Okay, so I'm going to do this a bit informally, but hopefully it will be clear.

To begin with, since I did the maximum code, I can get to V_{end} as a function of N_k (also M, for the two-parameter models). Since the mechanics of the inflationary period have not changed, that stands. Now, I also need N_{re}, which I believe I can find using this equation:

$$N_{re} = -N_k - N_{RD} + \ln\frac{a_{eq}H_{eq}}{a_0 H_0} + \ln\frac{H_k}{H_{eq}} - \ln\frac{k}{a_0 H_0} \tag{1}$$

Now, I want N_k, that's what I'm looking for, and I know k, I can just declare that N_{RD} is counted from nucleosynthesis, so I know that (I think?), and I can get H_k in terms of N_k, so I can just get N_{re} as a function of N_k (probably also M, but you get my point.) Note that this is independent of the equation of state, it's just a constraint on the number of e-folds that have to have happened, total. Next, I have this equation for T_{re}:

$$T_{re} = \exp\left[-\frac{3}{4}(1+w_{re})N_{re}\right]\left(\frac{3}{10\pi^2}\right)^{1/4}(1+\lambda)^{1/4}V_{end}^{1/4}, \tag{2}$$

where λ is defined as $\lambda = \frac{1}{3/\epsilon - 1}$, so $\lambda \approx 1/2$ at the end of inflation. Here is where w_{re}, the equation of state parameter during reheating, comes in, and straightforwardly, so that's good. At this point, T_{re} is a function of N_k, M, w_{re}, and λ (though that last is basically going to be 1/2 for pretty much every model, and isn't really something I need to worry about varying). Unless I've missed or forgotten something, then, that means that I can calculate the minimum number of e-folds with the same equation I used for the maximization,

$$N_k - \ln T_{re} = 68. \tag{3}$$

At least, it seems like that ought to work, as far as I can tell! I still need to track down values for some things (eg N_{RD}), but the outline seems reasonable.

I. NUMBERS

$$a_{eq} = 4.15 * 10^{-5}(\Omega_m h^2)^{-1} \tag{4}$$

$$\Omega_m h^2 = 0.128 \pm 0.008 \tag{5}$$

$$1 + z_{eq} = 2.4 * 10^4 \Omega_m h^2 \tag{6}$$

$$H_{eq}^2 = 2\Omega_m H_0^2 a_{eq}^{-3} \tag{7}$$

II. QUESTIONS

- What value of H_0 do I use?
- What value of Ω_m do I use? $\Omega_m h^2$ is an observable, but I seem to remember than h^2 depends on H_0 somehow.
- Do I actually need/want to use $N_k - \ln T_{re} = 68$, or just go with "And I know the temperature at the end of reheating [since I'm taking that to be nucleosynthesis]" and set T_{re} equal to that known value (which appears to be 0.05 MeV)?
- I actually can't quite figure out how to get N_{RD}, for reasons I will explain in detail below.

A. THE PROBLEM WITH N_{RD}

So my primary stumbling block here is that "number of e-folds of expansion" isn't actually a super sensible thing to talk about in a matter/radiation dominated universe, that is, when $a(t)$ is not proportional to e^{Ht}. I started by figuring that I would just do the standard $N = -\int H dt$ integration from nucleosynthesis to matter-radiation equality,