

Decentralized ML: Experiments on Topology and Privacy

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Abstract—Decentralized machine learning is an important type of machine learning used to address problems such as data locality, ownership, privacy and high communication costs. We contribute an empirical evaluation of the impact of topology and differential privacy on model convergence and error in a decentralized learning system.

I. INTRODUCTION

Centralized forms of machine learning have been thoroughly explored and are the current state of the art but have significant problems dealing with issues such as data locality, ownership, privacy and high communication costs. Decentralized machine learning addresses these problems by transferring control from a primary server to each individual edge device. This allows data to reside at the source, protecting privacy, and lowering communication costs by shifting the medium from the data itself to a compressed representation. While decentralized networks do enhance privacy there are still problems since sensitive data can be learned from something as simple as a shared gradient. This motivates our exploration of adding differential privacy to the communication to ensure maximum privacy.

A. Centralized, Federated & Decentralized machine learning

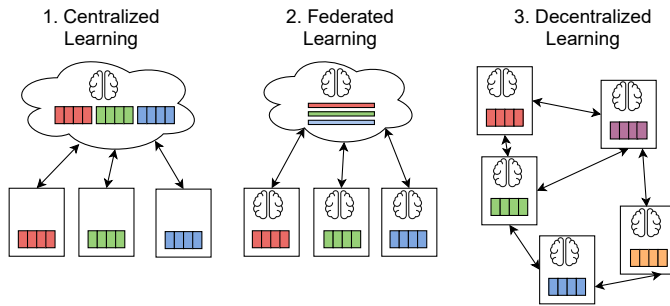


Fig. 1. Three types of organizational structures described graphically, where device units are depicted with rectangles, central units are depicted as clouds and arrows indicate communication between the units. Each brain represents a machine learning model that is trained by the corresponding unit. Each small colored box represents data residing on a device, and the colored rectangles represent a transformed representation of the data.

In a traditional centralized machine learning setup, as visualized in Figure 1.1, data is moved from remote devices to a central system where models can be classically trained using stochastic gradient descent (SGD) which for T iterations has runtime $O(1/\sqrt{T})$ if the target function is convex [1]. In federated learning, shown in Figure 1.2, each device collaboratively trains a global machine learning model and is organized by a central server. Data is not shared, and the

devices send updates to a server which aggregates these and in turn updates the shared models across the devices. The most well known optimization algorithm is federated averaging (parallel SGD/local SGD) with convergence $O(\frac{HM}{T} + \frac{\sigma}{\sqrt{TKM}})^1$ [2], [3]. Decentralized learning, depicted in Figure 1.3, does away with a central server, and replaces it with peer to peer communication between devices. There is no longer a global model such as in federated learning, as each device has its own unique model, however the process can be designed so that all the models converge. The main optimization algorithms for decentralized learning are gossip type (consensus) algorithms, which are designed for computation and information exchange in an arbitrarily connected network of nodes [4].

B. Topology

In decentralized learning, it has been shown that the convergence rate typically depends on the network topology. The network topology can be modeled as a graph $G = (V, E)$ with $\forall v \in V$ representing a device and $\forall \{i, j\} \in E$ representing a communication channel between two devices i and j . In particular the spectral gap $\delta \geq 0$ of the normalized adjacency matrix (W) of the communication topology graph is important in convergence.

Definition 1 (Spectral Gap) Assume $W \in [0, 1]^{n \times n}$ is a symmetric doubly stochastic matrix with eigenvalues $1 = |\lambda_1(W)| > |\lambda_2(W)| \geq \dots \geq |\lambda_n(W)|$ the spectral gap is

$$\delta = 1 - |\lambda_2(W)| \in (0, 1] \quad (1)$$

The reason for this is because of the spectral gaps relation to connectivity of the graph. For example, a fully connected graph has a $\delta = O(1)$ where a ring graph with node degree 2 has $\delta = O(1/n^2)$ on n nodes [5].

TABLE I
SPECTRAL GAPS FOR THE NORMALIZED, DOUBLY STOCHASTIC ADJACENCY MATRIX FOR OUR GRAPHS WITH 4, 16, AND 32 NODES.

