Fair Online Allocation of Perishable Goods and its Application to Electric Vehicle Charging

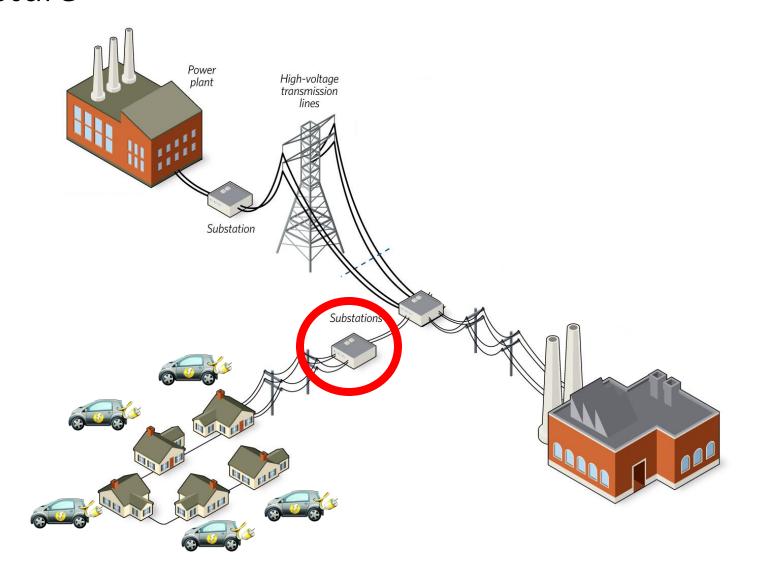
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EV proliferation will place growing strains on distribution infrastructure



Approach

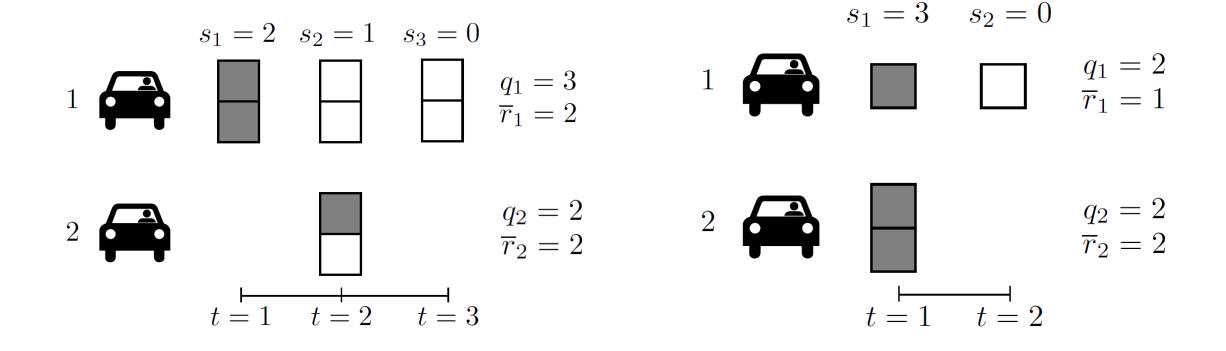
- Previous work has used auctions (mechanism design) to truthfully elicit EV owner's willingness to pay (valuations) for energy and use this in the online scheduling of EV charging
- In practice, people find it difficult to express their willingness to pay
- In this work we assume no preference information about the user's willingness to pay, but only their *scheduling constraints*
- The aim is to simultaneously satisfy multiple objectives and understand any trade-offs between them

Formal Model

- Finite time horizon with T discrete time steps, t.
- s_t discrete units of *perishable* supply available at timestep t
- On arrival, a_i , each EV $i \in A$ reports its requirements:
 - Departure time: d_i
 - Maximum charging rate: \bar{r}_i
 - Required energy: *q_i*
- The aim is to find an energy allocation for each EV, $x_{i.t}$:

 - MaxDelivered: maximises $\sum_{i \in A} \sum_{t=a_i}^{d_i} x_{i,t}$ MaxSatisfied: maximises $\sum_{i \in A} \{\sum_{t=a_i}^{d_i} x_{i,t} = q_i\}$
 - Online Envy Free: an allocation is envy free if agents do not prefer any other agent's allocations

Online Envy Free Examples



Both examples are online envy free (but not envy free in the traditional sense).

