Clase Teoría de Lenguajes: Pasaje de $AFND - \lambda$ a AFD y Minimización

- AFD para referirnos a un autómata finito determinísitico
- AFND para referirnos a un autómata finito no determinísitico, sin transiciones lambda
- AFND–λ para referirnos a un autómata finito determinísitico, con transiciones lambda

$2^{|Q|}$ si Q son los estados de AFND–λ

$$p \xrightarrow{a} q,\ p \xrightarrow{a} r,\ p \xrightarrow{a} s,\ p \xrightarrow{a} t \qquad \{p\} \xrightarrow{a} \{q, r, s, t\}$$

$$\{p, q\} \xrightarrow{a} \{r\} \qquad p \xrightarrow{a} r \text{ o bien } q \xrightarrow{a} r$$

AFND a AFD

$$N = \langle Q_N, \Sigma, \delta_N, q_0, F_N \rangle$$
$$D = \langle Q_D, \Sigma, \delta_D, \{q_0\}, F_D \rangle$$

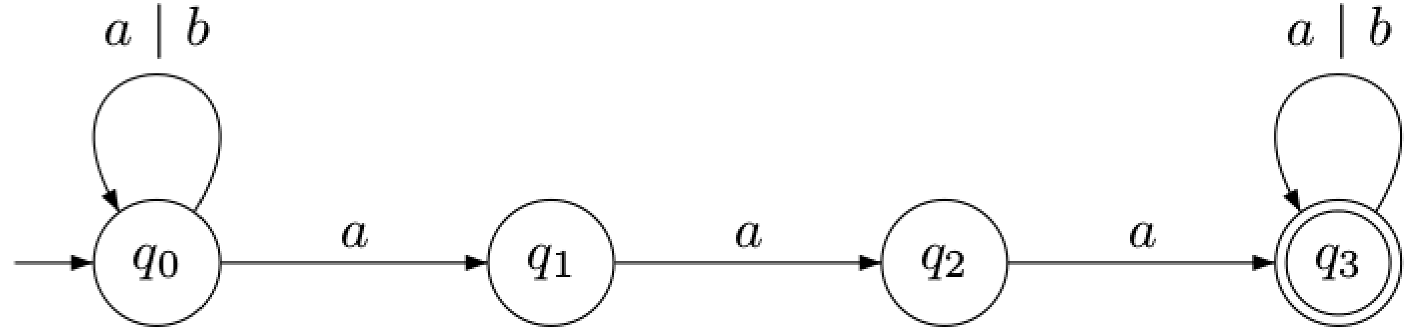
$$Q_D = P(Q_N)$$
$$F_D = \{\{q_1, \ldots, q_n\} \in Q_D : \{q_1, \ldots, q_n\} \cap F_N \neq \emptyset\}$$
$$\delta_D(\{q_1, \ldots, q_n\}, a) = \bigcup_{i=1}^{n} \delta_N(q_i, a)$$

$$Mover(T, a) = \bigcup_{t \in T} \delta(t, a)$$

Sea $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ un AFND, construyamos $M' = \langle Q', \Sigma, \delta', q_0', F' \rangle$ tal que $L(M) = L(M')$

1. Defino $\{q_0\}$ el estado inicial de M'
2. Inicializo $Q' := \{\{q_0\}\}$ donde Q' es marcable, y el estado inicial está sin marcar
3. Mientras exista $T \in Q'$ sin marcar:
   a) Marcar T.
   b) Para cada $a \in \Sigma$
      1) Hacer $U \leftarrow Mover(T, a)$.
      2) Si $U \notin Q'$ entonces agrego sin marcar U a Q'.
      3) Hacer $\delta'(T, a) = U$
   c) Fin Para.
4. Fin Mientras.
5. Hacer $F' \leftarrow \{X \in Q' | X \cap F \neq \emptyset\}$