	4	16	32
FullyConnectedGraph	.666	.933	.968
CycleGraph	1.0	.076	.019
RingOfCliques	1.0	.104	.028
Torus2D	1.0	.195	.051

C. Differential Privacy

Differential privacy (DP) is a method of publishing certain pieces of data in a privacy-preserving fashion. The fact that in DP the practitioners can quantitatively control the amount of

¹M devices, loss functions are H-smooth, stochastic gradients with variance at most σ^2 , T iterations, K local updates

privacy that will be leaked differentiates it from the concept of anonymization or the generation of synthetic data. Let us represent the DP (random) algorithm as a function \mathcal{A} and our original dataset as D , thus $\mathcal{A}(D)$ is the result of applying the DP mechanism to the original data. The algorithm \mathcal{A} is said to provide (ϵ, δ) -differential privacy if, for all datasets D_1 and D_2 that differ on a single element (i.e., the data of one person), and all subsets S of $\text{image}(\mathcal{A})$:

$$P[\mathcal{A}(D_1) \in S] \leq e^\epsilon P[\mathcal{A}(D_2) \in S] + \delta, \quad (2)$$

where the probability is taken with respect to the randomness used by mechanism \mathcal{A} . A common dilemma in privacy-preserving technologies is that the lower we set ϵ (the privacy budget), the "more random" \mathcal{A} has to be, resulting in data that possesses less utility, but at the same time leaking less information.

Coming back to the project at hand, training machine learning models often requires large, representative datasets, which may be crowdsourced and contain sensitive information. The models should not expose private information in these datasets, which is why we utilized the `opacus` library within python for training the models in a privacy-preserving fashion. `Opacus` alters the standard learning procedure by adding three extra steps in the backpropagation step. Instead of simply calculating the gradients and using them to update the corresponding parameters, `opacus` is based on DP-SGD algorithm, which we wrote out in Appendix B [6].

D. Related Work

We use the D-PSGD algorithm (note this is different from DP-SGD) where they provide a theoretical analysis to show that decentralized can outperform centralized algorithms for distributed stochastic gradient descent [7]. Specifically, in our Algorithm 1 we use the idea put forth in Lian et. al. Algorithm 1 to compute a neighborhood weighted average after the local update steps. Our algorithm runs in a loop, but the updates occur so that it mimics a parallel decentralized algorithm.

In Neglia et. al. the authors look at the contradiction between theory which states that worker communication topology should have a strong impact on the number of epochs needed to converge and experiments which show the opposite conclusion [8]. The authors note that in [9] and [10] they calculate the number of iterations T to approximate the minimum objective function by the desired error ϵ as

$$T \in O\left(\frac{1}{\epsilon^2 \delta}\right) \quad (3)$$

and conclude that a more connected network topology leads to faster convergence. However, their experiments show that when a dataset is statistically similar, the topology has little influence on the number of epochs needed to converge.

II. EXPERIMENTS

A. Data

To test the performance of the different architectures, we consider the task of predicting the class of a digit in the

MNIST dataset [11]. The training and test set consist of 60000 and 10000 samples, respectively, and we normalized the input data. The data is partitioned between the nodes either *randomly* or *uniformly*, i.e. each node gets a random amount of data or exactly the same amount. Each node receives a unique partition of the data. Furthermore, we have an *iid* or *non-iid* setting, where in the former, the classes are distributed randomly between each node, but in the latter, every node only gets data from certain classes.

B. Implementation

The fundamental building block of our implementation is the `Node` class. Each `Node` represents one of the agents within the decentralized network. Using it, we are able to define the data as well as all learning related parameters individually for each node. The artificial neural networks within each node is built with `PyTorch`. It is a convolutional network consisting of 2 convolutional and 2 fully connected layers, where we apply max-pooling and batch normalization after each convolutional layer. For each layer we use the `SeLU` activation function. The last fully connected layer is equipped with 10 activations (i.e. one per class) and the negative log-likelihood loss. Each `Node` possesses methods for training and testing the network and for sharing and receiving weights, such that all the weights of each individual's `PyTorch` model can be transmitted from one to the other.

