Riemannian Geometry (L24)

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This course is a possible natural sequel of the course Differential Geometry offered in Michaelmas Term. We shall explore various techniques and results revealing intricate and subtle relations between Riemannian metrics, curvature and topology. I hope to cover much of the following:

A closer look at geodesics and curvature. Brief review from the Differential Geometry course. Geodesic coordinates and Gauss' lemma. Jacobi fields, completeness and the Hopf–Rinow theorem. Variations of energy, Bonnet–Myers diameter theorem and Synge's theorem.

Hodge theory and Riemannian holonomy. The Hodge star and Laplace–Beltrami operator. The Hodge decomposition theorem (with the 'geometry part' of the proof). Bochner–Weitzenböck formulae. Holonomy groups. Interplays with curvature and de Rham cohomology.

Ricci curvature. Fundamental groups and Ricci curvature. Gromov's proof of Gallot's bound on the first Betti number (time permitting). The Cheeger–Gromoll splitting theorem.

Pre-requisite Mathematics

Manifolds, differential forms, vector fields. Basic concepts of Riemannian geometry (curvature, geodesics etc.) and basic familiarity with Lie groups. The course *Differential Geometry* offered in Michaelmas Term is an ideal pre-requisite.

Literature

1. S. Gallot, D. Hulin, J. Lafontaine, *Riemannian geometry.* Springer-Verlag, 1990.
2. M.P. do Carmo, *Riemannian geometry.* Birkhäuser, 1993.
3. I. Chavel, *Riemannian geometry, a modern introduction*, CUP 1995.
4. F.W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer-Verlag, 1983. Chapter 6.
5. P. Petersen, *Riemannian geometry*, Springer-Verlag, 1998.
6. A.L. Besse, *Einstein manifolds*, Springer-Verlag, 1987.

The first few chapters of do Carmo's text provide a good introductory reading.

Additional support

The lectures will be supplemented by four example classes, based on four example sheets. There will be a revision class in the Easter Term.