Sea M el AFND que reconoce cadenas de $\{a, b\}$ que contengan aaa:
$M = \langle \{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\} \rangle$



$$M' = \langle Q', \Sigma, \delta', q_0', F' \rangle$$

δ'	a	b
$\{q_0\}$		

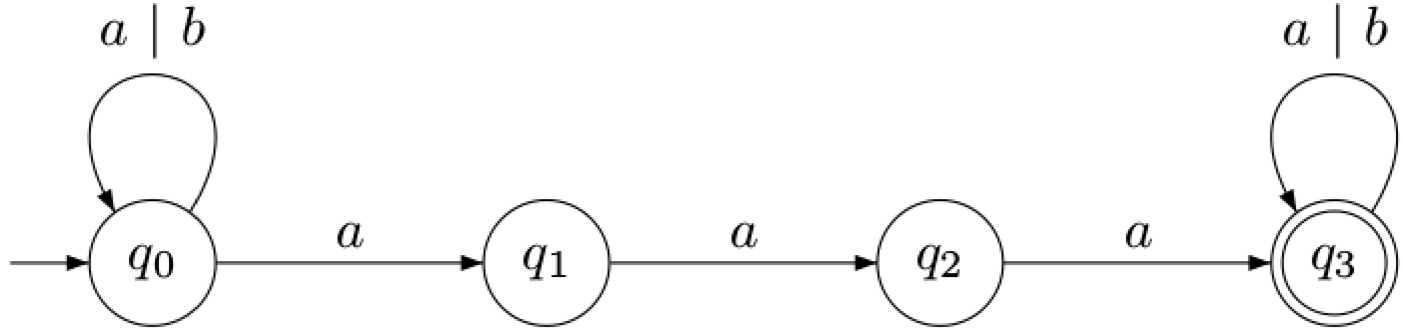
$$Mover(\{q_0\}, a) = \bigcup_{t \in \{q_0\}} \delta(t, a) = \{q_0, q_1\}$$
$$Mover(\{q_0\}, b) = \bigcup_{t \in \{q_0\}} \delta(t, a) = \{q_0\}$$

δ'	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		

