Chapter 5:

TRIANGULATION

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LIST OF ABBREVIATIONS

D-Card	Description Card
E	Easting
EDM	Electronic Distance Measurement
KUBH	Kathmandu University Boys Hostel
m	Meter
N	Northing
RL	Reduced Level
RO	Reference Origin

1. INTRODUCTION

1.1 Background

Triangulation is a method of Geodetic survey that provides a framework of survey points, whose relative positions in two or three dimensions are known to some prescribed degree of accuracy. In triangulation, the system consists of number of interconnected triangles in which the length of any one line, called the base line, and the angles of the net are measured very precisely. Knowing the length of one side and the angles of the net, the length of other sides can be computed, followed by calculation of bearing of those lines from the given true bearing of any one line. And, the coordinates of all triangulation stations are then determined from the coordinates of any one given station (Punmia et al., 2015).

Triangulation surveys are of much importance that the geodetic points determined from this procedure furnish the most precise control to which a more detailed survey of a country may be referred. Triangulation surveys are carried out to establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods and for photogrammetric surveys of large areas. Triangulation also assists in determining the shape and size of earth by making the observations for latitude, longitude and gravity. Besides this, determining accurate locations of points in engineering works such as fixing center line and abutments of long bridges over large rivers, detection of crustal movements, transferring the control points across wide sea channels, large water bodies, etc., triangulation has a vital role (Onoriode, n.d.).

This project was carried out on a geographically diversified topography of Kavrepalanchowk District. Six stations were established, namely, Chinatown, Eye Hospital, Talu Danda, Chaukot, KUBH and Dhaneshwor, located in Banepa, Dhulikhel and Panauti Municipality, whose horizontal and vertical cultural features (x, y, z) were determined via triangulation. For a group of 5 members, we were provided with J2-2 theodolite for carrying out fourth order triangulation.

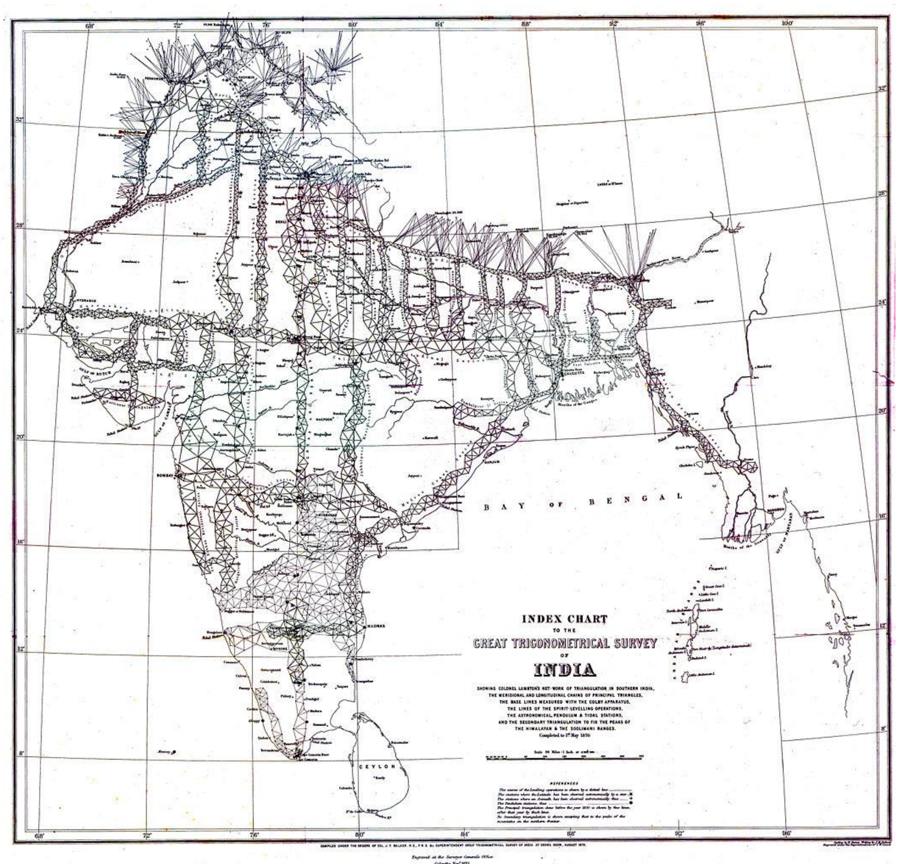


Figure 1. Triangulation Survey Layout in a Country

Source: Retrieved from (*Reddit*, 2016)

This project became a productive approach for us as the students of Geomatics Engineering as it helped in broadening the practical understanding of surveying, especially in the domain of establishment of control points in larger spectrum.

1.2 Objectives

The key objective of triangulation was

- To establish control points for larger area.

The secondary objectives achieved during the accomplishment of triangulation are pinpointed below:

- Determination of topographical diversification of broad area.
- Performing trigonometric levelling in order to determine RL of stations.
- Setting up monuments and preparation of D-Cards of monuments.

1.3 Scope

The course of triangulation was basically focused on establishing control points. It dealt with the scope of determining cultural features of the triangulation stations (i.e. (x, y, z) coordinates). The monumentation used for stations were of temporary nature (wooden pegs were used). So, permanent monumentation of control point is not under the scope of the project. Furthermore, the preparation of D-Cards for the established monuments was incorporated under the scope of the project. And, the triangulation was carried out following the fourth order specifications, so the accuracy greater than fourth order was not embraced under the scope of work.

2. METHODOLOGY

2.1 Theoretical Framework

Triangulation is based on the trigonometric proportion that if one side and three angles of a triangle is known, the remaining sides can be computed. Furthermore, the directions of remaining sides can be determined from the direction of one known side.

