Probability and Statistics

Haziq Hamid Ali
02-134191-071
BSCS-3A

Q1 a (a) Equally likely Events

When the likelihood of happening of two events are same they are known as Equally likely events. If we toss a coin there are equal chances of getting a head or tail.

Example: Find the probability of getting an odd number when a dice is rolled.

Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

$E = \{1, 3, 5\}$

$n(E) = 3$

$P(E) = \frac{3}{6} = \frac{1}{2}$

(b) Mutually Exclusive Events

Two events A and B are known as mutually Exclusive Events If they can't happen simultaneously. The probability of A and B ~~Exam~~, will be $P(A \text{ or } B) = P(A) + P(B)$.

Example: find the probability that if a card is picked randomly a King or a Queen will come.

P of getting King = 0.6

P of getting Queen = 0.2

Probability of getting King or Queen = $0.2 + 0.6$

$= 0.8$

Scanned with CamScanner

Q1 (b) $g(x) = x^2$ and $f(x) = \frac{(x+2)^2}{24}$, $x = 0, 1, 2, 3, 4, 5$

a) $E[g(x)] = E[x^2] = \sum_{x=0}^{5} g(x)f(x) = \sum_{x=0}^{5} (x^2)\frac{(x+2)^2}{24}$

$\cancel{\frac{(4)^2(4+2)^2}{24}} +$

$= \frac{(0)^2(0+2)^2}{24} + \frac{(1)^2(1+2)^2}{24} + \frac{(2)^2(2+2)^2}{24} +$

$\frac{(3)^2(3+2)^2}{24} + \frac{(4)^2(4+2)^2}{24} + \frac{(5)^2(5+2)^2}{24}$

$= (0)\left(\frac{1}{6}\right) + 1\left(\frac{3}{8}\right) + 4\left(\frac{2}{3}\right) + 9\left(\frac{25}{24}\right) + 16\left(\frac{3}{2}\right)$

$+ 25\left(\frac{49}{24}\right) = \frac{2099}{24}$

b) $E(x)$ for $f(x)$ and $E(x)$ for $g(x)$

$f(x)$

$Ex = \sum_{i}^{5} x_i f(x_i)$

$= 0 \frac{(0+2)^2}{24} + 1\frac{(1+2)^2}{24} + 2\frac{(2+2)^2}{24} + 3\frac{(3+2)^2}{24} + 4\frac{(4+2)^2}{24}$

$+ 5\left(\frac{5+2}{24}\right)^2 = 0 + \frac{3}{8} + \frac{4}{3} + \frac{25}{8} + 6 + \frac{245}{24}$

$= \frac{505}{24}$

$g(x)$

$E(x) = \sum_{i=0}^{5} x_i \, g(x_i)$

$= 0(0)^2 + 1(1)^2 + 2(2)^2 + 3(3)^2 + 4(4)^2 + 5(5)^2$

$= 225$

(c) $Var[x]$ for $f(x)$

$E(x)^2 = \sum_{i=0}^{5} x_i^2 \, f(x_i)$

$$= (0)^2 \frac{(0+2)^2}{24} + (1)^2 \frac{(1+2)}{24} + (3)^2 \frac{(3+2)^2}{24} + (4)^2 \frac{(4+2)^2}{24} +$$

$$(5)^2 \frac{(5+2)^2}{24} = \cancel{0 + \frac{3}{8} + \frac{8}{3} + \frac{75}{8} + 24 + \frac{1225}{24}} = \frac{2099}{24}$$

$$\cancel{\sigma^2 = E(x^2) - [E(x)]^2}$$

$$\cancel{= \frac{2099}{24} - 422.75}$$

$$= \cancel{-355.291} = \frac{3}{8} + 6 + \frac{363}{8} + 216 + \frac{6075}{8}$$

(d) ~~$Var[x]$ for $g(x)$~~

$$\sigma^2 = \frac{8217}{8} - \left(\frac{505}{24}\right)^2$$

$$\sigma^2 = 584.373.$$

Q1b (d) Var [x] for g (x)

