

SUBMISSION: 125

TITLE: Realizing temporal graphs from fastest travel times

AUTHORS: Nina Klobas, George Mertzios, Hendrik Molter and Paul Spirakis

----- Overall evaluation -----

This paper studies a distance-based realization problem in temporal graphs (graphs where each edge appears in only a subset of the times). A *periodic* temporal graph is a temporal graph where every edge e appears at time $x_e + i\Delta$ for some offset x_e , global period Δ , and integer i . For every two nodes u and v , we are given a value $d(u,v)$ and our goal is to build a periodic temporal graph such that the shortest *temporal* path (which they call the fastest path) between u and v has duration $d(u,v)$. They show that this problem is NP-hard and is $W[1]$ -hard parameterized by the feedback vertex set (and thus also the treewidth), but that it can be solved in polynomial time when the underlying graph is a tree and in FPT time parameterized by the feedback edge set.

Overall, my view is that this paper does quite a bit of technical work on a problem that is not well-motivated, and to prove results that are not particularly strong. So I do not think that it should be accepted to ICALP.

Strengths:

- There is a lot of nontrivial technical work. From a technical point of view, this is above the bar for ICALP.
- Temporal graphs are interesting objects.

Weaknesses:

- My main issue with this paper is the motivation for the problem. While temporal graphs are interesting, the authors do not motivate the realization from durations problem, particularly in the context of the periodic restriction. I don't understand why we should care about this problem as stated. First, why care about the periodic setting? The examples they use are things like satellite networks where there is natural periodicity, but this periodicity is with respect to the *nodes* – there is no way in satellite networks that every *edge* has the exact same period. So I could see motivation for every edge having *some* period, but I don't see any reason to study the setting where every edge has the *same* period. Second, why do we care about realization from durations? If we're trying to design "good" temporal graphs, then why would we want to realize durations as opposed to something like "minimize the maximum or average duration under some type of cost constraint"? The authors seem to think that saying "realization has been studied in the static setting, so it's interesting to study in the temporal setting" is enough motivation. In my view, this is not sufficient motivation.

- My second major set of issues is with the results. Again, there are issues both with the motivation for the results and the results themselves. For motivation, why care about parameterized complexity for this problem with respect to "tree-ness"? The result showing that it can be solved for trees is reasonably nice (albeit somewhat simple), and the authors seem to think that this naturally motivates the study of parameterized complexity w.r.t. treewidth, FVS, and FES. But they don't actually motivate this, other than stating that they follow a "distance from triviality" paradigm. Why study parameterized complexity to begin with? Why not approximation algorithms? If we do study parameterized complexity, why parameterize by anything related to the underlying graph rather than by the structure of the durations, or by a restriction on the realized graph (e.g., requiring that the graph does not change too much between adjacent times)? If we do parameterize by the underlying graph, why distance from trees? And if we do parameterize by distance from trees, what

about other parameters like pathwidth or arboricity? It just seems like there's no motivation for these particular problems.

- The main upper bound, that the problem is FPT parameterized by the FES (which forms the bulk of the technical work in the paper), seems extremely weak to me. When I see FPT algorithms, the first question I always ask is "why is it reasonable to expect this parameter to be small?" I see no reason why I would expect this parameter to be small. For example, if the underlying graph has m edges, then the FES has size at least $m-n$. Since FPT algorithms are only interesting when the parameter is logarithmic or less (in order to get polynomial time overall), this means that this upper bound only applies to graph with at most $n + O(\log n)$ edges. That's a pretty strong limitation, and the authors do not even justify or motivate it. This seems **particularly** problematic to me in temporal graphs, where the edges of the underlying graph actually represent "every edge that could exist in at least one timepoint", so I would expect the underlying graph to actually be **much** denser than a normal "real-world" graph. As just one example, temporal graphs are often used to study financial flows for anti-money laundering. So the underlying graph would include an edge between every two entities that have **ever** exchanged money. This seems like it would be quite dense, and certainly would not have small FES.

- Related to the motivation, the authors also seem to have missed the large amount of work on "dynamic graphs", which are a different name for the same object. I would suggest the authors look at the slightly out of date but still very good review article: "Dynamic networks: models and algorithms" by Kuhn and Oshman in SIGACT News '11. In dynamic networks we typically think of the graph as changing arbitrarily between time points, and seek to design good algorithms even in this setting. There are typically restrictions we can place to make the problems easier, e.g., restricting how much it can change between time points, requiring that there's a stable backbone, etc. To me, the assumptions and parameterizations made in that literature seem far better motivated than either the periodic restriction of the small FES restriction.