$$Mover(\{q_0, q_1\}, a) = \bigcup_{t \in \{q_0, q_1\}} \delta(t, a) = \delta(\{q_0\}, a) \bigcup \delta(\{q_1\}, a) = \{q_0, q_1, q_2\}$$

$$Mover(\{q_0, q_1\}, b) = \bigcup_{t \in \{q_0, q_1\}} \delta(t, a) = \delta(\{q_0\}, b) \bigcup \delta(\{q_1\}, b) = \{q_0\}$$

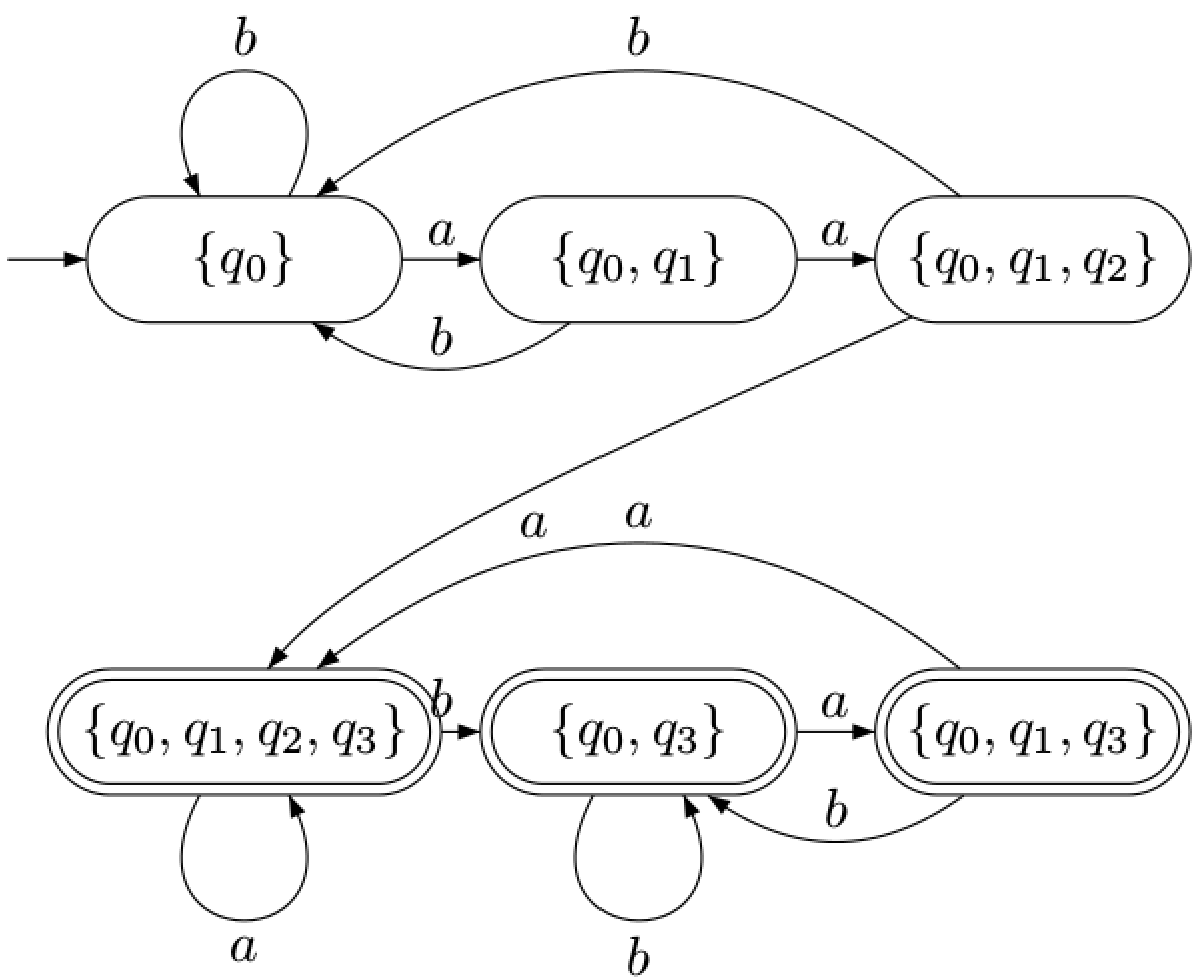
Sea M el AFND que reconoce cadenas de $\{a, b\}$ que contengan aaa:
$M = \langle \{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\} \rangle$



