Gaussian Processes and Measures (M16)

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The study of Gaussian processes and measures is concerned with infinite-dimensional notions of the 'normal', or 'Gaussian' distribution, and their properties in general vector spaces. They are fundamental building blocks in several areas of modern mathematics, ranging from probability and analysis to statistics and machine learning. In this course we will give a rigorous account of some of the classical results of the theory.

The course will roughly be structured as follows:

I. Basic definitions and properties of Gaussian processes with abstract index sets. Gaussian measures and random variables on separable Banach spaces. Concentration properties of norms of Gaussian processes and variables, zero-one law, Fernique's theorem. Dudley's metric entropy inequality. Examples.

III. Reproducing kernel Hilbert spaces and Bochner integrals. Karhunen-Loève series expansions. Cameron-Martin theorem. Small deviation and support properties of Gaussian measures. Radonifying maps. Examples. Application to random partial differential equation models.

Pre-requisites

Backgound in basic probability and measure theory is necessary, e.g., [1]. The approach taken in this course avoids the introduction of stochastic calculus techniques altogether but instead relies on basic ideas from functional analysis, background in which will be helpful but can be acquired as we move forward. The course will be loosely based on Chapter 2 of [2].

Additional support

Next to the lectures there will be three examples classes and a revision class for this course.

Literature

1. R.M. Dudley, *Real analysis and probability*, Cambridge University Press, Cambridge 2002

2. E. Giné & R. Nickl, *Mathematical foundations of infinite-dimensional statistical models*, Cambridge University Press, Cambridge 2016