For implementing the differential privacy mechanism in our nodes, we employed the `opacus` library, which is written as an extension on top of the standard `PyTorch` library, allowing us to define the previously discussed ϵ and δ parameter.

One step higher in the hierarchy is the `DecentralizedNetwork` class, which unifies all nodes in a complete decentralized network. The connectivity matrix between the nodes can be specified via the `graph_type` argument; all the graph implemented topologies are based on the `networkx` python library [12]. The way in which weights are shared between the `Nodes` is based on a synchronous approach, i.e., after each batch of iterations all nodes perform one optimization step, share their weights with all the neighbors and take the arithmetic mean over their own and the received weights.

C. Experimental setup

For our experiments we decided to compare four different topologies: the fully connected, ring, circle of cliques and torus graph, where we let the number of nodes vary between [4, 16, 32]. We ran the tests for all data combinations of [iid, non-iid] and [uniform, random] and furthermore once in a non-private and once in a private setting with parameters $\epsilon = 1.1$ and $\delta = 1e - 6$. In regard to the learning rate, for each setup we tried $\gamma \in [0.0001, 0.001, 0.01, 0.1, 0.2]$ and settled for the one which led to the best convergence, respectively.

In every epoch, each sample is used for training exactly once by a node. In the *iid uniform* setting with [4, 16, 32] nodes, each node gets [15000, 3750, 1875] samples, respectively, in all other settings the nodes differ in their available amount of

Algorithm 1: Decentralized Stochastic Gradient Descent based on (D-PSGD) [7]

```

initial weights  $\omega_{0,n}$ , step size  $\gamma$ , weight matrix  $W$ ,
data matrix  $X$ 
for  $e \in [Epochs]$  do
  for  $b \in [Batches]$  do
    for  $n \in [Nodes]$  do
      Compute a local stochastic gradient on
      local data  $X_{e_b,n} \rightarrow \nabla F_{e_b,n}(\omega_{e_b,n}; X_{e_b,n})$ 
      Update the local optimization weights
       $\omega_{e_b+\frac{1}{2},n} \leftarrow \omega_{e_b,n} - \gamma \nabla F_{e_b,n}(\omega_{e_b,n}; X_{e_b,n})$ 
    end
    Communicate weights between neighboring
    nodes
    Compute the neighborhood weighted average
    and update weights for every node
     $\omega_{e_b+1,n} = \sum_{j=1}^{|nodes|} W_{n,j} \omega_{e_b,j}$ 
  end
end

```

samples. When a node has fewer samples than its neighbors and it has used all its batches in an epoch, it stops sharing its weights but still updates its model with the weight that the other nodes share with it, until the epoch is completed.

Our results are summarized in Table II, III and IV. For each setup we mention the learning rate γ , and for each graph we list three convergence statistics. We say that a configuration converged in x epochs if the difference of the training accuracy in epoch i to epoch $i + 1$ does not change more than $\epsilon = 0.0001$. For this epoch, we state the training loss and test accuracy. If a model does not meet this condition in the available 50 epochs, we denote it with ' > 50 '.

III. DISCUSSION

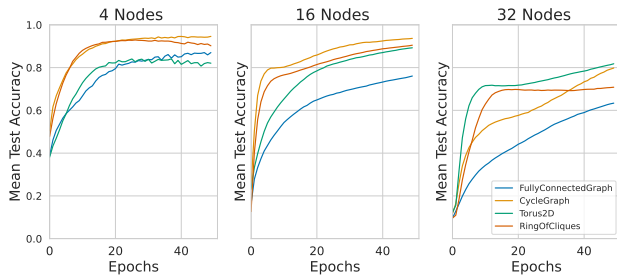


Fig. 2. Convergence plots from *non-iid uniform* and no differential privacy setup.

