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DEPARTMENT OF GEOMATICS ENGINEERING

A Report on 'Levelling and Errors Prominent in Levelling'

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[LEVELLING AND ERRORS PROMINENT IN LEVELLING]

ABSTRACT

Errors are inevitable. The discrepancies exist in all forms of surveying and measurement. They persist for sure, but with certain precautions and adjustment techniques, their aftermath could be altered. In the process of establishment of vertical control points, called levelling, various systematic and random errors ascend up. Systematic errors are cumulative in nature. They can be constant or variable throughout the observation and are generally attributable to a known circumstance. These can be expressed by some functional relationships. Earth Curvature Error, Atmospheric Refraction, Instrument Maladjustment, etc. are considered as the Systematic errors, which can be minimized by following proper field procedures, or they can be modeled and corrected computationally. Random errors on the other hand, like Instrument Levelling Error, Distance Observations Error, Incorrect graduated scale reading/ Rod Plumbing Error, etc. are compensating in nature and they can be minimized by taking redundant observations or adjustment by distribution.

1. INTRODUCTION

1.1 Background

Levelling is the method of establishing vertical control points. It is the process of determining difference in elevation of various points on the earth's surface (Shrestha, 1988). This procedure involves measurement in vertical direction to find out the elevation of given points with respect to given or assumed datum and to establish points at a given elevation or at different elevations with respect to a given or assumed datum (Punmia et al., 2015a).

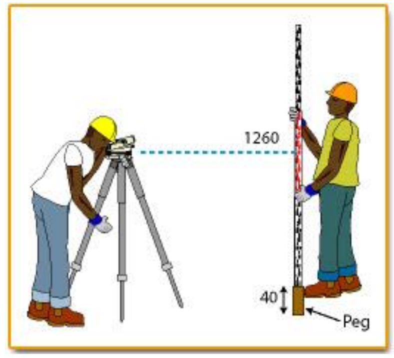

Figure 1. Field Procedure of Levelling

Levelling has a wide application in various fields of geospatial domain. Levelling helps in determining the topographical diversification in an area. Besides this, the levelling procedure helps in locating the gradient lines for drainage characteristics of an area and it also helps in designing and laying out various construction projects like highways, canals, railways, etc. Levelling assists in calculating volume of earthwork and reservoir as well.

1.2 Principle of Levelling

Levelling is based on the principle of making the line of sight horizontal (Roy, 1999). When the level machine is set up correctly and leveled, the bubble tube axis and the line of sight will be horizontal and the vertical axis of the instrument will be vertical, thus ensuing the line of sight in a horizontal plane, wherever the telescope is rotated.

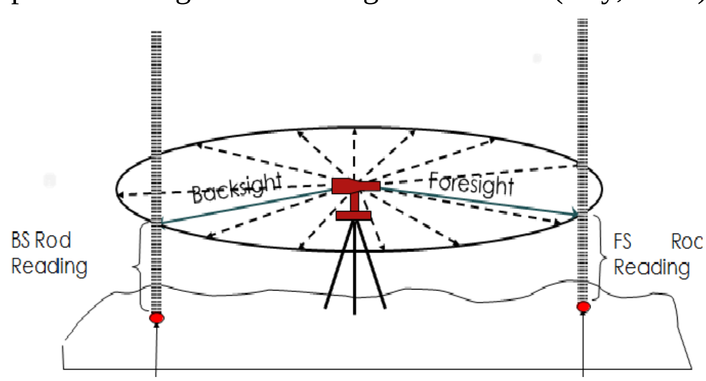

Figure 2. Principle of Levelling

When a level machine is setup correctly and levelled approximately midway between the benchmark (represented by A in the figure) and the point (represented by B in the figure), the elevation of the point can be ascertained by levelling procedure. From the staff held at the benchmark, a backsight is taken and the height of instrument is determined.

$$\mathrm{HI} = \text{Elevation of Benchmark} + \text{Backsight reading}$$

On turning the telescope to bring into the staff held on point B, a foresight is taken to formulate the elevation of that point.

$$\text{Elevation of the point B} = \mathrm{HI} - \text{Foresight}$$

1.3 Methods of Levelling

i) Direct Levelling (Spirit Levelling)

