

6) (i) Absurdo clásico $\tau = w$

$$\begin{array}{c}
 \frac{\frac{\frac{}{\Delta x} \quad \frac{}{\Delta x}}{\Gamma \vdash w \Rightarrow \perp \quad \Gamma \vdash w} \Rightarrow_e}{\Gamma \vdash w \Rightarrow \perp, \Gamma w \vdash \perp} \text{PBC} \\
 \frac{}{w \vdash \perp \vdash w} \Rightarrow_i \\
 \hline
 (\Gamma w \Rightarrow \perp) \Rightarrow w
 \end{array}$$

(ii) Ley de Peirce

$$\begin{array}{c}
 \frac{\frac{\frac{}{\Delta x} \quad \frac{}{\Delta x}}{\Gamma', t, \Gamma, P \vdash \tau \quad \Gamma', t, \Gamma, P \vdash \neg \tau} \Rightarrow_e}{\Gamma', t, \Gamma, P \vdash \perp} \text{PBC} \\
 \frac{}{\Gamma', t \vdash P} \Rightarrow_i \\
 \frac{\frac{}{\Delta x} \quad \frac{\Gamma', \tau \vdash (t \Rightarrow P) \Rightarrow \tau \quad \Gamma', \tau \vdash (\tau \Rightarrow P)}{\Gamma', \tau \vdash t} \Rightarrow_e}{\Gamma, \tau \vdash t} \Rightarrow_e \\
 \frac{\Gamma, \tau \vdash t \quad \Gamma, \tau \vdash \perp}{\Gamma, \tau \vdash \perp} \Rightarrow_e \\
 \frac{}{\Gamma \vdash (\tau \Rightarrow P) \Rightarrow \tau \vdash \tau} \text{PBC} \\
 \hline
 ((\tau \Rightarrow P) \Rightarrow \tau) \Rightarrow \tau
 \end{array}$$

$\Gamma' = (t \Rightarrow P) \Rightarrow \tau$
 τ

(iii) Tercero excluido odio tau.

$$\begin{array}{c}
 \frac{}{\sigma \wedge \neg \sigma \vdash \sigma \wedge \neg \sigma} \Delta x \\
 \frac{}{\neg(\sigma \vee \neg \sigma) \vdash \sigma \wedge \neg \sigma} \Delta x \\
 \frac{\frac{}{\sigma \wedge \neg \sigma \vdash \sigma \wedge \neg \sigma} \Delta x \quad \frac{}{\neg(\sigma \vee \neg \sigma) \vdash \sigma \wedge \neg \sigma} \Delta x}{\neg(\sigma \vee \neg \sigma) \vdash \sigma \wedge \neg \sigma} \Delta x \\
 \frac{}{\neg(\sigma \vee \neg \sigma) \vdash \perp} \Rightarrow_e \\
 \hline
 \phi \vdash \sigma \vee \neg \sigma \text{ PBC}
 \end{array}$$

Por ej $\neg(\sigma \vee \neg \sigma) \equiv \sigma \wedge \neg \sigma$
 S. Morgan y
 doble \neg
 $\neg(\sigma \vee \neg \sigma) \equiv \sigma \wedge \neg \sigma$

(iv) Consecuencia milagrosa



$$\begin{array}{c}
 \frac{\frac{}{\Delta x} \quad \frac{}{\Delta x}}{\Gamma \vdash P \Rightarrow P \quad \Gamma \vdash P} \Rightarrow_e \\
 \frac{}{\Gamma \vdash P \Rightarrow P, \Gamma P \vdash P} \Rightarrow_e \\
 \frac{\Gamma P \Rightarrow P, \Gamma P \vdash \perp}{\Gamma P \Rightarrow P \vdash P} \text{PBC} \\
 \hline
 (\Gamma P \Rightarrow P) \Rightarrow P \Rightarrow_i
 \end{array}$$

(v) Contraposición clásica

[illegible]

(VI) Anàlisi de casos

$$\begin{array}{c}
 \text{cases} \\
 \frac{}{\Gamma \vdash t \Rightarrow p} \text{ax} \quad \frac{}{\Gamma, \pi \vdash \pi} \text{ax} \\
 \frac{\Gamma, \neg t \vdash \neg p \Rightarrow q \quad \Gamma, \pi \vdash \neg p}{\Gamma, \neg t \vdash p} \text{ax} \\
 \frac{\Gamma, \neg t \vdash p \quad \Gamma, \pi \vdash \neg p}{\Gamma, \neg t \vdash \perp} \neg e \\
 \frac{}{\Gamma \vdash t \Rightarrow p} \text{ax} \quad \frac{\Gamma \vdash t}{\Gamma \vdash \neg t} \text{PBC} \\
 \frac{\Gamma \vdash p \Rightarrow_e \quad \Gamma \vdash \neg p \Rightarrow_x}{\Gamma \vdash p \vee \neg p} \neg e \\
 \Gamma \equiv (t \Rightarrow p), (\neg t \Rightarrow p), \neg p \vdash \perp \\
 \frac{(t \Rightarrow p), (\neg t \Rightarrow p) \vdash p}{(t \Rightarrow p), (\neg t \Rightarrow p) \vdash \perp} \text{PBC} \\
 \frac{t \Rightarrow p \vdash (\neg t \Rightarrow p) \Rightarrow p}{(t \Rightarrow p) \Rightarrow ((\neg t \Rightarrow p) \Rightarrow p)} \Rightarrow_i
 \end{array}$$

(VII) Implicación vs. disyunción

$$(1) (t \Rightarrow p) \Rightarrow (\neg t \vee p)$$

$$(2) (\neg p \vee q) \Rightarrow (t \Rightarrow p)$$