$$M' = \langle Q', \Sigma, \delta', q_0', F' \rangle$$

δ'	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_3\}$

$$M = \langle \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_1, q_2\}, \{q_0, q_1, q_2, q_3\},$$
$$\{q_0, q_3\}, \{q_0, q_1, q_3\}\}, \{a, b\}, \delta', \{q_0\}, \{q_0, q_1, q_2, q_3\}, \{q_0, q_3\}, \{q_0, q_1, q_3\}\rangle$$

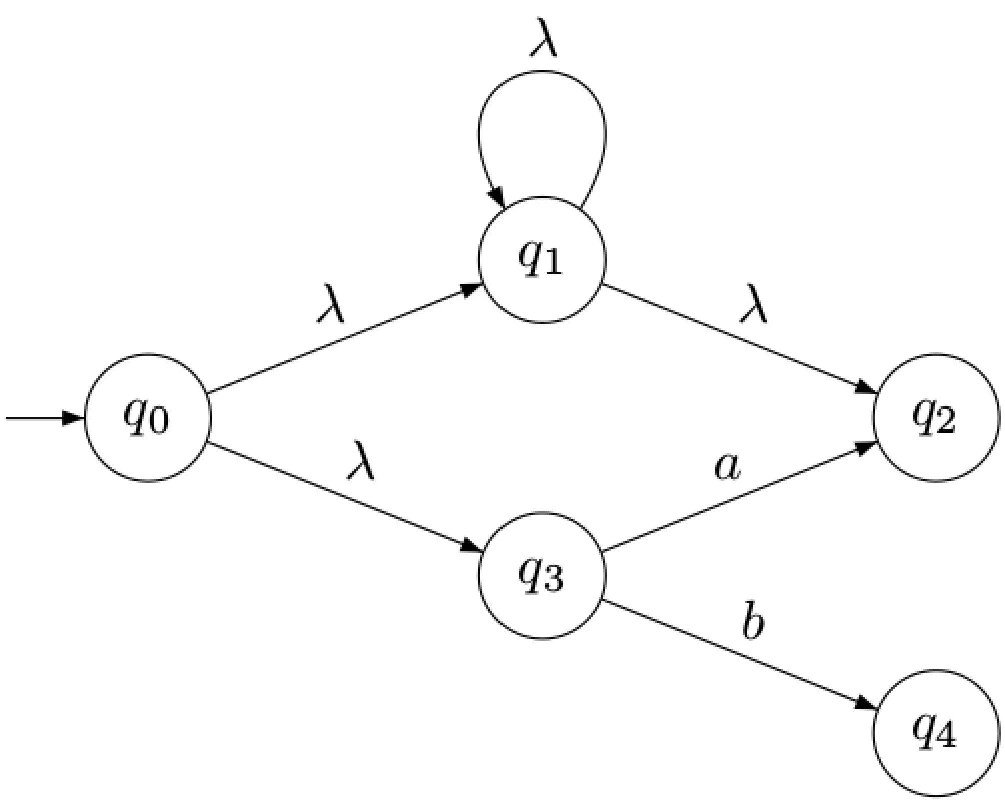


$ababaaaabaaa$	✔
aaa	✔
$baaa$	✔
$baaab$	✔
$\lambda,\ a,\ aa$	✗
$ababa$	✗
$bbbb$	✗

AFND–λ a AFD

Si Q son los estados del AFND–λ de entrada, sea $Clausura_\lambda : \mathbf{P}(Q) \to \mathbf{P}(Q)$:

$$Clausura_\lambda(K) = \{x \in Q | \exists q \in K \wedge (q, \lambda) \overset{*}{\vdash} (x, \lambda)\}$$



$$Clausura_\lambda(\{q_0\}) = \{q_0, q_1, q_2, q_3\}$$
$$Clausura_\lambda(\{q_1\}) = \{q_1, q_2\}$$
$$Clausura_\lambda(\{q_2\}) = \{q_2\}$$
$$Clausura_\lambda(\{q_3\}) = \{q_3\}$$
$$Clausura_\lambda(\{q_4\}) = \{q_4\}$$

$Mover : \mathbf{P}(Q) \times \Sigma \to \mathbf{P}(Q)$:

$$Mover(T, a) = Clausura_\lambda(\bigcup_{t \in T} \delta(t, a))$$

1. Hacer $Clausura_\lambda(\{q_0\})$ el estado inicial de M' (q_0').
2. Hacer $Q' = \{Clausura_\lambda(\{q_0\})\}$ donde Q' es marcable y $Clausura_\lambda(\{q_0\}$ está sin marcar.
3. Mientras exista $T \in Q'$ sin marcar:
    a) Marcar T.
    b) Para cada $a \in \Sigma$
        1) Hacer $U \leftarrow Mover(T, a)$.
        2) Si $U \notin Q'$ entonces agrego sin marcar U a Q'.
        3) Hacer $\delta'(T, a) = U$
    c) Fin Para.
4. Fin Mientras.
5. Hacer $F' \leftarrow \{X \in Q' | X \cap F \neq \emptyset\}$

Minimización de AFD

Indistinguibilidad

Estados Indistinguibles: Sea el AFD M, y $q_p, q_r \in Q$ dos estados.

$$q_p \equiv q_r \quad \text{si } L_p = L_r$$

lenguaje generado a partir de un estado q_i:

$$L_i = \{\alpha | \exists q_f \in F (q_i, \alpha) \vdash (q_f, \lambda)\}$$

$$q_p \text{ es indistinguible de } q_r$$

$$\forall \alpha \in \Sigma^* (\exists q_f \in F \mid (q_p, \alpha) \overset{*}{\vdash} (q_f, \lambda) \Longleftrightarrow \exists q'_f \in F \mid (q_r, \alpha) \overset{*}{\vdash} (q'_f, \lambda)$$