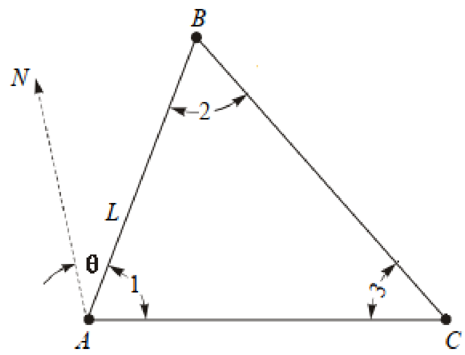


Figure 2. Principle of Triangulation

Let us consider, in the triangle ABC, all the angles and the length (L) of the side AB have been measured. Also, the azimuth θ of AB has been measured at the station A, whose coordinates (X_A, Y_A) are known.

The coordinates of the stations B and C can be determined by the method of triangulation, for which length of all the lines have to be computed first, using sine law:

$$\frac{AB}{sin3} = \frac{BC}{sin1} = \frac{CA}{sin2}$$

Since,

$$AB = L = l_{AB}$$

$$BC = \frac{Lsin1}{sin3} = l_{BC}$$

$$CA = \frac{Lsin2}{sin3} = l_{CA}$$

Moreover, calculation of azimuth proceeds as follows:

Azimuth of AB = θ = θ_{AB}

Azimuth of AC = $\theta + \angle 1$ = θ_{AC}

Azimuth of BC = $\theta + 180° - \angle 2$ = θ_{BC}

From the known lengths of the sides and the azimuths, the consecutive coordinates can be computed as below.

Latitude of AB = $l_{AB}cos\theta = L_{AB}$ Departure of AB = $l_{AB}sin\theta_{AB} = D_{AB}$

Latitude of AC = $l_{AC}cos\theta = L_{AC}$ Departure of AC = $l_{AC}sin\theta_{AC} = D_{AC}$

After all these, the desired coordinates of the triangulation stations B and C are:

X coordinate of B, $X_B = X_A + D_{AB}$ Y coordinate of B, $Y_B = Y_A + L_{AB}$

X coordinate of C, $X_C = X_A + D_{AC}$ Y coordinate of C, $Y_C = Y_A + L_{AC}$

Classification of Triangulation Network:

Based on the arrangement of triangles, also called layouts, the triangulation network can be classified as follows:

i) Single Chain of Triangles: This system of arrangement is used when the control points are required to be established in a narrow strip of terrain such as a valley between ridges. Simple triangles of a triangulation system provide only one route through which distances can be computed, and hence, this system does not provide any check on the accuracy of observations.

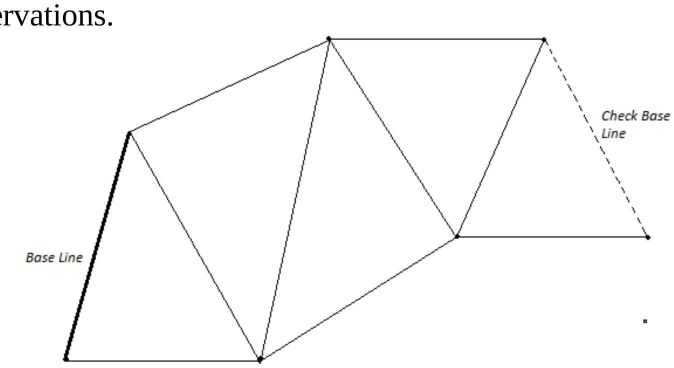


Figure 3. Single Chain of Triangles

ii) Double Chain of Triangles: If the belt is a bit wider and single chain cannot cover it, the double chain of triangle is used.

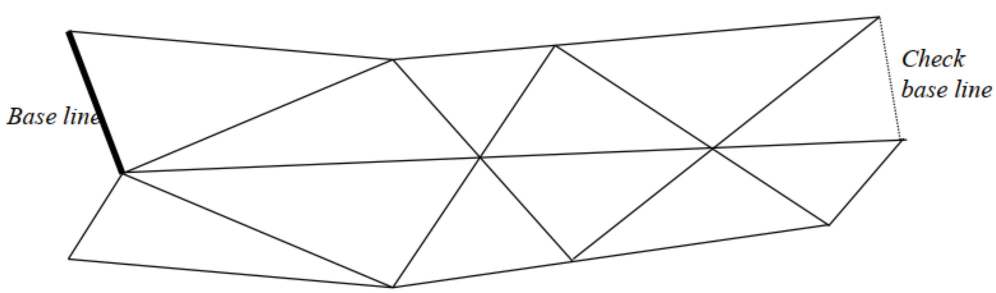


Figure 4. Double Chain of Triangles

iii) Braced Quadrilaterals: This system consists of quadrilaterals with diagonals, whose four corner stations are observed but not the intersection of the diagonals. Being composed by four overlapping triangles, this system is treated to be the strongest and best arrangement of triangles, and it provides a means of computing the length of the sides using different routes.

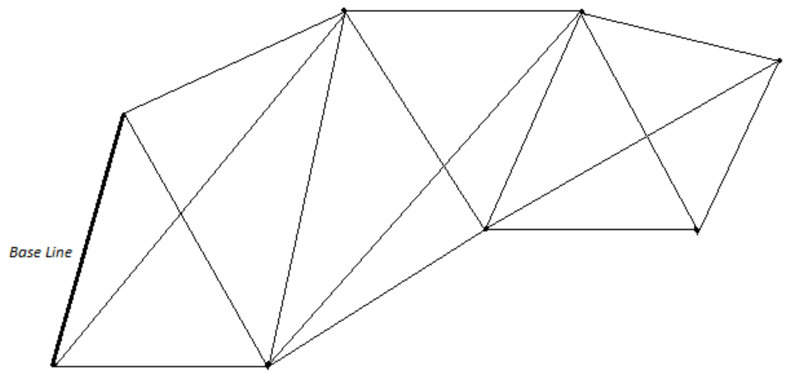


Figure 5. Braced Quadrilaterals

iv) Centered Figures: In this system, centered polygons, the polygons, the polygons with the interior stations, are used to form the triangulation net. The interior station should be the common vertex for all the triangles of the polygon. This system is used when the area to be covered is extended in all directions from a central location.

