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ASSIGNMENT NO. 1

Decision Theory

Q1. Payoffs Table :

Type of shampoo	Estimated sales (In units)		
	15000	10000	5000
Egg shampoo	30	10	10
Clinic shampoo	40	15	5
Deluxe shampoo	55	20	3

(i) Maxmin principle

Types of shampoo	Estimated sales (In Units)			Minimum pay-off
	15000	10000	5000	
Egg shampoo	30	10	10	10
Clinic shampoo	40	15	5	5
Deluxe shampoo	55	20	3	3

$\text{Max}(10, 5, 3) = 10$. Since the maximum pay-off is 10 units, the marketing manager has to choose Egg shampoo by Maxmin rule.

(ii) Minimax principle

Types of shampoo	Estimated sales (In Units)			Maximum pay-off
	15000	10000	5000	
Egg shampoo	30	10	10	30
Clinic shampoo	40	15	5	40
Deluxe shampoo	55	20	3	55

$\min(30, 40, 55) = 30$. Since the minimum-payoff is 30 units, the marketing manager has to choose Egg shampoo by minimax rule.

(iii) Laplace principle

Types of shampoo	Estimated sales (In units)			Average Pay-off
	15000	10000	5000	
Egg shampoo	30	10	10	16.67
Clinic shampoo	40	15	5	20
Deluxe shampoo	55	20	3	26

$\max(16.67, 20, 26) = 26$. Since the maximum payoff is 26 units, the marketing manager has to choose Deluxe shampoo by Laplace rule.

(iv) Hurwicz principle

$\therefore \text{Weighted average} = \alpha \times (\text{max in row}) + (1-\alpha) \times (\text{min in row})$
 consider $\alpha = 0.75$

Types of shampoo	Estimated sales (In units)			Weighted Average
	15000	10000	5000	
Egg shampoo	30	10	10	23
Clinic shampoo	40	15	5	31.25
Deluxe shampoo	55	20	3	42

$\max(23, 31.25, 42) = 42$. Since the maximum payoff is 42 units, the marketing manager has to choose Deluxe shampoo by Hurwicz rule.

(v) Regret Table principle

Types of Shampoo	Estimated sales (in unit)		
	15000	10000	5000
Egg Shampoo	25	10	0
clinic Shampoo	15	5	5
Deluxe Shampoo	0	0	7

Q2. Payoffs Table :

Rainfall	Estimated conditional Profit (Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	3000
Medium	4000	4000	5000
Low	2000	5000	4000

(i) Maxmin principle

Rainfall	Estimated conditional Profit (Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	3000
Medium	4000	4000	5000
Low	2000	5000	4000
Minimum payoff	2000	3500	4000

$\text{Max} (2000, 3500, 4000) = 4000$. Since the maximum payoff is 4000 Rs, the farmer should plant Crop C using Maxmin rule.

(ii) Laplace principle

	Estimated (conditional) Profit (Rs.)		
Rainfall	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	3000
Low	2000	5000	4000
Average Payoff	4833.33	4333.33	4666.67

$\text{Max}(4833.33, 4333.33, 4666.67) = 4833.33$. Since the maximum payoff is 4833.33 Rs, the farmer should plant the crop A using Laplace rule.

Fuzzy set and Relations.

Q1. $A = \{(1, 0.1), (2, 0.5), (3, 0.8), (4, 1.0), (5, 0.7), (6, 0.2)\}$

$B = \{(1, 1), (2, 0.8), (3, 0.4), (4, 0.1)\}$

Union : Comfortable house for a 4 person family or small

$$\begin{aligned} A \cup B &= \{(1, \max(0.1, 1)), (2, \max(0.5, 0.8)), (3, \max(0.8, 0.4)), \\ &\quad (4, \max(0.1, 1)), (5, \max(0.7, 0)), (6, \max(0.2, 0))\} \\ &= \{(1, 1), (2, 0.8), (3, 0.8), (4, 1), (5, 0.7), (6, 0.2)\} \end{aligned}$$

