Revisar, capaz faltan temas!

(IV) $\forall xs :: [a] . \forall f :: (a \to b) .$ length (map f xs) = length xs

$\Rightarrow)$

Caso base $P([\,]) = \text{length}(\text{map } f\ [\,]) \underset{m0}{=} \text{length } [\,] \underset{l0}{=} 0$ (1)

$(\Leftarrow)$ length $[\,] = 0$ (2)

(1) ≡ (2)

Caso inductivo

$HI \equiv \forall ys :: [a] . \forall f :: a \to b$ length (map f ys) = length ys

con $|ys| < |(x:xs)|$

Qvq $P(ys) \Rightarrow P((x:xs))$

$P((x:xs)) = \text{length}(\text{map } f\ (x:xs)) \underset{m1}{=} \text{length}(f\,x : \text{map } f\ xs)$

$\underset{L1}{=} 1 + \text{length}(\text{map } f\ xs) \underset{HI}{=} 1 + \text{length } xs \underset{L1\ izq}{=} \text{length}(x:xs)$ q.e.d

∴ Vimos que la propiedad vale para cualquier lista. y función $f :: a \to b$

(V) $\forall xs :: [a] . \forall p :: a \to Bool . \forall e :: a .$ *

$P =$ elem e (filter p xs) $\Rightarrow$ (elem e xs)

* Asumir Eq a.

Nota: Estoy usando filter con su def. recursiva explícita

Caso Base: $P([\,])$

$P([\,]) =$ elem e (filter p $[\,]$) $\underset{f0}{=}$ elem e $[\,]$ $\underset{e0}{=}$ false.

(False → *) Es trivialmente cierto por falsedad del antecedente en la implicación

Caso Inductivo: P ((x:xs))

"Si el elemento e estaba en la lista filtrada. Entonces, estaba en la original"

HI ≡ elem e (filter p ys) ⇒ (elem e ys)

$p \to q \equiv \neg p \vee q$ ¿Es más fácil probar eso?

P((x:xs)) = elem e (filter p (x:xs)) =

f1: separo en casos V y F.

P x : True

elem _ [] = F {e1}

elem x (y:ys) = x==y || elem x ys {e2}

= elem e (x : filter p xs) = e == x || elem e (filter p xs)

e2

= x == e || elem e xs = elem e (x:xs) (1)

¿HI?

e2 izq

P x : False

(2)

= elem e (filter p xs) ⇒ (elem e xs) ya valía.

HI

∴ Por el caso base, (1) y (2) del paso inductivo la prop. vale

Dudoso cómo apliqué HI.

VI. $\forall\ xs :: [a]\ .\ \forall\ x :: a\ .\ \text{ponerAlFinal}\ x\ xs = xs \mathbin{++} (x : [\,])$

$P([\,]) = \text{ponerAlFinal}\ x\ [\,] \underset{P0}{=} \text{foldr}\ (:)\ (x : [\,])\ [\,] \underset{f1}{=} (x : [\,]) \underset{[\,]\ \text{elem neutro}}{=} [\,] \mathbin{++} (x : [\,])$

$P(x : xs) = \text{ponerAlFinal}\ x'\ (x : xs) \underset{P0}{=} \text{foldr}\ (:)\ (x' : [\,])\ (x : xs) \underset{f1}{=}$

$= x : \text{foldr}\ (:)\ (x' : [\,])\ (xs) \underset{P0}{=} x : \text{ponerAlFinal}\ x'\ xs = x : (xs \mathbin{++} x')$

1Z

$= (x : xs) : x'$

$= (x : xs) \mathbin{++} [x']$

No compro. Rehacer.