Alright, I'm skipping the bits that are unchanged from the Nmax code, starting where I pick up from there.

A. Code setup: What I Am Doing And How

I have H_k as a function of N_k and M, just the standard formula, only I have an m_{pl} in front because I had been working in units of m_{pl} as they all dropped out in the end when I was just getting N_{max}, but this is one of the places where they matter.

Next, I have a bunch of known parameters (I took my values from the most recent Planck results,[1]). My formulae for a_{eq} and H_{eq} come from [2], though I used the updated values for things like $\Omega_m h^2$ and H_0, and converted all non-dimensionless values to eV. As an aside, I can get three different values for a_{eq}: First, by using the formula in my code with the modern value of $\Omega_m h^2$, and that gives me about $3.5 * 10^{-4}$, a second by using the generally accepted/cited value of z_{eq}, namely 3400, which gives $2.9 * 10^{-4}$, and a third by using the formula $a_{eq} = \frac{\rho_{r0}}{\rho_{m0}}$ and the more recent value for the current matter density of the universe given by the Planck data, which gives me $a_{eq} = 2.6 * 10^{-4}$. I don't think that these numbers are different enough to cause problems, however.

If I go back to the original paper ([3]), I find that they have N_{RD} defined as $\ln \frac{a_{eq}}{a_{re}}$, where a_{re} is the value of a at the end of reheating, which I'm taking to be nucleosynthesis. Since they actually started with $\log \frac{a_{eq}}{a_{re}}$ and then substituted N_{RD} in for that, that is clearly the formula to use. Taking the $t[\frac{a}{a_{eq}}]$ formula from [2], I trial-and-error'd to find that, at 250s after the big bang (which appears to be dead in the middle of the 200-300s range for nucleosynthesis), $\frac{a}{a_{eq}} = 0.00001$, so that I have established that $\frac{a_{re}}{a_{eq}} = 0.00001$, and I can just take the natural log of that to get 11.513.

DEEP BREATH. Now we get to the "matching e-folds" part, which is where I start to get a little less confident. The equation, for reference, is:

$$N_{re} = -N_k - N_{RD} + \ln \frac{a_{eq} H_{eq}}{a_0 H_0} + \ln \frac{H_k}{H_{eq}} - \ln \frac{k}{a_0 H_0} \tag{1}$$

Now, of these terms, only two of them will vary depending on the inflationary potential: N_k and $\ln \frac{H_k}{H_{eq}}$. The rest are going to be independent of inflationary potential–in fact, fixed, known values. Thus, I can simplify this equation to:

$$N_{re} = -N_k + \ln \frac{H_k}{H_{eq}} - 13.0807. \tag{2}$$

So that we now have N_{re} as a function of N_k. Next, in order to get T_{rh}, I plug that into an equation taken directly from [3]:

$$T_{re} = \exp\left[-\frac{3}{4}(1+w_{re})N_{re}\right]\left(\frac{3}{10\pi^2}\right)^{1/4}(1+\lambda)^{1/4}\,V_{end}^{1/4}, \tag{3}$$

where λ is defined as $\lambda = \frac{1}{3/\epsilon - 1}$, so $\lambda \approx 1/2$ at the end of inflation. Also, in my code, it is necessary to add a coefficient of m_{pl} because I had dropped it (since the log method I had been using for the N_{max} has this cancel immediately, as it calls for $\ln \frac{T_{re}}{m_{pl}}$).

Anyway, I now have two methods to try to get from T_{rh} to N_{min}, the first being the one I used to get N_{max}, which is not working. I am disregarding that method for the moment.

The second method takes the approach that "I know what the temperature was at nucleosynthesis, I shall declare that the reheat temperature must be this". Thus, I simply use FindRoot to find the value of N_k for which T_{rh} is 50,000eV. This appears to be working, if imperfectly.

B. Notes on what exactly is going wrong and what I think may be the problem

1. *Results*

Method 1 is not working. It produces a lot more error messages than method 2, and after 384 seconds (I used Timing) it produced a result of $N_k = -69$.

Method 2 appears to be working, but gives a larger range of potential N_{min} values than expected; if I use the smallest possible value of w, $w = -1/3$, I get values of N_{min} around 11, which does not seem to agree with the normal limiting range. Note, however, that if I restrict myself to $w \geq 0$, I get much more reasonable results, though still a bit low. If I increase w above about .4 (at least for the potential I'm working with at the moment) I start getting $N_{min} \approx 60$. If I continue to increase w, at some point the function breaks down and it just returns my initial starting value.

I also observe that I'm pretty consistently getting that $\ln \frac{H_k}{H_{eq}}$ is something around 115, for a couple of different potentia, so that you have a comprable number or in fact *more* e-folds of expansion during reheating as during inflation. This may or may not be bad.

2. Possible Sources of Error

And this part is now me hypothesizing/speculating as to what could be causing my issues. The first possibility is that some assumption I am making is unsound; perhaps there is some reason why I shouldn't make reheating last all the way to nucleosynthesis in all cases, or something. The second possibility is that there is a bound or constraint of some kind that I am not taking into account. Both of these assume I haven't made any mistakes otherwise; I am not certain that this is the case, though if it isn't, I can't find my mistake.

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[1] Most recent Planck results, `https://www.aanda.org/articles/aa/pdf/2016/10/aa25830-15.pdf`.

[2] Caltech prof's lecture notes, had useful formulae for me,`http://www.tapir.caltech.edu/~chirata/ph217/lec06.pdf`

[3] That first paper I took my methodology from, `https://arxiv.org/pdf/1412.0656.pdf`