Práctica 2 - induccion sobre listas
Ej 3
I. ∀ xs::[a] . length (duplicar xs) = 2 * length xs

$$P(ys) \ = \ \text{length (duplicar ys) = 2 * length ys}$$

$$\text{Tenemos que ver P([]) y P((x : xs))}$$

$$\text{Caso P([])} : length\ (duplicar\ [])\ =_{D0}\ length\ []\ =_{L0}\ 0\ (1)$$

$$2 * length\ []\ =_{L0}\ 2 * 0\ =\ 0\ (2)$$

$$\text{Luego (1)} \ \equiv\ (2)$$

$$\text{Caso P((x : xs))}$$

$$Sabemos\ que\ la\ propiedad\ vale\ para\ P(xs),\ qvq\ P(xs)\ \Longrightarrow\ P(x : xs).$$
$$Tenemos :\ HI \equiv P(ys)\ =\ \text{length (duplicar ys) = 2 * length ys}$$

$$Veamos\ las\ ecuaciones$$

$$\text{length (duplicar (x : xs))}\ =_{D1}\ length\ (x : x : duplicar\ xs)\ =_{L1\ der}\ 1\ +\ length\ (x : duplicar\ xs)$$
$$=_{L1\ der}\ 1\ +\ 1\ +\ length\ (duplicar\ xs)\ =_{HI}\ 2 + 2{*}length(xs)$$
$$=\ 2(1 + length\ xs)\ =_{\text{L1 Izq}}\ 2 * \text{length (x : xs)}$$
$$\text{q. e. d}$$

ii. ∀ xs::[a] . ∀ ys::[a] . length (xs ++ ys) = length xs + length ys
Queremos ver que vale para toda concatenación de listas. Hago inducción sobre xs. En general es sobre xs porque si hacés inducción sobre ys, las ecuaciones no te dicen qué podés hacer. Bueno,

$$\forall xs,\ ys\ ::\ [a].\ Length\ (xs\ ++\ ys)\ =\ length\ xs\ +\ length\ ys$$

$$\text{P([])} :\ length\ ([] ++ ys) =_{++0} length\ ys\ \ +\ 0\ =\ sumo\ 0\ como\ elemento\ neutro\ de\ la\ suma$$
$$la\ suma\ es\ conmutativa\ =\ 0\ +\ length\ ys\ =_{L0\ izq}\ length\ []\ +\ length\ ys$$

$$\text{HI} \equiv\ \forall xs',\ ys'\ ::\ [a].\ Length\ (xs'\ ++\ ys')\ =\ length\ xs'\ +\ length\ ys'$$
$$con\ |xs'|\ <\ |x : xs|,\ xs' :: [a],\ ys' :: [a]$$

$$P(x : xs) length\ ((x : xs)\ ++\ ys)\ =_{++1} length\ (x : (xs ++ ys)) =_{L1} 1 + length\ (xs ++ ys)$$
$$=_{HI}\ 1\ +\ length\ xs\ +\ length\ ys\ =_{L1\ izq}\ length(x : xs)\ +\ length\ ys$$
$$q. e. d$$

iii. ∀ xs::[a] . ∀ x::a . append [x] xs = x:xs

Recuerdo

$$foldr :: (a -> b -> b) -> b -> [a] -> b$$
$$foldr\ f\ e\ [] = e$$
$$foldr\ f\ e\ (x:xs) = f\ x\ (foldr\ f\ e\ xs)$$

Mmmm me perturba tener dos variables, experimento, a ver qué sale.

$$Para\ x :: a\ cualquiera,\ \mathrm{P}([]) = append\ [x]\ [] =_{A0} foldr\ (:)\ []\ [x] =$$
$$reescribo\ [x] \equiv x:[]$$
$$= foldr\ (:)\ []\ (x:[]) =_{f1} x : foldr\ (:)\ []\ [] =_{f0} (x:[])\ mmm\ dijo\ la\ muda$$

$$\mathrm{HI} \equiv \forall\ xs' :: [a]\ .\ \forall\ x' :: a\ .\ append\ [x']\ xs' = x' : xs'$$
$$P(x:xs) = append\ [x*]\ (x:xs) =_{A0} foldr\ (:)\ (x:xs)\ [x*] =_{f1}$$
$$x* : foldr\ (:)\ (x:xs)\ [] =_{f1} x* : (x:xs)\ \text{Consultar}$$

iv. ∀ xs::[a] . ∀ f::(a->b) . length (map f xs) = length xs

Inducción sobre xs,

$$\forall f :: a \to b,\ \forall xs :: [a]$$
$$P([]) = length\ (map\ f\ []) =_{m1} length\ [] =_{l1} 0 = length\ []$$

$$\mathrm{HI} \equiv \forall\ xs' :: [a]\ .\ \forall\ f :: (a -> b)\ .\ length\ (map\ f\ xs') = length\ xs'$$

$$|x:xs| > |xs'|$$
$$\mathrm{P}(x:xs)$$