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COMP 5531/4 Bipin C. Desai Winter 2022 Assignment #3

Q1

a) Consider a relation scheme R(A,B,C,D) with functional dependencies {A→B, B→C, C→D, D→A}.

1.What are the non-trivial dependencies that follow from the given dependencies?

2. What are the keys of R?

3. What are all super keys of R that are not keys?

Solution:

1. **{ A→BCD, B→CDA, C→DAB, D→ABC}**
2. **Keys of R: A,B,C,D**
3. **Super keys that are not keys:** ∅

b) Say whether each of the following statements about the given functional dependencies is TRUE or FALSE. If true, prove it using Armstrong's axioms. If false, give a counterexample, i.e. give a relation instance (With at least 2 rows) that satisfies the given dependencies but not the one that allegedly follows:

1. If AB→C and A→C then B→C
2. If AB→C, then A→C or B→C
3. If A→B, and BC→D then AC→D
4. If AB→C and B→D then AD→C
5. If A→C and B→C then A→B
6. If A→B and AB→C then A→C

Solution:

1. **FALSE**

A	B	C
1	3	5
2	3	7
3	4	9
4	5	9

B does not determine C because for the same value of B, we have different values of C

2. FALSE

A	B	C
1	3	4
1	5	6
2	3	9

A does not determine C because for the same value of A, we have different values of C. B does not determine C because for the same value of B, we have different values of C.

3. TRUE

Pseudo transitivity tells us that if $A \to B$ and $BC \to D$ holds then $AC \to D$

4. FALSE

A	B	C	D
1	3	5	1
1	2	7	1

AD does not determine the value of C because for the same value of AD, we have different values of C.

5. FALSE

A	B	C
1	5	3
1	6	3

A does not determine B because for the same value of A, we have different values of B.

6. TRUE

Through pseudo transitivity we know that if $A \to B$ and $AB \to C$ holds then $A \to C$ holds as well.

Q2 We say a set of attributes X is closed (With respect to a given set of FD's) If X+ = X. Consider a relation with schema R(A,B,C,D) and an unknown set of FD's. If we are told which set of attributes are closed, we can discover the FD's. What are the FD's if

a) All sets of the four attributes are closed b) The only closed sets are and {A,B,C,D} c) The closed sets are ,{A,B},and {A,B,C,D}
b) The only closed sets are and {A,B,C,D}
c) The closed sets are ,{A,B},and {A,B,C,D}

Solution:

a) The empty set ∅
b) {A → B, B→ C, C →D, D → A}
c) {C→ ABD, D → ABC, A→ B, B→ A}

Q3 Suppose we have a relation R(A,B,C,D,E), with some set of FD's, and we wish to project those FD's onto relation S(A,B,C) .Give the FD's that hold in S if the FD's for R are:

a) AB→DE, C→E, D→C ,and E→A

Solution:

From AB→DE we can get AB→D,AB→E by projectivity. And we know D→C, we can get AB→C by transitivity.
From C→E and E→A, we can get C→A by transitivity
So the final set of FDs for S are { AB→C, C→A }

b) A→D, BD→E, AC→E, and DE→B.

Solution:

From A→D and DE→B, we can get AE→B by Pseudo-trans.
From AC→E and AE→B, we can get AC→B by Pseudo-trans.

So the final set of FDs for S are { AC→B }

c) AB→D, AC→E, BC→D , D→A and E→B

Solution:

From AC→E and E→B, we can get AC→B by transitivity.
From BC→D and D→A, we can get BC→A by transitivity
So the final set of FDs for S are { AC→B, BC→A }

d) A→B, B→C, C→D , D→E, and E→A
Solution:

We have A→B and B→C
From C→D, D→E, and E→A we can get C→A by transitivity
So the final set of FDs for S are { A→B, B→C, C→A}

Q4 Consider the relation scheme R {A, B, C, D, E} in which the following set of functional dependencies hold { AB→C, D→E, CE→D, B→D}. 1. Is R in BCNF ? 2. Mention all the functional dependencies that violate BCNF. Using the closure test, show that these dependencies are in violation. 3. Decompose R into sub schemas which are in BCNF. 4. List all the dependencies that hold in the sub schemes that you have produced. Does the union of all these dependencies contain the same information as in the original set of functional dependencies? Why or why not?

Solution:

1. **R is not in BCNF**

2.
- **D→ E violates BCNF**
  D^+ = DE
  So D is not a candidate key of R
- **CE →D violates BCNF**
  CE^+ = CED
  So CE is not a candidate key of R
- **B →D violates BCNF**
  B^+ = BDE
  So B is not a candidate key of R

3. Decomposition:

- **$D \rightarrow E$**

  $D^{+} = DE$
  So, R1 (DE) and R2(ABCD)

  R1: $D \rightarrow E$ (in BCNF)
  R2: ABCD
  $AB \rightarrow C$ (in BCNF)
  $B \rightarrow D$ (violates BCNF)

- **$B \rightarrow D$ violates BCNF**

  R3 (BD), and R4(ABC)
  R3: $B \rightarrow D$ (in BCNF)
  R4: $AB \rightarrow C$ (in BCNF)

So the final decomposition:
R1 (DE): $D \rightarrow E$
R2 (BD): $B \rightarrow D$
R3 (ABC): $AB \rightarrow C$

4. The dependencies are

R1 (DE): $D \rightarrow E$
R2 (BD): $B \rightarrow D$
R3 (ABC): $AB \rightarrow C$

The decomposition is not dependency preserving. The dependency $CE \rightarrow D$ from the original relation cannot be inferred from the union of the dependencies of the relation in BCNF.

Q5 Consider R (A B C D E G H), F {$AB \rightarrow C$, $AC \rightarrow B$, $AD \rightarrow E$, $B \rightarrow D$, $BC \rightarrow A$, $E \rightarrow G$} Which of the following decomposition of R is dependency preserving and/or lossless join. a) {AB,BC,ABDE,EG} b) {ABC,ACDE,ADG} c) {BCH,ABC,EG}

Solution:
a)

Not lossless

Because: loss H

Not dependency preserving

Because: can not preserve AB→C, AC→B, BC→A

b)

Not Lossless

Because: loss H

Not dependency preserving

Because: can not preserve B→D, E→G

c)

Not Lossless

Because:

	A	B	C	D	E	G	H
R1	α	α	α				α
R2	α	α	α				
R3					α	α	

Lossy table

Not dependency preserving

Because: can not preserve AD→E , B→D