$\sigma^2 = E(X^2) - [E(x)]^2$

$E(X) = 225$

$E(X^2) = \sum_{x=0}^{5} x_i^2 \, g(x)$

$= 0 + (1^2)(1^2) + (2^2)(2^2) + (3^2)(3^2) + (4^2)(4^2) + (5^2)(5^2)$

$= 1 + 16 + 81 + 256 + 625$

$= 979$

$\sigma^2 = 979 - (225)^2$

$= -49646$

Scanned with CamScanner

Q1 e Prove the following.

a) $E[aX] = a E[X]$

Proof:-

x	x_1	x_2	....	x_n
$F(x)$	$f(x_1)$	$f(x_2)$	--------	$F(x_n)$

$a \in$ Constant

ax	ax_1	a_2	--------	ax_n
$f(x)$	$f(x_1)$	$f(x_2)$	--------	$f(x_n)$

$E(ax) = aF(x) = ax_1 f(x_1) + ax_2 f(x_1)$ ------- $ax_n f(x)_n$.

$= a[\Sigma x f(x)]$

$E(ax) = a E(x)$.

Q1c

b) $E[X \pm Y] = E[X] \pm E[Y]$

Proof:-

let random variable X with m value So there will be another variable Y having n Value then we will have following b. variable Probability distribution function.

$x \backslash y$	y_1	y_2	- - - - - - - - -	y_n	Sum of raw
x_1	$f(x_1, y_1)$	$f(x_1, y_2)$	- - - - - - - -	$f(x_1, y_n)$	$f(x_1)$
x_2	$f(x_2, y_1)$	$f(x_2, y_2)$	- - - - - - - -	$f(x_2, y_n)$	$f(x_2)$
:	:	:	:	:	:
:	:	:	:	:	:
:	:	:	:	:	:
:	:	:	:	:	:
x_m	$f(x_m, y_1)$	$f(x_m, y_2)$		$f(x_m, y_n)$	$f(x_m)$
Sum of Colom	$f(y_1)$	$f(y_2)$		$f(y_n)$	$\Sigma f(xy)$.

hence $f(x_i, y_i) = f(x_i)$ and $f(y_i)$ are joint and marginal Probability respectively at $X = Y_i$ and $X = y_i$ and also

$$f(x_i) = \sum_{j=1}^{n} f(x_i y_j);$$

$$f(y_j) = \sum_{i=1}^{n} f(x_i, y_j)$$

$$\therefore E(X+Y) = \sum_{i=1}^{m}\sum_{j=1}^{n} (x_i + y_j) f(x_i y_j)$$

$$\therefore \sum_{i=1}^{m}\sum_{j=1}^{n} [x_i f(x_i y_j) + y_j f(x_i y_j)]$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} [x_i f(x_i y_j)] + \sum_{i=1}^{m}\sum_{j=1}^{n} y_i f(x_i, y_j)$$

$$= \sum_{i=1}^{m} x_i [\sum_{j=1}^{n} f x_i y_j] + \sum_{j=1}^{n} y_j [\sum_{j=1}^{m} f(x_i y_j)]$$

$$= \sum_{i=1}^{m} x_i f(x_i) + \sum_{j=1}^{n} y_j f(y_j)$$

$$E(X+Y) = E(X) + E(Y)$$

Proved

$$\therefore E(X-Y) = E(X) - E(Y).$$

Q2 a)

Consider x = no of heads 4 tosses of a fair Coin.

Total no of Sample Points $= 2^4 = 16$

i.e $S = \{(H,H,H,H), (H,H,H,T), (H,H,T,H) (H,T,H,H),$
$(T,H,H,H), (H,T,H,T), (H,T,T,H), (H,T,T,T),$
$(T,H,H,T), (T,H,T,H), (T,H,T,T), (T,T,H,H),$
$(T,T,H,T), (T,T,T,H), (T,T,T,T)\}$.

