CS 466: Design and Analysis of Algorithms

Michael Noukhovitch

Spring 2016, University of Waterloo

Notes written from Mark Petrick's lectures.

Contents

1 Introduction

1.1 Travelling Salesman

Given a graph $G = (V, E)$ with weights on edges $w : E \to \mathbb{R}^{+ve \cup \{0\}}$, find a **TSP Tour**:

- a cycle C that visits evert vertex exactly once
- has min $\sum_{e \in C} w(e)$

This is NP-Complete

1.2 Metrick TSP

Create an approximation problem with TSP by defining distances in a space:

- symmetry: $d(u, v) = d(v, u)$
- triangle inequality: $d(u, v) \leq d(u, w) + d(w, v)$

Approximation Algorithm (1977)

- find a minimum spanning tree and walk the tour
- take shortcuts to avoid re-visiting the same vertices
- by triangle inequality, the length is $\leq 2 * l_{MST}$ and therefore total $l \leq 2 * l_{TSP}$ (2-approx)
- ends up with runtime of MST, O(m log n)

1.3 Further Approximation

- TSP General: no constant factor approx
- Metric space: 1.5-approx, proven limit is 1.0045
- Euclidean space: $(1 + \epsilon)$-approx in polytime