MOCNINY S CELOČÍSELNÝM EXPONENTOM (Riešené príklady 2)

Na zopakovanie:

1)
$$a^0 = 1$$

$$a \neq 0, a \in R$$

2)
$$a^1 = a$$

$$a \neq 0, a \in R$$

3)
$$a^{-n} = \frac{1}{a^n}$$

$$\alpha \neq 0, \alpha \in R, n \in Z$$

4)
$$a^n . a^m = a^{n+m}$$

$$a \in R$$
; $n, m \in Z$

5)
$$a^n : a^m = a^{n-m}$$

$$a \in R$$
; $n, m \in Z$

6)
$$(a^n)^m = a^{n.m}$$

$$a \in R$$
; $n, m \in Z$

7)
$$(a.b)^n = a^n.b^n$$

$$a,b \in R$$
; $n \in Z$

8)
$$\left(\frac{a}{h}\right)^n = \frac{a^n}{h^n}$$

$$a \neq 0$$
; $n, m \in Z, n > m$

Príklady:

1. Vypočítajte (použite pravidlá pre počítanie s mocninami):

a)
$$6^4$$
: $6^2 = 6^2 = 36$

b)
$$2^3 \cdot 3^2 = 8.9 = 72$$

c)
$$2^{-5} - 3^2 + 4^{-1} - 2^3 = \frac{1}{2^5} - 9 + \frac{1}{4^1} - 8 = \frac{1}{3^2} + \frac{1}{4} - 17 = \frac{1+8}{3^2} - 17 = \frac{9-17.32}{3^2} = \frac{-535}{3^2}$$

$$d) (2^{-2})^{-3} \cdot (2^2)^{-3} \cdot (2^{-4})^{-3} = 2^6 \cdot 2^{-6} \cdot 2^{12} = 2^{12} = 4096$$

e)
$$(3a^2b^{-4}c^3)^{-2} = 3^{-2}a^{-4}b^8c^{-6} = \frac{b^8}{3^2 \cdot a^4 \cdot c^6} = \frac{b^8}{9 \cdot a^4 \cdot c^6}$$
 P1: $\underline{a \neq 0}$ P2: $\underline{c \neq 0}$

f)
$$2^{-3} - 4^{-2} - 5^2 + 20^2 = \frac{1}{2^3} - \frac{1}{4^2} - 25 + 400 = \frac{1}{8} - \frac{1}{16} + 375 =$$

$$=\frac{2-1}{16}+\frac{375}{1}=\frac{1+6000}{16}=\frac{6001}{16}$$

$$g) (3^2)^{-2} \cdot (3^{-3})^2 \cdot (3^{-3})^{-3} = 3^{-4} \cdot 3^{-6} \cdot 3^9 = 3^{-1} = \frac{1}{3}$$

h)
$$(5x^{-2}y^3z^4)^{-3} = 5^{-3}x^6y^{-9}z^{-12} = \frac{1}{5^3} \cdot x^6 \cdot \frac{1}{y^9} \cdot \frac{1}{z^{12}} = \frac{x^6}{125 \cdot y^9 \cdot z^{12}}$$

P1:
$$y \neq 0$$
 P2: $z \neq 0$

2. Vypočítajte (použite pravidlá pre počítanie s mocninami)

a)
$$\left(\frac{5}{4}\right)^{-1} = \frac{5^{-1}}{4^{-1}} = \frac{\frac{1}{5}}{\frac{1}{4}} = \frac{4}{5}$$
 verzia2: $\left(\frac{5}{4}\right)^{-1} = \frac{\frac{1}{1}}{\frac{5}{4}} = \frac{4}{5}$

$$b)\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

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c)
$$\frac{2^{5} \cdot 2^{7}}{2^{10}} = \frac{1 \cdot 2^{7}}{2^{5}} = 2^{7} : 2^{5} = 2^{7-5} = 2^{2} = 4$$

$$d)\frac{(-3)^3 \cdot (-3)^6}{(-3)^5 \cdot 3^2} = (-3)^4 \cdot 3^2 = +81.9 = 729$$

e)
$$\frac{15^3.5^{-2}}{3^2.2^{-1}} = \frac{(3.5)^3.5^{-2}}{3^2.2^{-1}} = \frac{3^3.5^3.5^{-2}}{3^2.2^{-1}} = \frac{3.5}{\frac{1}{2}} = 15.2 = 30$$

$$f) \frac{9^{-3}.45^{2}.6^{-2}}{5.4^{-3}} = (D.\acute{u}.)$$

$$g) \left[\frac{5^3 \cdot (2 \cdot 3^3)^2}{3 \cdot (2 \cdot 3 \cdot 5)^3} \right]^2 =$$

$$h) 25 \cdot \left(\frac{5}{2}\right)^{-2} \cdot \left(-2^{-3}\right)^{-1} =$$

3. Zjednodušte výrazy s mocninami:

a)
$$4ab^3c \cdot 3a^3c^5 = 12 \cdot a^4 \cdot b^3 \cdot c^6$$

(pri kladnej mocnine sa nedelí, preto nepotrebujeme podmienky)

b)
$$(12a^8b^{14}c^2)$$
: $(6a^2b^7c^2) = 2.a^6.b^7$ P1: $\underline{a \neq 0}$ P2: $\underline{b \neq 0}$ P3: $\underline{c \neq 0}$

(tu sa už delí, preto potrebujeme podmienky)

c)
$$\frac{a^{-1}b^3}{c^{-2}d} = \frac{\frac{1}{a}b^3}{\frac{1}{c^2}d} = \frac{c^2b^3}{ad}$$
 P1: $d \neq 0$ P2: $c \neq 0$ P3: $a \neq 0$

(tu tiež potrebujeme podmienky, lebo záporná mocnina je vlastne delenie)

d)
$$(7a^6b^{-3}c^{-2}d).(8a^{-3}b^{-5}c^3d^{-1}) = (D.ú.)$$

$$e) \frac{a^{-6}.b^7}{d^{-3}c^4} \cdot \frac{a^5c^3}{d^7b^7} = (D.\acute{u}.)$$

$$f) \frac{abc}{b^{-1}c^{-1}d^{-1}} : \frac{1}{a} = (D.\acute{u}.)$$

g)
$$\left[\left(\frac{a^2b^{-5}}{c^3} \cdot \frac{d^{-1}}{c} \right)^{-1} \right]^2 = (D.ú.)$$

$$h)\, \frac{5a^{-2}bc^3d^{-4}}{3a^{-3}b^3c^5d^{-7}} \cdot \frac{21a^5b^{-2}cd^2}{105a^5b^{-4}c^2d^3} =$$

$$\frac{9a^{-3}b^{-2}}{16cd^{-3}}:\frac{8c^{-3}d}{(-3)^3(ab)^2} =$$

$$\int \left(\frac{x^{-2}y^2z^{-2}}{x^0y^{-8}}\right)^{-2}:\frac{x^2z^3}{x^{-4}y^7}=$$

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(Riesene
$$k) \left(\frac{a}{2b^{-1}}\right)^{-3} \left(\frac{4b^{-2}}{3a^{-3}}\right)^{-1} \left(\frac{a^0b}{3^2b^{-2}}\right)^{-2} =$$