----- REVIEW 2 -----

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----- Overall evaluation -----

The paper considers a realization problem for temporal graphs. In this problem, one is given a matrix of lengths of fastest temporal paths in a graph, and the objective is to create a periodic temporal graph (with a single label per edge) that agrees with this matrix, or determine that such a temporal graph does not exist. One result of the paper is that this problem is polynomially solvable when the underlying static graph is a tree. The other results try to determine the complexity of the problem when parameterized by parameters measuring the distance of the graph from being a tree.

- When the parameter is the vertex feedback number (or the tree-width) the problem is $W[1]$ -hard.
- When the parameter is the edge feedback number the problem is FPT.

While realization problems for static graphs have been studied extensively before, the paper claims to be the first to study such a problem for temporal graphs. Such an extension seems to be natural, but unfortunately, the results of the paper show that this task is very difficult even for a relatively restricted set of temporal graphs. Moreover, getting both the FPT and $W[1]$ -hardness results of the paper is very technically complicated, and involves much case analysis (leading to full version of 56 pages in the appendix of the paper). Thus, I doubt that the paper will be of interest to a significant audience: only experts on realization problems of temporal graphs are likely to take the trouble of reading such an involved paper, and such experts do not really exist as the field is new and seems to

be of little potential for positive results.

Due to the large length of the paper, the main part of the paper consists mostly of “teasers” that give very partial information, tend to be highly technical, and include very basic intuition if any. This is unfortunate as it gives the reader a feeling of a zero knowledge proof. Perhaps it would have been better if the authors had devoted all the available space to one of the two considered parameters (with the result for the other parameter appearing only in the appendix). In this way, it might have been possible to give a substantial explanation within the main part of the paper regarding the proofs related to the chosen parameter.

Given all the above, I feel that this paper is below the bar for ICALP, especially in its current form. Perhaps a more specialized conference such as IPEC will be a better fit for the paper.

Minor remark: Definition 13 assumes that the segment $S_{\{u, v\}}$ between vertices u and v is unique. This does not seem to be true if one allows a segment to be an arbitrary path in G' . Maybe the segments should be restricted to paths in the spanning tree?

----- REVIEW 3 -----

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----- Overall evaluation -----

The authors present a first result for the graph realization problem in a temporal setting. The problem asks for the construction of a periodic temporal graph that satisfies the temporal distance property of a given distance matrix. They refer to this problem as temporal distance realization (TGR). They present an NP-hardness proof for TGR in the general case, and a polynomial algorithm for the case that the underlying graph is a tree or a cycle. Furthermore, they generalize the cyclical case to an FPT-algorithm parameterized by feedback edge number, and a $W[1]$ -hardness proof regarding the feedback vertex number. Note that in their setting the temporal graph repeats periodically with a frequency of Δ and each edge can occur only at one time step.

First the authors show the NP-hardness proof of the problem, which is based on a reduction from 3-SAT NAE. They show NP-hardness even in the case that Δ is a small constant. For the $W[1]$ -hardness proof they use a reduction from MULTICOLORED CLIQUE. This proof works in two steps, as they first show $W[1]$ -hardness for the case that Δ is infinity. Using a secondary result which reduces the infinity-case to the finite case. For both proofs the authors provide nice figures that give a good intuition.

Their most interesting result in my opinion is the FPT algorithm parameterized by feedback edge number, as it combined multiple techniques such as ILP and structural properties. Intuitively, they first give a polynomial time result for trees, which is based on the property that in temporal graphs where the underlying graph is a tree, any shortest temporal path between two vertices is also a shortest path to all other vertices on this temporal path. This property does not hold in non-tree temporal graphs, and is one of the difficulties that arise in the construction of their FPT result, which generalizes their result to graphs with a bound feedback edge set.

Overall: The paper is well written and structured, the results are introduced in an intuitive fashion before detailed descriptions of technical realizations follow. The techniques the authors present are of independent interest. The distance realization problem is well-researched in the static setting, extending it to the temporal setting is a very natural and interesting extension.

I suggest accepting the paper.

Comments: Question: How are connector gadgets feedback vertex vertices? Add sketched edges if they are connected to other gadgets not shown in the picture. Note: this is examined in the appendix, but the addition of such sketched edges would make this much more intuitive.

Minor Comments:

I suggest defining the MULTICOLORED CLIQUE problem instead of referring to a publication that discusses this problem, as this is quite inconvenient for a reader.

Line 45: variations >of< this problem

Line 308+310: satisfying conditions of the simple TGR problem

Best wishes,

Kousha Etessami and Uriel Feige

Program Committee chairs for ICALP 2023