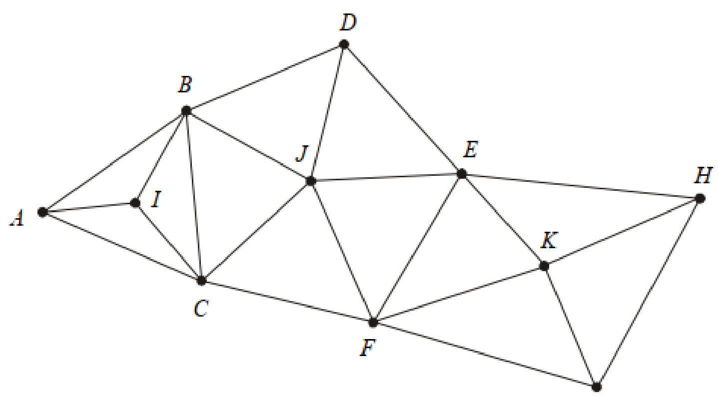


Figure 6. Centered Figures

Basic terms related to Triangulation:

i) Main Stations: The triangulation stations that are used to carry forward the network of the triangulation are known as main stations.

ii) Subsidiary Stations: The triangulation stations that are used only to provide additional rays to intersected points are known as subsidiary stations. These stations are not used to carry forward the triangulation network.

iii) Satellite Station: The stations which are selected closed to main triangulation stations to avoid the intervening obstruction are known are as satellite stations, also called eccentric or false stations.

iv) Laplace Station: The triangulation station at which astronomical observations are made for azimuth is called Laplace station.

v) Pivot Stations: Those stations at which no observations are made but the angles at them are used for the continuity of the triangulation series are known as pivot stations.

vi) Base Line: A line whose length is measured in the field during triangulation is called base line.

v) Well Conditioned Triangle: The triangles of such a shape, in which any error in angular measurement has a minimum effect upon the computed lengths, is known as well-conditioned triangle. The best shape of an isosceles triangle is that triangle whose base angles are 56°14' each. However, from practical considerations, an equilateral triangle may be treated as a well-conditional triangle.

v) Strength of Figures: The accuracy of the system due to shape of the figure in the net is measured in terms of the strength of figures. The U.S. Coast and Geodetic Surveys has developed a convenient method of evaluating the strength of a triangulation figure which is based on the expression for the square of the probable error (L^2), that would occur in the sixth place of the logarithm of any side, if the computations are carried from a known side through a single chain of triangles after the net has been adjusted for the side and angle conditions.

vi) Control Points: They are the points located on the ground by precise surveying.

vii) Bearing: The bearing of a line is the horizontal angle which it makes with a reference meridian.

vii) Meridian: The fixed reference line about which bearing is measured is called meridian.

viii) Collimation Error: If the axis of telescope is not parallel to the line of collimation, the error introduced is called collimation error.

ix) Index error: When the telescope of the properly setup theodolite is exactly horizontal, the zenithal reading must read $(90°00'00")$ in face left observation and $(270°00'00")$ in face right observation and on summing them up, it must results $360°$. If certain deviation is observed in this case, the discrepancy is called Index error.

(Punmia et al., 2005) (Punmia et al., 2015) (Onoriode, n.d.) (Duggal, 2013)

2.2 Study Area

Triangulation was carried out in Kavrepalanchowk District, with six stations established in larger extent, forming the net of braced quadrilateral. Station 1001 (Dhaneshwor) and station 1005 (Chaukot) is located in Panauti Municipality. Station 1002 (Chinatown) and station 1003 (Eye Hospital) is located in Banepa Municipality whereas station 1004 (Talu Danda) and station 1006 (KUBH) is located in Dhulikhel Municipality (*OpenStreetMap*, n.d.).