In our *iid uniform* setup, we expected to see similar results as in [13], where the authors noticed that convergence of average accuracy tends to happen faster for sparser networks. Our results corroborate these results, and we see that no matter the topology the average node converges to a similar accuracy in the end, see Figure 4 in Appendix C. Jiang et. al. then state that sparse topologies may be faster for convergence, but

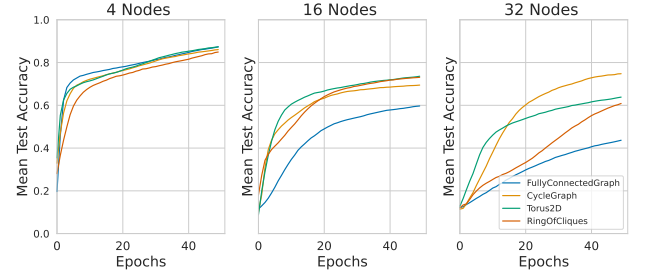


Fig. 3. Convergence plots from *non-iid uniform* and differential privacy setup.

that dense topologies are more stable and therefore crucial for a decentralized learning paradigm. By introducing two new paradigms, a *non-iid* data setting and a privacy-conscious setting, we interrogate this claim. In the *non-iid* data setting, we give each node only 3 out of the 10 classes from MNIST and expect sparser topologies to struggle with learning, while denser topologies that are averaging more weights should be able to incorporate information of the other classes faster. We expected the cycle graphs with node degree 2 to learn the worst, as amongst 3 nodes they only cover at most 9 out of the 10 class labels. As can be seen in Figure 2 in the *non-iid* setting, the test curve of the fully connected graph with 16 and 32 nodes lags behind the other sparser graphs. The cycle graph does well in general, even being the top performing topology in the 16 node scenario. We postulate that potentially our *non-iid* scenario was not difficult enough, which means that even in *non-iid* scenarios, sparsity may be desired if the data is shuffled to some extent. For further experiments, there is room to make the *non-iid* setting much more difficult by either restricting the number of classes or trying a more difficult task such as CIFAR-10 [14].

In the privacy setting, our expectation was that generally all models would converge slower and not reach the same level of test accuracy because of the introduction of noise. This is exactly what happened and can clearly be seen in the difference between Figure 2 and Figure 3. Comparing the final accuracies from Table II, the introduction of noise seems to hinder learning more in the more difficult setting of *non-iid random* and when increasing the number of nodes. Regarding further experiments, it would be worth investigating the performance of the network over multiple values of the privacy budget ϵ , as seen in [15].

IV. CONCLUSION

To conclude, we built a framework that allows to simulate different scenarios in a decentralized machine learning setting. The main factors we adjusted were the type of topology, the number of nodes, the independence and identically distributed assumption of the data, and adding differential privacy. Our experiments show that sparse graphs, as measured by the spectral gap of the topology, perform unexpectedly well even in difficult settings, while more densely connected graphs learn more slowly as the number of nodes increases. This contradicts previous theory but falls in line with other experimental results.

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APPENDIX A CODE

To see our implementation and the code that produced our tables and plots, visit our *GitHub Repository*.

APPENDIX B ALGORITHM

We state the DP-SGD algorithm on which the *opacus* library is based on.

Algorithm 2: DP-SGD

Input: Examples $\{x_1, \dots, x_N\}$, loss function $\mathcal{L}(\theta) \frac{1}{N} \sum_i \mathcal{L}(\theta, x_i)$. Parameters: learning rate η_t , noise scale σ , group size L , gradient norm bound C .

Initialize θ_0 randomly;

for $t \in [T]$ **do**

 Take a random sample L_t with sampling probability $\frac{L}{N}$;

Compute gradient: For each $i \in L_t$, compute $g_t(x_i) \leftarrow \nabla_{\theta_t} \mathcal{L}(\theta_t, x_i)$;

Clip gradient: $\tilde{g}_t(x_i) \leftarrow \frac{g_t(x_i)}{\max(1, \frac{\|g_t(x_i)\|_2}{C})}$;

Add noise: $\tilde{g}_t \leftarrow \frac{1}{L} (\sum_i \tilde{g}_t(x_i)) + \mathcal{N}(0, \sigma^2 C^2 I)$;

Descent: $\theta_{t+1} \leftarrow \theta_t - \eta_t \tilde{g}_t$

end

Output: θ_T and compute the overall privacy cost (ϵ, δ) using a privacy accounting method

APPENDIX C FIGURES

In our report we present the convergence plots for the *non-iid uniform* setup. For completeness, we will also list the plots for the *iid uniform*, *iid random* and *non-iid random* setup.