$x = 0, 1, 2, 3, 4$ Can be considred random Variables.

$P(X = x = 0) = P(\text{No head}) = \dfrac{\{(T,T,T,T)\}}{16} = \dfrac{1}{16}$.

$P(X=1) = P(\text{1 head}) = \dfrac{\{(H,T,T,T), (T,H,T,T), (T,T,H,T), (T,T,H,T)\}}{16} = \dfrac{4}{16} = \dfrac{1}{4}$.

$P(X=2) = P(\text{2 head}) = \{(H,H,T,T), (H,T,H,T), (H,T,T,H)$
$(T,H,H,T), (T,H,T,H), (T,T,H,H)$

$= \dfrac{6}{16} = \dfrac{3}{8}$.

$$P(3\ \text{head}) = \{(H,H,H,T), (H,H,T,H), (H,T,H,H)$$
$$(T,H,H,H)\} = \frac{4}{16} = \frac{1}{4}.$$

$$P(4\ \text{head}) = \{(H,H,H,H)\} = \frac{1}{16}.$$

x	0	1	2	3	4
$f(x)$	1/16	1/4	3/8	1/4	1/16.

$$P(x) = \frac{C_x}{8} ;\quad x = 0, 1, 2, 3, 4.$$

Scanned with CamScanner

Q2 b

(a) The two normal distributions have the same shape.
TRUE.
Two normal distributions have the same shape because the normal distribution curve is symmetric about it's mean, and the mean characterizes the distribution.

(b) The two normal distributions are centered at the same place.
FALSE.
They willn't centre at the same place because the increase of mean will move the curve through axis towards left or right.

Q3 (a) Binominal Distribution, Geometric Distribution, Hypergeometric Distribution, Multinomial Distribution, Negative Binomial Distribution and Poisson Distribution.

Q3 (b)

① Binomial Random Variable: refers to the number of successes in a binomial experiment.

② Binomial Experiment : is a statistical experiment that has following properties

- The experiment consists of 'n' repeated trials.
- Each trial can result in just two possible outcomes.
- The probability of success denoted by 'p' is the same for on every trial.
- The trials are independent.

③ Binomial Probability Mass function: The probability of getting exactly 'K' successes in 'n' independant bernoulli trials is given by the probability mass function.

④ Binomial Distribution: The probability distribution of a binomial random variable.

Binomial Distribution versus Poisson Distribution

The difference between two is that while both mejor the number of certain random events or successes within a certain frame. The Binomial is based on discrete events while the poisson is based on conthious events.

Q3 (c)

$p = 0.42$
$q = 1 - p$
$q = 1 - 0.42$
$q = 0.58$

Total no of patients = $n = 18$
find probability of 'x' no of patients, $x = 5$

$P(X = x) = {}^{n}C_{x} \, (p)^{x} \, (q)^{n-x}$

$\Rightarrow 8568 \, (0.01306) \, (8.405 \times 10^{-4})$
$\boxed{= 0.094}$

The probability of recovery of 5 patients out of 18 is 0.09%.

Q4 (a)

Sample Space for two dices

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

x	P_x	$f(x)$
1	1/36	$x<2 = 1/36$
2	2/36	$x<3 = 1/36 + 2/36 = 3/36$
3	2/36	$x<4 = 3/36 + 2/36 = 5/36$
4	3/36	$x<5 = 5/36 + 3/36 = 8/36$
5	2/36	$x<6 = 8/36 + 2/36 = 10/36$
6	4/36	$x<7 = 10/36 + 4/36 = 14/36$
–	–	
8	2/36	$x<9 = 14/36 + 2/36 = 16/36$
9	1/36	$x<10 = 16/36 + 1/36 = 17/36$
10	2/36	$x<11 = 17/36 + 2/36 = 19/36$
11	–	
12	4/36	$x<13 = 19/36 + 4/36 = 23/36$
–	–	
–	–	
15	2/36	$x<16 = 23/36 + 2/36 = 25/36$
16	1/36	$x<17 = 25/36 + 1/36 = 26/36$