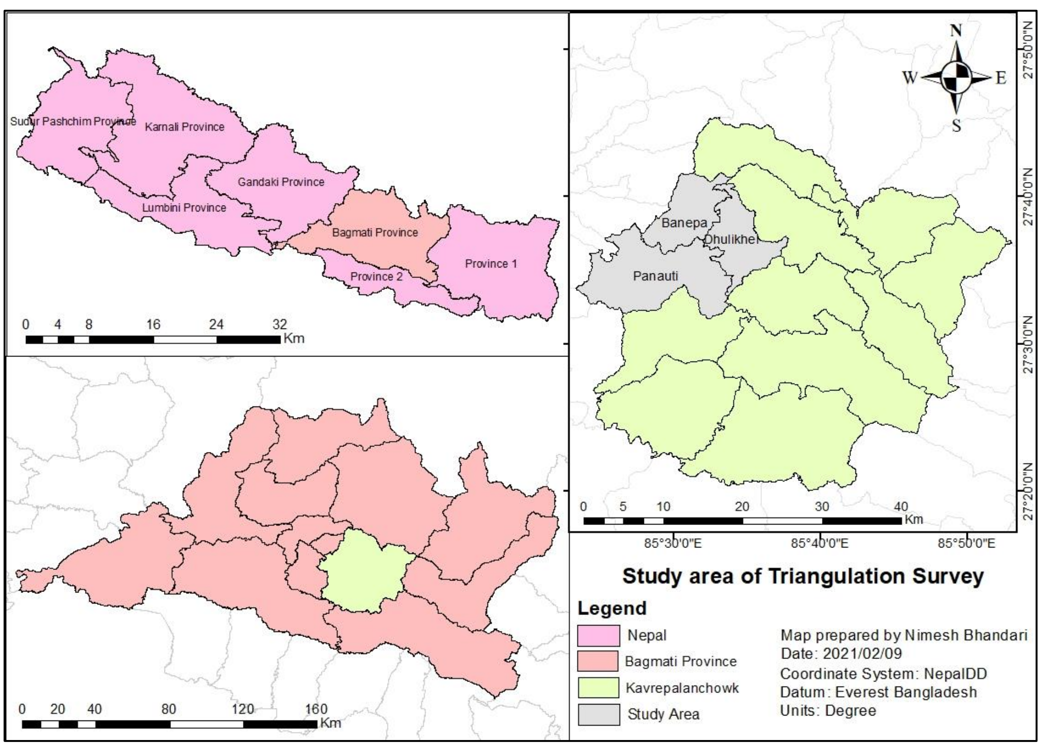


Figure 7. Map Showing Study Area

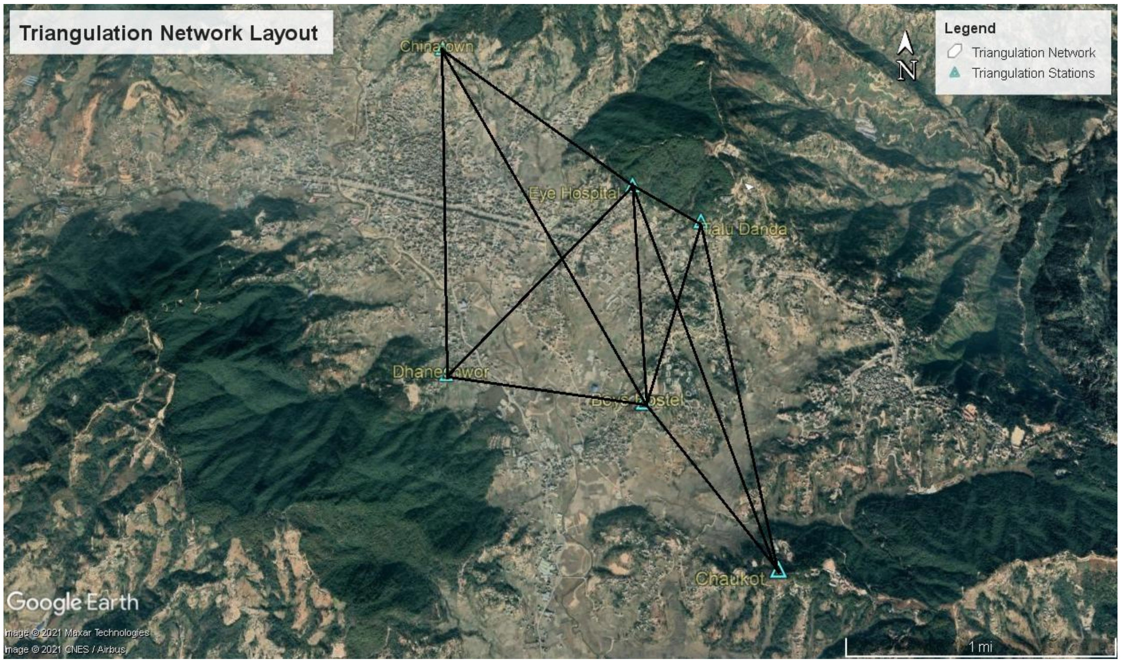


Figure 8. Triangulation Network Layout in Satellite Imagery View
(Retrieved from (*Google Earth*, n.d.))

2.3 Specifications

i. Order of Work: Fourth Order

ii. Measurement Unit: Degree, minutes, second for Angular measurement. Meters for Linear measurement.

iii. Angular Measurement: J2-2 Theodolite

iv. Linear Measurement (Base Line): Total Station

v. Number of sets: 3, setting RO as $(000°10'00'')$, $(060°10'00'')$ and $(120°10'00'')$

vi. Set to Set Discrepancy: 60^{cc}

vii. Allowable Face to Face Misclosure: 2^{c}

viii. Index Error Tolerance: 5^{c}

ix. Permissible Horizontal Misclosure: 30^{cc}

x. Recommended side length: 0.2 – 2 Km

x. Observation Time: Whole Day

2.4 Study Method and its Workflow

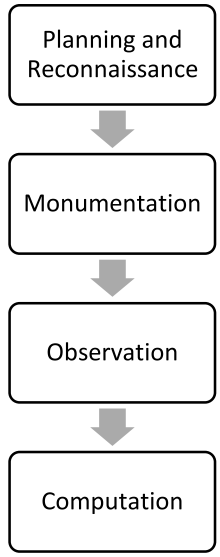


Figure 9. Methodological Flow Diagram

A) Planning and Reconnaissance

Plan about human resources, time availability, instrument resources, area to be surveyed were carried out under the supervision of faculties of the department. It was planned to establish six stations for carrying out triangulation survey.

Reconnaissance is fundamentally the first field operation and prominent stage. In this phase, field to be surveyed was fully inspected by one of the members of the project group, in each station. Intervisibility of the stations were checked by the use of binoculars witnessing from each station.

B) Monumentation

After ascertaining the intervisibility of the stations, monumentation was done. Wooden pegs were pegged around the permanent witness structure so that, they could be recovered at any time for further use. Stability of the monuments and their possible security threats were duly considered during monumentation.