A. iid uniform setup

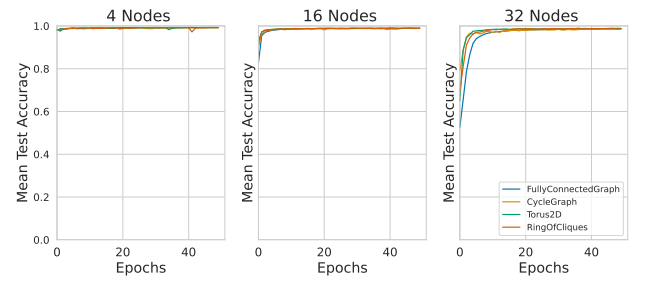


Fig. 4. Convergence plots from the *iid uniform* and no differential privacy setup.

For the *iid uniform* setup, we can see in Figure 4 that for all graphs we achieve a perfect mean test accuracy over all devices after only a couple of epochs. Only for the graphs with 32 nodes, convergence takes slightly longer. If adding differential privacy as depicted in Figure 5, we are not able to achieve a perfect test accuracy, but for 4 and 16 nodes we still manage to

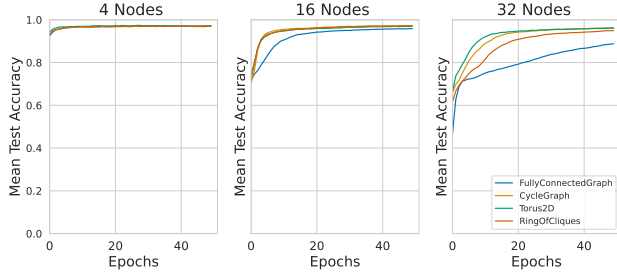


Fig. 5. Convergence plots from the *iid uniform* and differential privacy setup.

converge. For 32 nodes on the other hand convergence happens slower, and especially the fully connected graph struggles to achieve a test accuracy as good as the others.

B. iid random setup

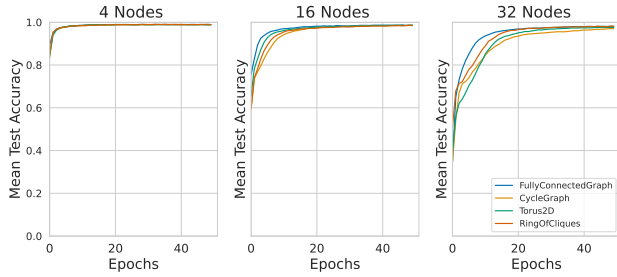


Fig. 6. Convergence plots from the *iid random* and no differential privacy setup.

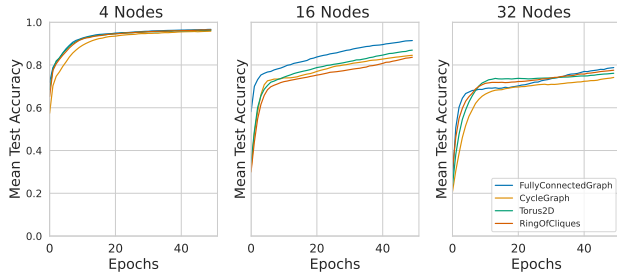


Fig. 7. Convergence plots from the *iid random* and differential privacy setup.

In the *iid random* setup, where each device only gets data from a certain class, we see a similar but slightly slower convergence behavior compared to the *iid uniform* setup. This time the fully connected graph performs as good or even better than the other topologies. In the differential private configuration with 32 nodes, the networks start struggling to achieve a top test accuracy and stagnate before 80%.

