

Designing and Evaluating a Likelihood-Ratio Test for IRT models

Methodology and Statistics for the Behavioural,

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1 Introduction

In organisations, resources tend to be dedicated to researching important organisational variables, such as job performance and personality traits. These types of variables are latent constructs. This entails that they are not directly observable. Thus, to measure these constructs we have to infer them based on responses to a test or questionnaire. To achieve this, Item Response Theory (IRT) is often used. A large issue in IRT, however, is that you must use a model that correctly describes the answers to test items in order to gain correct conclusions. This means that you should investigate whether the model you use, fits the data you acquired. If a wrong model is used, it can lead to dire consequences such as faults in the validity of your measurement (**consq2**; **consq3**; Zhao & Hambleton, 2017) and by extension your conclusion. In an organisation, this could for example translate into issues such as discharging the wrong people. Hence, it is vital that you test the appropriateness of your model (i.e., model-fit).

Currently in IRT research, there are mainly two procedures that can be applied to every model to measure model-fit. Namely, a χ^2 -difference test and the Pearson's χ^2 -test. These tests both suffer from issues. Pearson's test, for example, will not follow a χ^2 -distribution when many score pattern frequencies are missing or low. An issue with the χ^2 -difference test is that it is difficult to use for the three-parameter (3PL) model. This is because the 3PL model can only be nested under the four-parameter logistic (4PL) model, which suffers from consistency and estimation issues (Barton & Lord, 1981; Loken & Rulison, 2010). Another concern with the χ^2 -difference test is that when sample size is large, the test will increase in power and it will reject models that are still reasonable (Curran et al., 1996). For this reason, fit indices were introduced to investigate whether a model might still be reasonable. Fit indices are standardised indicators of model-fit. For an overview of current fit indices in IRT and their limitations, see Nye et al. (2020). Besides these indices, fit indices from Structural Equation Modeling can also be used in IRT. However, hardly any research has been done towards the use of the Tucker-Lewis Index (TLI; Tucker and Lewis, 1973) and the Comparative Fit Index (CFI; Bentler, 1990) in IRT. We found only one paper examining the CFI (Yang, 2020) and two papers investigating the TLI (Cai et al., 2021; Yang, 2020). However, we believe that the calculations for the CFI and TLI by Yang are incorrect due to a wrong baseline model, which would affect

the results. To summarise, there are (a) too few goodness-of-fit tests available in IRT to test the 3PL model and (b) insufficient research on the CFI and TLI, which we will address.

Our proposed research would (a) develop and test the performance of a goodness-of-fit Likelihood Ratio (LR) test applicable to all IRT models and (b) test the performance of the CFI and TLI with a complete-independence baseline model. More specifically, we will answer the following three research questions:

1. Under which conditions will the χ^2 -test associated with our LR perform well?
2. How does the performance of our developed test compare to the performance of a χ^2 -difference and Pearson's χ^2 -test?
3. What is the performance of the TLI and CFI with a complete-independence baseline model in IRT?

2 Analytic strategy

In order to answer the research questions, we will conduct a simulation study. When simulating data, we will vary the following four variables:

- Test length
- Sample size
- Model types
- Number of groups

For a complete overview of the conditions that will be used for the different variables, see *Table 1*.

Table 1

Overview of Simulation Conditions for Each Variables

Variable	Conditions	Description
Test length	5 - 10 - 20	The total number of items that the test will consist of
Sample size	20 - 50 - 100 - 200 - 500	The total number of observations that will be available for each item
Model type	1PL - 2PL - 3PL	The models that we will use as the basis for both data generation and model-fitting
Number of groups	2 - 3 - 4	The number of groups that the total dataset gets divided into for the LR calculations

Note. 1PL = one-parameter logistic model; 2PL = two-parameter logistic model; 3PL = three-parameter logistic model.

Each particular condition will be replicated 500 times and in each replication the following LR will be calculated:

$$\frac{\max(L_0)}{\prod_{j=1}^g \max(L_j)} \quad (1)$$

where L_0 is the likelihood for the whole dataset and L_j is the likelihood for each group in g , gained by randomly assigning the observations to g groups. According to Wilk's theorem (Wilks, 1938), this LR will then asymptotically follow a χ^2 -distribution with the following formula:

$$-2\ln \frac{\max(L_0)}{\prod_{j=1}^g \max(L_j)} \rightarrow \chi^2(\Delta) \quad (2)$$

where Δ is the difference in the number of estimated parameters between the numerator and denominator. This entails that the LR can be used for Null Hypothesis Significance testing. Performance of the test can then be operationalised by both Type I error and Power. Power can be calculated by fitting and testing a different model to the data than the model used to generate the data. Type I error can be calculated when fitting and testing the model that was used to generate the data. With these values, we can study what sample size and test length is necessary for our χ^2 -test to have enough Power. Furthermore, we can compare multiple tests with one another, where a test with either higher Type I error or lower Power will be noted

as performing worse. To measure the performance of the TLI and CFI, we can calculate the proportion of times that the fit indices improve when the correct model is used compared to the other models. Finally, we will provide an empirical example by testing the LR test, CFI and TLI on the online available LSAT data and interpreting their results.

The proposed research would be conducted in R (R. C. Team, 2021) through the use of RStudio (R. Team, 2020) with the MASS (Venables & Ripley, 2002), lavaan (Rosseel, 2012) and ltm (Rizopoulos, 2006) packages. Ethical registration for the simulated data has been approved and approval for the example dataset has been requested.

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