D-Cards were also prepared for each of the stations. D-Card incorporated one general reference sketch and other scaled sketch of the station depicting nearby locations such as villages, accessible roads, etc. The measurement of station from at least three permanent witness structure were mentioned on the D-Card. Further information like grid sheet number, municipality, land owner, type of monument, direction and dimensions from nearby places, etc were illustrated on the D-Card for the benchmarks.

D) Observation

Six stations were established for carrying out triangulation initiating from 1001, running up to 1006. Two different types of observations were made in each of the stations as follows and also a base line measurement was taken in one of the lines.

i) Horizontal Angle

ii) Vertical Angle

iii) Length of a line (Base line measurement)

For the angular measurement, following procedures were followed:

- Setting up a tripod at a convenient height for observer.
- Setting up the theodolite followed by centering with optical plummet.
- Levelling up initially with circular bubble followed by levelling with plate level.
- Cross Hair Focusing.
- Image Focusing.
- Sighting towards the target.

i) Horizontal Angle:

In the fourth order triangulation survey, three sets of horizontal reading were taken from each station. For the first set, the RO was set to 0°10’00”. Similarly, for the second and third set of reading, the RO was set to 60°10’00” and 120°10’60” respectively. Reiteration method of observing horizontal angles was adopted in which the signals at triangulation stations are bisected successively and the angular values from RO to those stations are noted down. After observing the last visible station, the telescope was again brought back to sight RO and RO (horizontal angle) was closed.

Moreover, in each set, two face readings were taken; left face reading and right face reading. This was done by swinging the theodolite clockwise followed by transiting, to change the orientation of vertical circle in theodolite.

ii) Vertical Angle:

Zenithal angles were actually observed in the field using transit theodolite. Zenithal angle is preferred over vertical angle because it is difficult to record and compute negative values (less than $0°$ or maybe $359°$ or something like that) that may arise while reading depressions in vertical measurement. Also, vertical angle is the compliment of zenithal angle and hence we can easily derive vertical angle. Vertical angle was measured to determine the elevation of trigonometrical stations.

iii) Measurement of Base Line: Since, the accuracy of the entire triangulation system depends on that attained in the measurement of the base line, the measurement of base line forms the most important part of the triangulation operations. Length of the line joining the stations 1006-1001 was measured using EDM under the supervision of faculties of the department.

E) Computation

i) Arithmetic Check:

Arithmetic check was carried out in the field after observation.

For this, all the observed angles for each set were summed up and multiple of 360 was deducted from each if the sum exceeds more than $360°$. Again, the results from each set were summed up and deducted by the multiple of 360 if the sum exceeds more than $360°$ (Case 1).

The next procedure done was summing up the mean angles on RO for all sets and multiplying the sum by number of stations observed counting RO twice.

Also, the mean angles of all sets for each station were summed up and multiplied by the number of sets. The latter two outcomes were summed up again and the exceeding value from $360°$ was deducted with the multiple of 360 (Case 2).

The reductions are considered fine if the difference of the results from case 1 and case 2 lies within 2^{cc}.

A triangulation form showing angular observations and arithmetic check is shown below.

ii) Distribution of Horizontal Misclosure: The difference between the opening and closing readings to the RO was divided by the number of observations made from that station, counting RO just once (say, x). The correction was applied in cumulative manner, like, –x for first station, -2x for second station and –nx for nth station.

In the observed horizontal angles from station 1005 (Chaukot), the horizontal misclosure found was $000°00'3.67"$.

Table 1. Distribution of Horizontal Misclosure of Observed Angles at Station 1005

Station	Angular Observations	Corr^{n} applied	Corrected Values
KUBH	$000°00'00"$	$000°00'00"$	$000°00'00"$
Eye Hospital	$018°25'25"$	- $000°00'1.22"$	$018°25'23.78"$
Taludanda	$027°26'44.67"$	- $000°00'2.66"$	$027°26'42.01"$
KUBH	$000°00'3.67"$	- $000°00'3.67"$	$000°00'00"$

iii) Figural Adjustment of Triangulation Network:

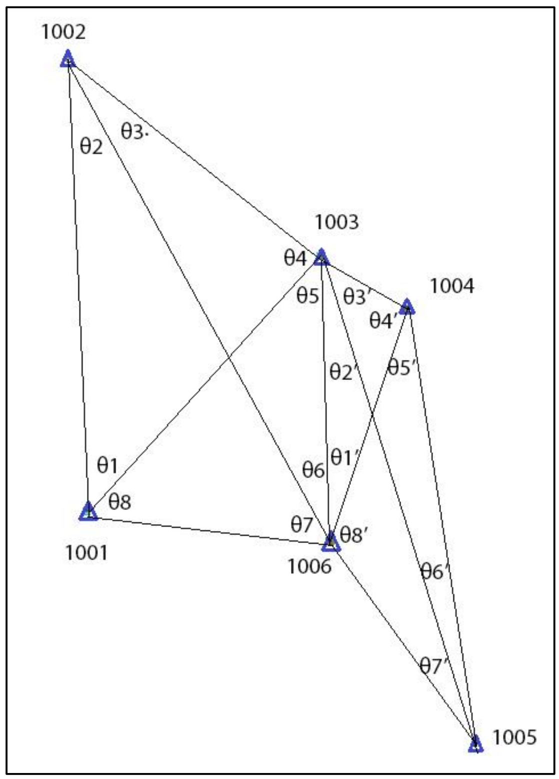


Figure 10. Triangulation Network Layout Demonstrating Position of Stations

Figural adjustment techniques were adopted for observed horizontal angles in braced quadrilaterals by following the subsequent mentioned ways:

- $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8$
- $\theta_1 + \theta_2 = \theta_5 + \theta_6$
- $\theta_3 + \theta_4 = \theta_7 + \theta_8$

Also, the side condition was confirmed using the relation;

$$\sum \log\sin \text{odd angles} = \sum \log\sin \text{even angles}$$

Table 2. Angular Adjustment for First Braced Quadrilateral formed by Stations 1001, 1002, 1003 and 1006

Ang.	Obs. values	1^{st}Corr^{n}	Value after 1^{st} corr^{n}	$\theta_1+\theta_2,\ \theta_3+\theta_4$ $\theta_5+\theta_6, \theta_7+\theta_8$	2^{nd} Corr^{n}
θ_1	45°16′8.11"	1′7.6"	45°17′15.71"	71°11′51.12"	+15.32”
θ_2	25°53′27.78"	1′7.6"	25°54′35.41"		+15.32”
θ_3	23°00′32.33"	1′7.6"	23°01′39.83"	108°41′39.45"	+2’59.4”
θ_4	85°38′52.02"	1′7.6"	85°39′59.62"		+2’59.4”
θ_5	44°50′42.01"	1′7.6"	44°51′49.61"	71°12′52.4"	-15.32”
θ_6	26°19′55.19"	1′7.6"	26°21′2.79"		-15.32”
θ_7	53°53′24.93"	1′7.6"	53°54′32.53"	108°53′37.03"	+2’59.4”
θ_8	54°57′56.9"	1′7.6"	54°59′04.5"		+2’59.4”
Sum	359°50′17"	+9′0.83"	360°00′00"		

Ang.	Values after 1^{st} & 2^{nd} cor^{n}	log sin odd ∠	log sin even ∠	Difference for 10” arc	Corr^{n}	Final Values
θ_1	45°17′31.03"	1.851687		0.000021	-1	45°17′30.03"
θ_2	25°54′50.73"		1.640504	0.000043	+2	25°54′52.73"
θ_3	23°04′39.22"	1.593260		0.000049	-2	23°04′37.22"
θ_4	85°42′59.02"		1.998785	0.000002	+0	85°42′59.02"
θ_5	44°51′34.29"	1.848418		0.000021	-1	44°51′33.29"
θ_6	26°20′47.47"		1.647186	0.000043	+2	26°20′49.47"
θ_7	53°51′33.14"	1.907180		0.000015	-1	53°51′32.14"
θ_8	54°56′5.1"		1.913018	0.000015	+1	54°56′06.1"
Sum	360°00′00"	7.200545	7.199493	0.000209		360°00′00"

In the last section, correction was applied with the intention that:

To change 0.000209, angle is changed to 10”.

To change 1, angle is changed to 10/0.000209

So, to change 0.000021 (in θ_1), angle is changed to 10”*0.000021/0.000209

$$= 10 * \frac{21}{209} = 1.00$$

Same technique was adopted for second quadrilateral.

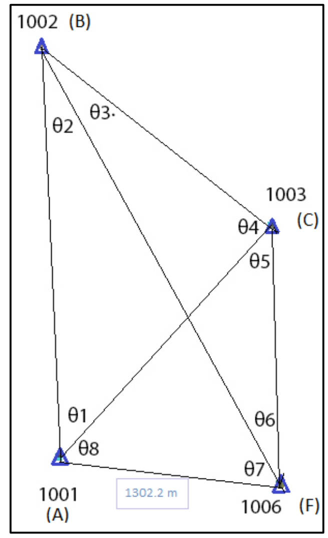
Table 3. Angular Adjustment for Second Braced Quadrilateral formed by Stations 1003, 1004, 1005, 1006

Ang.	Obs. values	1^{st}Corr^{n}	Value after 1^{st} corr^{n}	$\theta_1+\theta_2$, $\theta_3+\theta_4$ $\theta_5+\theta_6$, $\theta_7+\theta_8$	2^{nd} Corr^{n}
$\theta_1{}'$	$20°43'23.79"$	$3'43.36"$	$20°39'40.44"$	$37°02'2.42"$	+18.33”
$\theta_2{}'$	$16°26'5.34"$	$3'43.36"$	$16°22'21.98"$		+18.33”
$\theta_3{}'$	$43°08'13"$	$3'43.36"$	$43°04'29.65"$	$143°11'47.06"$	+7’13.07”
$\theta_4{}'$	$100°11'0.77"$	$3'43.36"$	$100°07'17.41"$		+7’13.07”
$\theta_5{}'$	$28°09'35.22"$	$3'43.36"$	$28°05'51.87"$	$37°03'15.74"$	-18.33”
$\theta_6{}'$	$09°01'7.23"$	$3'43.36"$	$08°57'23.87"$		-18.33”
$\theta_7{}'$	$18°25'33.78"$	$3'43.36"$	$18°21'50.42"$	$142°42'54.78"$	-7’13.07”
$\theta_8{}'$	$124°24'47.72"$	$3'43.36"$	$124°21'4.36"$		-7’13.07”
Sum	$360°29'46.88$	$-29'46.88"$	$360°00'00"$		

