^2 = 6(s+a)$

$= s^2 + as - 2a^2 = (s-a)(s+2a)$

$$F(s) = \frac{6(s+a)}{(s-a)(s+2a)}$$

$$\frac{6(s+a)}{(s-a)(s+2a)} = \frac{A}{s-a} + \frac{B}{s+2a}$$

$$\frac{6(s+a)}{(s-a)(s+2a)} = \frac{A(s+2a) + B(s-a)}{(s-a)(s+2a)}$$

$$\frac{6(s+a)}{(s-a)(s+2a)} = \frac{As + 2aA + Bs - Ba}{(s-a)(s+2a)}$$

$6(s+a) = As + 2aA + Bs - Ba$

$6s + 6a = As + Bs + 2aA - Ba$

$6s + 6a = s(A + B) + a(2A - B)$

Adding equations

$6 = A + B$

$6 = 2A - B$

$12 = 3A$

$A = 12/3$

$A = 4$ Putting in (1)

$6 = 4 + B$

$\boxed{B = 2}$

$$f(s) = \frac{4}{s-a} + \frac{2}{s+2a}$$

$$\boxed{= 4e^{at} + 2e^{-2at}}$$

Q6

a) Find the z-transform of $\{f(k)\}$ where,

$$f(k) = \begin{cases} 5^k, & k < 0 \\ 3^k, & k \geq 0 \end{cases}$$

$$z[f(k)] = \sum_{k<0}^{-1} 5^k z^{-k} + \sum_{k\geq 0}^{\infty} 3^k z^{-k}$$

$$= [\cdots + 5^{-3} z^{-3} + 5^{-2} z^{-2} + 5^{-1} z^{-1}] + [1 + (3)^1 z^{-1} + (3)^2 z^{-2} + (3)^3 z^{-3} + \cdots]$$

$$= \left[\cdots + \left(\frac{z}{5}\right)^3 + \left(\frac{z}{5}\right)^2 + \left(\frac{z}{5}\right)\right] + \left[1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \left(\frac{3}{z}\right)^2 + \cdots\right]$$

$$= \frac{z/5}{1 - z/5} + \frac{1}{1 - 3/z} \Rightarrow \frac{z/5}{5 - z/5} + \frac{1}{z - 3/z}$$

$$= \frac{z}{5 - z} + \frac{z}{z - 3}$$

$$= \frac{z(z-3) + z(5-z)}{(5-z)(z-3)} = \frac{z^2 - 3z + 5z - z^2}{5z - 15 - z^2 + 3z}$$

$$= \frac{2z}{-z^2 + 8z - 15}$$

$$= z[f(k)] = \frac{-2z}{z^2 - 8z + 15}$$

Q5b.

$$f(k) = \left\{0, \cos\frac{0}{2}, \cos\frac{1}{2}, \cos\frac{2}{2}, \ldots\right\}$$

$$f(k) = \{0, 1, 0.9999, 0.9998, 0.9996, \ldots\}$$

$$f(k) = f(z)$$

$$z\{f(k)\} = \frac{0}{z^0} + \frac{1}{z^0} + \frac{0.9999}{z^1} + \frac{0.9998}{z^2} + \ldots$$

Scanned with CamScanner

Q6

a) i) $\cos^2 t = \frac{1}{2}\left[\frac{s}{s^2+4} + \frac{1}{s}\right]$

L.H.S

$\cos^2 x = \frac{1}{2} + \cos 2x \cdot \frac{1}{2}$

$\cos^2 t = \frac{1}{2} + \frac{1}{2}\cos 2t$

Applying Laplace

$L(1/2) + 1/2\, L(\cos 2t)$

$\frac{1}{2s} + \frac{1}{2}\left[\frac{s}{s^2+4}\right]$

$\frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$

ii) $\sin 2t \cos 3t = \frac{2(s^2-5)}{(s^2+1)(s^2+25)}$

$\sin a \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right]$

$= \frac{1}{2}\left[\sin(2t+3t) + \sin(2t-3t)\right]$

$= \frac{1}{2}\left[\sin(5t) + \sin(-t)\right]$

$= \frac{1}{2}\left(\frac{5}{s^2+25} + \frac{(-1)}{s^2+1}\right]$

Scanned with CamScanner

$= \frac{1}{2}\left[\frac{s}{s^2+2s} - \frac{1}{s^2+1}\right]$

$= \frac{1}{2}\left[\frac{s(s^2+1) - 1(s^2+2s)}{(s^2+2s)(s^2+1)}\right] = \frac{1}{2}\left[\frac{5s^2+5-s^2-2s}{(s^2+2s)(s^2+1)}\right]$

$= \frac{1}{2}\left[\frac{4s^2-20}{(s^2+2s)(s^2+1)}\right]$

$= \frac{1}{2}\left[\frac{4(s^2-5)}{(s^2+2s)(s^2+1)}\right] \Rightarrow \frac{2(s^2-s)}{(s^2+2s)(s^2+1)}$

iii) $\frac{1}{t}(1-\cos t)\} = \frac{1}{2}\log(s^2+1) - \log s$

$L(1/t\, f(t)) = \int_s^\infty f(s)\, ds$

$L(1) = 1/s, \quad L(1/t \cdot 1) = \int_s^\infty \frac{1}{s}\, ds = [\ln s]_s^\infty$