18	2/36	$x<19 = 26/36 + 2/36 = 28/36$
20	2/36	$x<21 = 28/36 + 2/36 = 30/36$
24	2/36	$x<25 = 30/36 + 2/36 = 32/36$
25	1/36	$x<26 = 32/36 + 1/36 = 33/36$
30	2/36	$x<31 = 33/\cancel{36} + 2/36 = 35/36$
36	1/36	$x<36 = 35/36 + 1/36 = 36/36$

Q4b

n	X	Y	XY	X^2	Y^2
1	5	85	425	25	7225
2	4	103	412	16	10609
3	6	70	420	36	4900
4	5	82	410	25	6724
5	5	89	445	25	7921
6	5	98	490	25	9604
7	6	66	396	36	4356
8	6	95	570	36	9025
9	2	169	338	4	28561
10	7	70	490	49	4900
11	7	48	336	49	2304
Total	58	975	4732	326	96129

a) $$r = \frac{11(4732) - (58)(975)}{\sqrt{(11)(326) - (58)^2}\sqrt{(11)(96129) - (975)^2}}$$

$$= \frac{52052 - 56550}{\sqrt{3586 - 3364}\sqrt{1057419 - 950625}}$$

$$= \frac{-\cancel{514498}}{\cancel{\sqrt{222}\sqrt{106794}}} \qquad r = -0.923$$