It is a method of levelling in which the vertical distances with respect to a horizontal line of sight can be used to determine the relative difference in elevation between two adjacent points. It is the most precise method of determining elevations. Direct levelling can be further classified into following types based on different working methods:

- <u>Simple Levelling:</u> Only one setup is done for the measurement of height difference between two points.
- <u>Differential Levelling:</u> More than one setup is maintained to determine the difference in elevation of two points. This method is adopted when two points who difference is elevation to be measured are far apart for each other.
- <u>Profile Levelling:</u> In this method of direct levelling, the elevations of the points along a given line are measured at a certain interval in order to obtain the profile of the surface of the ground along than line.
- <u>Cross-Sectioning:</u> Cross-Sectioning is the process of taking levels on both the lateral sides of a main line at right angles to that line, in order to determine a vertical cross section of the surface of the ground.
- <u>Reciprocal Levelling:</u> The difference in elevation between two points is accurately determined by two sets of reciprocal observations when it is not possible to setup the level between the two points.

- Precise Levelling: It is the method of levelling in which high degree of precision is maintained using special equipment and special precautions, to eliminate, as far as possible, all sources of error.

ii) Indirect Levelling

It is used when direct levelling becomes difficult in easily inaccessible points such as peaks, top of towers, etc. The principal methods of indirect levelling are viz.

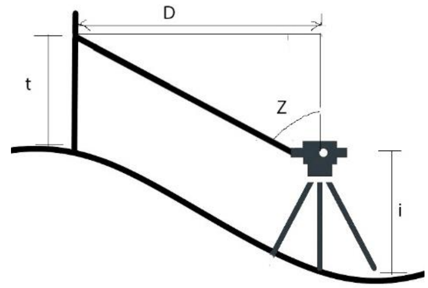

Figure 3. Trigonometric Levelling to determine the RL of a spot

- Trigonometric Levelling: It is the process in which the elevation of the points is computed from the vertical angles and the horizontal distances, just similar to the computation of length of any side in a triangle with proper trigonometric relations.
- Barometric Levelling: Barometric Levelling is based on the phenomenon that the difference in elevation between two points is proportional to the difference in atmospheric pressure at these points. As the atmospheric pressure doesn't remain constant in the course of a day even at a particular place, this method is relatively inaccurate, unreliable and is less used such as in cases of reconnaissance work.
- Hypsometric Levelling: In this method, difference between elevation is determined by noting down the temperature at which the water starts boiling. Boiling point of water decreases as the altitude of the place increases. It is very rough method and is rarely used.

(Punmia et al., 2015a) (Punmia et al., 2015b) (Roy, 1999)

2. Errors in Levelling

As presented in the aforementioned methods of levelling, differential levelling and trigonometric levelling are the two most commonly employed procedures for finding the elevation difference between stations among which, both are subjected to systematic and random errors.

Table 1. Categorization of Systematic and Random Errors

Systematic Errors	Random Errors
→ Earth Curvature Error	→ Instrument Levelling Error
→ Atmospheric Refraction	→ Distance Observations Error
→ Instrument Maladjustment	→ Incorrect graduated scale reading/ Rod Plumbing Error

The effects of these systematic errors can be minimized by following proper field procedures, or they can be modeled and corrected computationally. And the random errors, if detectable, are removed by omitting the observations comprising blunders. Otherwise, being compensating in nature, they can be minimized by taking redundant observations or adjustment by distribution. These must be treated according to the theory of random errors (Ghilani, 2017).

2.1 Systematic Errors in Differential Levelling

Systematic Errors are cumulative in nature and can be expressed by some functional relationships. It should always be presumed that these errors are present in differential leveling observations, and thus corrective field procedures, should be followed to minimize their effects.

i) Collimation Error (Instrument Maladjustment)

Collimation Error occurs when the line of sight of an instrument is not truly horizontal. The error due to non-parallelism of line of collimation and axis of bubble tube could be diminished by keeping the sight distances short and balanced (Punmia et al., 2015a).