$L(\cos t) = \frac{s}{s^2+1}, \quad L\left(\frac{1}{t}\cos t\right) = \int_s^\infty \frac{s}{s^2+1}\, ds = \left(\frac{1}{2}\ln\right.$

$L\left(\frac{1}{t}\cdot(1-\cos t)\right) = [\ln s]_s^\infty - \left[\frac{1}{2}\ln(s^2+1)\right]_s^\infty$

$\lim \ln s - \ln s - \frac{1}{2}\lim_{s\to\infty}\ln(s^2+1) + \frac{1}{2}\ln(s^2+1)$

$\frac{1}{2}\ln(1+s^2) - \ln s + \lim_{s\to\infty}\ln\left[\frac{s}{\sqrt{s^2+1}}\right]$

- $\frac{1}{a}\ln b = \ln(b)^{1/a}$ $\quad\quad \ln a - \ln b = \ln\left(\frac{a}{b}\right)$

Scanned with CamScanner

$$\frac{1}{2}\ln(s^2+1) - \ln s + \lim_{s\to 0} \ln\left(\frac{1}{\sqrt{1+1/s^2}}\right) = \frac{1}{2}\ln(s^2+1) - \ln s + \ln 1.)$$

$$= \frac{1}{2}\ln(s^2+1) - \ln s \quad \text{Proved!}$$

Q6 b.

i). $$\frac{s-4}{4(s-3)^2+16} = \frac{1}{4}e^{3t}\cos 2t - \frac{1}{8}e^{3t}\sin 2t$$

$$\frac{s-4}{4(s-3)^2+16} = \frac{1}{4} \cdot \frac{s-4}{(s-3)^2+(2)^4}$$

$$= \frac{1}{4}\left(\frac{s-3-1}{(s-3)^2+(2)}\right)$$

$$= \left(\frac{1}{4}\frac{(s-3)}{(s-3)^2+(2)}\right) - \left(\frac{1}{2}\frac{1-2}{(s-3)^2+(2)^2}\right)$$

$$= \frac{1}{4}\left(e^{3t}\cos 2t - \frac{1}{2}e^{3t}\sin 2t\right)$$

$$= \frac{1}{4}e^{3t}\cos 2t - \frac{1}{8}e^{3t}\sin 2t$$

ii) $\frac{3s-8}{4s^2+25} = \frac{3}{4}\cos\frac{5t}{2} - \frac{4}{2}\sin\frac{5t}{2}$

$\frac{3s-8}{4s^2+25} = \frac{3s-8}{4(s^2+25/4)}$.

$= \frac{1}{4} \frac{3s-8}{s^2+(5/2)^2}$

$= \left(\frac{3}{s^2+(5/2)^2} - \frac{8 \times 5/2 / 5/2}{s^2+(5/2)^2}\right)$

$= \frac{1}{4}\left(3\cos\frac{5}{2}t - \frac{16}{5}\sin\frac{5}{2}t\right)$

$= \left(\frac{3}{4}\cos\frac{5}{2}t - \frac{4}{5}\sin\frac{5}{2}t\right)$

iii) $\frac{s+4}{s(s-1)(s^2+4)} = -1+e^t - \frac{1}{2}\sin 2t$

$\frac{s+4}{s(s-1)(s^2+4)}$

$\frac{A}{s} + \frac{B}{s-1} + \frac{Cx+D}{s^2+4}$

Scanned with CamScanner

$(A)(s-1)(s^2+4) + (B)(s)(s^2+4) + ((Cn+D)(s)(s-1))$

$= A(s^3+4s-s^2-4) + B(s^3+4s) + (n+D(s^2-s))$

$= As^3 + 4As - As^2 - 4A + Bs^3 + 4Bs + Cs^3 - Cs^2 + Ds^2 - Ds$

$= s^3(A+B+C) + s^2(-A-C+D) + s(4A+4B-D)$

$A+B+C=0$

$-A-C-D=0$

$4A+4B-D=1$

$-4A=4$

$A=-1,\ B=1,\ C=0,\ D=-1$

$$\frac{-1}{s} + \frac{1}{s-1} + \frac{(0)s+(-1)}{s^2+4}$$

$$-1 + e^t - \frac{1}{s^2+4}$$

Scanned with CamScanner

Q1

b). fourier series for:

$f(x) = e^{-x}$ in interval

$0 \leq x \leq 2\pi$

f(x) :-

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} \sin nx$$

where ;

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)\, dx$$

$$= a_0 = \frac{-1}{2\pi} \left[ e^{-2\pi} - e^0 \right] = \frac{1}{2\pi} \left(1 - e^{-2\pi}\right)$$

Therefore,

$$a_0 = \frac{1}{2\pi} \left(1 - e^{-2\pi}\right) \rightarrow \quad (2)$$

Now,

$$= \frac{1}{\pi} \left( \frac{e^{-x}}{1+n^2} \left(-\cos nx + n \sin nx\right) \right]_0^{2\pi}$$

Scanned with CamScanner

Substitute for n=1, 2, 3...

Then;

$$e^{-x} = \left(\frac{1-e^{-2\pi}}{\pi}\right)\left(\frac{1}{2} + \frac{1}{2}\cos x + \frac{1}{5}\cos 2x + \frac{1}{10}\cos 3x + \dots + \frac{1}{2}\sin x + \frac{2}{5}\sin 2x + \frac{3}{10}\sin 3x + \dots\right)$$

Proved!