Theoretical Results (offline): MaxDelivered vs MaxSatisfied

- An MaxDelivered solution does not imply it is MaxSatisfied
- A MaxSatisfied solution does not imply it is MaxDelivered
- Proposition 1: There always exists an (offline) solution which maximizes both MaxDelivered and MaxSatisfied.

What about online (unknown future arrivals)?

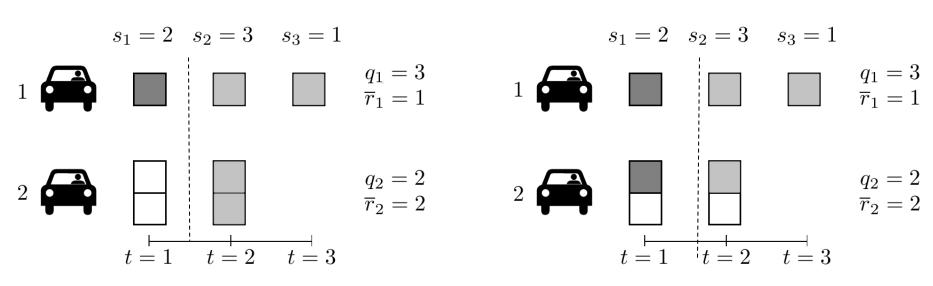
The MaxDelivered objective seems easy by simply using up as much resource as possible at each time step, irrespective of who gets it. However, this is not the case:

Theorem 1. No online mechanism exists which always optimises MaxDelivered

Theorem 2. No online mechanism exists which always optimises MaxSatisfied

OnlineMaxDelivered/Satisfied Algorithms

At each timestep, optimise MaxDelivered/MaxSatisfied allocation given current agents, but subject to the constraint that, in the current timestep, the maximum number of possible units should be allocated



Possible *OnlineMaxSatisfied* solution without constraint

Possible *OnlineMaxSatisfied* solution *with* constraint

Theoretical Results

Theorem OnlineMaxDelivered runs in polynomial time.

Proof by polynomial-time reduction to a feasible flow of a flow network with upper capacity constraints.

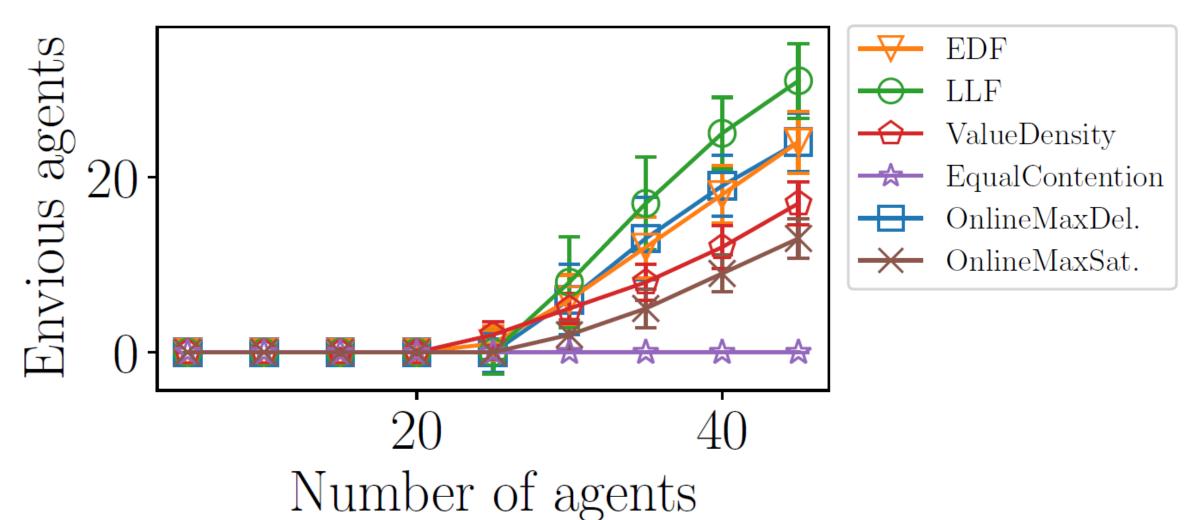
Theorem Computing a MaxSatisfied allocation is NP-hard, even when $s_t = 2$ at every time step, and we have that, for all $i: \bar{r}_i = 1$.