Indistinguibilidad de orden k

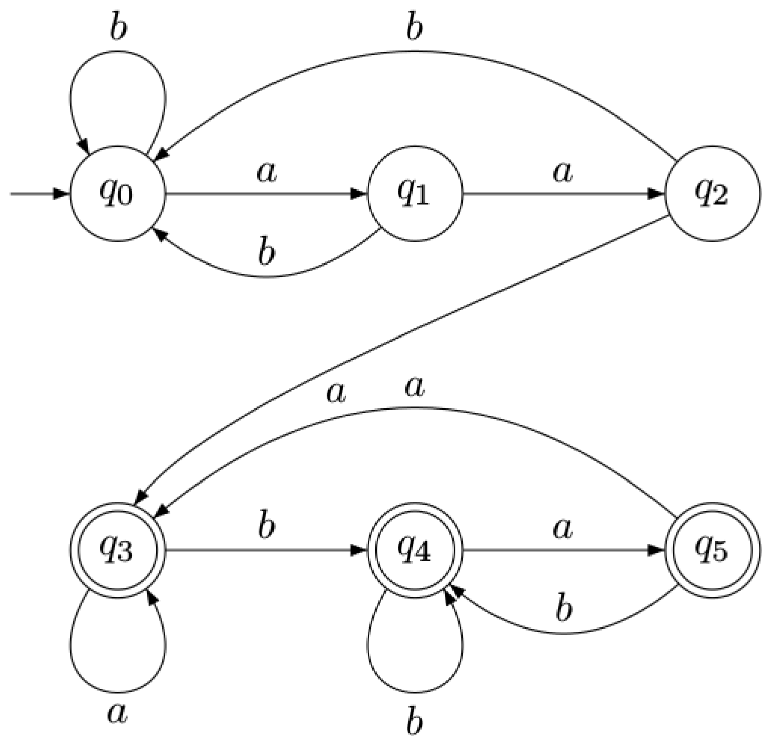
Sea el AFD M, y $q, r \in Q$ dos estados $\qquad q \equiv_k r$ si

$$\forall \alpha \in \Sigma^* \wedge |\alpha| \leq k \; (\exists q_f \in F \mid (q, \alpha) \overset{*}{\vdash} (q_f, \lambda) \Longleftrightarrow \exists q'_f \in F \mid (r, \alpha) \overset{*}{\vdash} (q'_f, \lambda)$$

Propiedades

1. $\equiv_k$ es relación de equivalencia.
2. $\equiv_{k+1} \subseteq \equiv_k$
3. $Q/\equiv_0 = \{Q - F, F\}$
4. $p \equiv_{k+1} r \Longleftrightarrow (p \equiv_k r \wedge \forall a \in \Sigma, (\delta(p, a) \equiv_k \delta(r, a))$
5. $(\equiv_p) = (\equiv_{p+1}) \Rightarrow \forall k > 0 \; (\equiv_p) = (\equiv_{p+k})$

AFD $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \delta, q_0, \{q_3, q_4, q_5\} \rangle$



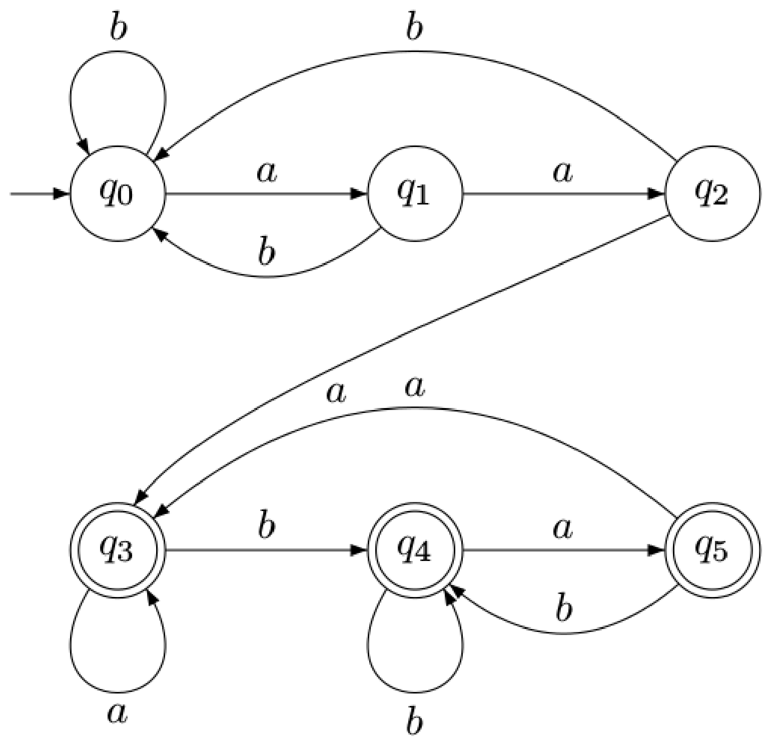
	$\equiv_0$	a	b
q_0	NF		
q_1	NF		
q_2	NF		
q_3	F		
q_4	F		
q_5	F		

