Commutative Algebra (M24)

Dr N. J. Williams

This course will provide an introduction to the theory of commutative rings and modules over these rings. It should be viewed as a foundational course for Algebraic Geometry and Algebraic Number Theory.

The course will cover a selection of topics including:

- Localisation of rings, local rings, and local properties
- Tensor products and flatness
- Primary decomposition
- Integral extensions, going up and going down theorems
- Noetherian and Artinian rings, Hilbert's basis theorem
- Dedekind domains and discrete valuation rings
- Direct and inverse limits, completion, and the Artin–Rees Lemma
- Dimension theory and Hilbert functions

Prerequisites

You will have attended a first course in ring theory, such as the IB course Groups, Rings and Modules. Experience of more advanced material such as Part II courses Galois Theory, Representation Theory, Algebraic Geometry or Number Fields is desirable (contributing to your level of algebraic maturity) but not essential.

Literature

1. M. F. Atiyah and I. G. Macdonald. *Introduction to commutative algebra.* Addison-Wesley Publishing Co., 1969
2. R. Y. Sharp. *Steps in commutative algebra.* London Mathematical Society Student Texts. Cambridge University Press, 2000,
3. H. Matsumura. *Commutative ring theory.* Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1986
4. O. Zariski and P. Samuel. *Commutative algebra. Vol. 1.* Graduate Texts in Mathematics. Springer-Verlag, 1975
5. O. Zariski and P. Samuel. *Commutative algebra. Vol. II.* Graduate Texts in Mathematics. Springer-Verlag, 1975

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.