Intersection : Comfortable house for a 4 person family and small

$$\begin{aligned} A \cap B &= \{(1, \min(0.1, 1)), (2, \min(0.5, 0.8)), (3, \min(0.8, 0.4)), \\ &\quad (4, \min(0.1, 1)), (5, \min(0.7, 0)), (6, \min(0.2, 0))\} \\ &= \{(1, 0.1), (2, 0.5), (3, 0.4), (4, 0.1), (5, 0), (6, 0)\} \end{aligned}$$

Q2. $A = B = \{1, 2, 3, 4\}$

Relation $R \Rightarrow R = \{(a, b) \mid b = a+1, a, b \in A, B\}$

$\therefore R = \{(1, 2), (2, 3), (3, 4)\}$

$$\text{Relation matrix} = M_R = \begin{matrix} & \begin{matrix} d_1 & d_2 & d_3 & d_4 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$B3. \quad R = \begin{matrix} & \begin{matrix} d_1 & d_2 & d_3 & d_4 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} 0.6 & 0.6 & 0.9 & 0.8 \\ 0.1 & 0.2 & 0.9 & 0.8 \\ 0.9 & 0.3 & 0.4 & 0.8 \\ 0.9 & 0.8 & 0.4 & 0.2 \end{bmatrix} \end{matrix}$$

$$T = \begin{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{matrix} & \begin{bmatrix} 0.1 & 0.2 & 0.7 & 0.9 \\ 1 & 1 & 0.4 & 0.6 \\ 0 & 0 & 0.5 & 0.9 \\ 0.9 & 1 & 0.8 & 0.2 \end{bmatrix} \end{matrix}$$

(ii) Max min composition

$$\therefore U(p, s) = \max_d (\min \{ R(p, d), T(d, s) \})$$

U	s_1	s_2	s_3	s_4
p_1	0.8	0.8	0.8	0.9
p_2	0.8	0.8	0.8	0.9
p_3	0.8	0.8	0.8	0.9
p_4	0.8	0.8	0.7	0.9

(iii) Max product composition

$$\therefore U(p, s) = \max_d (R(p, d), T(d, s))$$

\cup	s_1	s_2	s_3	s_4
P_1	0.72	0.8	0.64	0.81
P_2	0.72	0.8	0.64	0.81
P_3	0.72	0.8	0.64	0.81
P_4	0.8	0.8	0.63	0.81

Q4. $A = \{ \frac{1}{2} + 0.65/4 + 0.5/6 + 0.35/8 + 0/10 \}$

$B = \{ 0/2 + 0.35/4 + 0.5/6 + 0.65/8 + 1/10 \}$

(i) $A \cup B = \max \{ A(u), B(u) \}$

$= \{ \frac{1}{2} + 0.65/4 + 0.5/6 + 0.65/8 + 1/10 \}$

(ii) $A \cap B = \min \{ A(u), B(u) \}$

$= \{ 0/2 + 0.35/4 + 0.5/6 + 0.35/8 + 0/10 \}$

(iii) $\bar{A} = 1 - A(u)$

$= \{ 0/2 + 0.35/4 + 0.5/6 + 0.65/8 + 1/10 \}$

(iv) $\bar{B} = 1 - B(u)$

$= \{ \frac{1}{2} + 0.65/4 + 0.5/6 + 0.35/8 + 0/10 \}$

(v) $\bar{A} \cap \bar{B} = \min \{ \bar{A}(u), \bar{B}(u) \}$

$= \{ 0/2 + 0.35/4 + 0.5/6 + 0.35/8 + 0/10 \}$

(vi) $\bar{A} \cup \bar{B} = \max \{ \bar{A}(u), \bar{B}(u) \}$

$= \{ \frac{1}{2} + 0.65/4 + 0.5/6 + 0.65/8 + 1/10 \}$

(vii) $\overline{A \cap B} = \bar{A} \cup \bar{B}$ (De Morgan's Law)

$= \{ 0/2 + 0.35/4 + 0.5/6 + 0.35/8 + 0/10 \}$