$$\cancel{= -105.66}$$

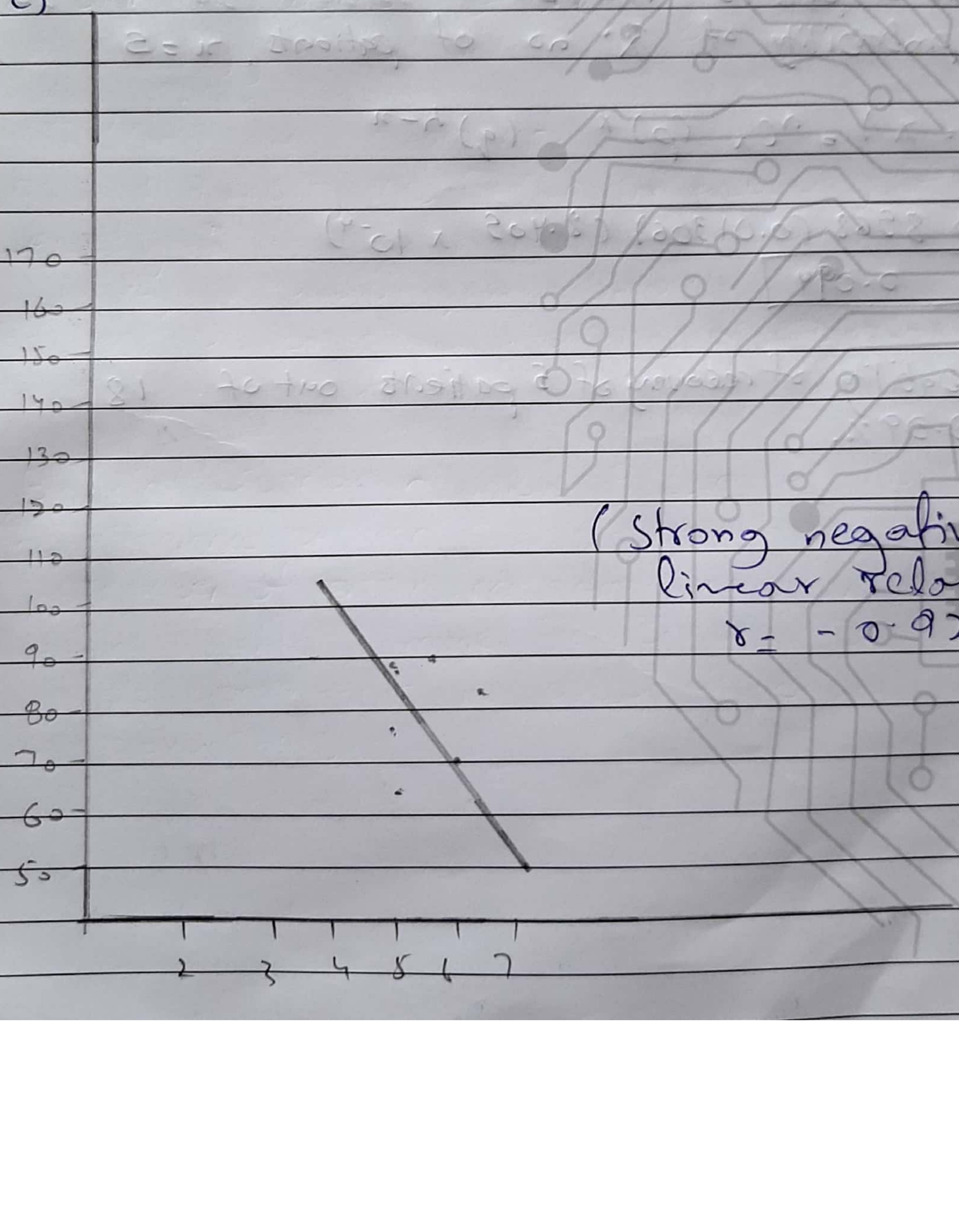
b) Since $r = -0.923$

it lies between $-1 < r < -0.5$

$-1 < -0.923 < -0.5$

There exists highly negative co-relation between age and price of orians

c)



(Strong negative linear relation $r = -0.9237$)

Q5 a)

Sample Space for 2 dice role:

[1][1], [1][2], [1][3], [1][4], [1][5], [1][6].
[2][1], [2][2], [2][3], [2][4], [2][5], [2][6].
[3][1], [3][2], [3][3], [3][4], [3][5], [3][6].
[4][1], [4][2], [4][3], [4][4], [4][5], [4][6]
[5][1], [5][2], [5][3], [5][4], [5][5], [5][6]
[6][1], [6][2], [6][3], [6][4], [6][5], [6][6].

Q5 b)

$$P(X=x) = \frac{e^{-mt}(mt)^x}{x!}$$

a) $t=1$

$mt = (7)(1) = 7$

$$P(x \geq 5) = \sum_{x=0}^{5} \frac{e^{-7}(7)^5}{5!} = 0.1277.$$

b) $t=3$

$mt = (7)(3) = 21$

$$P(x \geq 6) = 1 - \left[e \sum_{i=0}^{6} \frac{e^{(-21)}(21)^6}{6!} \right]$$

$$= 1 - \left[\frac{e^{-21}(21)^0}{0!} + \frac{e^{-21}(21)^1}{1!} - - - - \right]$$

$= 0.999.$

Q5 c) Normal Distribution: is a symmetric distribution where most of the observations cluster around the central peak and the probabilities for values farther away from the mean taper off equally in both directions.
Standard Normal Distribution: is the distribution that occurss when a normal random variable has a mean of zero and a standard deviation of one.

a) $P(0.15 \leq x \leq 1.151)$
As we know
$x = z$ in standard Normal distribution

$P(0.15 \leq z \leq 1.51)$
$\sigma(b) - \sigma(a)$
$\sigma(1.51) - \sigma(0.15)$
$\boxed{= 0.37486}$

b) $P(0.88 \leq x \leq 2.24)$
$= \sigma(b) - \sigma(a)$
$= \sigma(2.24) - \sigma(-0.88)$
$= 0.98745 - 0.18943$
$\boxed{= 0.79802}$