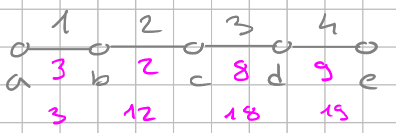
George, Nina, Paul - 25.10.2022

Cycles, every edge has the same period Delta and gets exactly one label per edge.

CLAIM: We can find the desired labeling in polynomial time (if it exists).

DEF: $d = \lambda_{max} - \lambda_{start} + 1$

λ_{max}: last label, λ_{start}: first label

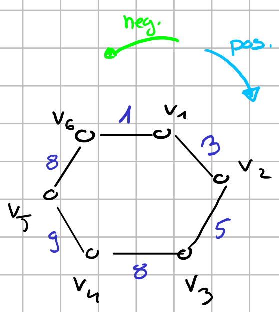


$d(a,e) = 4 - 1 + 1 = \underline{4}$

$\rightarrow d(a,e) = 19 - 3 + 1 = \underline{17}$

example: $C_6,\ \Delta = 10$

$$D = \begin{array}{c|cccccc|} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 3 & 6 & 7 & 1 \\ v_2 & 1 & 0 & 1 & 4 & 5 & 9 \\ v_3 & 9 & 1 & 0 & 1 & 2 & 11 \\ v_4 & 13 & 8 & 1 & 0 & 1 & 10 \\ v_5 & 4 & 6 & 8 & 1 & 0 & 1 \\ v_6 & 1 & 3 & 5 & 2 & 1 & 0 \end{array}$$



$v_1 \to v_3$ $5-3+1=3$ | $v_1 \to v_3$ $18-1+1=18$
$v_1 \to v_4$ $8-3+1=6$ | $v_1 \to v_4$ $9-1+1=9$
$v_1 \to v_5$ $9-3+1=7$ | $v_1 \to v_5$ $8-1+1=8$

$v_2 \to v_4$ $8-5+1=4$ | $v_2 \to v_4$ $19-3+1=17$
$v_2 \to v_5$ $9-5+1=5$ | $v_2 \to v_5$ $18-3+1=16$
$v_2 \to v_6$ $18-5+1=14$ | $v_2 \to v_6$ $11-3+1=9$

$v_3 \to v_5$ $9-8+1=2$ | $v_3 \to v_5$ $28-5+1=24$
$v_3 \to v_6$ $18-8+1=11$ | $v_3 \to v_6$ $21-5+1=17$
$v_3 \to v_1$ $21-8+1=14$ | $v_3 \to v_1$ $13-5+1=9$

$v_4 \to v_6$ $18-9+1=10$ | $v_4 \to v_6$ $31-8+1=24$
$v_4 \to v_1$ $21-9+1=13$ | $v_4 \to v_1$ $23-8+1=16$
$v_4 \to v_2$ $23-9+1=15$ | $v_4 \to v_2$ $15-8+1=8$

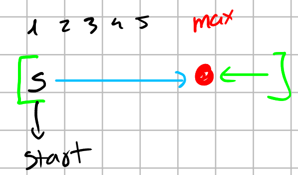
$P_3 = 6-3 = 3$

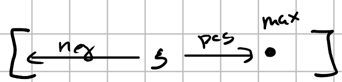
$P_2 = 3-1 = 2$

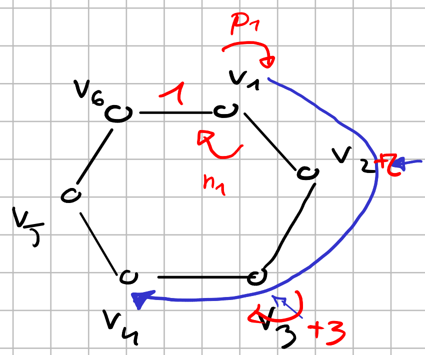
$n_2 = 8$

$P_2 = 2$

$$D = \begin{array}{c|cccccc|} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 3 & 6 & 7 & 1 \\ v_2 & 1 & 0 & 1 & 4 & 5 & 9 \\ v_3 & 9 & 1 & 0 & 1 & 2 & 11 \\ v_4 & 13 & 8 & 1 & 0 & 1 & 10 \\ v_5 & 4 & 6 & 8 & 1 & 0 & 1 \\ v_6 & 1 & 3 & 5 & 2 & 1 & 0 \end{array}$$







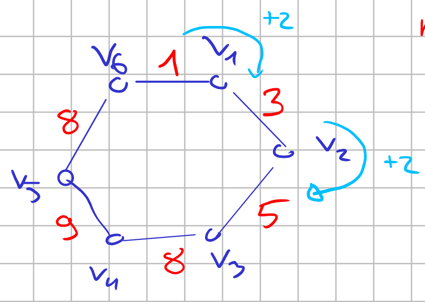
p_i = waiting time at vertex i in pos. direction

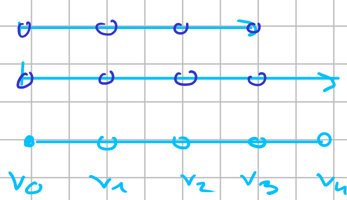
n_i = —||— neg. direction

$+\ p_1 + n_1 = \Delta$

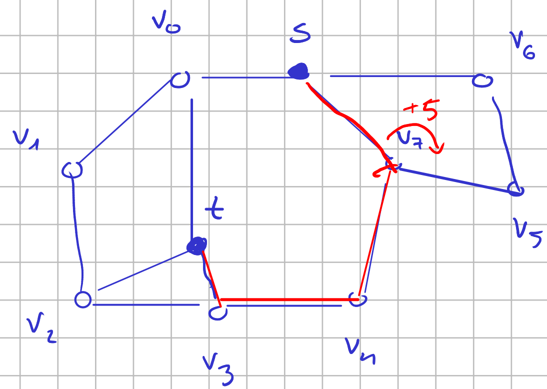
$+\ p_i,\ n_i$

$p_1 = 2$
$p_2 = 2$
$p_3 = 3$
$p_4 = 1$
$p_5 = 9$
$p_6 = 3$



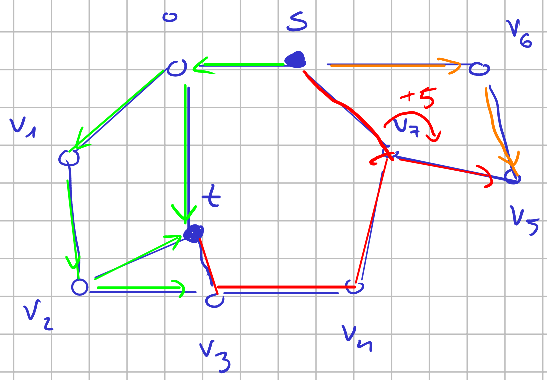


	s	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	t
row of s	0	1	1	3	12	10	6	1	1	



r_i
o_i
g_i

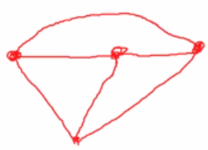
row of s	s	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	t
	0	1	1	3	12	10	6	1	1	



	A	B	C	D	F	J	G
A	0	1	5	9	11	12	13
J			7	4	1	0	1

IDEA:
1. go from a vertex and see how far you can reach using one of the paths.
2. There is a vertex where/when we don't know which path we used to go there.

Try the procedure for:
- theta (cycle with a path)
- for general (?) graph



If we are not given Delta, can we find it?
INPUT: D, G ... Can we set any bound for Delta as a function of n (number of vertices) and c (max value in the matrix D)?