C. non-iid random setup

The *non-iid random* setup is the hardest one for our decentralized models to learn. Not only does each device only get data from a certain class, but also a random amount of data assigned. If the node ends up having few samples for a label,

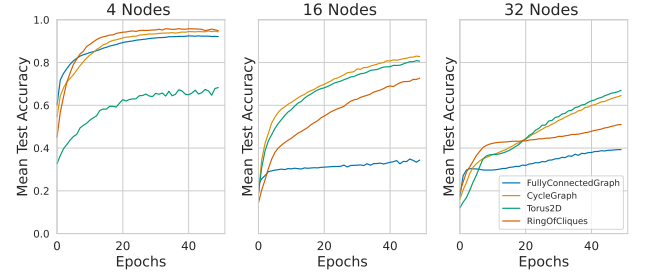


Fig. 8. Convergence plots from the *non-iid random* and no differential privacy setup.

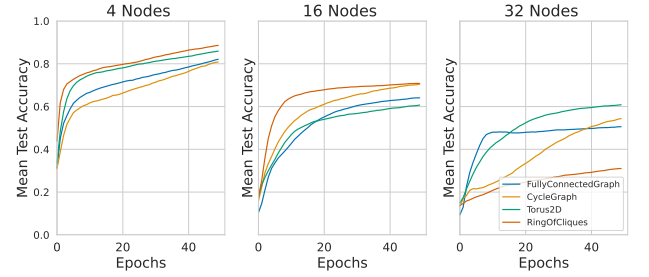


Fig. 9. Convergence plots from the *non-iid random* and differential privacy setup.

this label will be underrepresented and the network will have a hard time achieving a good test performance. This can be one explanation for why in Figure 8 and 9 certain graphs exhibit a poor test performance. Also at the end of the available 50 epochs, some models still seem to be improving and would need more training epochs.

APPENDIX D

TABLES

Corresponding to the setups and figures from Appendix C, we show the convergence behavior of our models in Table II, III and IV, where we list all the results for the models with 4, 16, and 32 nodes, respectively.

TABLE II
4 NODESTABLE III
16 NODES

Setup	γ	Graphs	Epoch	Train Loss	Test Acc.	Setup	γ	Graphs	Epoch	Train Loss	Test Acc.
IID Uniform	0.01	FullyConnectedGraph	23	0.0002	0.9864	IID Uniform	0.01	FullyConnectedGraph	8	0.0628	0.9840
		CycleGraph	26	0.0013	0.9848			CycleGraph	15	0.0156	0.9842
		Torus2D	7	0.0143	0.9845			Torus2D	20	0.0097	0.9842
		RingOfCliques	7	0.0148	0.9844			RingOfCliques	9	0.0307	0.9904
IID Uniform Private	0.01	FullyConnectedGraph	15	0.1520	0.9509	IID Uniform Private	0.01	FullyConnectedGraph	26	0.1849	0.9339
		CycleGraph	19	0.1531	0.9579			CycleGraph	15	0.1413	0.9532
		Torus2D	9	0.1403	0.9572			Torus2D	41	0.1315	0.9744
		RingOfCliques	30	0.1557	0.9556			RingOfCliques	14	0.1571	0.9583
IID Random	0.0001	FullyConnectedGraph	13	0.0595	0.9779	IID Random	0.0001	FullyConnectedGraph	24	0.0795	0.9795
		CycleGraph	38	0.0621	0.9837			CycleGraph	30	0.0942	0.9775
		Torus2D	15	0.0617	0.9810			Torus2D	10	0.3016	0.9421
		RingOfCliques	8	0.0830	0.9712			RingOfCliques	18	0.1559	0.9777
IID Random Private	0.0001	FullyConnectedGraph	>50	0.1457	0.9537	IID Random Private	0.0001	FullyConnectedGraph	>50	0.2699	0.8851
		CycleGraph	36	0.1803	0.9361			CycleGraph	24	1.1281	0.7429
		Torus2D	>50	0.1453	0.9469			Torus2D	>50	0.4870	0.8490
		RingOfCliques	13	0.2455	0.9001			RingOfCliques	14	1.5346	0.7073
Non-IID Uniform	0.0001	FullyConnectedGraph	29	0.0901	0.7229	Non-IID Uniform	0.0001	FullyConnectedGraph	34	1.9198	0.6763
		CycleGraph	27	0.2853	0.9257			CycleGraph	>50	0.6539	0.9192
		Torus2D	>50	0.1283	0.9327			Torus2D	47	1.0500	0.8441
		RingOfCliques	>50	0.1400	0.9443			RingOfCliques	18	1.4788	0.7902
Non-IID Uniform Private	0.0001	FullyConnectedGraph	>50	0.4352	0.8417	Non-IID Uniform Private	0.0001	FullyConnectedGraph	>50	2.0603	0.5462
		CycleGraph	>50	0.8369	0.8376			CycleGraph	>50	1.8268	0.6672
		Torus2D	>50	0.4450	0.8413			Torus2D	44	1.9188	0.7086
		RingOfCliques	>50	0.7288	0.8308			RingOfCliques	45	1.6598	0.6988
Non-IID Random	0.0001	FullyConnectedGraph	>50	0.0730	0.9225	Non-IID Random	0.0001	FullyConnectedGraph	40	1.4010	0.3601
		CycleGraph	20	0.1564	0.8665			CycleGraph	>50	0.3703	0.8278
		Torus2D	16	0.1390	0.5032			Torus2D	27	1.0256	0.7018
		RingOfCliques	32	0.0622	0.9197			RingOfCliques	3	2.2882	0.3376
Non-IID Random Private	0.0001	FullyConnectedGraph	16	1.4456	0.6528	Non-IID Random Private	0.0001	FullyConnectedGraph	>50	2.0750	0.6549
		CycleGraph	>50	0.6959	0.7704			CycleGraph	>50	1.5378	0.7234
		Torus2D	9	1.2229	0.6922			Torus2D	26	1.4966	0.5668
		RingOfCliques	>50	0.3553	0.8464			RingOfCliques	27	1.9804	0.6530