$q_1 \xrightarrow{b} q_0$ $\quad$ q_0 es NF $\quad$ $[q_1,b]$ ponemos NF

$q_2 \xrightarrow{a} q_3$ $\quad$ q_3 es F $\quad$ $[q_2,a]$ ponemos F

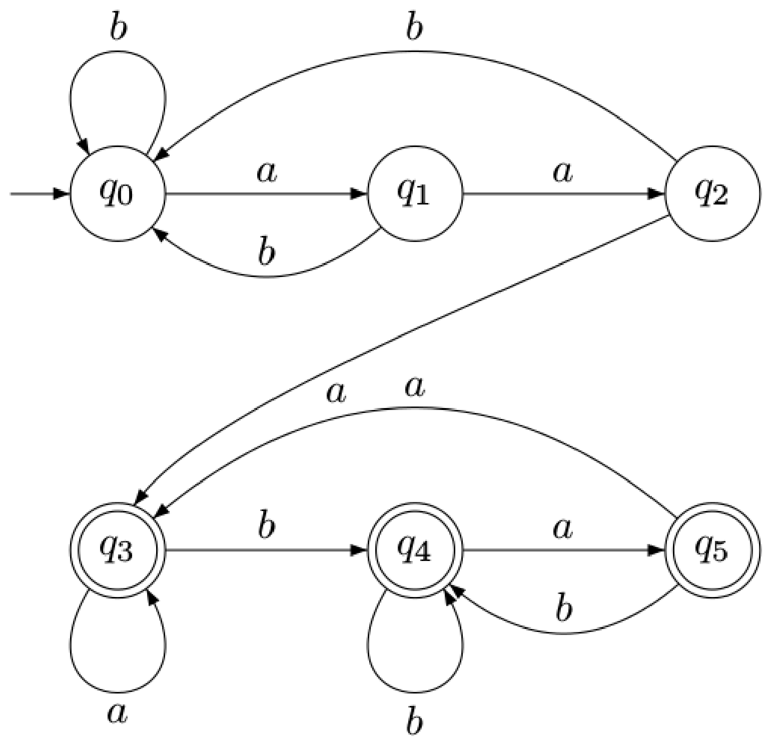
$q_4 \xrightarrow{a} q_5$ $\quad$ q_5 es F $\quad$ $[q_4,a]$ ponemos F

AFD $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \delta, q_0, \{q_3, q_4, q_5\} \rangle$



	$\equiv_0$	a	b
q_0	NF	NF	NF
q_1	NF	NF	NF
q_2	NF	F	NF
q_3	F	F	F
q_4	F	F	F
q_5	F	F	F

AFD $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \delta, q_0, \{q_3, q_4, q_5\} \rangle$



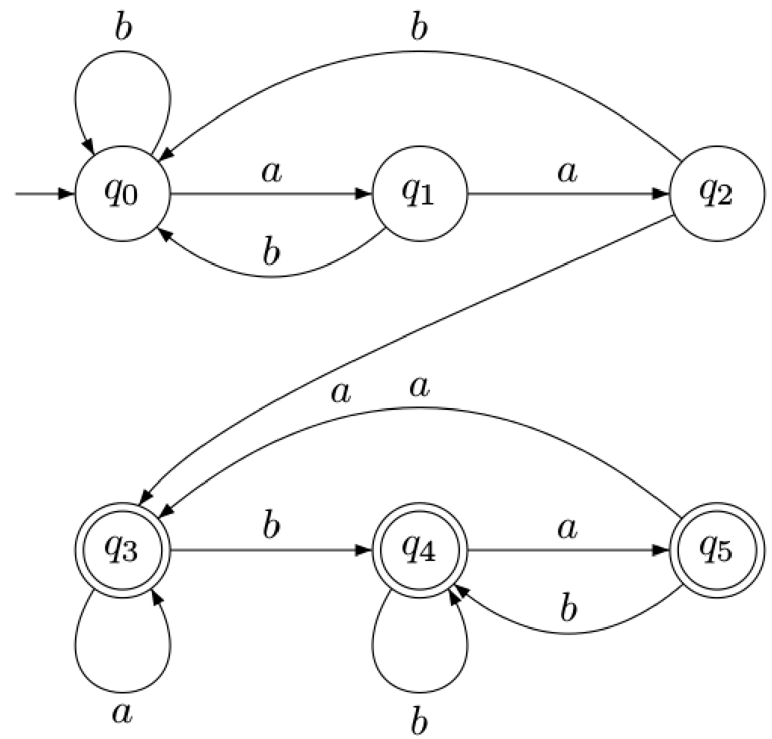
	$\equiv_0$	a	b	$\equiv_1$	a	b
q_0	NF	NF	NF	♡		
q_1	NF	NF	NF	♡		
q_2	NF	F	NF	♣		
q_3	F	F	F	♢		
q_4	F	F	F	♢		
q_5	F	F	F	♢		