Ang.	Values after 1^{st} & 2^{nd} cor^{n}	log sin odd ∠	log sin even ∠	Difference for 10” arc	Corr^{n}	Final Values
$\theta_1{}'$	$20°39'58.77"$	1.547682		0.000056	-2	$20°39'58.75"$
$\theta_2{}'$	$16°22'40.31"$		1.450204	0.000072	2	$16°22'40.33"$
$\theta_3{}'$	$42°57'16.58"$	1.833414		0.000023	-1	$42°57'16.57"$
$\theta_4{}'$	$100°00'4.34"$		1.993350	-0.000004	0	$100°00'4.34"$
$\theta_5{}'$	$28°05'33.54"$	1.672928		0.000039	-1	$28°05'33.53"$
$\theta_6{}'$	$08°57'5.54"$		1.192007	0.000134	4	$08°57'5.58"$
$\theta_7{}'$	$18°29'03.49"$	1.501121		0.000063	-2	$18°29'03.47"$
$\theta_8{}'$	$124°28'17.43"$		1.916142	-0.000014	0	$124°28'17.43"$
Sum	$360°00'00"$	6.555145	6.551703	0.000369		$360°00'00"$

iv) <u>Best Route Finding:</u>

The next step was to compute the distance of the legs from observed angles. For this, it necessitates finding the best route, that relies on strength of triangulation layout. We were provided with the distance of line from station 1006-1001 as 1302.2 m. Also, we know, for braced quadrilateral, R = 0.6



Computation of strength of first quadrilateral ABCF (1001, 1002, 1003, 1006) for all the routes by which the length BC can be computed from known length of line AF proceeds as follows:

Route-1, using Δ^s AFC and ABC with common side AC.

For ΔAFC, the distance angles of AF and AC are $\theta_5 = 44°51'33.29"$ and $\theta_6 + \theta_7 = 26°20'49.47 + 53°51'32.14 = 80°12'21.61"$, respectively.

$$\delta_A = 2, \delta_B = 0$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 4$$

For ΔABC, the distance angles of AC and BC are $\theta_2 + \theta_3 = 25°54'52.73" + 23°04'37.22" = 48°59'29.95"$ and $\theta_1 = 45°17'30.03"$ respectively.

$$\delta_A = 2, \delta_B = 2$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 12$$

$R_1 = 0.6 * (4 + 12) = 9.6$

Route-2, using Δ^s AFB and ACB with common side AB.

For ΔAFB, the distance angles of AF and AB are $\theta_2 = 25°54'52.73"$ and $\theta_7 = 53°51'32.14"$, respectively.

$$\delta_A = 4, \delta_B = 2$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 28$$

For ΔACB, the distance angles of AB and BC are $\theta_4 = 85°42'59.02"$ and $\theta_1 = 45°17'30.03"$ respectively.

$$\delta_A = 0, \delta_B = 2$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 4$$

$R_2 = 0.6 * (28 + 4) = 67.2$

Route-3, using Δ^s AFC and FCB with common side FC.

For ΔAFC, the distance angles of AF and AB are $\theta_5 = 44°51'33.29"$ and $\theta_8 = 54°56'06.1"$, respectively.

$$\delta_A = 2, \delta_B = 1$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 7$$

For ΔFCB, the distance angles of AB and BC are $\theta_3 = 23°04'37.22"$ and $\theta_6 = 26°20'49.47"$ respectively.

$$\delta_A = 5, \delta_B = 4$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 28$$

$R_3 = 0.6 * (7 + 28) = 21$

Route-4, using Δ^s AFB and FCB with common side FB.

For ΔAFB, the distance angles of AF and FB are $\theta_2 = 25°54'52.73"$ and $\theta_1 + \theta_8 = 45°17'30.03" + 54°56'06.1" = 100°13'36.13"$, respectively.

$$\delta_A = 4, \delta_B = 0$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 16$$

For ΔFCB, the distance angles of AC and BC are $\theta_4 + \theta_5 = 85°42'59.02 + 44°51'33.29 = 130°34'31"$ and $\theta_6 = 26°20'49.47"$ respectively.

$$\delta_A = -2, \delta_B = 4$$

$$\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 12$$

$R_4 = 0.6 * (16 + 12) = 16.8$

→ Route 1 has the lowest value of R, $R_1 = 9.6$, so the best route to compute the length of BC is Route 1.

Now, distance computation using sine law proceeds as follows:

Given that, Distance of AF = 1302.2 m.

For distance of AC, using sine law in ΔAFC,

$$\frac{AC}{\sin(\theta_6 + \theta_7)} = \frac{AF}{\sin(\theta_5)} \rightarrow AC = 1819.22 \text{ m}$$

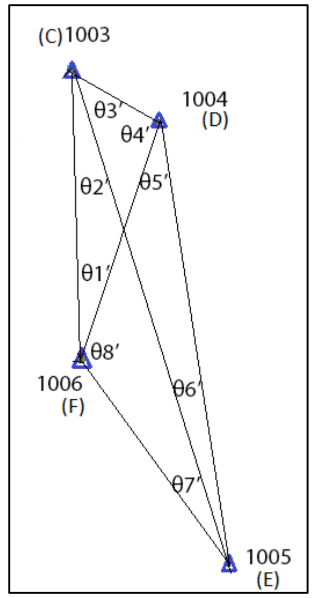
For distance of BC, using sine law in ΔACB,

$$\frac{BC}{\sin(\theta_1)} = \frac{AC}{\sin(\theta_2 + \theta_3)} \rightarrow BC = 1713.35 \text{ m}$$

For distance of CF, using sine law in ΔAFC,

$$\frac{\mathrm{CF}}{\sin(\theta_8)} = \frac{\mathrm{AF}}{\sin(\theta_5)} \rightarrow \mathrm{CF} = 1511.06\ \mathrm{m}$$

In a similar manner, strength of second quadrilateral CDEF (1003, 1004, 1005, 1006) was determined for all the routes by which the length DE can be computed from known length of line CF. It was found that the route that uses the Δs FDE and FCD with common side FD was found to be the best route.



From the known distance of line CF = 1511.06 m, distance of succeeding lines was determined using sine law.

