

# Restriction in Doubly-Modal Disjunctions

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## Preliminaries

Klinedinst and Rothschild (2012) (K&R henceforth): a disjunction ' $A$  or  $B$ ' is evaluated relative to an information state  $s$  s.t.  $B$  is relative to  $s[\neg A]$ :

$$(1) \llbracket A \text{ or } B \rrbracket^{w,s} = 1 \Leftrightarrow \llbracket A \rrbracket^{w,s} = 1 \vee \llbracket B \rrbracket^{w,s[\neg A]} = 1$$

K&R also assumes that a modal in a disjunct is **restricted by the relevant information state**, to capture so-called "non-tabular" readings of disjunction like the one in (2).

- (2) a. John is away at a conference, or he must be on campus.  
b.  $\Rightarrow$  If John is not away, he must be on campus.

## The prejacent problem

Sentences like (3a) pose a problem for K&R—we want the domain of the modal in the right disjunct to be restricted by the prejacent of the modal in the left disjunct.

- (3) a. John must have failed his exam, or he would be happy.  
b.  $\Rightarrow$  If John passed his exam, he would be happy.  
c.  $\Rightarrow$  If John might have passed his exam, he would be happy.

$$\begin{aligned} &\Box A \text{ or } \Diamond B \\ &\neg A \rightarrow \Diamond B \\ &\Diamond \neg A \rightarrow \Diamond B \end{aligned}$$

So, the entry in (1), incorrectly predicts that (3a) should have the following truth-conditions:

$$(4) \llbracket \Box A \text{ or } \Diamond B \rrbracket^{w,s} = 1 \Leftrightarrow \llbracket \Box A \rrbracket^{w,s} = 1 \vee \llbracket \Diamond B \rrbracket^{w,s[\Diamond \neg A]} = 1$$

The prejacent problem arises not just for epistemic modals, but also root modals.

- (5) a. John has to write a paper, or he will get a C.  
b.  $\Rightarrow$  If John doesn't write a paper, he will get a C.  
c.  $\Rightarrow$  If John doesn't have to write a paper, he will get a C.

## Klinedinst and Rothschild's patch

K&R propose that the right disjunct can be evaluated relative to an information state updated with a **subclause** of the first disjunct (Klinedinst and Rothschild, 2012; Meyer, 2015).

As Cariani (2017) observes, this doesn't work—given a sentence of the form ' $(\Diamond A \text{ or } \Diamond B) \text{ or } \Box C$ ', we want  $\Box C$  to be restricted by ' $\neg A$  and  $\neg B$ '; however, ' $A$  or  $B$ ' is not a sub-clause of ' $\Diamond A \text{ or } \Diamond B$ '.

- (6) a. Jones might be eating lunch or might be teaching, or he would be in his office.  
b.  $\Rightarrow$  If Jones wasn't getting lunch and wasn't teaching, he would be in his office.

K&R don't offer any principle way of determining when to restrict by a subclause rather than the full clause.

## Our novel observation

**Observation:** in the root case, if the restriction comes from the entirety of the negation of the left disjunct, the **restricted modal claim is pragmatically infelicitous**.

- (7) a. John has to write a paper, or he will get a C.  
b.  $\Rightarrow$  If John doesn't have to write a paper, he will get a C. (infelicitous)  
The conditional paraphrase seems intuitively **false** if receiving a C is a penalty.  
In contrast, when the negation of the entirety of the left disjunct doesn't lead to (pragmatic) infelicity, we restrict with the entire left disjunct.  
(8) a. (Either) John is required to finish his paper, or he would come to the party tonight.  
b.  $\Rightarrow$  If John wasn't required to finish his paper, he would come to the party tonight.

**Hypothesis:** In ' $\Box_1 A \text{ or } \Box_2 B$ ', restriction of  $\Box_2 B$  with  $\neg A$  is forced (to avoid triviality) iff restriction of  $\Box_2 B$  with  $\neg \Box_1 A$  leads to a contextual contradiction.

## Test case #1

**Prediction:** restriction varies by context; if receiving a C is not a penalty, restriction with the negation of the left disjunct should be possible.