$q_1 \xrightarrow{b} q_0$ q_0 es ♡ $[q_1,b]$ ponemos ♡

$q_2 \xrightarrow{a} q_3$ q_3 es ♢ $[q_2,a]$ ponemos ♢

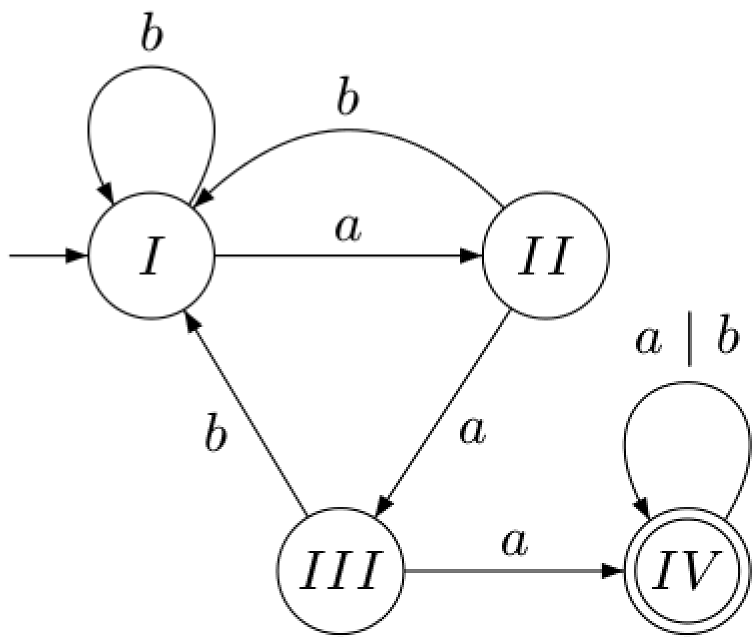
$q_4 \xrightarrow{a} q_5$ q_5 es ♢ $[q_4,a]$ ponemos ♢

AFD $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \delta, q_0, \{q_3, q_4, q_5\} \rangle$



	$\equiv_0$	a	b	$\equiv_1$	a	b	$\equiv_2$	a	b	$\equiv_3$
q_0	NF	NF	NF	$\heartsuit$	$\heartsuit$	$\heartsuit$	$\maltese$	$\star$	$\maltese$	I
q_1	NF	NF	NF	$\heartsuit$	$\clubsuit$	$\heartsuit$	$\star$	$\circledcirc$	$\maltese$	II
q_2	F	F	NF	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\circledcirc$	$\boxdot$	$\maltese$	III
q_3	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV
q_4	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV
q_5	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV

$$M = \langle \{I, II, III, IV\}, \{a, b\}, \delta', I, \{IV\} \rangle$$



$ababaaaabaaa$	✔
aaa	✔
$baaa$	✔
$baaab$	✔
$\lambda,\ a,\ aa$	✗
$ababa$	✗
$bbbb$	✗

	$\equiv_0$	a	b	$\equiv_1$	a	b	$\equiv_2$	a	b	$\equiv_3$
q_0	NF	NF	NF	$\heartsuit$	$\heartsuit$	$\heartsuit$	$\maltese$	$\star$	$\maltese$	I
q_1	NF	NF	NF	$\heartsuit$	$\clubsuit$	$\heartsuit$	$\star$	$\circledcirc$	$\maltese$	II
q_2	F	F	NF	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\circledcirc$	$\boxdot$	$\maltese$	III
q_3	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV
q_4	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV
q_5	F	F	F	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\boxdot$	$\boxdot$	$\boxdot$	IV