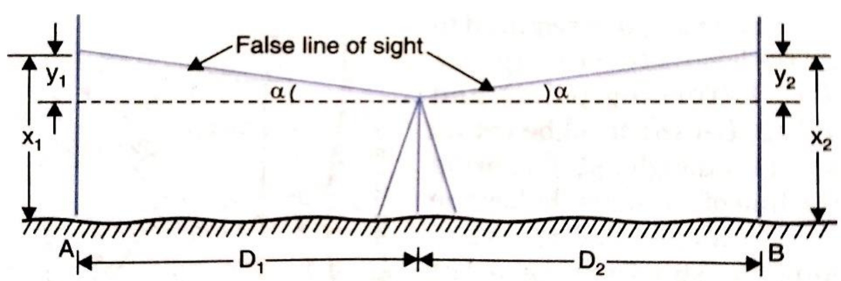

Figure 4. Balancing BS and FS

As depicted in the figure above, for an individual setup,

$$\tan\alpha = \frac{y_1}{D_1} \Rightarrow y_1 = D_1 \tan\alpha$$

For very small angle α, say, $\tan\alpha \sim \alpha$.

So, $y_1 = D_1\, \alpha$

For two step setups, $e_c = y_1 - y_2 = D_1\,\alpha - D_2\,\alpha$, where e_c is the error in elevation due to presence of collimation error.

Therefore, the collimation error for a line of levels can be expressed as:

$$e_c = \alpha[(D_1 - D_2) + (D_3 - D_4) + (D_5 - D_6) + \cdots + (D_{n-1} - D_n)]$$

$$\longrightarrow e_c = \alpha[\sum BS_{distance} - \sum FS_{distance}] \qquad \ldots\ldots\ldots \text{Equation (A)}$$

The collimation error determined from the $Equation\ (A)$ is treated as a correction, and thus subtracted from the observed elevation difference to obtain the corrected value.

ii) Earth Curvature and Refraction

From the definition of a level surface and a horizontal line, it is evident that the horizontal line departs from a level surface because of the curvature of the earth. Again, in the long sights, the horizontal line of sight doesn't remain straight but slightly bends downwards having concavity towards earth surface due to refraction. The combined effect of Earth Curvature and the refraction on an individual sight always causes a rod reading to be too high.

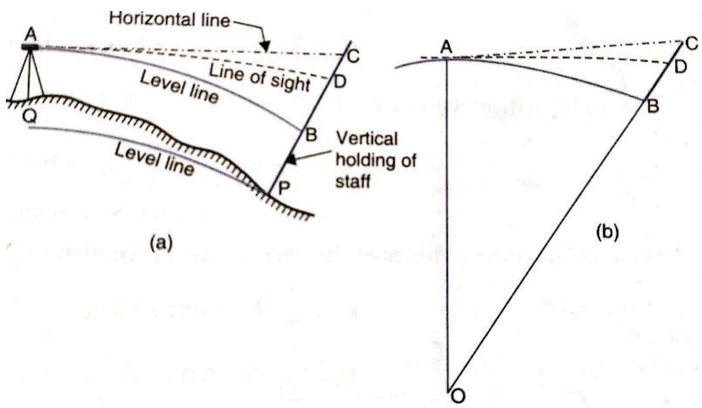

Figure 5. Curvature and Refraction

In the figure,

$$OC^2 = OA + AC^2$$

Let,

$BC = C_c = \text{Correction for Curvature}$

$AB = d = \text{horizontal distance between A and B}$

$AO = \text{Radius of earth in same unit as that of d (approx. 6371 km)}$

$\therefore (R + C_c)^2 = R^2 + d^2$

$\text{or}, C_C(2R + C_C) = d^2$

$$\text{or}, C_c = \frac{d^2}{2R + C_C} \sim \frac{d^2}{2R}(-\text{ve}), (\text{Neglecting } C_C \text{ in comparision to } 2R)$$

$$= 0.0785\ d^2 \text{ meters, d being in km}$$

The refraction curve is irregular because of varying atmospheric conditions, but for average conditions, it is assumed to have a diameter about seven times that of the earth (Punmia et al., 2015a).

The correction in refraction, C_r is therefore, given by:

$$C_r = \frac{1}{7}\frac{d^2}{2R}(+\text{ve})$$

$$= 0.01121\ d^2 \text{ meters, when d is in km}$$

The combined effect due to curvature and refraction will be given by

$$C = \frac{d^2}{2R} - \frac{1}{7}\frac{d^2}{2R} = \frac{6}{7}\frac{d^2}{2R}(\text{subractive})$$

$$= 0.06728\ d^2 \text{ meters, d being in km}$$

iii) Combined Effects of Systematic Errors on Elevation Differences

Furthermore, the combined effect of systematic errors on elevation difference is given by:

$$\Delta h = (x_1 - x_2) - \alpha(D_1 - D_2) - 0.06728(D_1^2 - D_2^2)$$

Where, x_1 is the backsight rod reading and x_2 is the foresight rod reading.