TABLE IV
32 NODES

Setup	γ	Graphs	Epoch	Train Loss	Test Acc.
IID Uniform	0.01	FullyConnectedGraph	13	0.1109	0.9855
		CycleGraph	45	0.0024	0.9896
		Torus2D	24	0.0123	0.9914
		RingOfCliques	17	0.0316	1.0000
IID Uniform Private	0.01	FullyConnectedGraph	>50	0.3892	0.9062
		CycleGraph	42	0.1660	0.9587
		Torus2D	37	0.1718	0.9561
		RingOfCliques	>50	0.2026	0.9535
IID Random	0.0001	FullyConnectedGraph	20	0.1668	0.9699
		CycleGraph	28	0.3504	0.9449
		Torus2D	26	0.2075	0.9725
		RingOfCliques	21	0.1560	0.9810
IID Random Private	0.0001	FullyConnectedGraph	10	2.0063	0.6856
		CycleGraph	43	1.5771	0.7426
		Torus2D	>50	1.1874	0.7786
		RingOfCliques	29	1.3188	0.7374
Non-IID Uniform	0.0001	FullyConnectedGraph	>50	2.2148	0.6272
		CycleGraph	>50	1.3124	0.8504
		Torus2D	18	2.0856	0.7269
		RingOfCliques	18	2.1649	0.6778
Non-IID Uniform Private	0.0001	FullyConnectedGraph	>50	2.2620	0.4382
		CycleGraph	>50	2.0726	0.7578
		Torus2D	41	2.1279	0.5871
		RingOfCliques	8	2.2980	0.2347
Non-IID Random	0.0001	FullyConnectedGraph	4	2.3635	0.3233
		CycleGraph	>50	1.2416	0.8311
		Torus2D	47	1.1472	0.7857
		RingOfCliques	>50	1.7693	0.3880
Non-IID Random Private	0.0001	FullyConnectedGraph	>50	1.7446	0.5298
		CycleGraph	6	2.2925	0.2671
		Torus2D	23	2.2218	0.6097
		RingOfCliques	>50	2.1935	0.3787