For distance of FD, using sine law in ΔFCD,

$$\frac{\mathrm{FD}}{\sin(\theta_2{}' + \theta_3{}')} = \frac{\mathrm{CF}}{\sin(\theta_4{}')} \rightarrow \mathrm{FD} = 1315.21\ \mathrm{m}$$

For distance of DE, using sine law in ΔFDE,

$$\frac{\mathrm{DE}}{\sin(\theta_8{}')} = \frac{\mathrm{FD}}{\sin(\theta_6' + \theta_7{}')} \rightarrow \mathrm{DE} = 2353.24\ \mathrm{m}$$

v) <u>Bearing Calculation:</u>

We're provided the bearing of the line 1001-1006, i.e. 106°

With the help of observed angles, bearing of the remaining lines were calculated.

Bearing of line 1001-1003 = 51°03′53.9"

Bearing of line 1003-1002 = 316°46′52.92"

Bearing of line 1006-1004 = 26°52′20.36"

Bearing of line 1004-1006 = 178°46′46.83"

vi) <u>Calculation of Elevations:</u>

We had observed the zenithal angle of each stations and horizontal distance between two stations, we followed Trigonometric Levelling Procedure to determine RL of stations as shown in figure below.

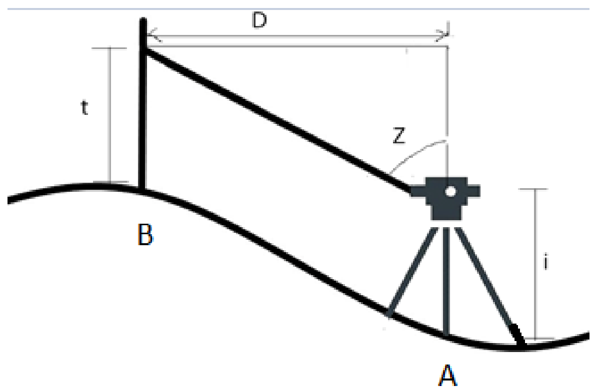


Figure 11. Trigonometric Levelling to Determine RL of a Station

As shown in figure above, the elevation of station B can be computed by using the relation;

$$\text{Elevation of B} = \text{Elevation of A} \pm (\mathrm{D}\cot \mathrm{Z} + \mathrm{i} - \mathrm{t})$$

Where, D = Distance between two stations

i = Instrument Height

t = Height of Target

This technique was implemented for determining RL of Triangulation Stations.

2.4.1 Data Used

We've used the self-measured data collected in the field. However, the length of a line connecting the stations 1006 and 1001, coordinates of the station 1001 and bearing of that line 1001-1006 were provided by the faculties of the department.

2.4.2 Instruments Used

- Theodolite: A theodolite is a precision instrument for measuring angles in the horizontal and vertical planes. Horizontal between the stations and Zenithal angles were observed in the field using J2-2 Theodolite.
- Measuring Tape: A measuring tape is a flexible ruler used to measure size or distance and is a ubiquitous measuring tool. We used measuring tape to measure Instrument Height in the field.

2.5 Measures Adopted for Controlling Error during Triangulation

i. Proper centering of the theodolite was done using optical plummet to eliminate the possible errors like deviated reading from actual horizontal reading caused by improper centering.

ii. Proper levelling of the instrument was done using circular bubble and plate bubble attached to the theodolite.

iii. Two face reading (face left and face right reading) was taken for each observed stations to eliminate the trouble that might be yield by collimation error. This also helps in diminishing the index error.

iv. Three sets of reading were taken setting RO as ($000°10'00"$), ($060°10'00"$) and ($120°10'00"$) and mean of the observed angles of each station were taken to minimize the possible errors of unequal graduations.

3. RESULTS

As per out objective, coordinates of the stations were furnished viz.

Coordinates of 1001: (354257.84, 3055865.18)

Bearing of line 1001-1006 (β): $106°$

Distance of line 1001-1006: 1302.2 m

ΔE of line 1001-1006 : $dsin\beta$ = 1302.2*sin($106°$) = 1251.75

ΔN of line 1001-1006 : $dcos\beta$ = 1302.2*cos($106°$) = -358.93

$So,\ E_6 = E_1 + \Delta E = 354257.84 + 1251.75 = 355509.59$ m

$N_6 = N_1 + \Delta N = 3055865.18 - 358.93 = 3055506.25$ m

Similarly, cultural features (x, y, z) of other stations were derived. S

Table 4. Final Coordinates of Stations determined by Triangulation Survey

Station	Easting	Northing	Elevation
1001	354257.84	3055865.18	1507.70
1002	354499.67	3058257.05	1595.90
1003	355672.94	3057008.25	1565.91
1004	356104.07	3056679.44	1583.78
1005	356154.19	3054326.73	1579.09
1006	355509.59	3055506.25	1597.96

4. LIMITATIONS

Hence, triangulation survey was carried out. Besides all the precautions adopted, there are some limitations of our project which are pinpointed below:

i. As the specifications for fourth order survey were implemented, it doesn't provide the accuracy of greater than fourth order.

ii. Due to site conditions, few of the stations were so established that the concept of well-conditioned triangle was not satisfactorily implemented.

iii. There were not sufficient permanent structures for witness near some of the stations because of which D-Card preparation and relocating them became difficult.

iv. Line of the sights passing from major city area might have hampered the data taken in some way.

5. CONCLUSION AND RECOMMENDATIONS

Fourth order triangulation survey was done for determining the cultural features (x, y, z) of the established stations. Multiple set and multiple face observations were adopted for diminishing probable errors and ensuring the precision of the observed data. From the bearing of a line and the coordinates of a station, coordinates of rest of the station were determined. As a recommendation, it is suggested that, it would have better, if the project was carried out on sites away from urban premises and the stations to be observed were located on north facing hills.

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