For a level of lines, the corrected elevation difference is:

$$\Delta h = \sum x_{BS} - \sum x_{FS} - \alpha(\sum D_{BS} - \sum D_{FS}) - 0.06728(\sum D_{BS}^2 - D_{FS}^2)$$

2.2 Random Errors in Differential Levelling

Differential levelling is subjected to several sources of random errors, which have no functional relationship based on a deterministic system, and are usually modelled by the probability theory.

i) Reading Errors

The estimated error in rod readings is usually expressed as a ratio of the estimated standard error in the rod reading per unit sight distance length. For instance, if an observer's ability to read a rod is within ±0.01 m per 200 m, then $\sigma_{r/D}$ is $\pm 0.01/200$

$\pm$0.00005 m/m. Using this, rod reading errors for any individual sight distance D can be estimated as

$$\sigma_r = D\sigma_{r/D}$$

Where, $\sigma_{r/D}$ is the estimated error in the rod reading per unit length of sight distance and D is the length of sight distance.

ii) Instrument Levelling Errors

It is generally given in the technical data for each instrument. For precise levels, this information is usually listed in arc seconds or as an estimated elevation error for a given distance. For example, the estimated error may be listed as $\pm$1.5 mm/km, which corresponds to $\pm(1.5/1000000)\rho = \pm 0.3''$.

iii) Rod Plumbing Error

Rod Plumbing Error is the error that should be modelled when computing the standard error in an elevation difference. For any rod reading, the rod plumbing error is approximated as:

$$\mathrm{e_{LS}} = \mathrm{r} - \mathrm{r'} = \frac{\mathrm{d}^2}{2\mathrm{r}}$$

Where, d is the linear amount that the rod is out of plumb at the location of rod reading, r. The size of d is dependent on the rod level bubble centering error and the reading location. If the rod bubble is out of level by β, then d is:

$$\mathrm{d} = \mathrm{r}\sin\beta$$

From the above two equations,

$$\mathrm{e_{LS}} = \frac{\mathrm{r}}{2}\sin^2\beta$$

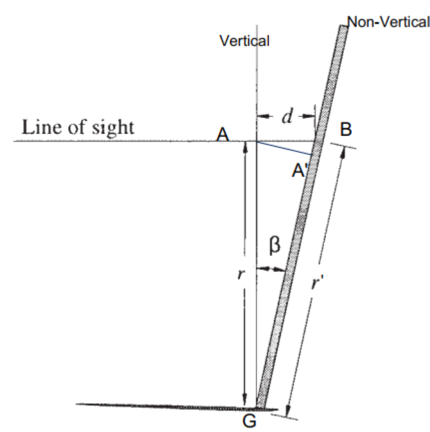

Figure 6. Non-Vertical Level Staff

In the above figure, AG = r = A’G, BG = r’.

In triangle ABG,

$$\mathrm{BG}^2 = \mathrm{AG}^2 + \mathrm{AB}^2$$

$$\mathrm{r'}^2 = \mathrm{r}^2 + \mathrm{d}^2$$

$$(\mathrm{r} + \mathrm{e_{LS}})^2 = \mathrm{r}^2 + \mathrm{d}^2$$

$$2\mathrm{re_{LS}} = \mathrm{d}^2 \implies \mathrm{e_{LS}} = \frac{\mathrm{d}^2}{2\mathrm{R}}$$

Since, d = $r\sin\beta$,

So, $$e_{LS} = \frac{r^2 \sin^2 \beta}{2}$$

3. Conclusion

As shown by the preceding discussions, the major random error sources in differential levelling are caused by random errors in rod readings and instrument levelling. Furthermore, the collimation error is considered to be systematic and is effectively negated by balancing eh backsight and foresight distances. However, no matter what method is used to observe the lengths of the sight distances, some random error in these lengths will be present. As the definition of the Random Error suggests, these are those variants which remain after all the errors have been removed. The random errors in the elevation differences is caused due to the effects of Earth Curvature, refraction, and instrumental collimation errors.

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