**Context:** John's grades are currently in the A-range. But if he needs to write a paper for the class, his grades will drop to B+. The professor might waive the paper requirement for everyone.

- (9) a. (Either) John has to write a paper, or he will get a A.  
b.  $\Rightarrow$  If John doesn't have to write a paper, he will get a A.

The contrast with (7a) supports **Hypothesis**.

## Test case #2

**Prediction:** even if both types of restriction lead to two non-trivial restricted modal claims, the negation of the whole left disjunct is the default.

- (10) a. John is required to get us Pepsi, or he would get us Coke.  
b.  $\Rightarrow$  If John was not required to get us Pepsi, he would get us Coke.

The other restriction, paraphrased as *If John doesn't get us Pepsi, he would get us Coke* is not odd, but dispreferred out of the blue.

## A note about conjunctive reading

It's tempting to read (7a) as a conjunction of both disjuncts (Meyer, 2015).

We think the conjunctive reading and restriction from prejacent are **separable** issues.

**Context:** We realized that John is not good at playing the guitar. But we also don't know whether he wants to be a pro.

- (11) a. (Either) John has to practice 6 hours a day, or he will only play in a mediocre band.  
b.  $\Rightarrow$  If John doesn't practice 6 hours a day, he will only play in a mediocre band.

## Extending the observation

Now we need to test **Hypothesis** in the epistemic case (3a).

$$(12) a. \llbracket \text{must}_f A \rrbracket^{w,s} = 1 \text{ iff } \forall w' \in \text{dom}(w, f) : \llbracket A \rrbracket^{w',s} = 1.$$

b. John must have failed his exam, or he would be happy.

c. Right disjunct with default restriction:  $\llbracket \Box \text{happy(John)} \rrbracket^{w,s[\neg \Box \text{fail(John)}]}$   
 $\Leftrightarrow$  If *pass* is epistemically **possible**, John would happy.

Intuitively, the **right disjunct with default restriction would be false**. Presumably, in some worlds where *pass* is possible, John actually fails and is unhappy.

## A possible implementation: selection function

The basic idea: disjunction checks whether the two disjuncts are trivial under the default restriction and can rescue the triviality by choosing an alternative restriction.

We can use a **selection function** *sel*: it outputs the alternatively restricted disjunction that's closest in structural complexity to the default **and** non-trivial.

$$(13) a. \llbracket A \text{ or } B \rrbracket^{w,s} = \text{sel}(\llbracket A \rrbracket^{w,s} \vee \llbracket B \rrbracket^{w,s[\neg A]}, N \cap \text{d-a1t}(\llbracket A \rrbracket^{w,s} \vee \llbracket B \rrbracket^{w,s[\neg A]}))$$

b.  $N = \{\phi \mid \phi \text{ doesn't have a trivially false disjunct in } c\}$

$$c. \text{d-a1t}(\llbracket A \rrbracket^{w,s} \vee \llbracket B \rrbracket^{w,s[\neg A]}) = \{\llbracket A \rrbracket^{w,s} \vee \llbracket B \rrbracket^{w,s[\neg p]} \mid p \in \mathcal{A}(A)\} \quad (\text{alternative restrictions})$$

d.  $\mathcal{A}(A)$  is the set of alternatives of  $A$ :  $\mathcal{A}(A) := \{A' : A' \preceq A\}$  where  $\preceq$  is a pre-order on strings which ranks them by their structural complexity à la Katzir (2007)

The selection function *sel* takes a default disjunction, and a set of alternatively restricted disjunctions, restricted by a non-triviality predicate  $N$ ; it outputs a member of the latter (**Inclusion**), and if the default restriction is non-trivial, it will be selected (**Centering**) (Stalnaker, 1968).

$$(14) a. \text{Inclusion: } \text{sel}(\phi, N \cap \text{d-a1t}(\phi)) \in N \cap \text{d-a1t}(\phi)$$

b. **Centering:** If  $\phi \in N \cap \text{d-a1t}(\phi)$ ,  $\text{sel}(\phi, N \cap \text{d-a1t}(\phi)) = \phi$

## References

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