Proof by reduction from the scheduling problem $P2|\mathrm{pmtn}, r_i| \sum U_i$

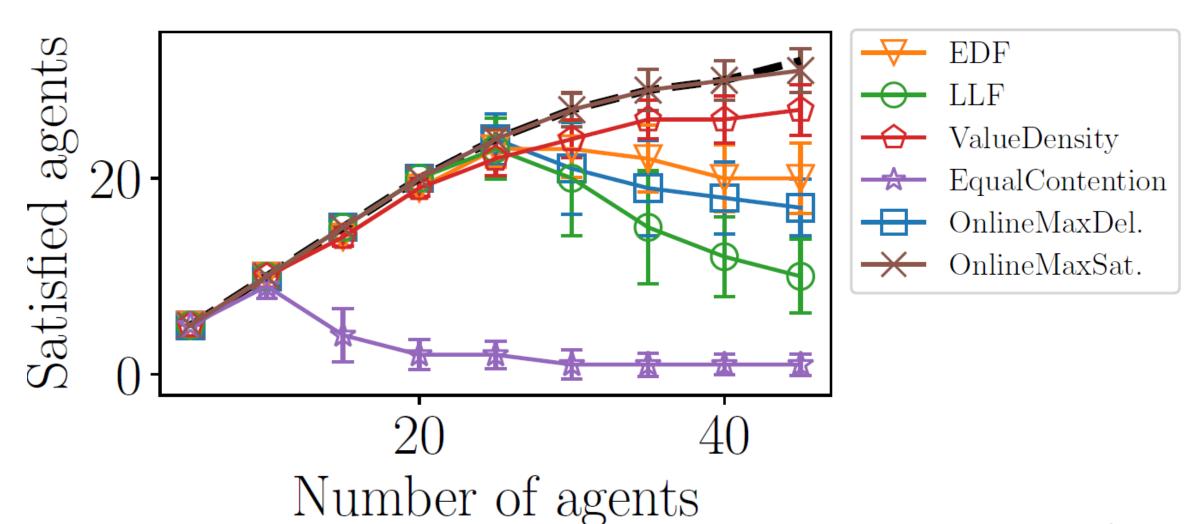
Other Algorithms

- Equal Contention: Online envy freeness can be trivially achieved by allocating all active agents an equal amount at each timestep (subject to \bar{r}_i and q_i and amount allocated so far)
 - Note: with more agents than supply, none of the agents receive any resource
- Value-Density: Prioritise agents according to $\frac{q_i \sum_{t' < t} x_{i,t'}}{(d_i t + 1) \cdot \overline{r}_i}$ Earliest Deadline First (EDF)
- Least Laxity First (LLF)

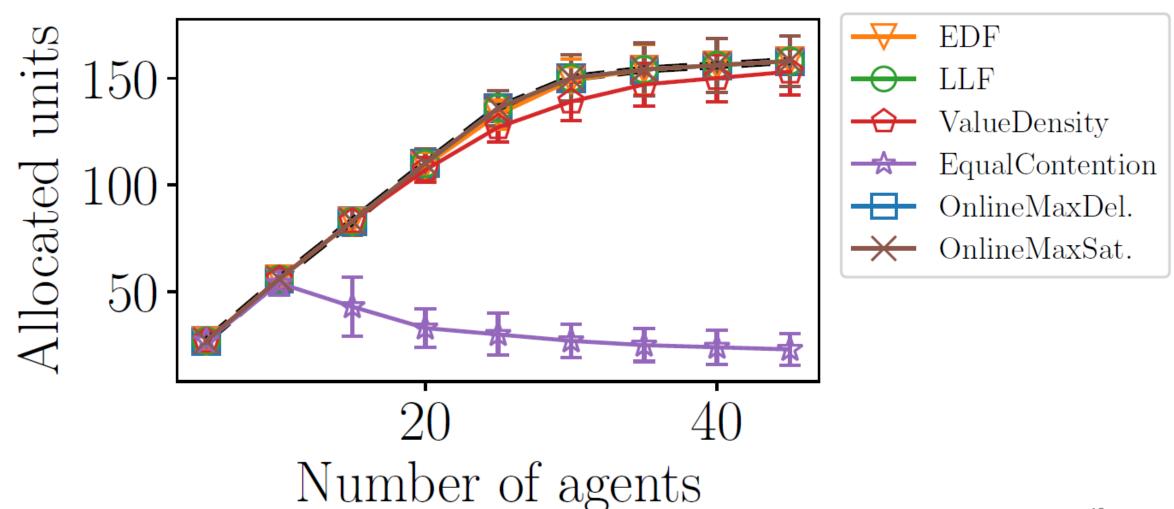
Empirical Results: Envy



Empirical Results: Satisfied



Empirical Results: Delivered



Main Contributions & Future Work

- Solutions exist which can satisfy both MaxDelivered and MaxSatisfied in offline settings, but MaxSatisfied is NP hard
- No algorithm exists which guarantees MaxDelivered or MaxSatisfied in online settings
- In practice, close to 100% performance in both can be obtained
- Equal contention satisfies online envy freeness, but performs very poorly in terms of the other objectives
- Future work:
 - Mechanism design without money: truthfully elicit constraints
 - Algorithms which improve envy freeness

MaxDelivered vs MaxSatisfied

MaxSatisfied (but not MaxDelivered)

 $s_1 = 1$ $s_2 = 1$ $s_3 = 2$









$$q_1 = 3$$
$$\overline{r}_1 = 2$$

MaxDelivered (but not MaxSatisfied)

 $s_1 = 1$ $s_2 = 1$ $s_3 = 1$

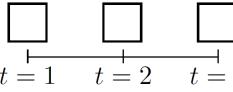










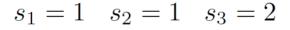






MaxDelivered vs MaxSatisfied

MaxSatisfied and MaxDelivered











$$q_1 = 3$$

$$\overline{r}_1 = 2$$

MaxDelivered (but not MaxSatisfied)

$$s_1 = 1$$
 $s_2 = 1$ $s_3 = 1$







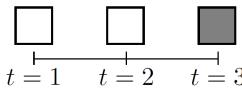






$$\begin{array}{c} 2 = 2 \\ 2 = 1 \end{array}$$



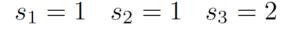


$$q_2 = 2$$

$$\overline{r}_2 = 1$$

MaxDelivered vs MaxSatisfied

MaxSatisfied and MaxDelivered











$$q_1 = 3$$

$$\overline{r}_1 = 2$$

MaxDelivered and MaxSatisfied

$$s_1 = 1$$
 $s_2 = 1$ $s_3 = 1$



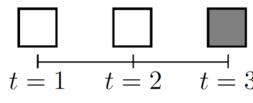






$$q_1 = 3$$
$$\overline{r}_1 = 2$$





$$q_2 = 2$$
$$\overline{r}_2 = 1$$



$$q_2 = 2$$

$$\overline{r}_2 = 1$$

Proof (MaxDelivered)

t=3

